

PQI-3401 – Engenharia de Reações Químicas II
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ANALYSIS OF NON-IDEAL REACTORS

ANÁLISE DE REATORES NÃO IDEAIS

Reinaldo Giudici



UNIVERSIDADE DE SÃO PAULO
ESCOLA POLITÉCNICA
DEPARTAMENTO DE ENGENHARIA QUÍMICA

Analysis of Nonideal Reactors

OBJECTIVES

- To study **deviations** from the **ideal reactors** (CSTR, PFR)
 - Identification/diagnosis (qualitative)
 - Models for nonideal reactors (quantitative)
- Ideal reactors
 - Limiting behavior (total mixing/no-mixing)
 - Easier to math. model
 - Fairly good approx. for many real systems

Ref.

H.S. Fogler,

Elements of Chemical Reaction Engineering,

3rd ed., Prentice-Hall, 1999.

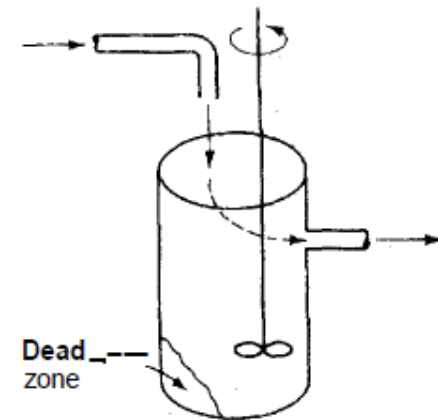
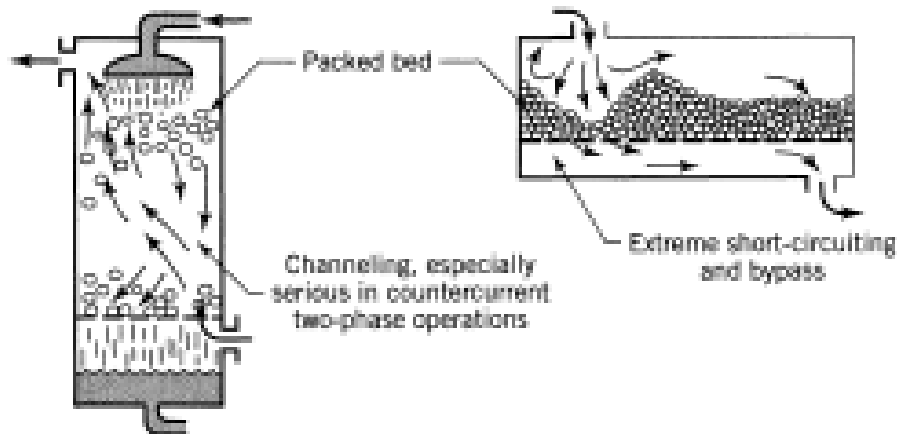
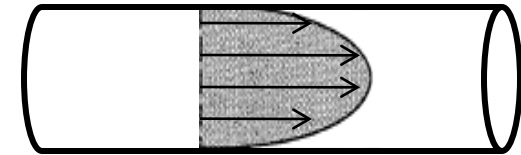
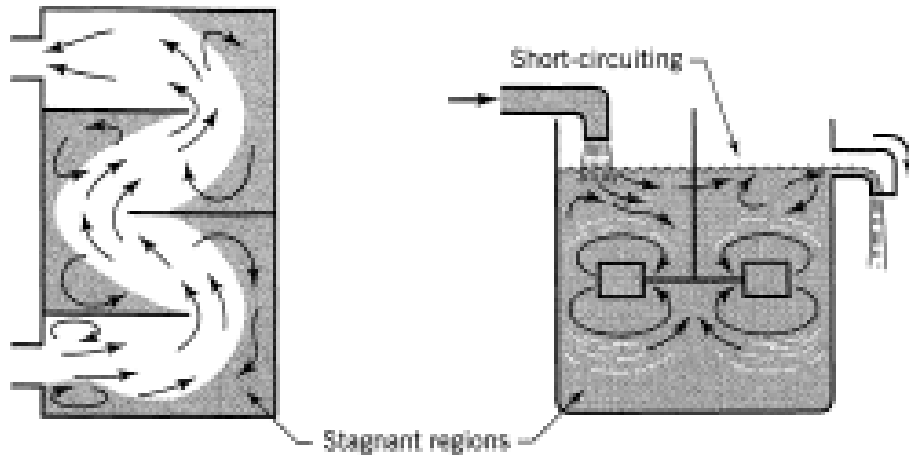
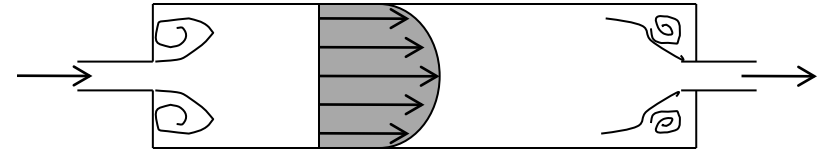
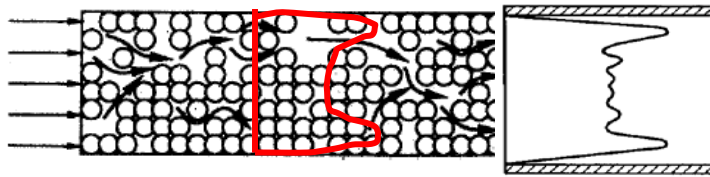
(Cap. 13 & 14)

O. Levenspiel,

Chemical Reaction Engineering,

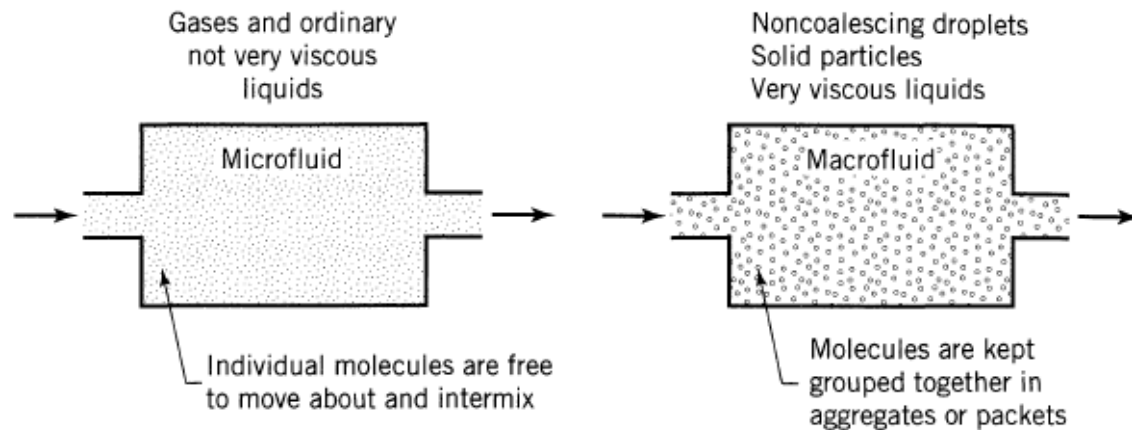
3rd ed., Wiley, 1999

Deviations of the ideal reactor behavior

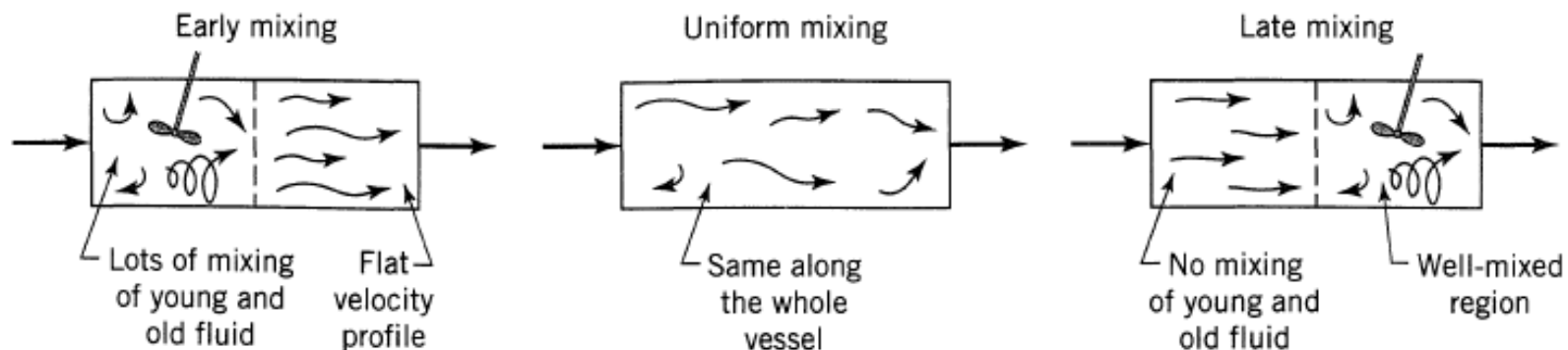


Factors affecting nonideal behavior

- the **RTD** or **residence time distribution** of material which is flowing through the vessel
- the **state of aggregation of the flowing material, its tendency to clump and** for a group of molecules to move about together



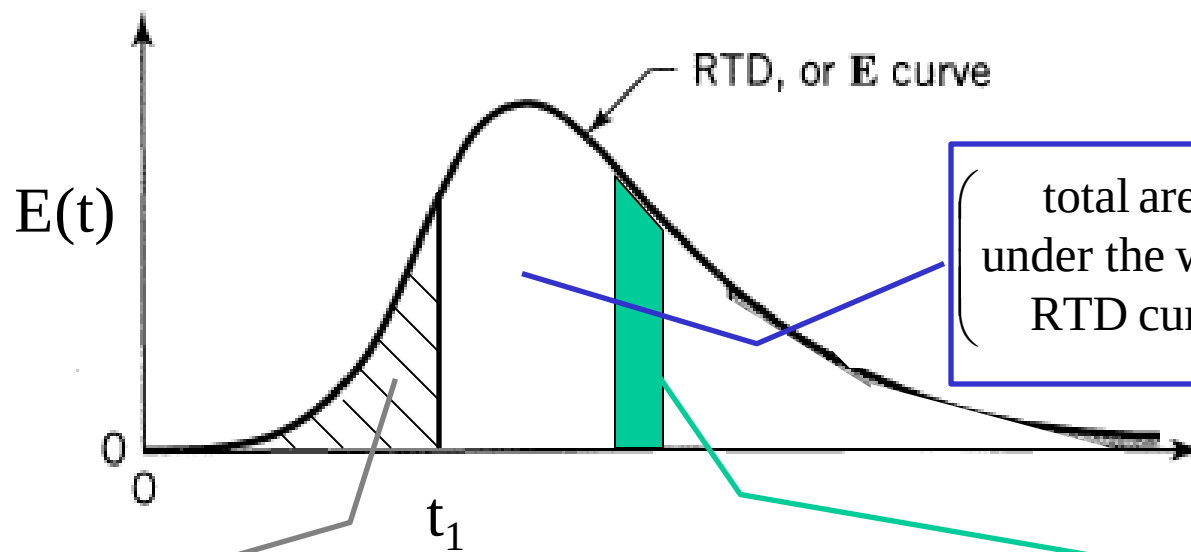
- the **earliness and lateness of mixing of material in the vessel.**





RTD

Residence Time Distribution



$$\left(\begin{array}{c} \text{total area} \\ \text{under the whole} \\ \text{RTD curve} \end{array} \right) = \int_0^{\infty} E(t).dt = 1$$

$$\left(\begin{array}{c} \text{fraction of the} \\ \text{exit stream with} \\ \text{RT} \leq t_1 \end{array} \right) = \int_0^{t_1} E(t).dt$$

$$\left(\begin{array}{c} \text{fraction of the} \\ \text{exit stream with} \\ \text{RT between} \\ t \text{ and } t + dt \end{array} \right) = E(t).dt$$

How to measure the RTD ?

Stimulus-response experiment using a **tracer**

Tracer requirements:

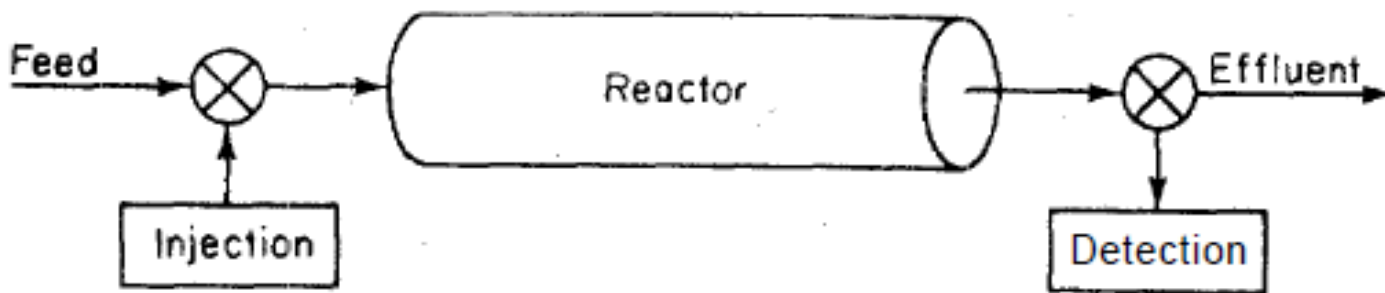
Physical properties similar to the reacting mixture

