

PQI-3401 – Engenharia de Reações Químicas II
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POLICONDENSAÇÃO

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Policondensação (“step-growth”)

- Monômeros tem (pelo menos) 2 grupos funcionais reativos, p.ex. -OH, -NH₂, -COOH, -COCl, etc.

Exemplos (tipo 1, tipo AB)

aminoácido → poliamida + H₂O



hidroxi-ácido carboxílico → poliéster + H₂O

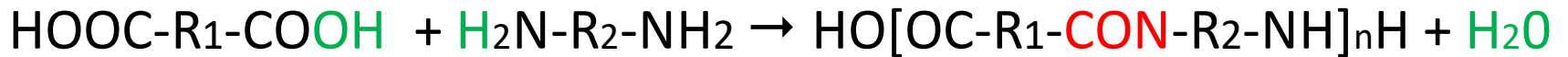


Policondensação (“step-growth”)

- Monômeros tem (pelo menos) 2 grupos funcionais reativos, p.ex. -OH, -NH₂, -COOH, -COCl, etc.

Exemplos (tipo 2, tipo AA+BB)

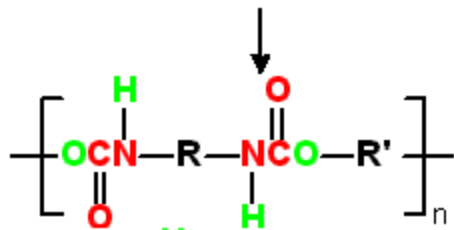
ácido di-carboxílico + diamina → poli**amida** + H₂O



ácido di-carboxílico + diálcool → poli**éster** + H₂O



di-isocianato + diálcool → poliuretano



Polycondensation ("step-growth")



Type I (AB)

nylon-6



ϵ -caprolactam

Type II (AA + BB)

nylon-6,6



hexamethylene diamine

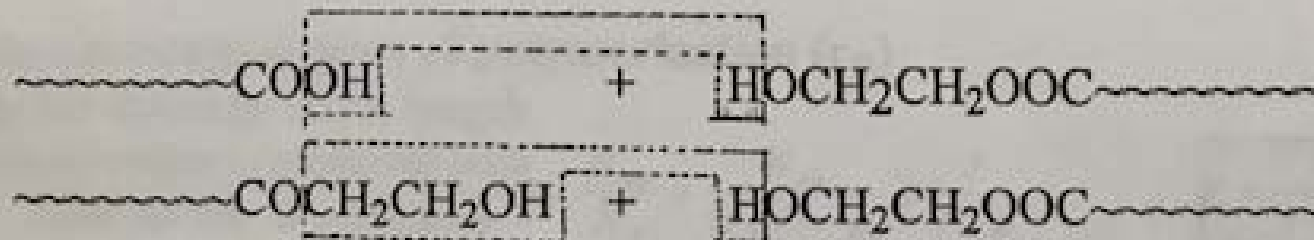


adipic acid

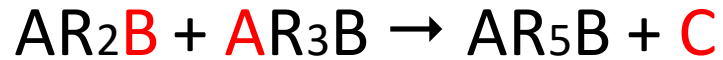
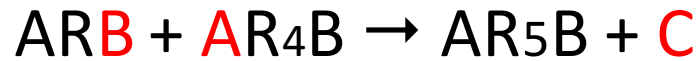
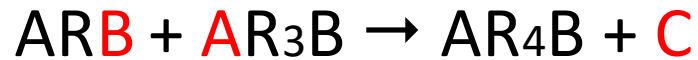
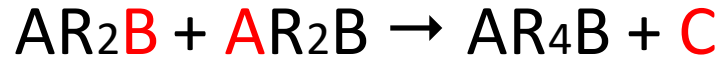
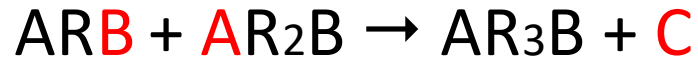
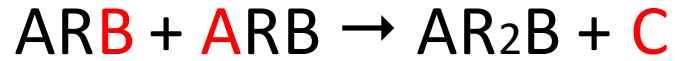


Type III (AA + BB, AA + AA)

PET



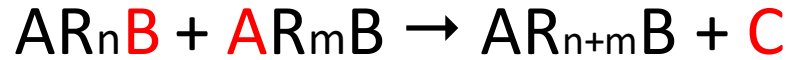
Policondensação tipo AB

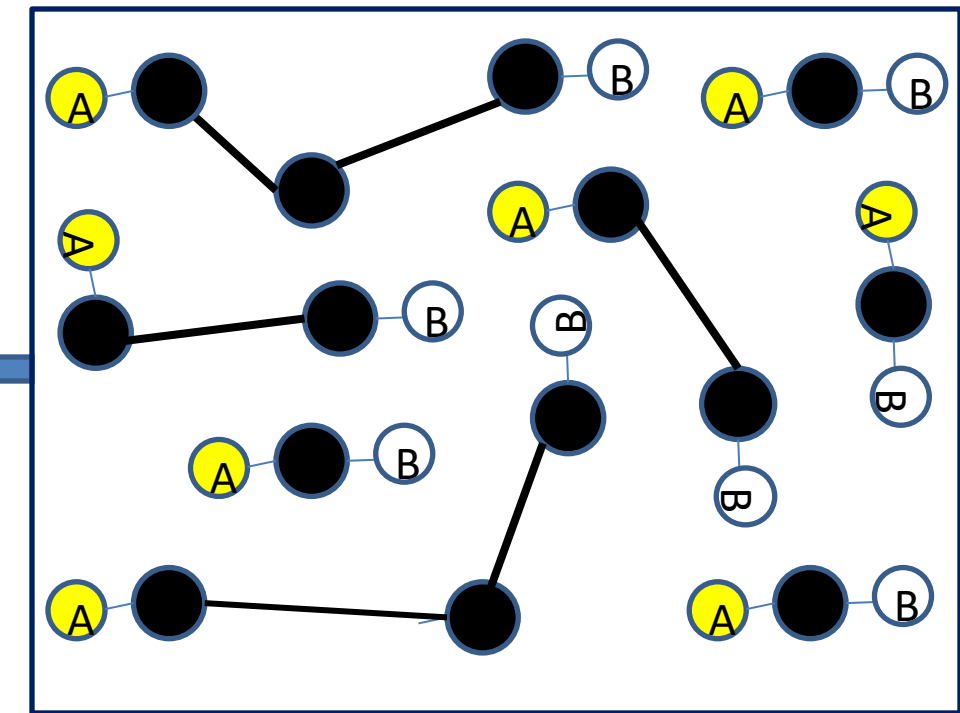
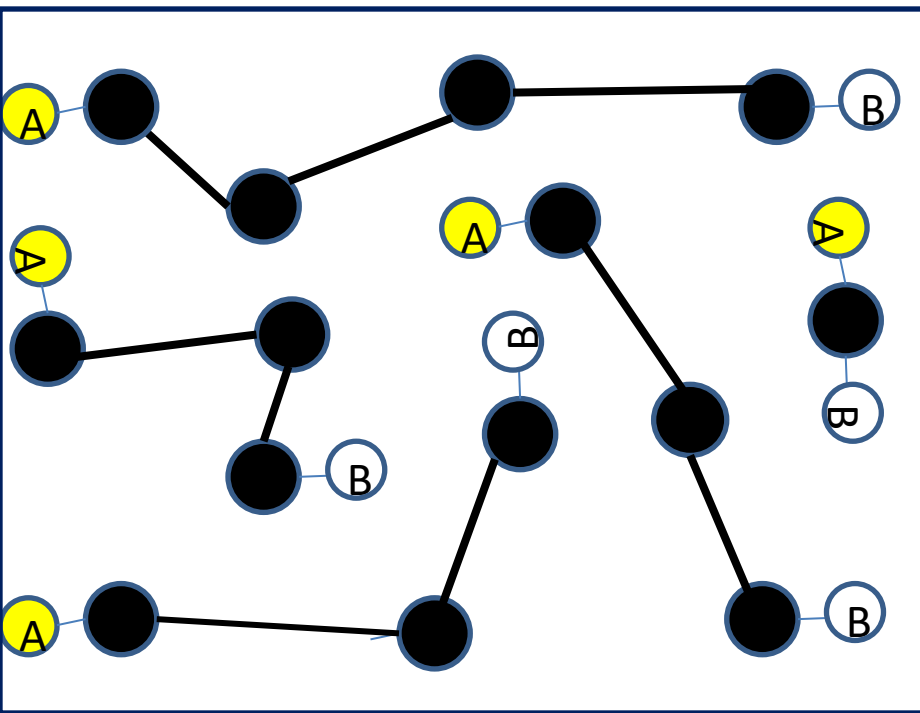
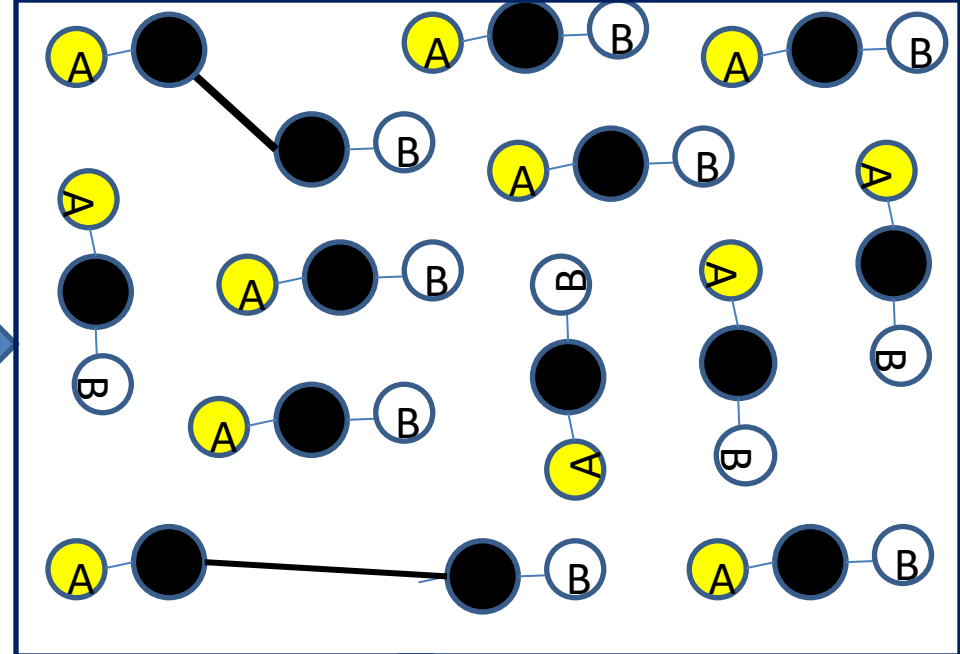
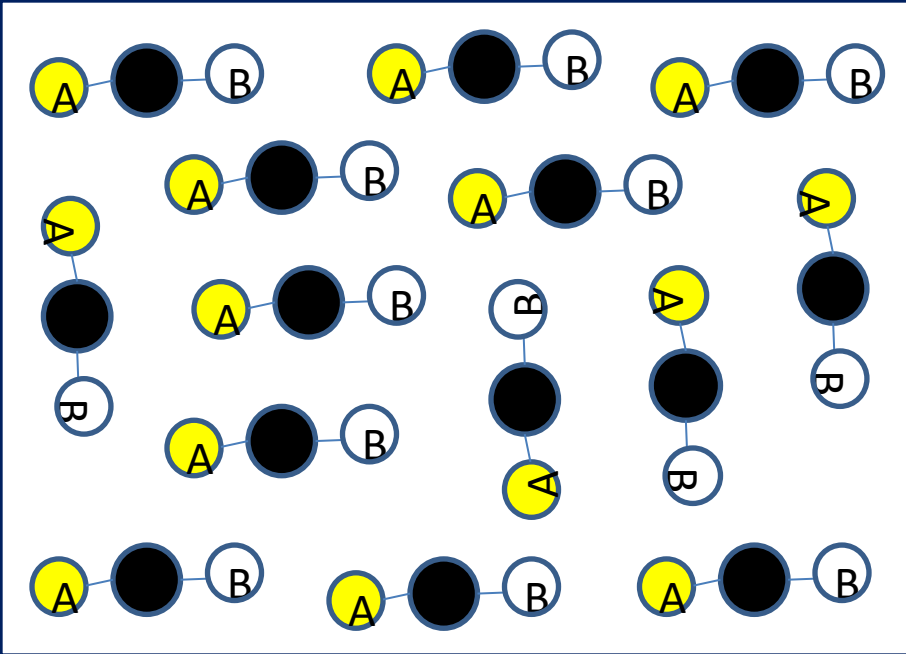