Completely soluble in the reacting mixture

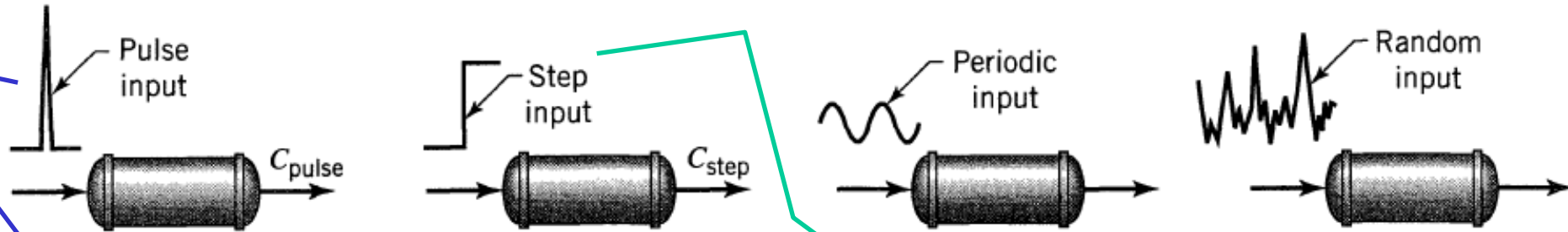
Should not adsorb on the reactor walls and other surfaces

Easily detectable

Usually nonreactive



How to measure the RTD ?



pulse = Dirac's delta function

$$\delta(t - t_0) = \begin{cases} 0 & \text{for } t \neq t_0 \\ \infty & \text{for } t = t_0 \end{cases}$$

$$\int_0^{\infty} \delta(t - t_0) dt = 1$$

a property of delta function :

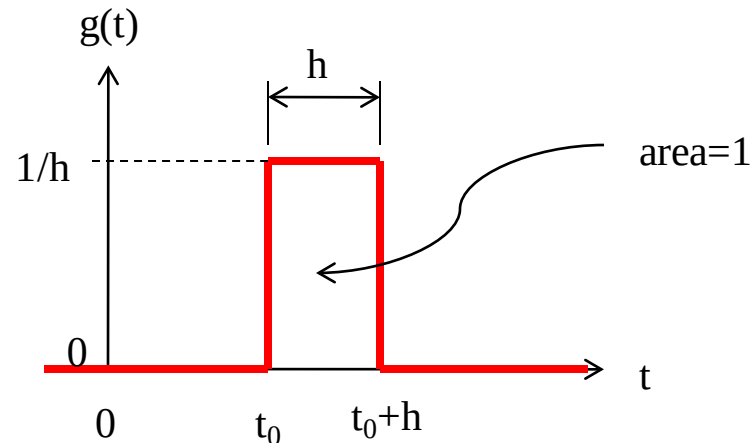
$$\int_a^b f(t) \delta(t - t_0) dt = \begin{cases} f(t_0) & \text{for } a \leq t_0 \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$g(t) = \begin{cases} (1/h) & \text{for } t_0 \leq t \leq t_0 + h \\ 0 & \text{otherwise} \end{cases}$$

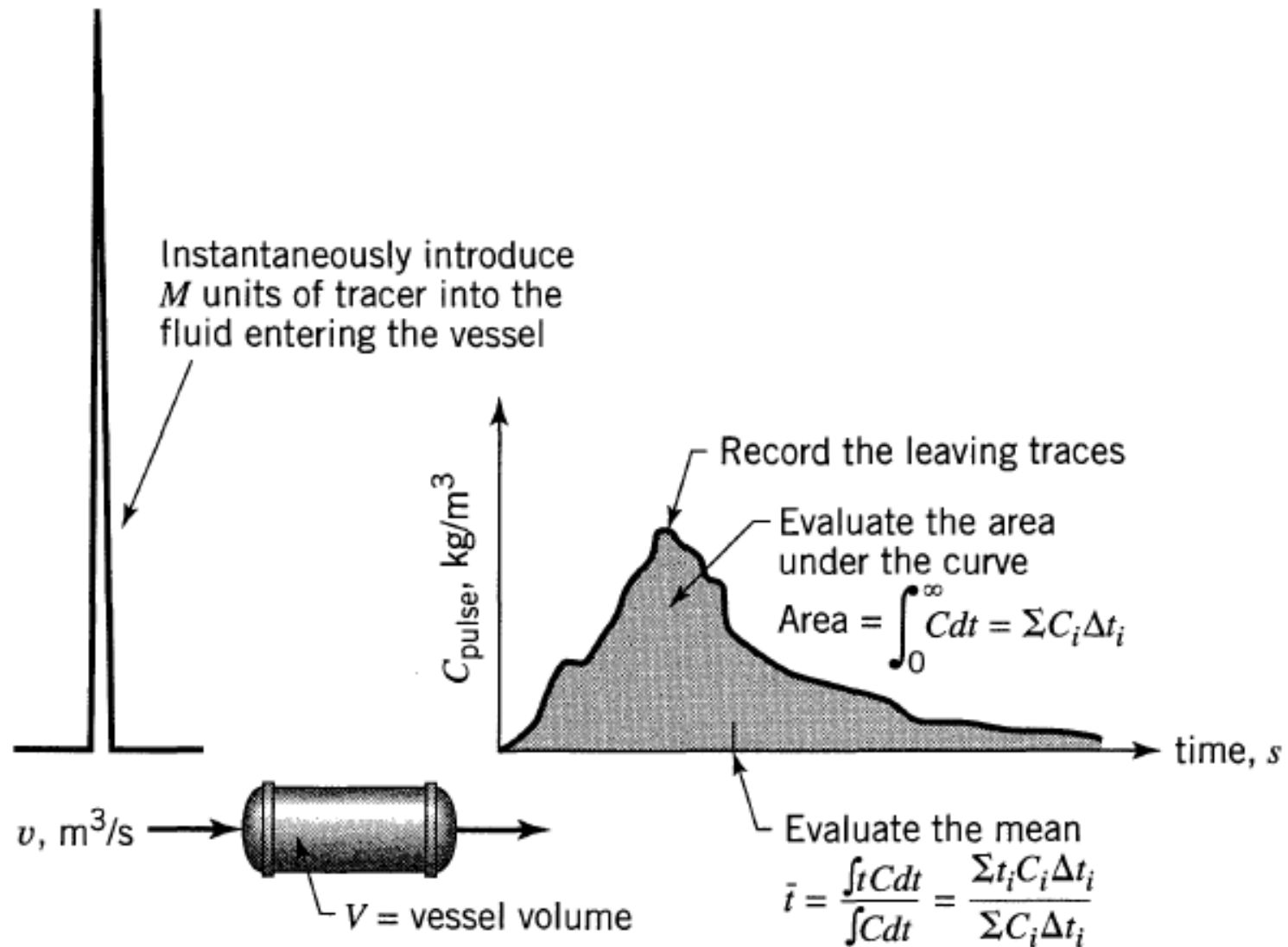
$$\delta(t - t_0) = \lim_{h \rightarrow 0} g(t)$$

step function

$$step(t - t_0) = \begin{cases} 0 & \text{for } t < t_0 \\ C_0 & \text{for } t \geq t_0 \end{cases}$$



The pulse experiment (input = pulse)



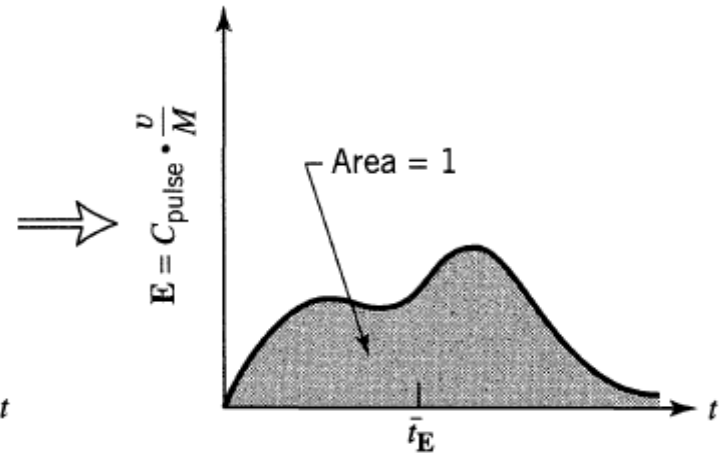
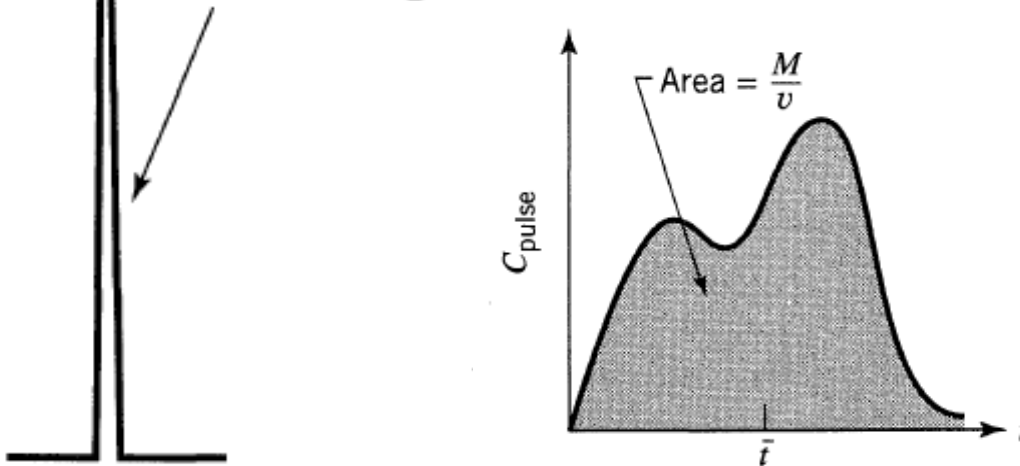
The pulse experiment

$$C_{input}(t) = \delta(t - 0) = \begin{cases} \infty & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

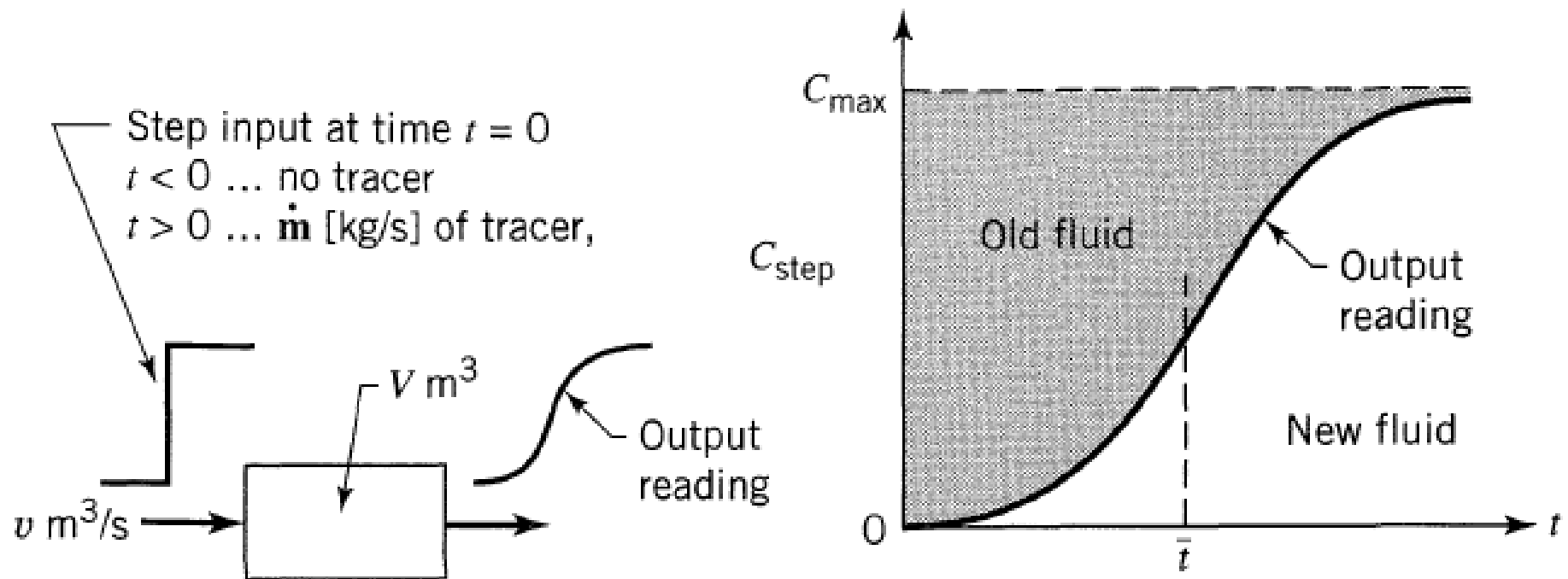
$$C_{exit}(t) = C_{pulse}(t)$$

$$C(t) = \frac{C_{pulse}(t)}{\text{area}} = \frac{C_{pulse}(t)}{\int_0^\infty C_{pulse}(t) dt}$$

Instantaneously introduce M units of tracer into the fluid entering the vessel



The step experiment (input = step)