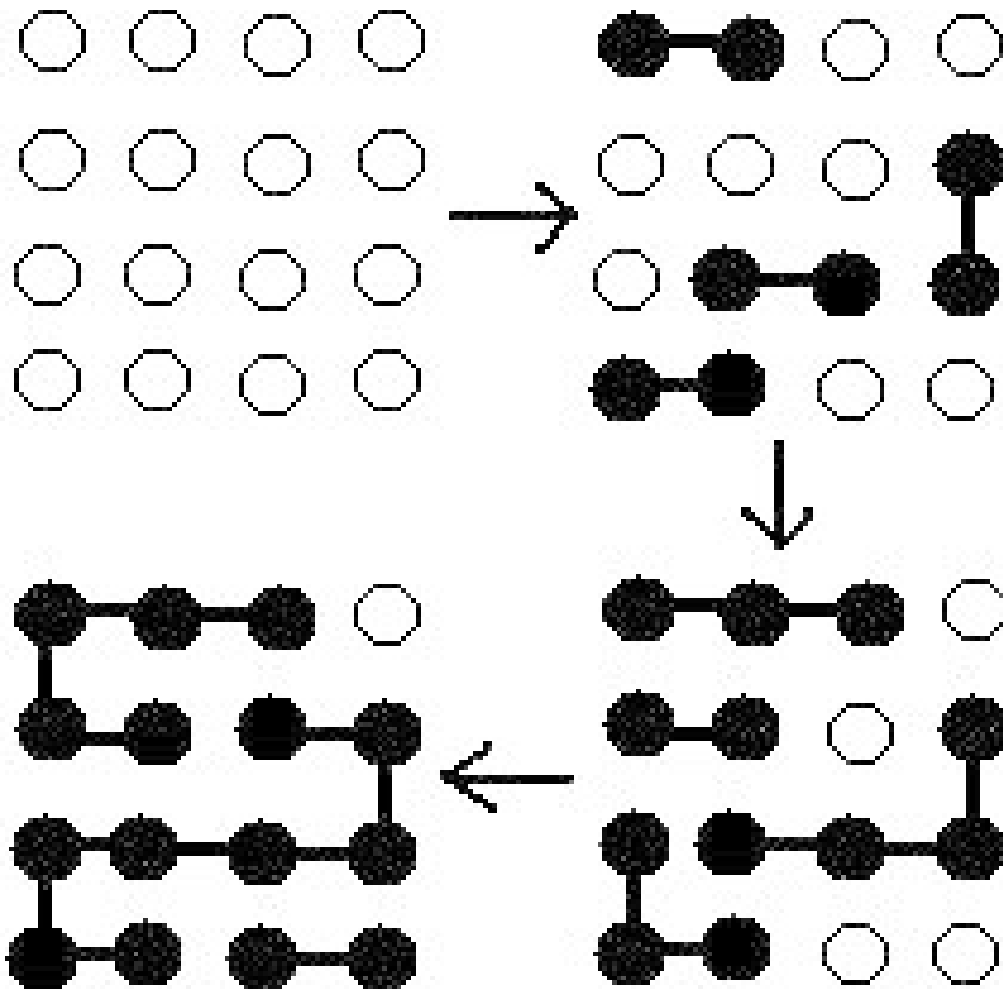
...

etc

...







Polycondensation (type AB)

Conversion (p) = fraction of end-groups that have reacted

$$p = \frac{N_{Ao} - N_A}{N_{Ao}} = 1 - \frac{N_A}{N_{Ao}}$$

N_{Ao} = initial number of mols of end-groups A

N_A = number of mols of end-groups A

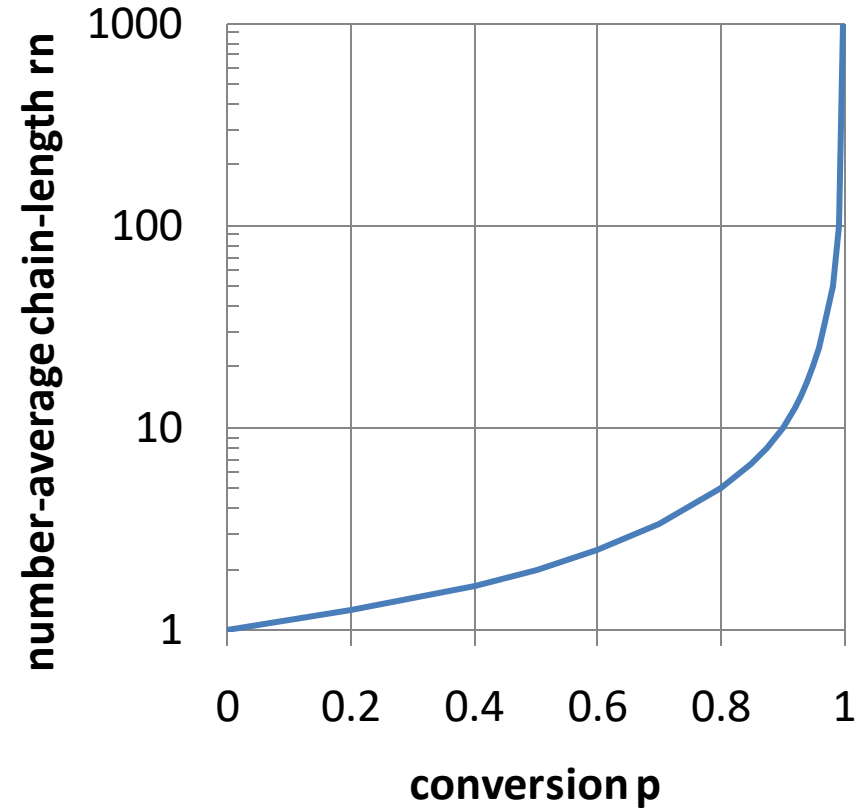
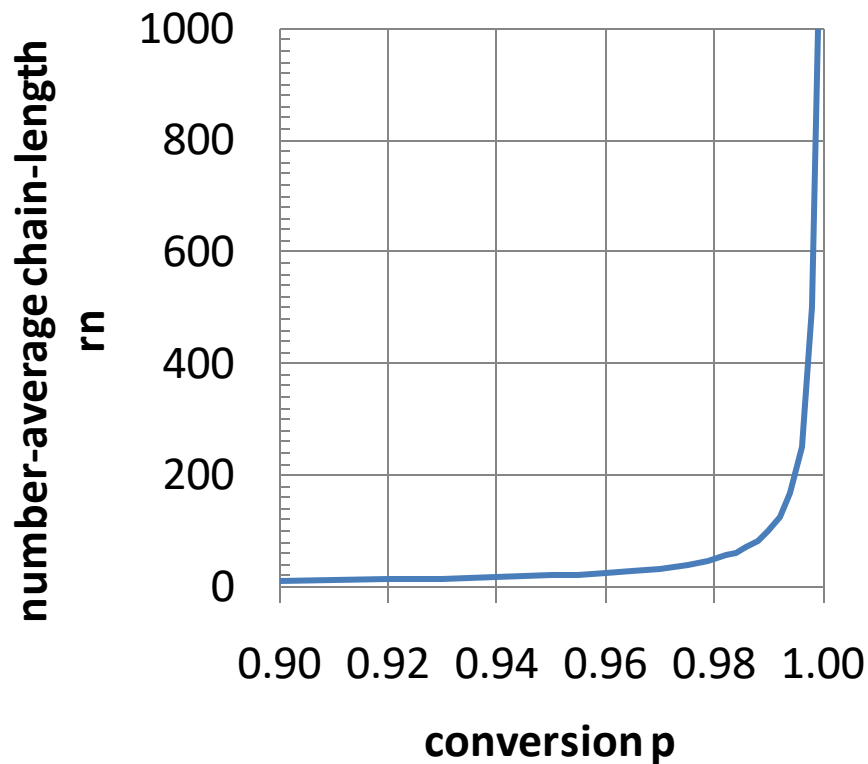
Number-average degree of polymerization (r_n)