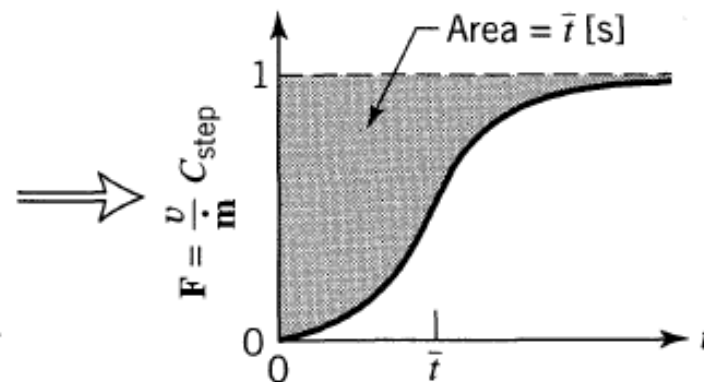
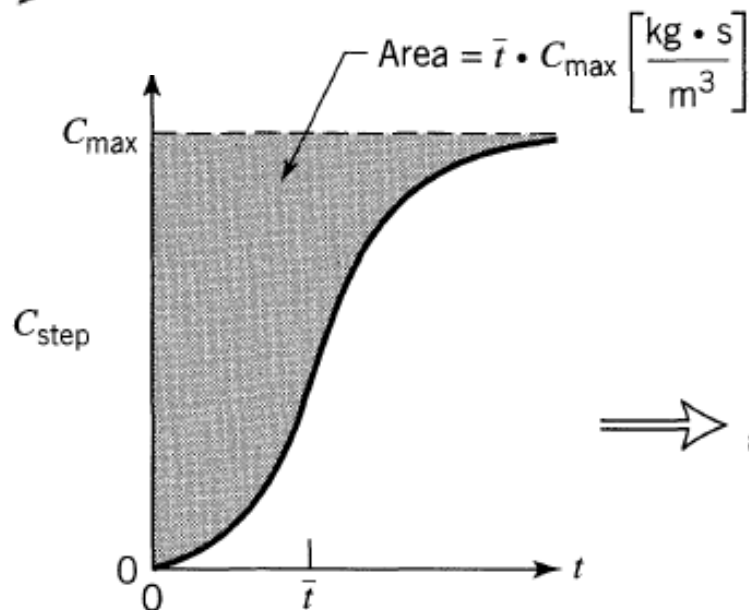
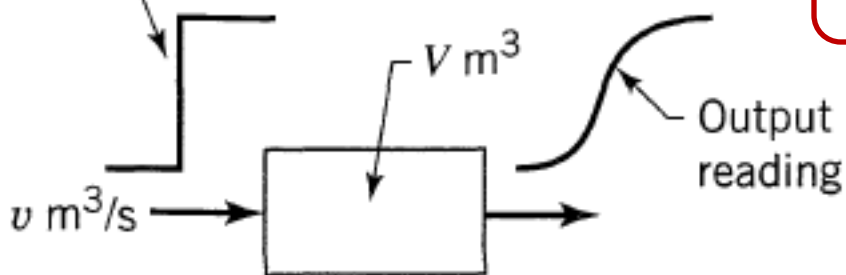
The step experiment

$$C_{input}(t) = \begin{cases} 0 & \text{for } t < t_0 \\ C_{max} & \text{for } t \geq t_0 \end{cases}$$

Step input at time $t = 0$
 $t < 0$... no tracer
 $t > 0$... $\dot{m}$ [kg/s] of tracer,

$$C_{exit}(t) = C_{step}(t)$$

$$F(t) = \frac{C_{step}(t)}{C_{max}}$$



Relationship between
 $C(t)$ = response to pulse
 $F(t)$ = response to step
 and
 $E(t)$ = RTD

PULSE EXPERIMENT

all tracer entered at $t = 0$
 then
 amount of tracer exiting the reactor
 at time t has $RT = t$

$$\left(\begin{array}{c} \text{fraction of tracer} \\ \text{exiting the reactor} \\ \text{at time } t \end{array} \right) = \left(\begin{array}{c} \text{fraction of} \\ \text{tracer that} \\ \text{has } RT = t \end{array} \right)$$

$$C(t) = E(t)$$

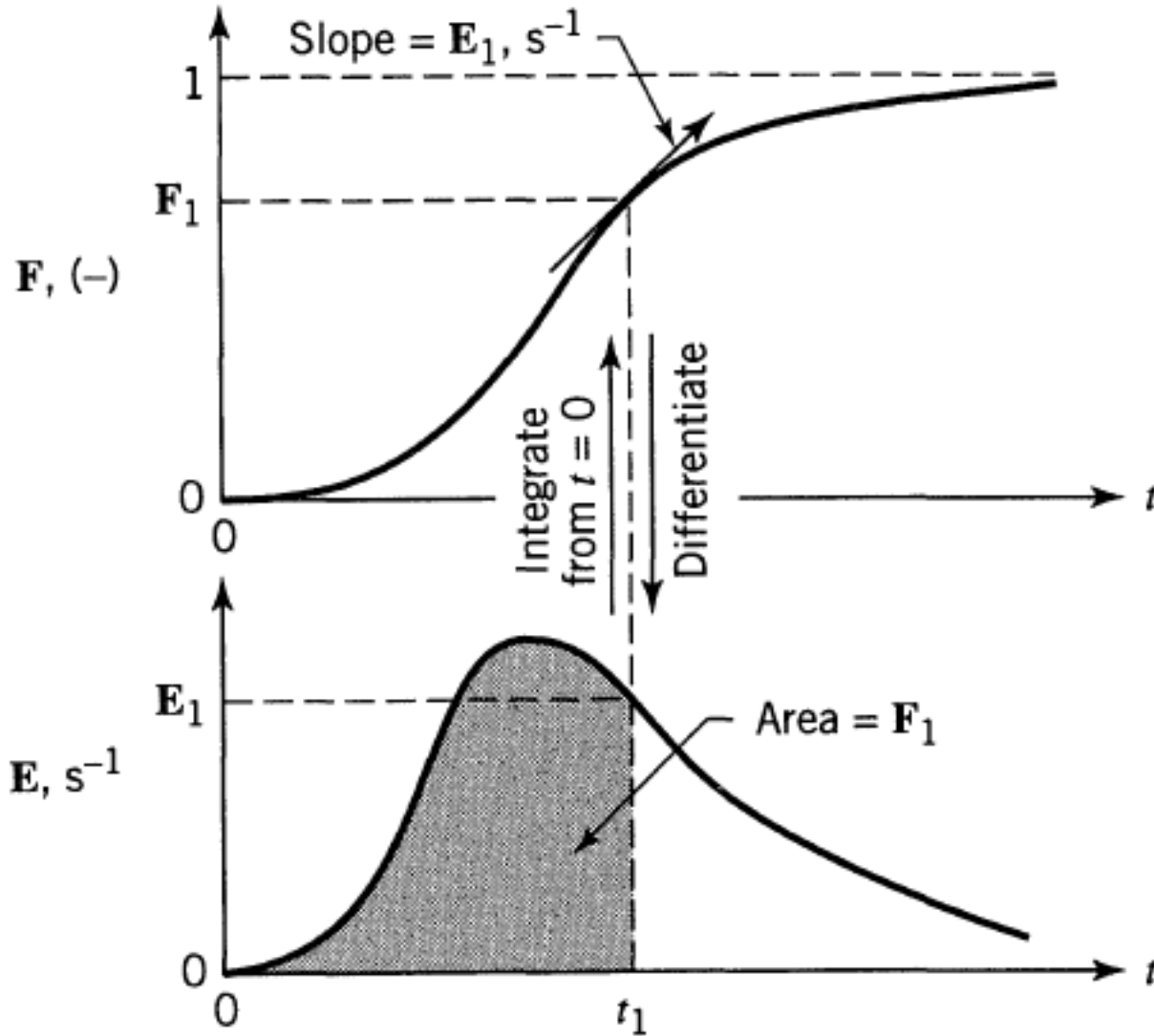
STEP EXPERIMENT

tracer exiting the reactor at time t_1
 entered in the reactor
 at a time between 0 and t_1

$$\left(\begin{array}{c} \text{fraction of tracer} \\ \text{exiting the reactor} \\ \text{at time } = t_1 \end{array} \right) = \left(\begin{array}{c} \text{fraction of} \\ \text{tracer} \\ \text{with } RT \leq t_1 \end{array} \right)$$

$$F(t_1) = \int_0^{t_1} E(t).dt$$

$$E(t) = \frac{dF(t)}{dt}$$



$$F(t_1) = \int_0^{t_1} E(t).dt$$

$$E(t)|_{t_1} = \left. \frac{dF(t)}{dt} \right|_{t_1}$$

Characteristics of the RTD curve

$$\left(\begin{array}{c} \text{mean} \\ \text{residence} \\ \text{time} \end{array} \right) = t_m = \frac{\int_0^{\infty} t.E(t).dt}{\int_0^{\infty} E(t).dt} = \int_0^{\infty} t.E(t).dt$$

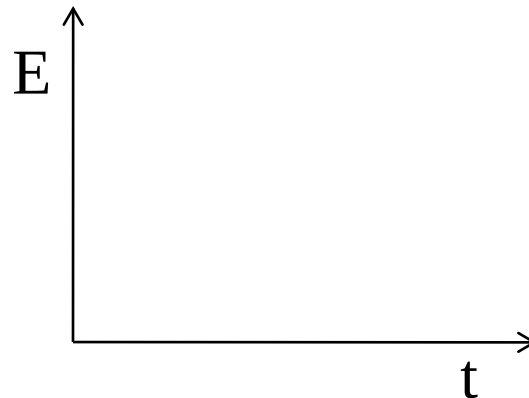
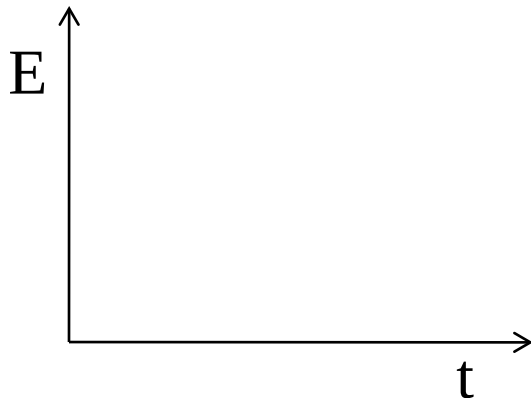
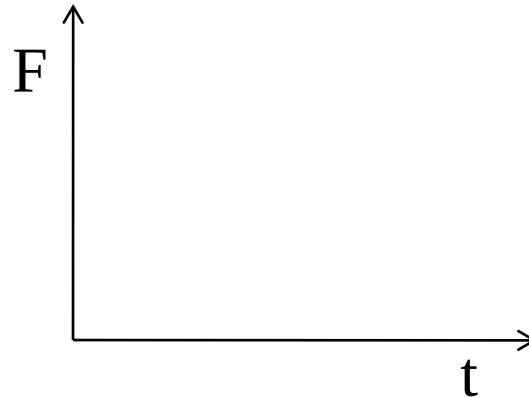
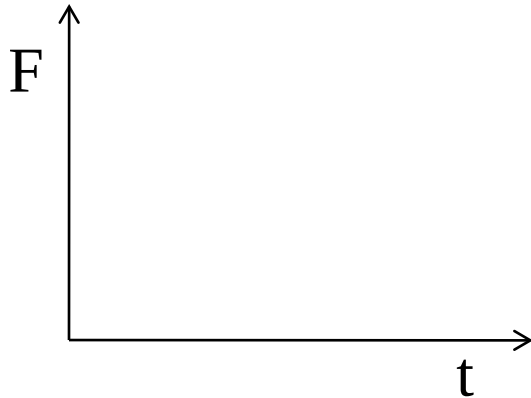
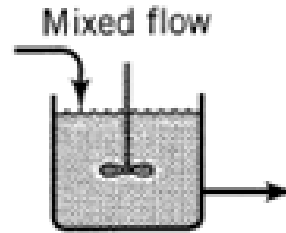
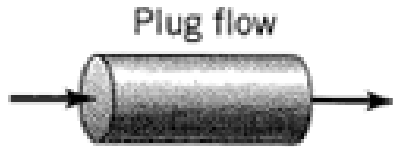
$$\text{variance} = \sigma^2 = \int_0^{\infty} (t - t_m)^2 .E(t).dt$$

$$\text{skewness} = s^3 = \frac{1}{\sigma^{3/2}} \int_0^{\infty} (t - t_m)^3 .E(t).dt$$

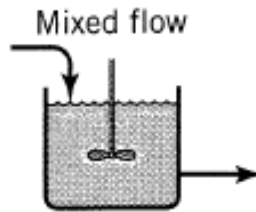
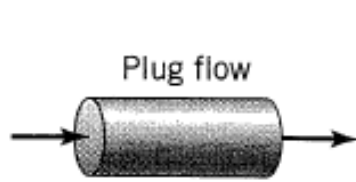
for a fluid of constant density
(constant volumetric flow rate)

$$\left(\begin{array}{c} \text{mean} \\ \text{residence} \\ \text{time} \end{array} \right) = t_m = \tau = \frac{V}{v_o}$$

RTD of the ideal reactors



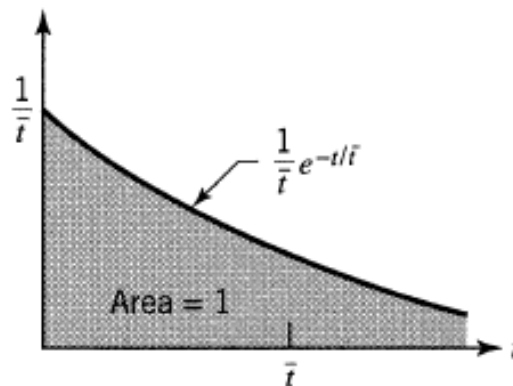
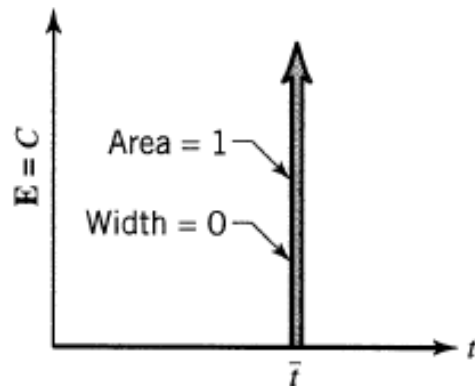
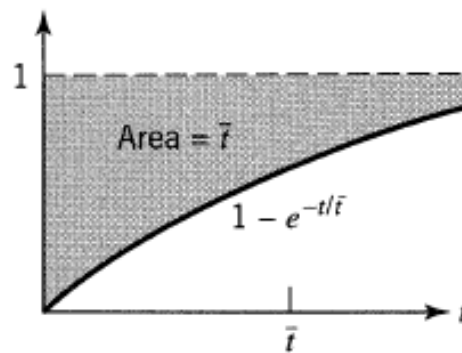
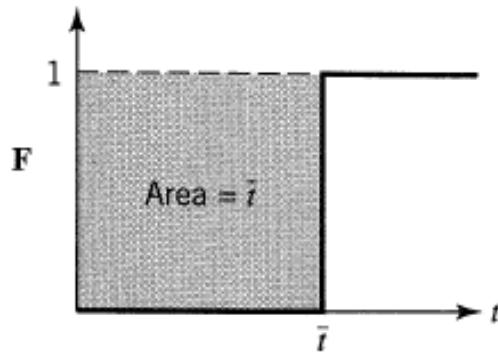
RTD of the ideal reactors