Number-average-chain length (r_n)

$$\bar{r}_n = \frac{\text{total number of monomeric units}}{\text{number of molecules}} = \frac{N_{Ao}}{N_A} = \frac{N_{Ao}}{N_{Ao}(1-p)} = \frac{1}{1-p}$$

Polycondensation (type AB)

$$\bar{r}_n = \frac{1}{1-p}$$



Conversion of end-groups (p) must be > 0.99 for average chain length of 100

KINETICS OF POLYCONDENSATION REACTIONS

Polycondensation reaction can be treated as a reaction between end-groups A and B.

For a batch reactor of constant volume (assuming the hypothesis of equal reactivity of the end-groups):

$$\frac{d[A]}{dt} = -k \cdot [cat] \cdot [A] \cdot [B]$$

where [A] = concentration of A groups (mol/L)

k = rate constant

[cat] = concentration of catalyst (acid, base, metal oxide)

For polycondensation type 1 (AB)

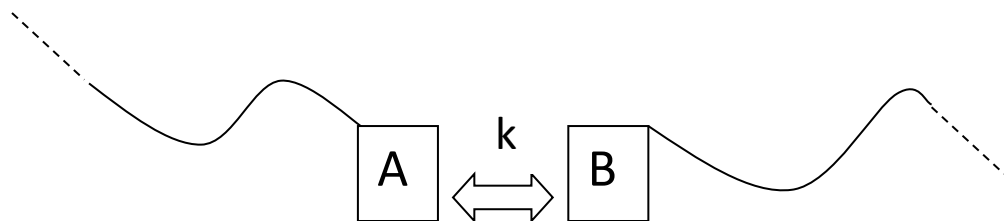
$[A] = [B] = M$ = concentration of all polymer molecules

$[A]_o = [B]_o = M_o$ = initial concentration of monomer AB

$$\frac{dM}{dt} = -kM^2 \Rightarrow M = \frac{M_o}{1 + M_o kt} \Rightarrow \frac{1}{1 - p} = 1 + M_o kt$$

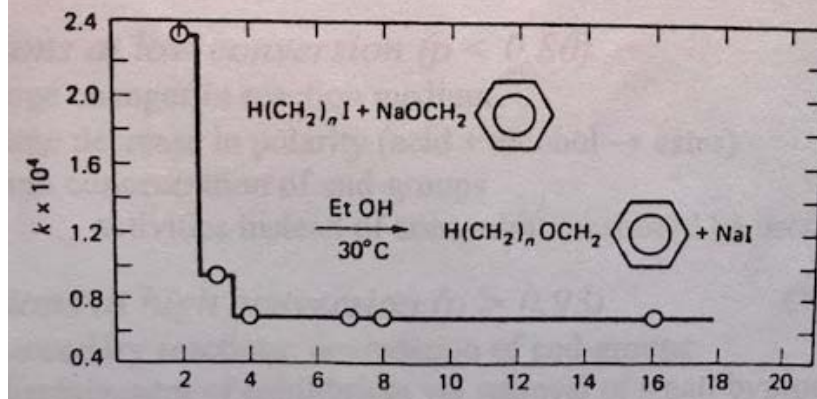
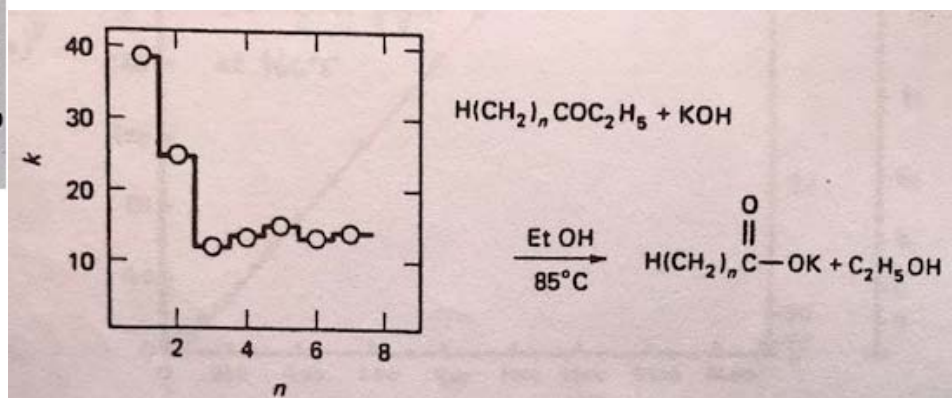
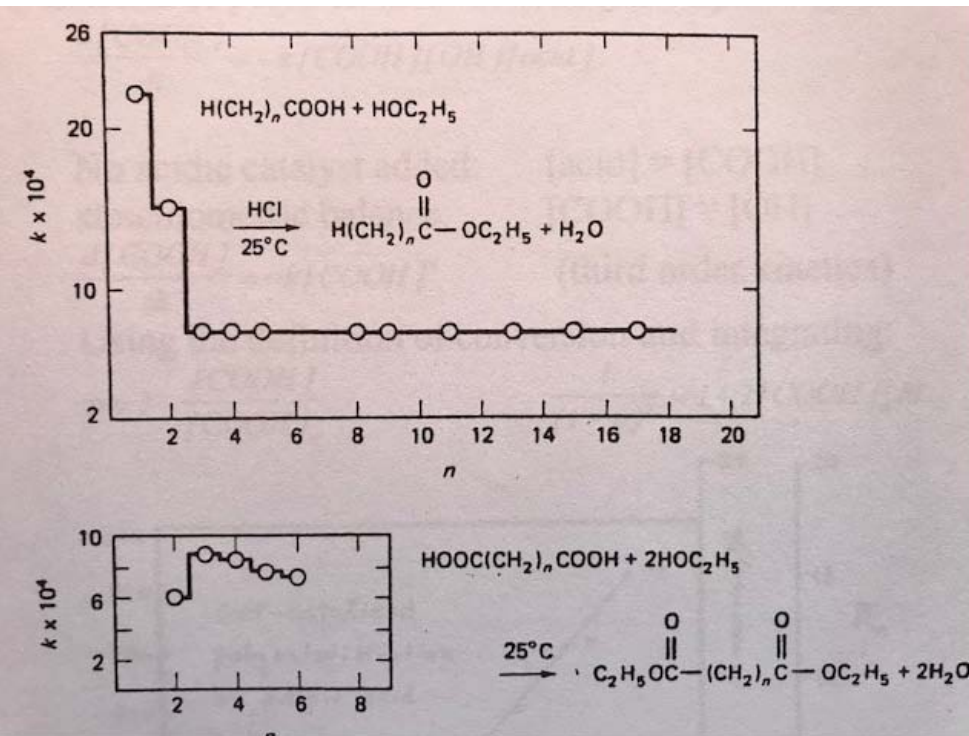
Hypothesis: *equal reactivity of end-groups*

Does the reactivity of an end-group depend on the length of the chain to which it is linked?



→experimental evidences from experiments with homologous series

Cinética de reações de séries homólogas



Dependence of rate constants of functional group reactions on molecular size
(P. J. Flory, *Principles of Polymer Chemistry*, Ithaca, N.Y., Cornell University Press, 1953, p. 70-71)

Example: kinetics of polyesterification (catalyzed by acids)

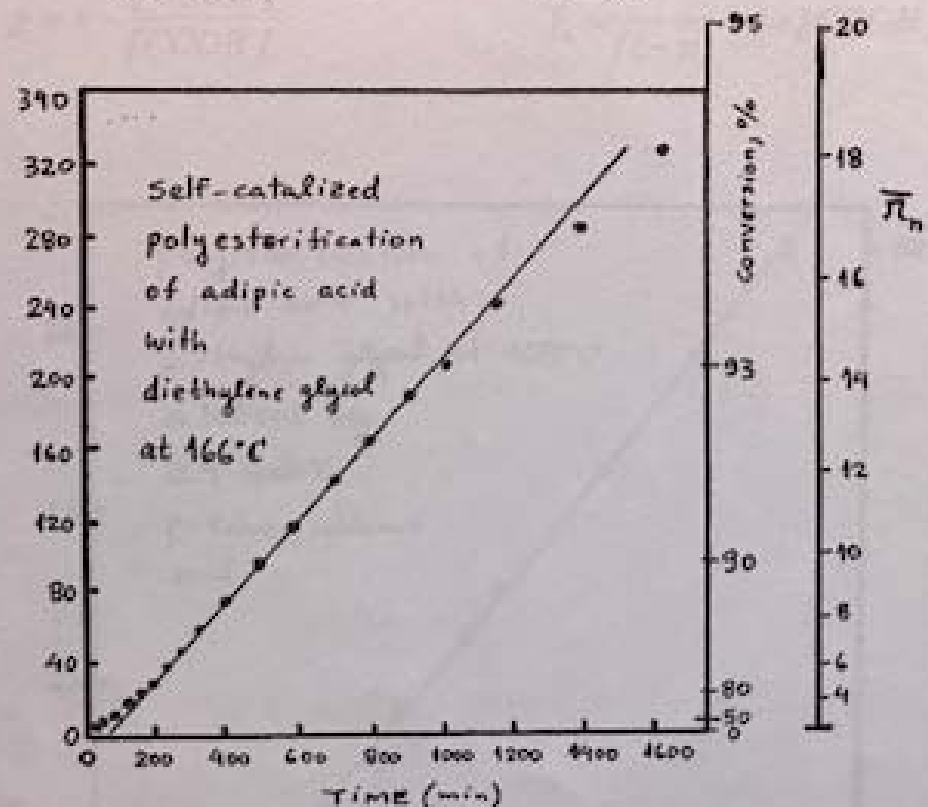
$$\frac{d[\text{COOH}]}{dt} = -k[\text{COOH}][\text{OH}][\text{acid}]$$

Case 1: No acidic catalyst added: $[\text{acid}] = [\text{COOH}]$
 stoichiometric balance: $[\text{COOH}] = [\text{OH}]$
 $\frac{d[\text{COOH}]}{dt} = -k[\text{COOH}]^2$ (third order kinetics)

Using the definition of conversion and integrating:

$$p = 1 - \frac{[\text{COOH}]}{[\text{COOH}]_0} \quad \frac{1}{(1-p)^2} = 1 + 2[\text{COOH}]_0^2 kt$$