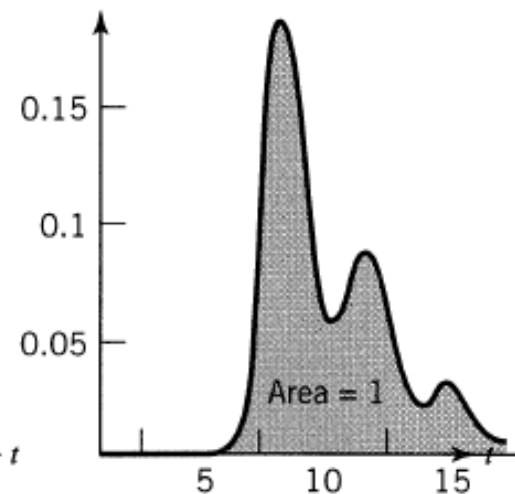
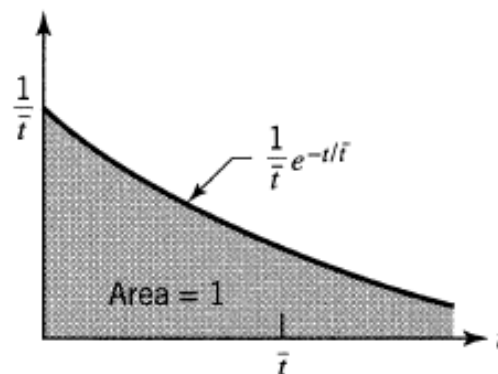
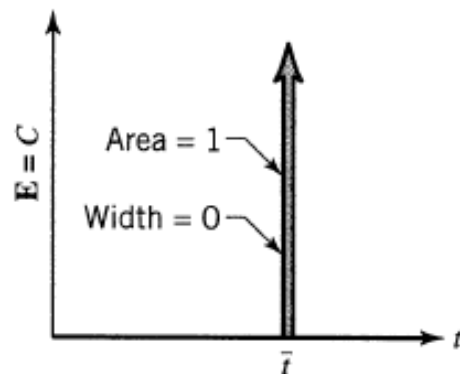
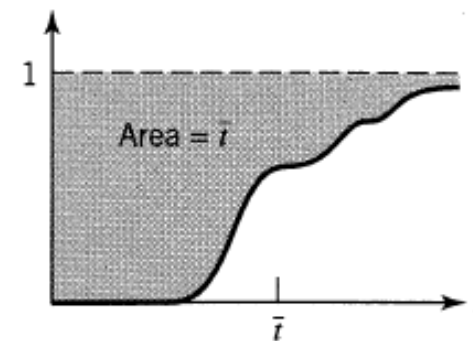
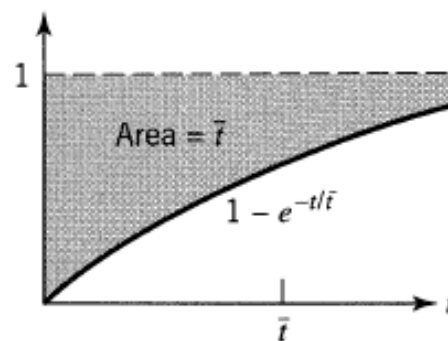
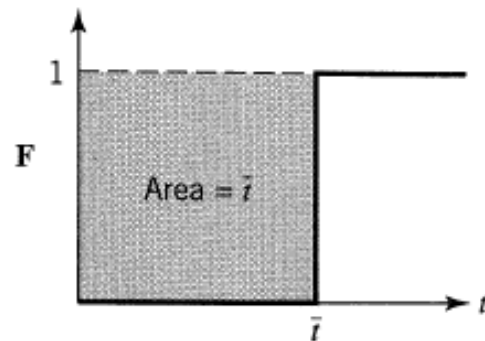
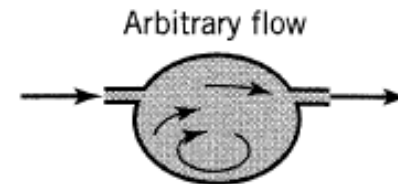
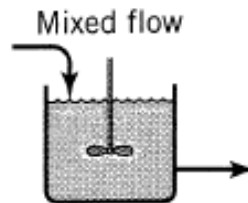
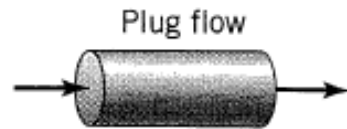
Assignment #1

Derive the RTD curve ($E(t)$) for the ideal reactors (PFR and CSTR)

Use “though experiment” and mass balances



Using RTD to detect flow nonidealities



Using RTD to detect flow nonidealities

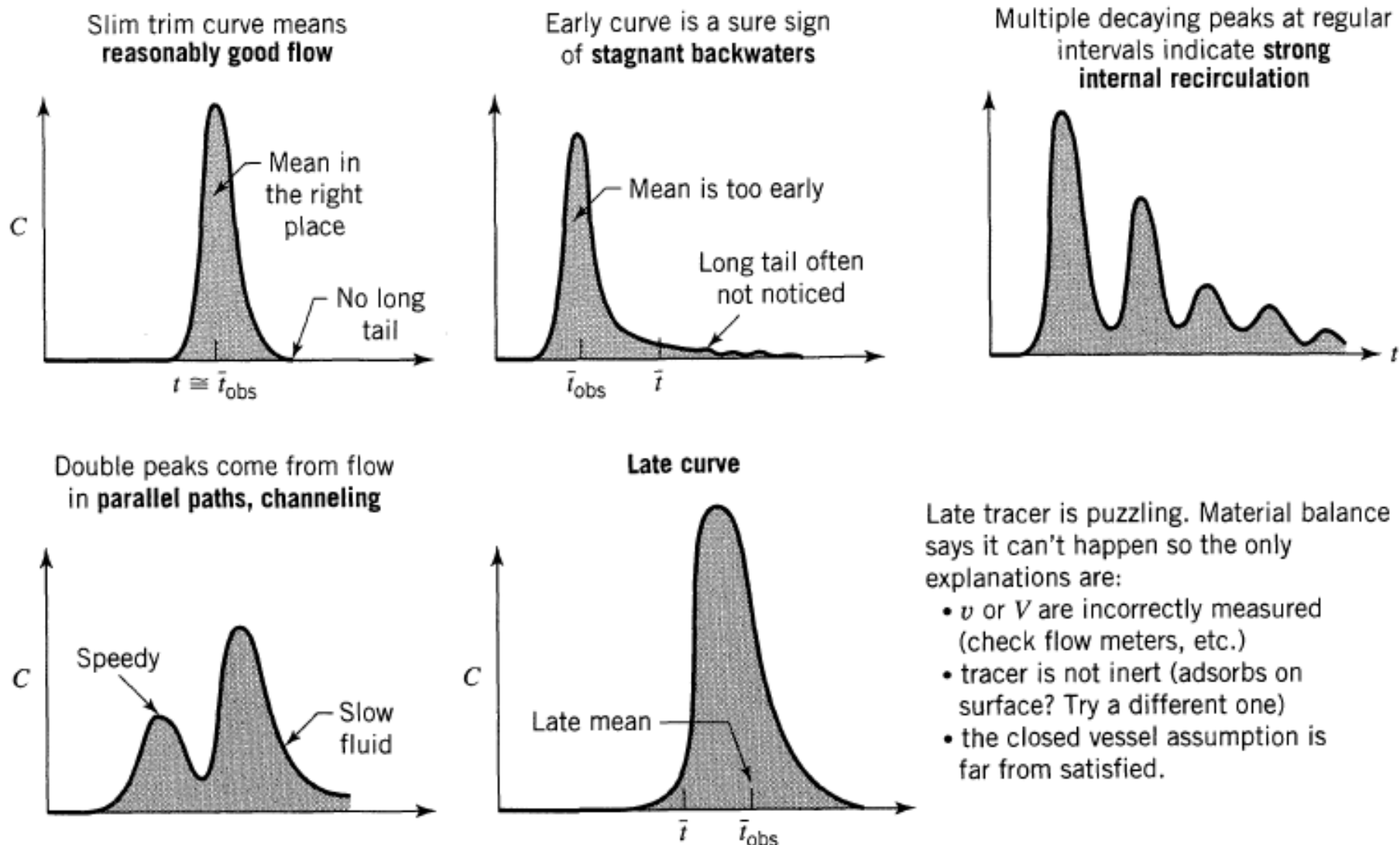


Figure 12.3 Misbehaving plug flow reactors.

Using RTD to detect flow nonidealities

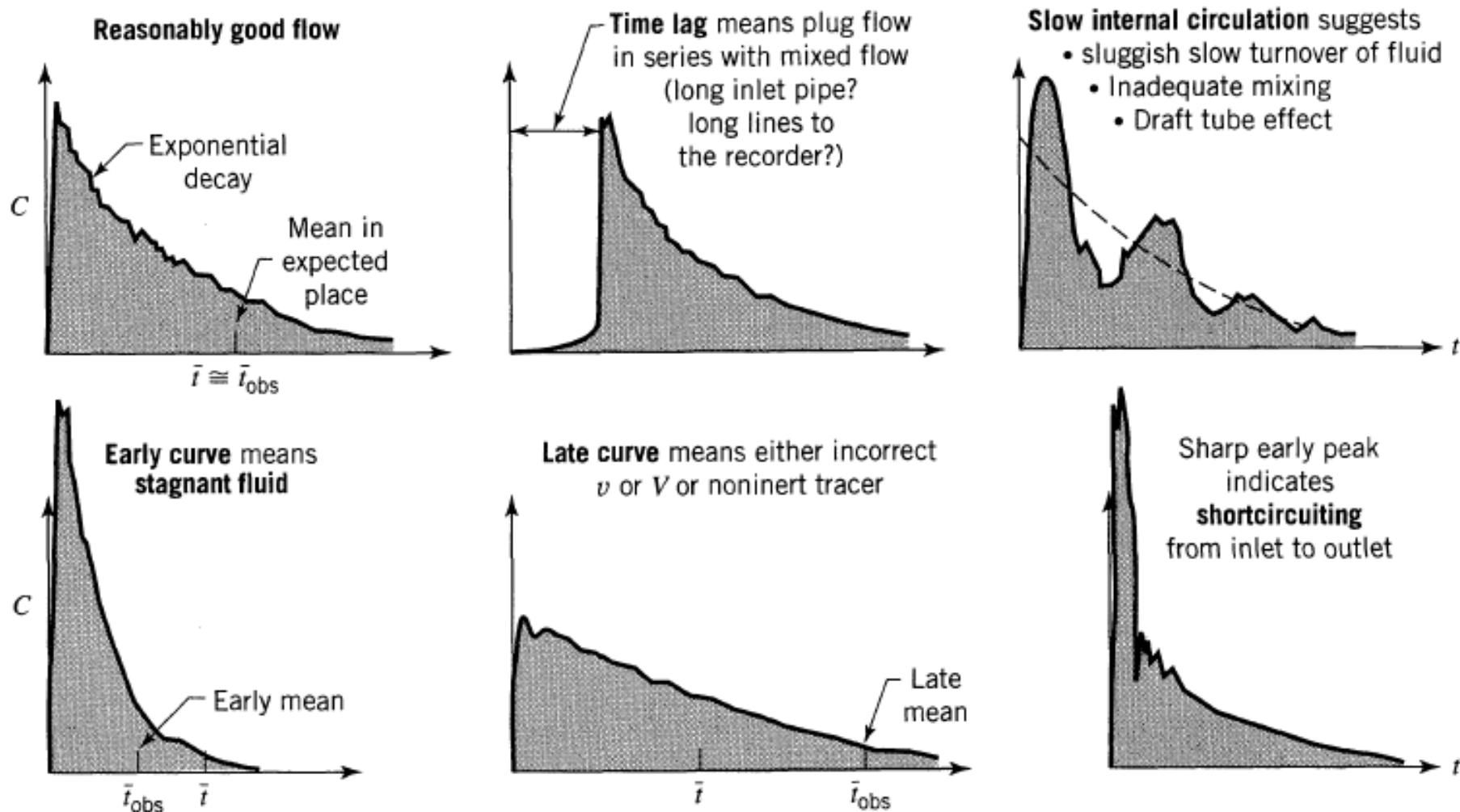


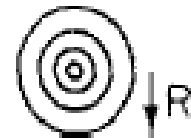
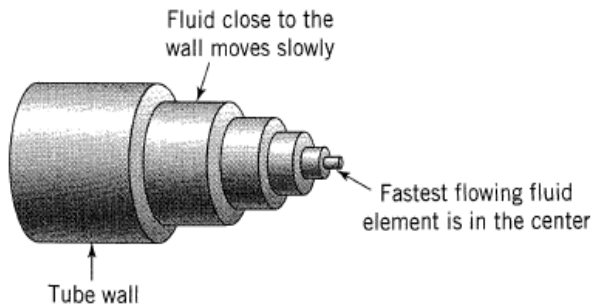
Figure 12.4 Misbehaving mixed flow reactors.