$$\frac{1}{(1-p)^2}$$



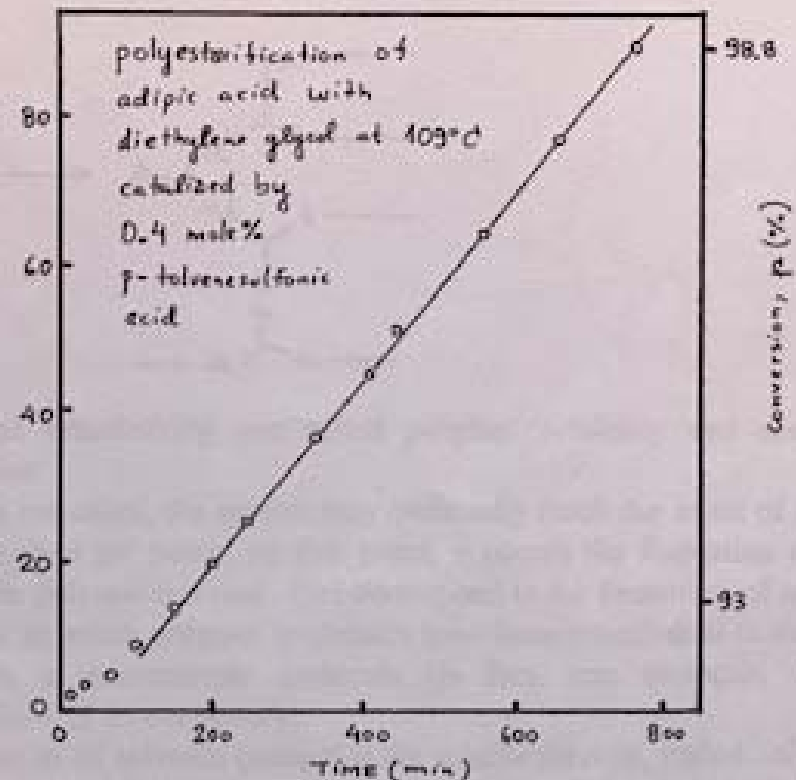
Case 2: Sulfuric acid or p-toluenesulfonic acid
 [acid] remains constant
 stoichiometric balance: $[\text{COOH}] = [\text{OH}]$

$$\frac{d[\text{COOH}]}{dt} = -k' [\text{COOH}]^2 \quad (\text{second order kinetics})$$

Using the definition of conversion and integrating:

$$p = 1 - \frac{[\text{COOH}]}{[\text{COOH}]_0} \quad \bar{\tau}_n = \frac{1}{(1-p)} = 1 + [\text{COOH}]_0 k' t$$

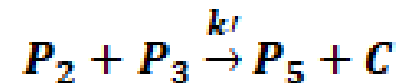
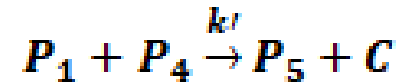
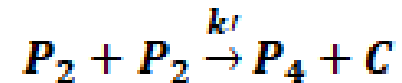
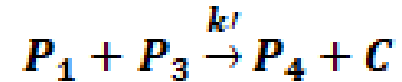
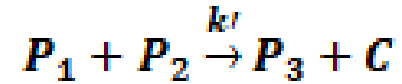
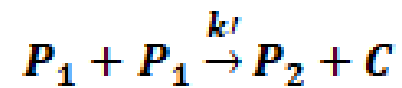
$$\bar{n}_n = \frac{1}{1-p}$$



Polycondensation

Molar mass distribution (chain length distribution)

$$[P_j] = f(j)$$



....

$$\frac{d[P_1]}{dt} = r_{P_1} = -2k'[P_1][P_1] - k'[P_1][P_2] - k'[P_1][P_3] - k'[P_1][P_4] - \dots$$

$$\frac{d[P_2]}{dt} = r_{P_2} = -k'[P_2][P_1] - 2k'[P_2][P_2] - k'[P_2][P_3] - k'[P_2][P_4] - \dots + k'[P_1][P_1]$$

$$\frac{d[P_3]}{dt} = r_{P_3} = -k'[P_3][P_1] - k'[P_3][P_2] - 2k'[P_3][P_3] - k'[P_3][P_4] - \dots + k'[P_1][P_2]$$

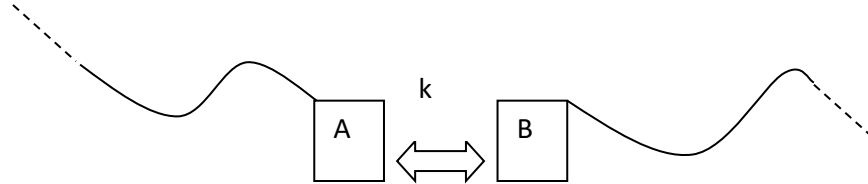
$$\frac{d[P_4]}{dt} = r_{P_4} = -k'[P_4][P_1] - k'[P_4][P_2] - k'[P_4][P_3] - 2k'[P_4][P_4] - \dots + k'[P_1][P_3] + k'[P_2][P_2]$$

$$\frac{d[P_5]}{dt} = r_{P_5} = -k'[P_5][P_1] - k'[P_5][P_2] - k'[P_5][P_3] - k'[P_5][P_4] - \dots + k'[P_1][P_4] + k'[P_2][P_3]$$

$$\frac{d[P_6]}{dt} = r_{P_6} = -k'[P_6][P_1] - k'[P_6][P_2] - k'[P_6][P_3] - k'[P_6][P_4] - \dots + k'[P_1][P_5] + k'[P_2][P_4] + k'[P_3][P_3]$$

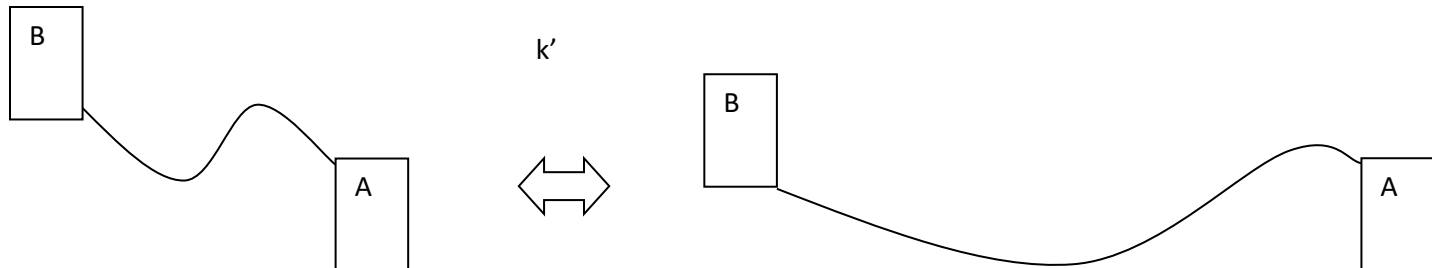
Qual a relação entre
 k (constante cinética da reação entre um grupo A e um grupo B)
e
 k' (constante cinética da reação entre duas moléculas P_n e P_m) ?

**Seja k a constante cinética da reação entre um grupo A e um grupo B
(não importando em que cadeia os grupos A e B estão ligados)**



**Como deve ser a constante k' da reação entre 2 moléculas de polímero
de tamanhos diferentes (p.ex. P_1 e P_5) ?**

**Como deve ser a constante k' da reação entre 2 moléculas de polímero
de mesmo tamanho (p.ex. P_5 e P_5) ?**



Como deve ser a constante k' da reação entre 2 moléculas de polímero de tamanhos diferentes (p.ex. P_1 e P_5) ?

Como deve ser a constante k' da reação entre 2 moléculas de polímero de mesmo tamanho (p.ex. P_5 e P_5) ?

A-(P₁)-B



A-(P₅)-B

A-(P₅)-B



A-(P₅)-B

$$k' = k \quad \text{se } n = m$$

$$k' = 2k \quad \text{se } n \neq m$$

??

Argumentos:

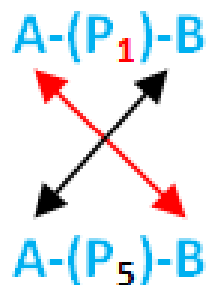
- Semântico
- Pictórico
- Matemático
- Probabilístico

$$k' = k \quad \text{se } n = m$$

$$k' = 2k \quad \text{se } n \neq m$$

Argumento Semântico:

descrever com palavras as “duas” possibilidades



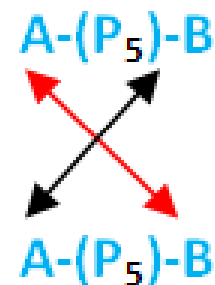
A do P1 reagindo com B do P5

$\neq$

A do P5 reagindo com B do P1

$\therefore$ 2 possibilidades diferentes

$$k' = 2k$$



A do P5 reagindo com o B do P5

$=$

A do P5 reagindo com o B do P5

$\therefore$ 2 possibilidades iguais, portanto
há apenas 1 possibilidade

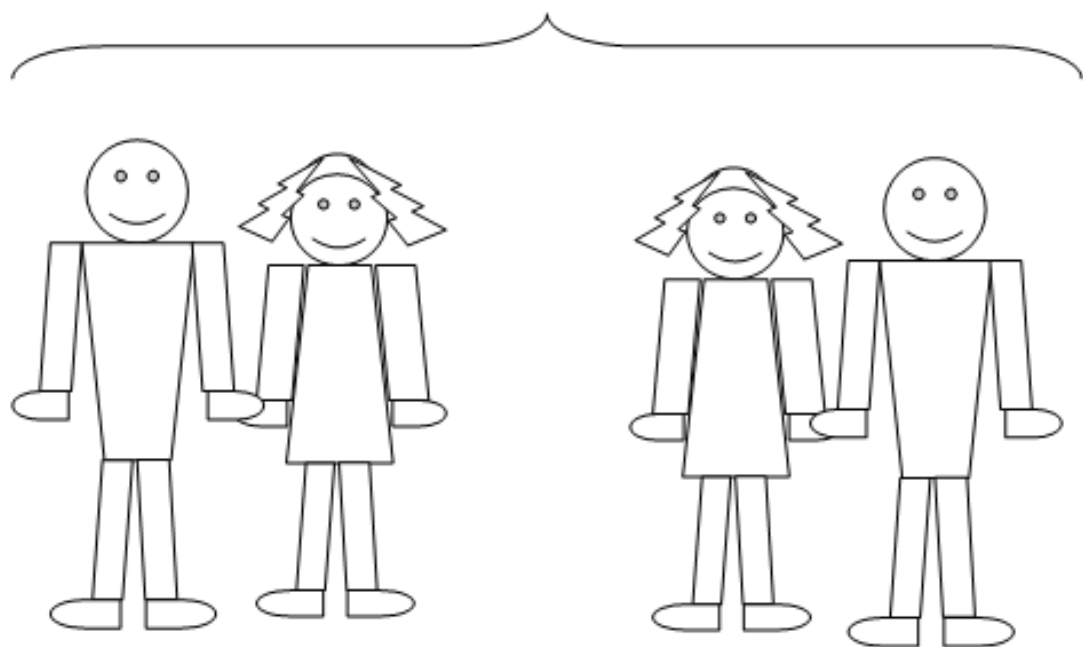
$$k' = k$$

$$k' = k \quad \text{se } n = m$$
$$k' = 2k \quad \text{se } n \neq m$$

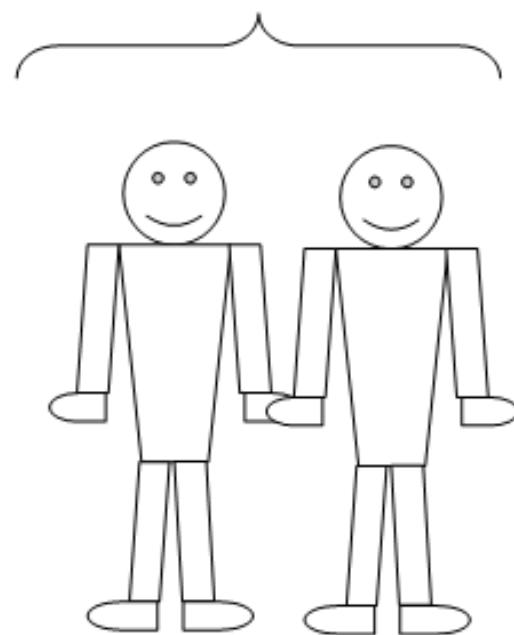
Argumento Pictórico:

desenhar as “duas” possibilidades

2 maneiras diferentes



2 maneiras iguais (indistinguíveis)



$$\begin{aligned} k' &= k & \text{se } n = m \\ k' &= 2k & \text{se } n \neq m \end{aligned}$$

Argumento Matemático:

fazer o balanço de todas as espécies P_i somadas juntas ($M = \sum_{i=1}^{\infty} [P_i]$)
e comparar com o balanço de C

pela estequiometria da reação da policondensação $P_n + P_m \rightarrow P_{n+m} + C$

$$M + [C] = \text{constante} \Rightarrow dM/dt + d[C]/dt = 0$$

(verificar, exercício)

$$\begin{aligned} k' &= k & \text{se } n &= m \\ k' &= 2k & \text{se } n &\neq m \end{aligned}$$

Argumento Probabilístico



Jogue 2 dados simultaneamente

-Qual a chance de sair o par 4 e 4 ?

Resposta: 1 resultado no universo de 36 resultados possíveis (prob = $1/36$)



-Qual a chance de sair o par 1 e 5 ?