MODELS FOR NONIDEAL REACTORS

Laminar flow reactor

USP



$$u(r) = u_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right] = 2u_{\text{medio}} \left[1 - \left(\frac{r}{R} \right)^2 \right] = 2 \frac{v_0}{\pi R^2} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

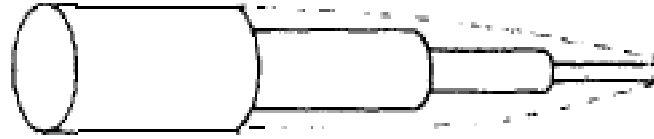
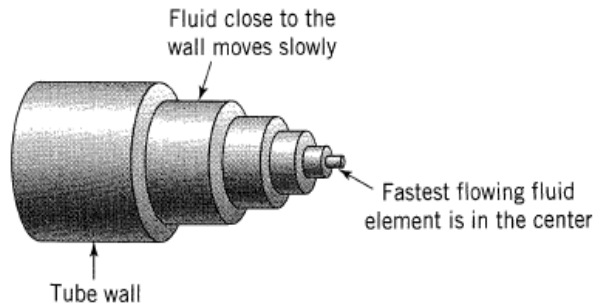
$$t(r) = \frac{L}{u(r)} = \frac{L \pi R^2}{2u_{\text{medio}} \pi R^2 \left[1 - (r/R)^2 \right]} = \frac{V}{2v_0 \left[1 - (r/R)^2 \right]} = \frac{\tau}{2 \left[1 - (r/R)^2 \right]}$$

$$dt = \frac{\tau}{2R^2} \frac{2.r.dr}{\left[1 - (r/R)^2 \right]^2} \frac{(\tau/2)^2}{(\tau/2)^2} = \frac{4}{\tau R^2} \left[\frac{\tau/2}{1 - (r/R)^2} \right]^2 r.dr = \frac{4t^2}{\tau R^2} r.dr$$

$$\left(\begin{array}{c} \text{fraction of fluid} \\ \text{flowing between} \\ r \text{ and } r + dr \end{array} \right) = \frac{dv}{v_o} = \frac{u(r).2\pi.r.dr}{v_o} = \frac{L}{t} \frac{2\pi}{v_o} \frac{\tau R^2}{4t^2} dt = \frac{\tau^2}{2t^3} dt$$

Laminar flow reactor

USP



$$\left(\begin{array}{c} \text{fraction of fluid} \\ \text{flowing between} \\ r \text{ and } r + dr \end{array} \right) = \frac{\tau^2}{2t^3} dt = \left(\begin{array}{c} \text{fraction of fluid} \\ \text{flowing with RT} \\ \text{between} \\ t \text{ and } t + dt \end{array} \right) = E(t).dt$$

minimum residence time (at the tube center) is

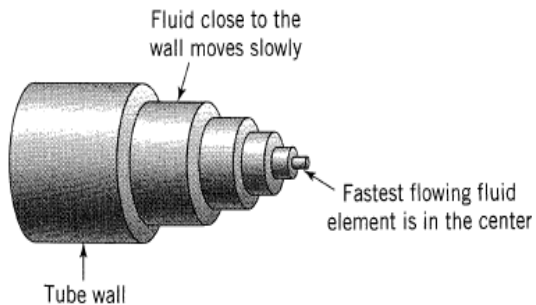
$$t_{\min} = \frac{L}{u_{\max}} = \frac{L}{2.u_{\text{medio}}} \left(\frac{\pi R^2}{\pi R^2} \right) = \frac{V}{2v_0} = \frac{\tau}{2}$$

$$E(t) = \begin{cases} 0 & \text{for } t < \frac{\tau}{2} \\ \frac{\tau^2}{2t^3} & \text{for } t \geq \frac{\tau}{2} \end{cases}$$

$$F(t) = \int_0^t E(t)dt = \begin{cases} 0 & \text{for } t < \frac{\tau}{2} \\ 1 - \frac{\tau^2}{4t^2} & \text{for } t \geq \frac{\tau}{2} \end{cases}$$

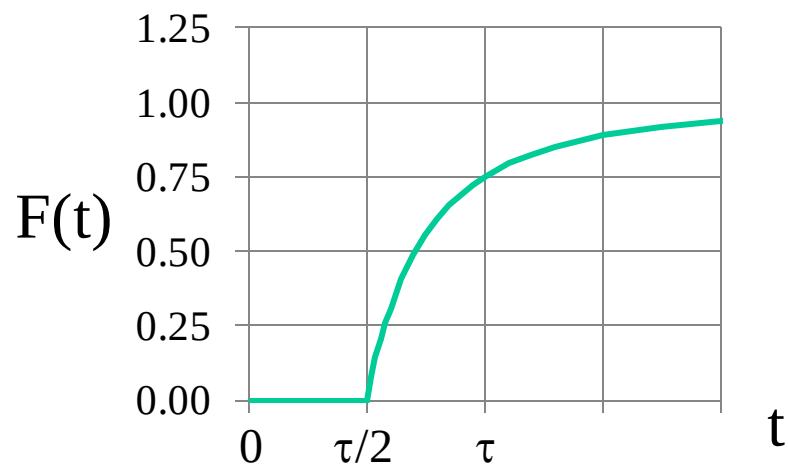
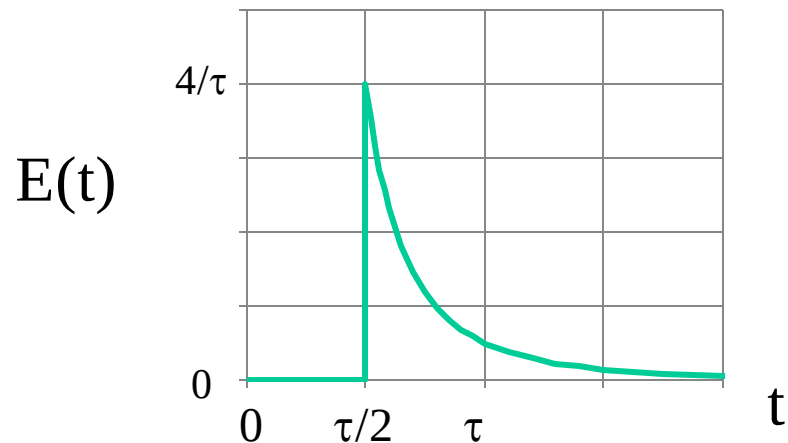
Laminar flow reactor

USP



$$E(t) = \begin{cases} 0 & \text{for } t < \frac{\tau}{2} \\ \frac{\tau^2}{2t^3} & \text{for } t \geq \frac{\tau}{2} \end{cases}$$

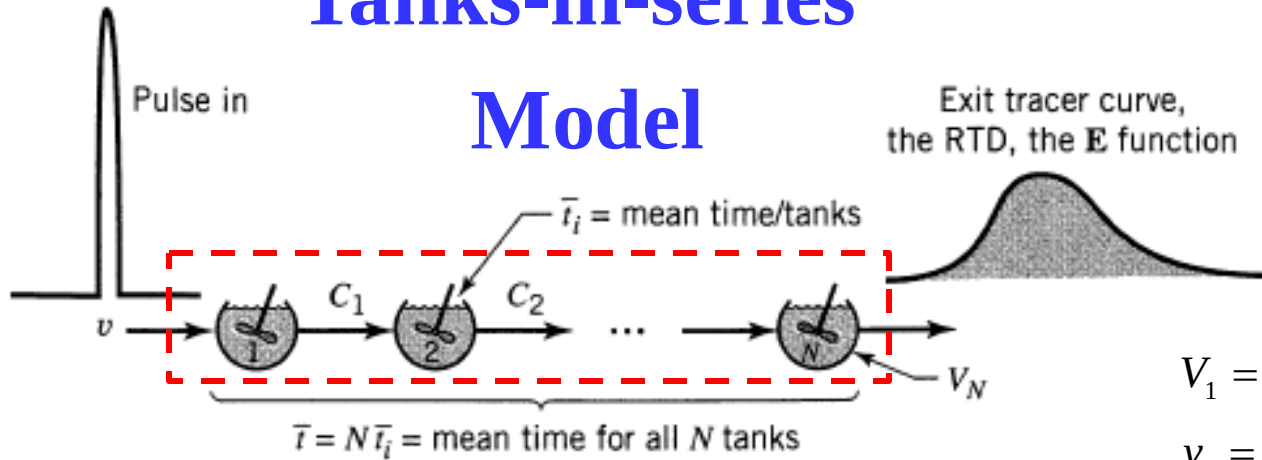
$$F(t) = \int_0^t E(t) dt = \begin{cases} 0 & \text{for } t < \frac{\tau}{2} \\ 1 - \frac{\tau^2}{4t^2} & \text{for } t \geq \frac{\tau}{2} \end{cases}$$



Tanks-in-series Model

Tanks-in-series

Model



$$V_1 = V_2 = V_3 = \dots = V_N = \frac{V}{N} = V_i$$

$$v_1 = v_2 = v_3 = \dots = v_N = v_o$$

$$\tau_1 = \tau_2 = \tau_3 = \dots = \tau_N = \frac{\tau}{N} = \tau_i$$

$$V_1 \frac{dC_1}{dt} = v_o (C_0 - C_1) \quad C_1(t=0) = N_o / V_1$$

$$V_2 \frac{dC_2}{dt} = v_o (C_1 - C_2) \quad C_2(t=0) = 0$$

$$V_3 \frac{dC_3}{dt} = v_o (C_2 - C_3) \quad C_3(t=0) = 0$$

$$\vdots$$

$$V_N \frac{dC_N}{dt} = v_o (C_{N-1} - C_N) \quad C_N(t=0) = 0$$

$$C_1 = \frac{N_o}{V_i} \exp\left(-\frac{t}{\tau_i}\right)$$

$$C_2 = \frac{N_o}{V_i} \left(\frac{t}{\tau_i}\right) \exp\left(-\frac{t}{\tau_i}\right)$$

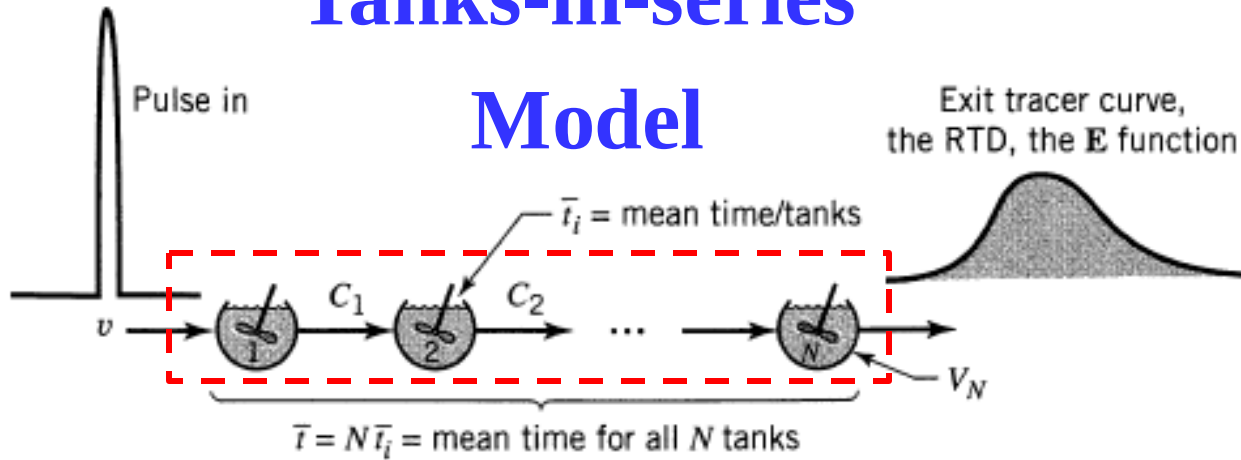
$$C_3 = \frac{N_o}{V_i} \frac{1}{2} \left(\frac{t}{\tau_i}\right)^2 \exp\left(-\frac{t}{\tau_i}\right)$$

$$\vdots$$

$$C_N = \frac{N_o}{V_i} \frac{1}{(N-1)!} \left(\frac{t}{\tau_i}\right)^{N-1} \exp\left(-\frac{t}{\tau_i}\right)$$

Tanks-in-series

Model



$$C_N = \frac{N_o}{V_i} \frac{1}{(N-1)!} \left(\frac{t}{\tau_i} \right)^{N-1} \exp\left(-\frac{t}{\tau_i} \right)$$

$$E(t) = \frac{C_N(t)}{\int_0^\infty C_N(t) dt} = \frac{C_N(t)}{N_o / v_o} = \frac{v_o C_N(t)}{C_o V} \Rightarrow E(t) = \frac{t^{N-1}}{(N-1)! (\tau / N)^N} \exp\left(-\frac{t}{\tau / N} \right)$$

$$\theta = \frac{t}{\tau}$$

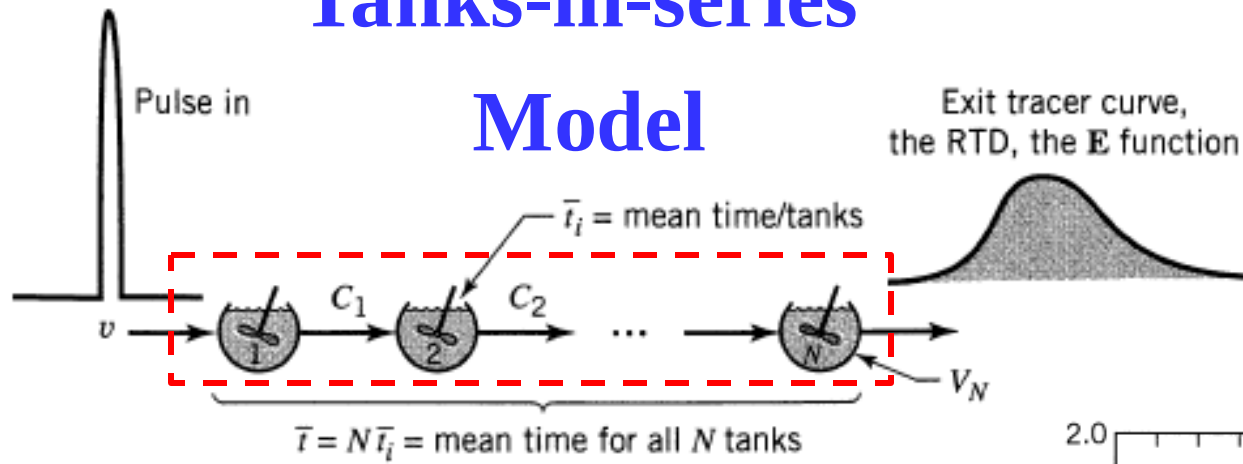
$$E_\theta = \tau \cdot E(t)$$

$$E_\theta = \frac{N(N\theta)^{N-1}}{(N-1)!} \exp(-N\theta)$$

$$\frac{\sigma^2}{\tau^2} = \frac{1}{N}, \quad N = \text{parameter that quantifies nonideality} \quad \begin{cases} N = 1 & \text{CSTR} \\ 1 < N < \infty & \text{real reactor} \\ N \rightarrow \infty & \text{PFR} \end{cases}$$

Tanks-in-series

Model



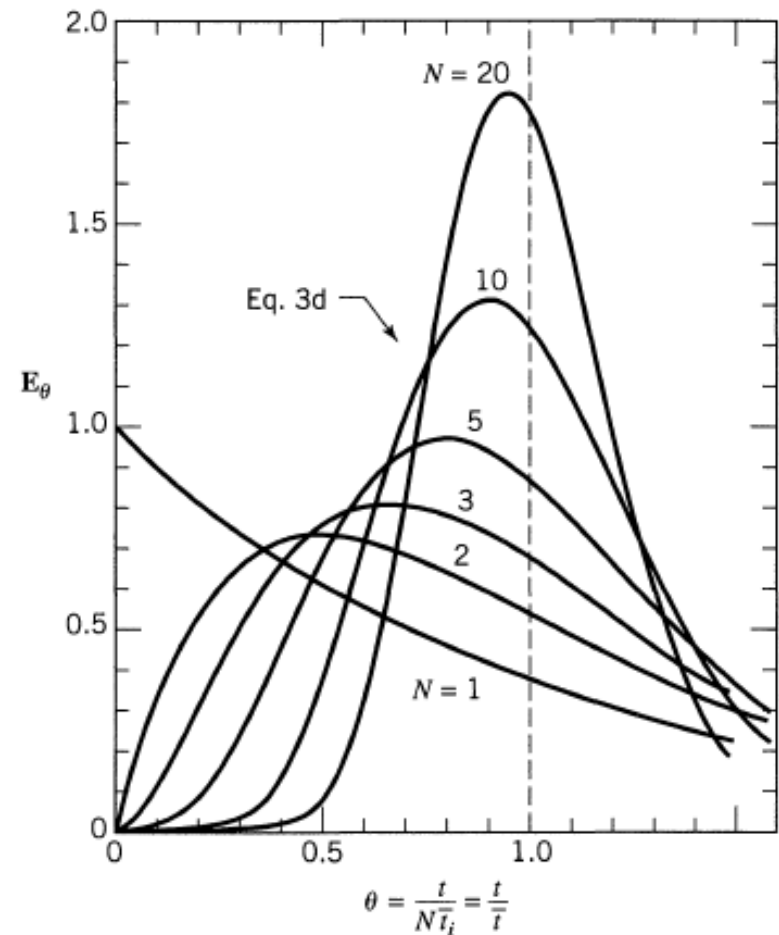
$$V_1 = V_2 = V_3 = \dots = V_N = \frac{V}{N} = V_i$$

$$v_1 = v_2 = v_3 = \dots = v_N = v_o$$

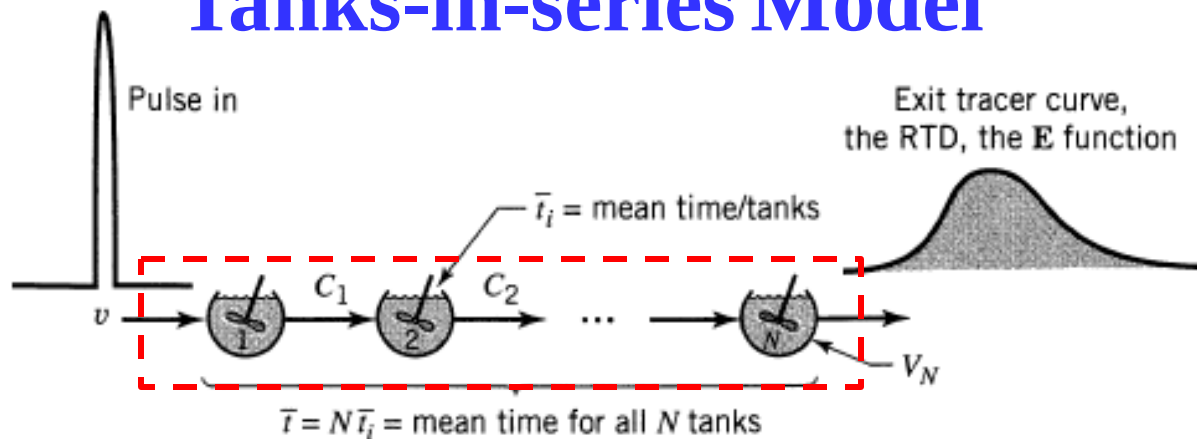
$$\tau_1 = \tau_2 = \tau_3 = \dots = \tau_N = \frac{\tau}{N} = \tau_i$$

$$E(t) = \frac{t^{N-1}}{(N-1)! \left(\tau/N\right)^N} \exp\left(-\frac{t}{\tau/N}\right)$$

$$E_\theta = \frac{N(N\theta)^{N-1}}{(N-1)!} \exp(-N\theta)$$

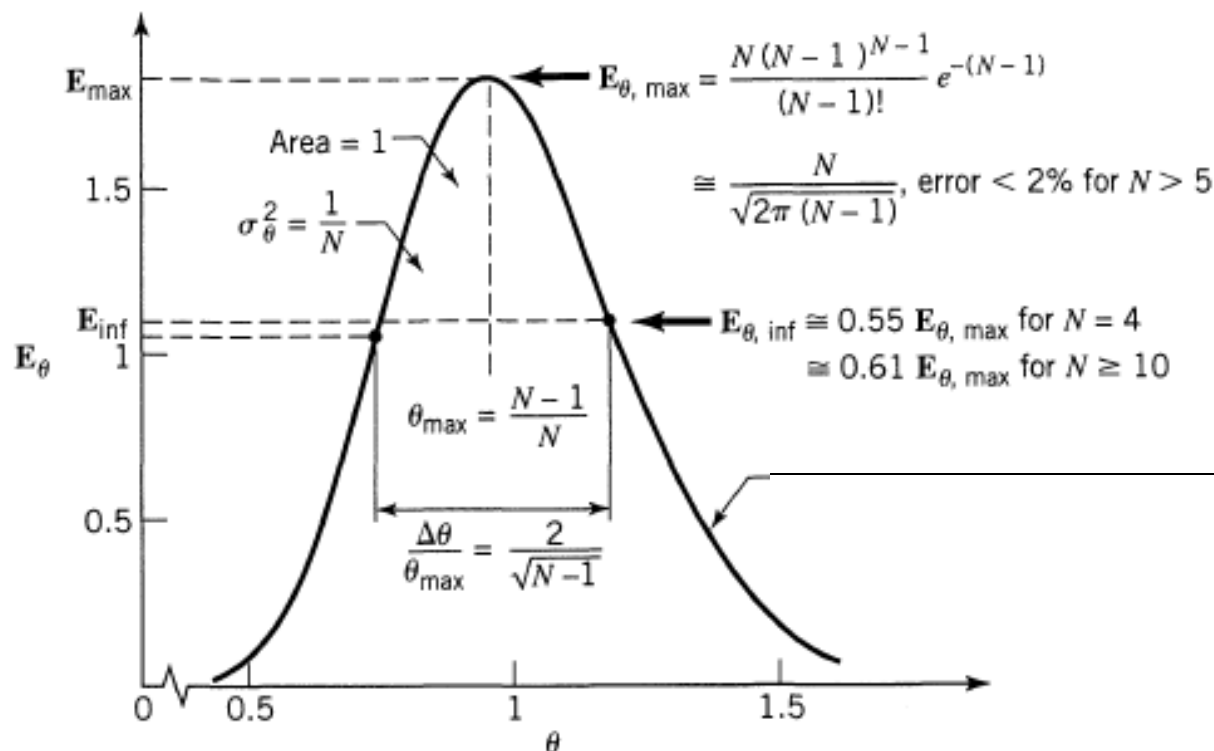


Tanks-in-series Model



$$E(t) = \frac{t^{N-1}}{(N-1)! (\tau/N)^N} \exp\left(-\frac{t}{\tau/N}\right)$$

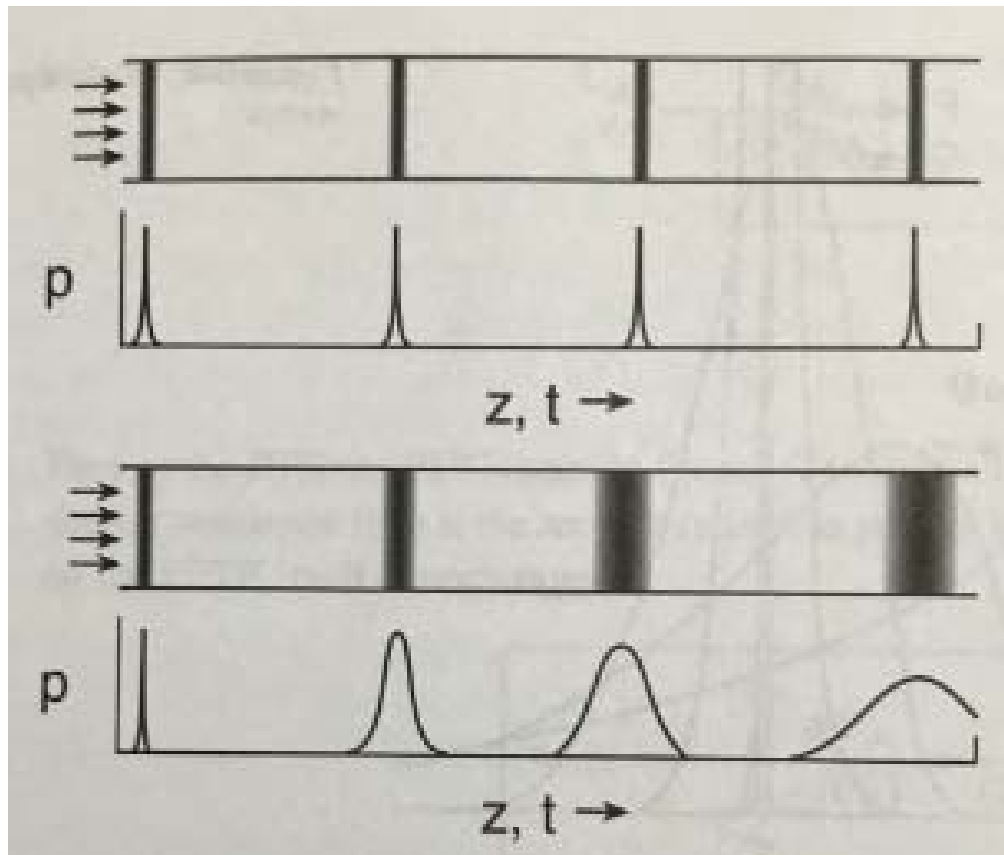
$$E_\theta = \frac{N(N\theta)^{N-1}}{(N-1)!} \exp(-N\theta)$$



O. Levenspiel,
Chemical Reaction
Engineering,
3rd ed.,
Wiley, 1999.

Figure 14.3 Properties of the RTD curve for the tanks-in-series model.

Axial dispersion Model

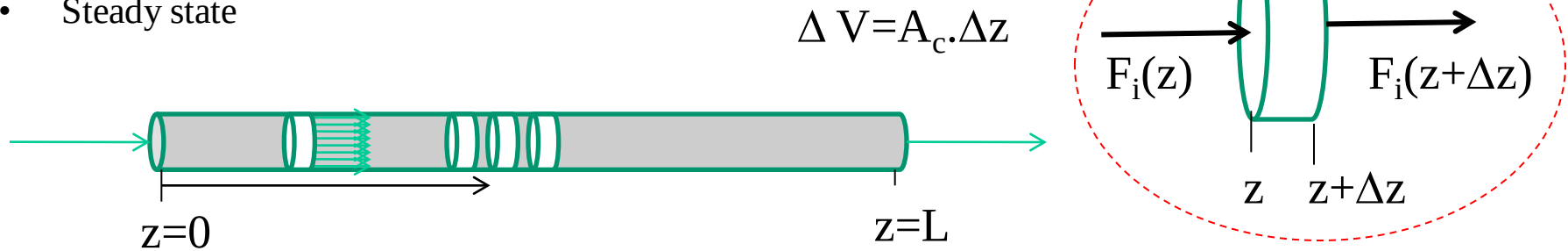


PFR

Axial
dispersion
model

Axially-Dispersed Tubular Reactor (ADPFR)

- No changes in radial and angular direction, only axial variations (z)
- Flow in direction z ($v_r=0$, $v_\theta = 0$) and **some mixing in axial direction (similar to diffusion)**
- Plug flow (piston-like velocity profile)
($v_z = \text{constant}$, no changes with r)
- Steady state



$$\frac{dF_i}{dV} = \frac{dF_i}{A_c \cdot dz} = r_i \Rightarrow \frac{dF_i}{dz} = A_c r_i$$

$$F_i = \underbrace{A_c u C_i}_{\text{convective flow}} + \underbrace{A_c J_i}_{\text{axial mixing flow}} \quad J_i = -D_A \frac{dC_i}{dz}$$

- Mixing flow in the axial direction
- Account for different phenomena
- D_A = effective axial dispersion coefficient

$$\frac{dF_i}{dz} = A_c \frac{d}{dz} \left(v_z C_i - D_A \frac{dC_i}{dz} \right) = A_c r_i$$

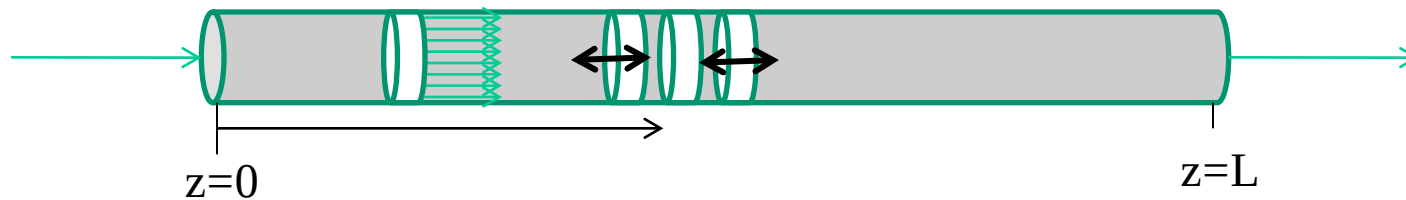
$$v_z \frac{dC_i}{dz} = D_A \frac{d^2 C_i}{dz^2} + r_i$$