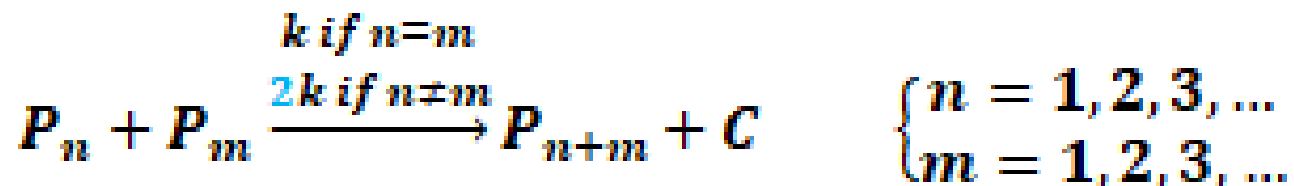
Resposta: 2 chances no universo de 36 resultados possíveis (prob = $2/36$)



Etapas (elementares) do mecanismo de policondensação



...



Balanços para cada espécie P_j

$$\frac{d[P_1]}{dt} = -\textcolor{red}{2}k[P_1][P_1] - \textcolor{blue}{2}k[P_1][P_2] - \textcolor{blue}{2}k[P_1][P_3] - \textcolor{blue}{2}k[P_1][P_4] - \dots$$

$$\frac{d[P_2]}{dt} = -\textcolor{blue}{2}k[P_2][P_1] - \textcolor{red}{2}k[P_2][P_2] - \textcolor{blue}{2}k[P_2][P_3] - \textcolor{blue}{2}k[P_2][P_4] - \dots + k[P_1][P_1]$$

$$\frac{d[P_3]}{dt} = -\textcolor{blue}{2}k[P_3][P_1] - \textcolor{blue}{2}k[P_3][P_2] - \textcolor{red}{2}k[P_3][P_3] - \textcolor{blue}{2}k[P_3][P_4] - \dots + \textcolor{blue}{2}k[P_1][P_2]$$

$$\frac{d[P_4]}{dt} = -\textcolor{blue}{2}k[P_4][P_1] - \textcolor{blue}{2}k[P_4][P_2] - \textcolor{blue}{2}k[P_4][P_3] - \textcolor{red}{2}k[P_4][P_4] - \dots + \textcolor{blue}{2}k[P_1][P_3] + k[P_2][P_2]$$

$$\frac{d[P_5]}{dt} = -\textcolor{blue}{2}k[P_5][P_1] - \textcolor{blue}{2}k[P_5][P_2] - \textcolor{blue}{2}k[P_5][P_3] - \textcolor{blue}{2}k[P_5][P_4] - \dots + \textcolor{blue}{2}k[P_1][P_4] + \textcolor{blue}{2}k[P_2][P_3]$$

$$\frac{d[P_6]}{dt} = -\textcolor{blue}{2}k[P_6][P_1] - \textcolor{blue}{2}k[P_6][P_2] - \textcolor{blue}{2}k[P_6][P_3] - \textcolor{blue}{2}k[P_6][P_4] - \dots + \textcolor{blue}{2}k[P_1][P_5] + \textcolor{blue}{2}k[P_2][P_4] + k[P_3][P_3]$$

...

$$\frac{d[P_j]}{dt} = r_{P_j} = -\textcolor{blue}{2}k[P_j] \sum_{i=1}^{\infty} [P_i] + k \sum_{i=1}^{j-1} [P_i] [P_{j-i}]$$

$$\frac{d[P_j]}{dt} = r_{P_j} = -\textcolor{blue}{2}k[P_j]M + k \sum_{i=1}^{j-1} [P_i] [P_{j-i}]$$

Resolvendo analiticamente os balanços para cada espécie P_j

$$\frac{d[P_1]}{dt} = -2k[P_1] \left(\frac{M_o}{1+M_o kt} \right) \longrightarrow [P_1] = M_o \left(\frac{1}{1+M_o kt} \right)^2$$

$$\frac{d[P_2]}{dt} = -2k[P_2] \left(\frac{M_o}{1+M_o kt} \right) + k[P_1][P_1] \longrightarrow [P_2] = M_o \left(\frac{1}{1+M_o kt} \right)^2 \left(\frac{M_o kt}{1+M_o kt} \right)$$

$$\frac{d[P_3]}{dt} = -2k[P_3] \left(\frac{M_o}{1+M_o kt} \right) + 2k[P_1][P_2] \longrightarrow [P_3] = M_o \left(\frac{1}{1+M_o kt} \right)^2 \left(\frac{M_o kt}{1+M_o kt} \right)^2$$

...

$$\frac{d[P_j]}{dt} = -2k[P_j] \left(\frac{M_o}{1+M_o kt} \right) + k \sum_{i=1}^{j-1} [P_i] [P_{j-i}] \longrightarrow [P_j] = M_o \left(\frac{1}{1+M_o kt} \right)^2 \left(\frac{M_o kt}{1+M_o kt} \right)^{j-1}$$

$$\frac{1}{1-p} = 1 + M_o kt \Rightarrow 1-p = \frac{1}{1+M_o kt} \Rightarrow p = \frac{M_o kt}{1+M_o kt}$$

$$[P_j] = M_o (1-p)^2 p^{j-1}$$

$$y_j = \frac{[P_j]}{\sum_{i=1}^{\infty} [P_i]} = \frac{[P_j]}{M_o (1-p)} = (1-p) p^{j-1}$$

$$w_j = \frac{j[P_j]}{\sum_{i=1}^{\infty} i[P_i]} = j(1-p)^2 p^{j-1}$$

MWD - distribuição tipo Flory

Distribuição tipo Flory

$$[P_j] = M_o(1-p)^2 p^{j-1}$$

$$y_j = \frac{[P_j]}{\sum_{i=1}^{\infty} [P_i]} = (1-p)p^{j-1}$$

$$w_j = \frac{j[P_j]}{\sum_{i=1}^{\infty} i[P_i]} = j(1-p)^2 p^{j-1}$$

$$\bar{r}_n = \sum_{i=1}^{\infty} i y_i = \frac{1}{1-p}$$

$$\bar{r}_w = \sum_{i=1}^{\infty} i w_i = \frac{1+p}{1-p}$$

$$D = \frac{\bar{M}_w}{\bar{M}_n} = \frac{\sum_{i=1}^{\infty} i w_i}{\sum_{i=1}^{\infty} i y_i} = 1 + p$$