Reator tubular com dispersão axial

- Mesmas hipóteses do reator PFR, exceto que agora se considera a ocorrência de mistura na direção z (dispersão axial). O fluxo de mistura axial é modelado como uma difusão:

$$\frac{\partial C_A}{\partial t} + \vec{\nabla} \cdot (\vec{v} C_A) = \vec{\nabla} \cdot (D_a \vec{\nabla} C_A) + r_A$$

$$\vec{j}_A = -D_A \vec{\nabla} C_A$$



$$\frac{\partial C_A}{\partial t} + v_r \frac{\partial C_A}{\partial r} + v_\theta \frac{\partial C_A}{\partial \theta} + v_z \frac{\partial C_A}{\partial z} = D_A \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 C_A}{\partial \theta^2} + \frac{\partial^2 C_A}{\partial z^2} \right] + R_{V_A}$$

$$\frac{\partial C_A}{\partial t} = -v_z \frac{\partial C_A}{\partial z} + D_A \frac{\partial^2 C_A}{\partial z^2} + r_i$$

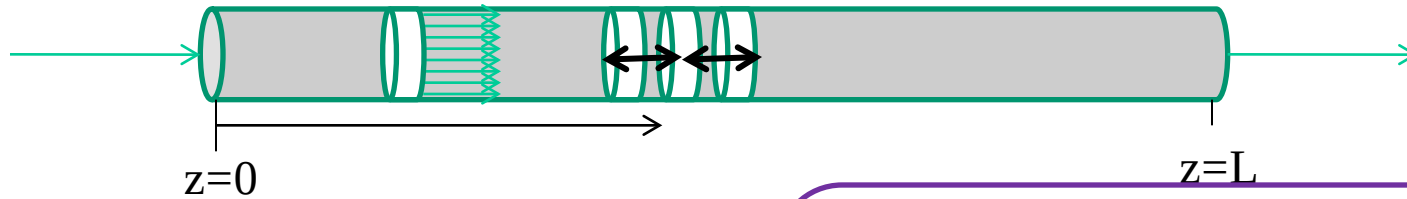
Esta equação diferencial tem, agora, derivadas de 2ª ordem, então requer 2 condições de contorno em z

Condições de contorno de Danckwerts

$$z = 0 \quad c_A \Big|_{z=0} = c_{A,e} + \frac{D_a}{v_z} \frac{dc_A}{dz} \Big|_{z=0}$$

$$z = L \quad \frac{dc_A}{dz} \Big|_{z=L} = 0$$

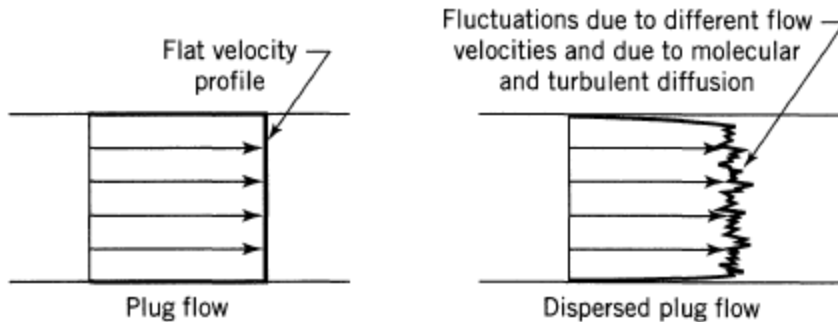
Axial Dispersion Model



$$\frac{\partial C_A}{\partial t} = -v_z \frac{\partial C_A}{\partial z} + D_A \frac{\partial^2 C_A}{\partial z^2} + r_i$$

$$z = 0 \quad c_A|_{z=0} = c_{A,e} + \frac{D_a}{v_z} \frac{dc_A}{dz} \Big|_{z=0}$$

$$z = L \quad \frac{dc_A}{dz} \Big|_{z=L} = 0$$



$$Pe = \frac{uL}{D_A}$$

The axial dispersion coefficient is the parameter that quantifies the nonideality

- $D_A = 0$ $Pe \rightarrow \infty$ PFR
- $D_A \rightarrow \infty$ $Pe = 0$ CSTR

Axial Dispersion Model

$$\frac{\partial C_A}{\partial t} = -u \frac{\partial C_A}{\partial z} + D_A \frac{\partial^2 C_A}{\partial z^2} + r_i$$

$$\begin{aligned} z = 0 \quad c_A|_{z=0} &= c_{A,e} + \frac{D_a}{u} \frac{dc_A}{dz} \Big|_{z=0} \\ z = L \quad \frac{dc_A}{dz} \Big|_{z=L} &= 0 \end{aligned}$$

for odd $i = 1, 3, 5, \dots$

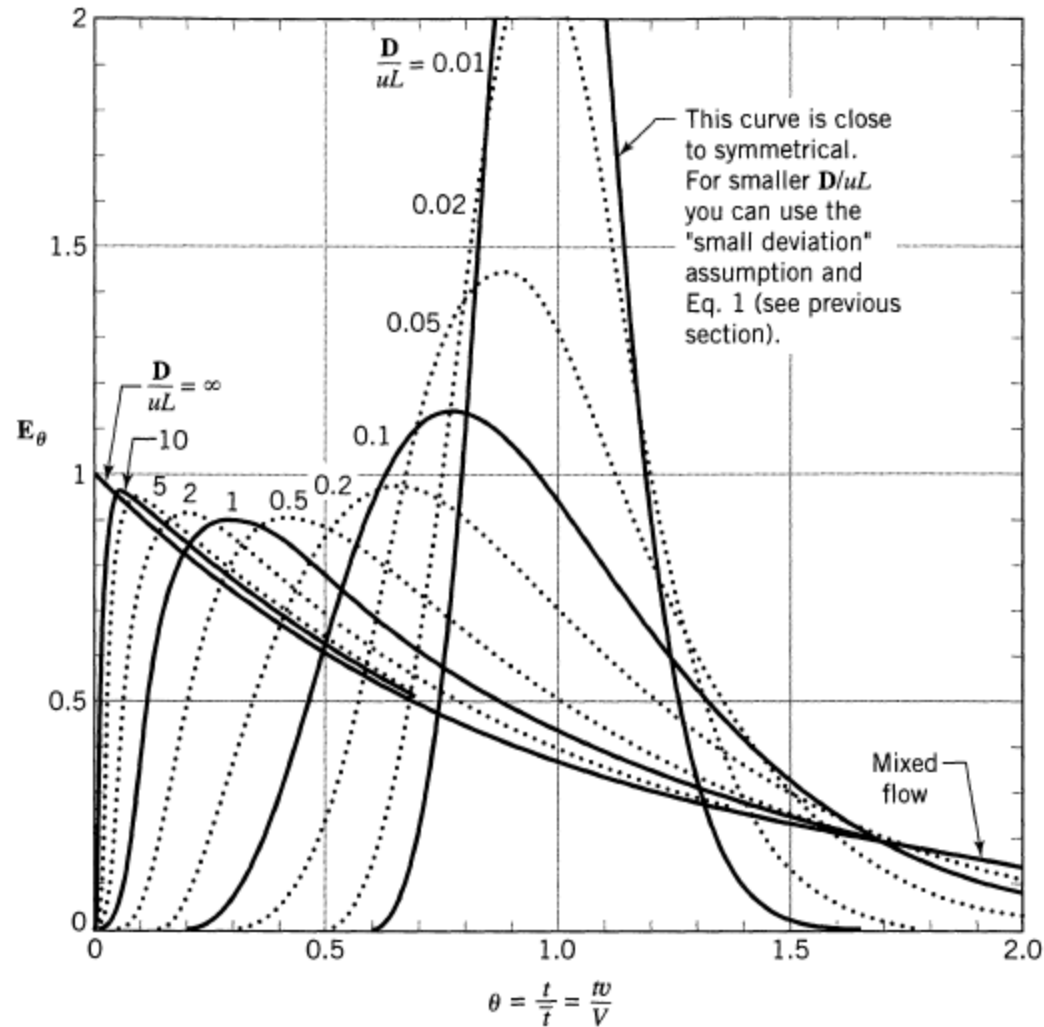
$$\tan\left(\frac{\alpha_i}{2}\right) = \frac{Pe}{4(\alpha_i/2)}$$

for even $i = 2, 4, 6, \dots$

$$\cotg\left(\frac{\alpha_i}{2}\right) = \frac{-Pe}{4(\alpha_i/2)}$$

$$Pe = \frac{uL}{D_A}$$

$$\frac{C_s(t)}{M/V} = E(\theta) = e^{Pe/2} \sum_{i=1}^{\infty} \frac{(-1)^{i+1} 8\alpha_i^2}{4\alpha_i^2 + 4Pe + Pe^2} \exp\left(\frac{-\theta(Pe^2 + 4\alpha_i^2)}{4Pe}\right)$$



Axial Dispersion Model

$$\frac{C_s(t)}{M/V} = E(\theta) = e^{Pe/2} \sum_{i=1}^{\infty} \frac{(-1)^{i+1} 8\alpha_i^2}{4\alpha_i^2 + 4Pe + Pe^2} \exp\left(\frac{-\theta(Pe^2 + 4\alpha_i^2)}{4Pe}\right)$$

for odd $i = 1, 3, 5, \dots$

$$\tan\left(\frac{\alpha_i}{2}\right) = \frac{Pe}{4(\alpha_i/2)}$$

for even $i = 2, 4, 6, \dots$

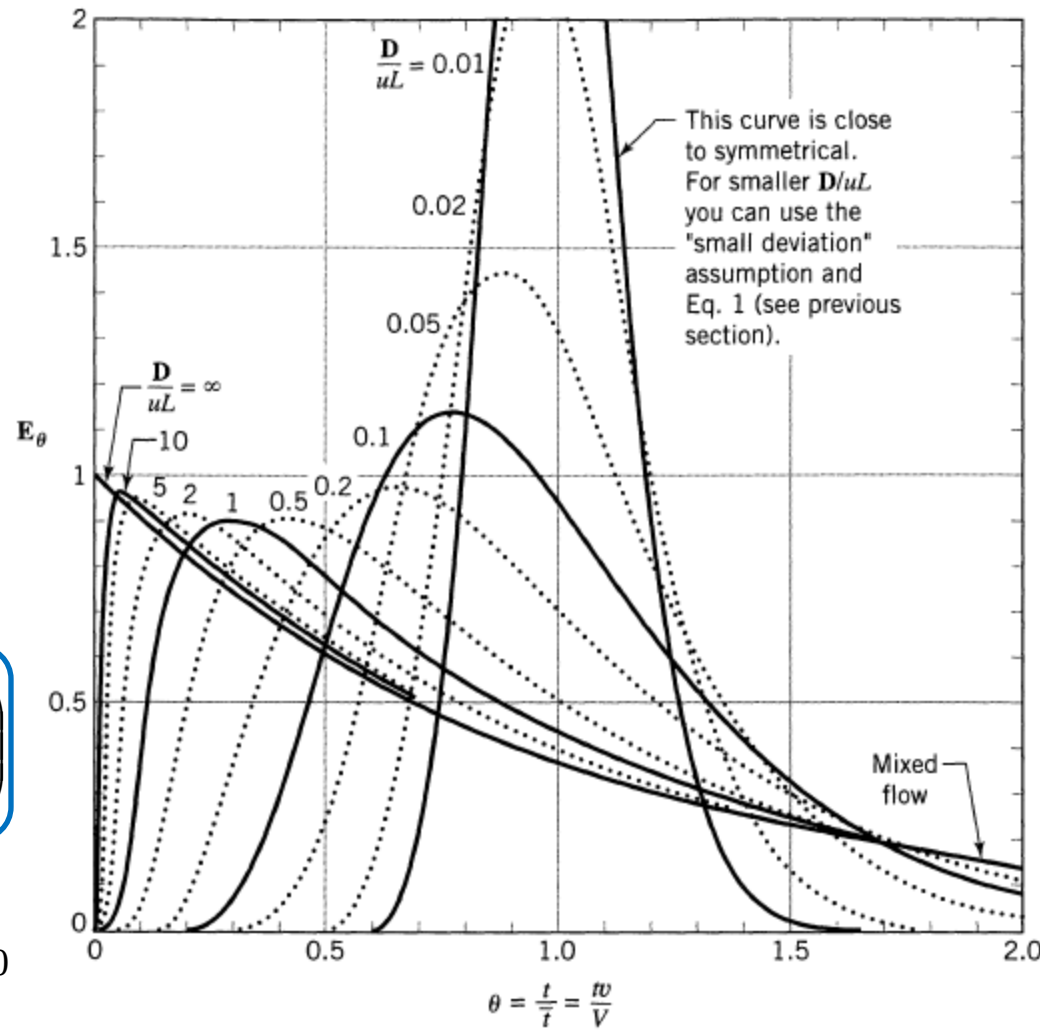
$$\cotg\left(\frac{\alpha_i}{2}\right) = \frac{-Pe}{4(\alpha_i/2)}$$

$\approx$

An excellent approximation for the infinite-series solution is

$$E(\theta) \cong \left(\frac{Pe+1}{4\pi\theta^3}\right)^{1/2} \exp\left(\frac{-(Pe+1)(1-\theta)^2}{4\theta}\right)$$

Gouvea, Park, & Giudici, Estimação de coeficientes de dispersão axial em leitos fixos. 18. Encontro sobre Escoamento em Meios Porosos. Nova Friburgo. 1990



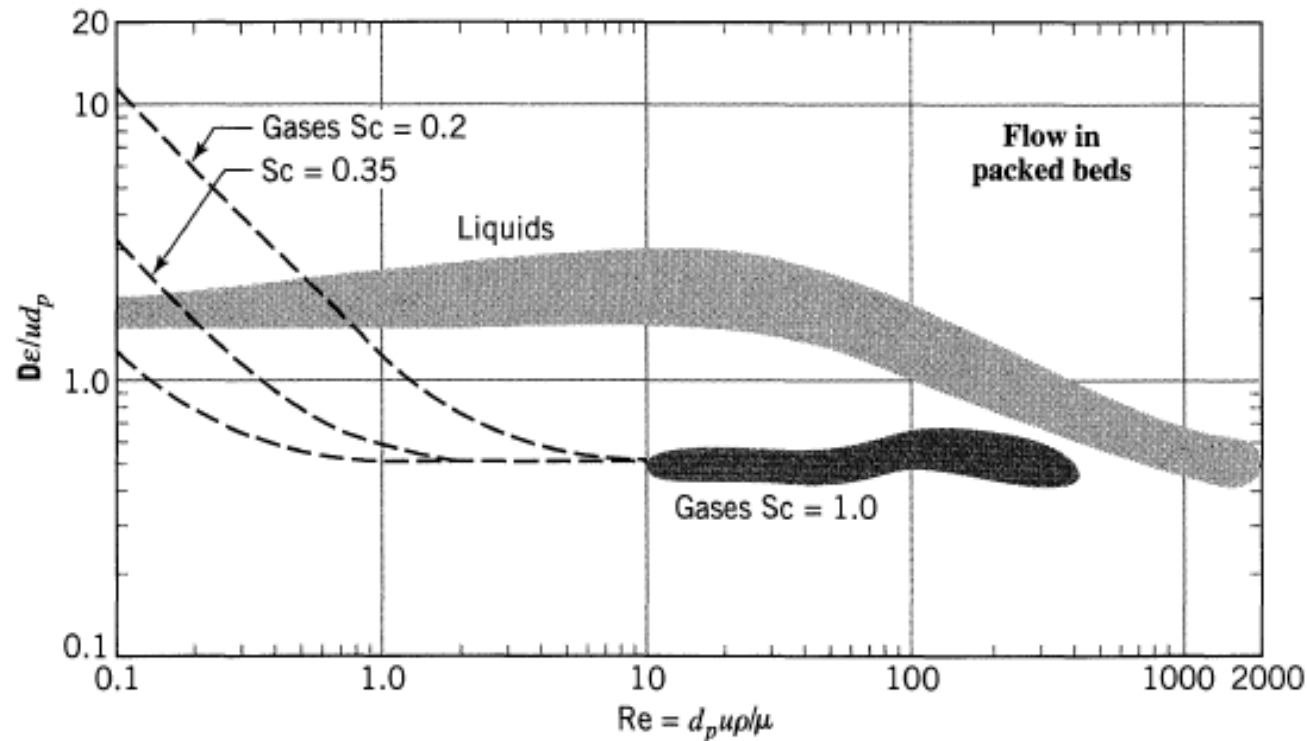


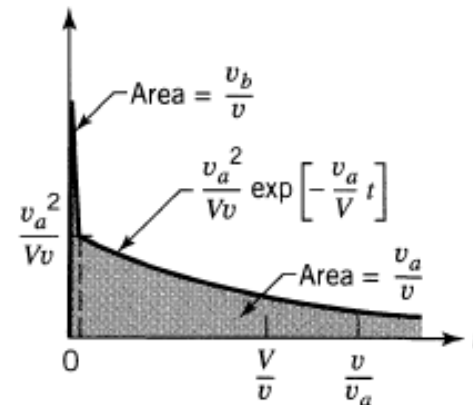
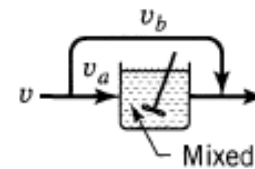
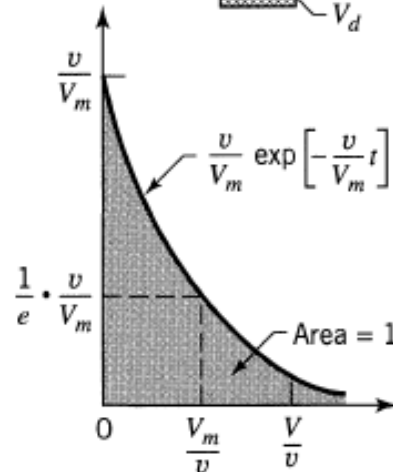
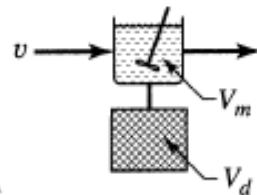
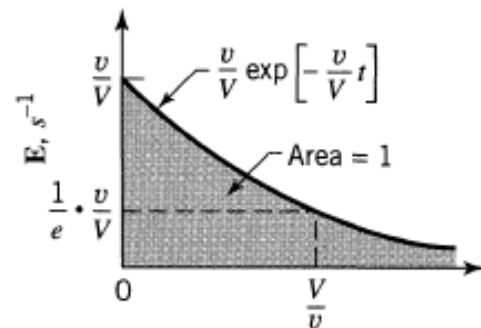
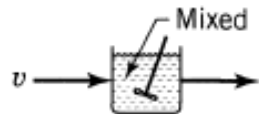
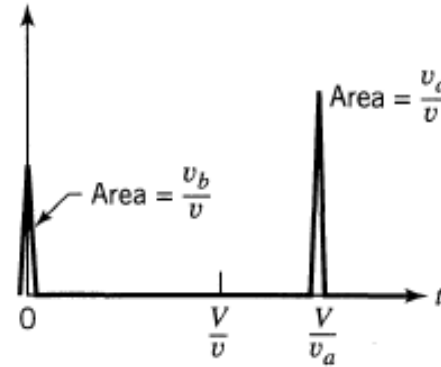
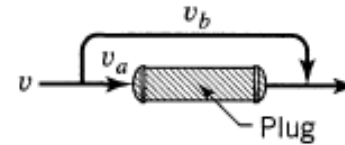
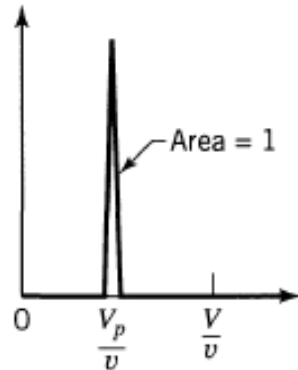
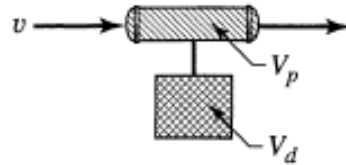
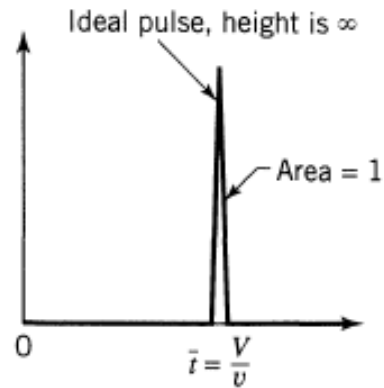
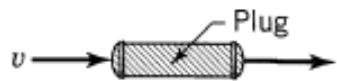
Figure 13.17 Experimental findings on dispersion of fluids flowing with mean axial velocity u in packed beds; prepared in part from Bischoff (1961).

Chung & Wen (1968) correlation

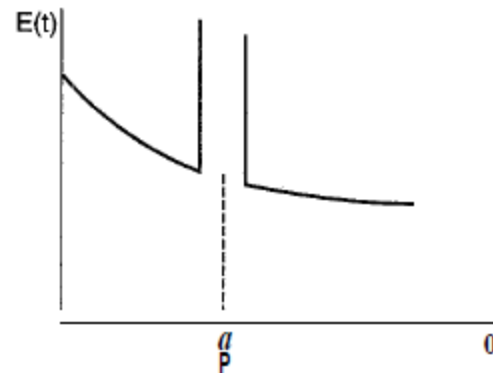
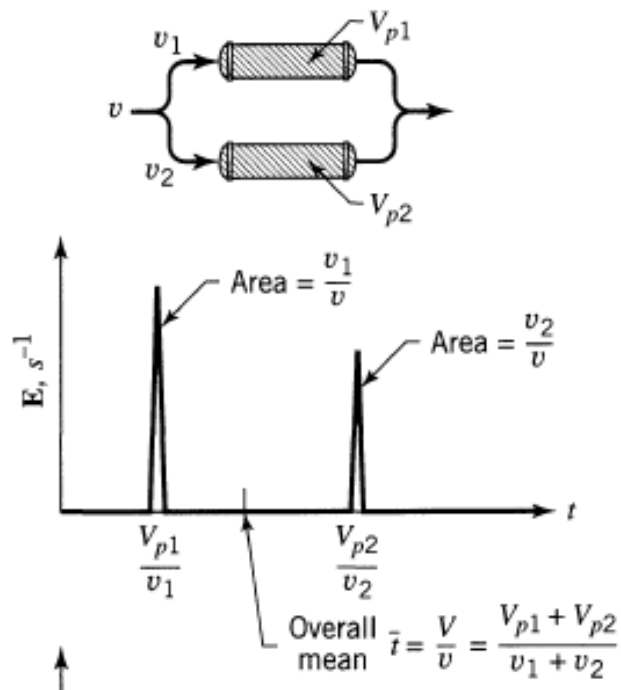
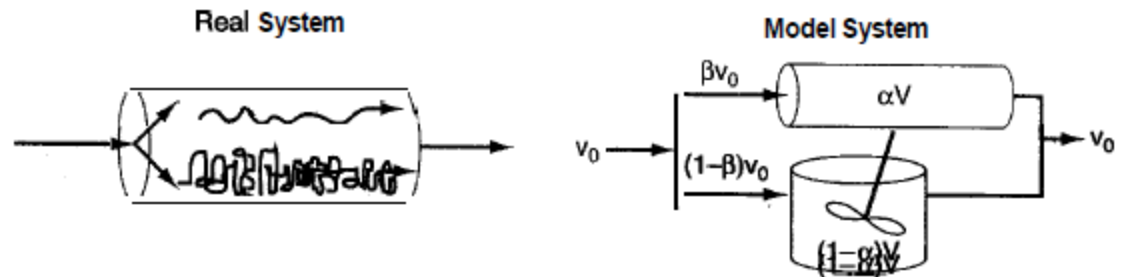
$$\varepsilon \cdot Pe_L \frac{d_p}{L} = \varepsilon \cdot \left(\frac{uL}{D_{ea}} \right) \frac{d_p}{L} = \varepsilon \cdot \left(\frac{ud_p}{D_{ea}} \right) = 0.2 + 0.008 Re^{0.48}$$

Compartment Models

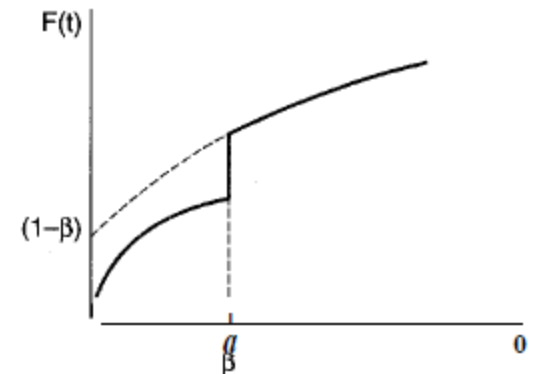
Compartment Models



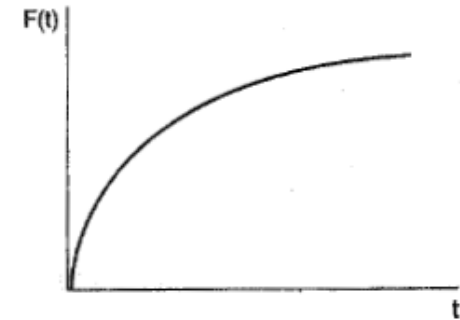
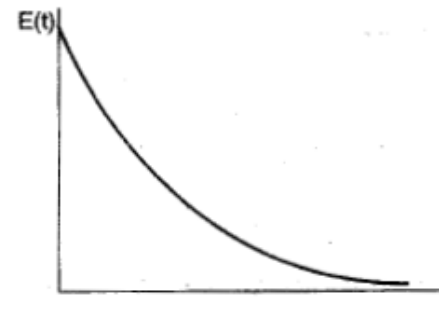
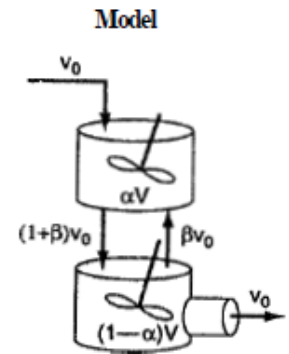
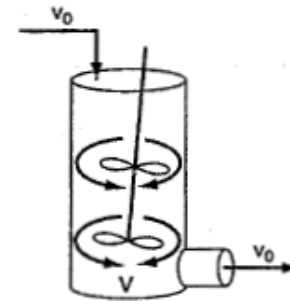
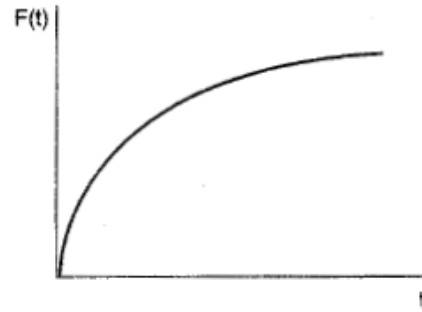
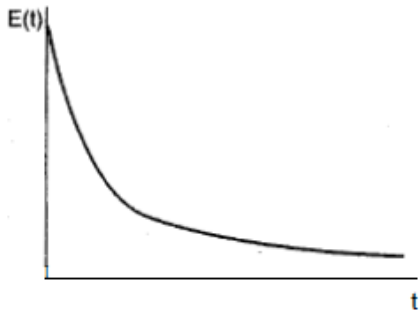
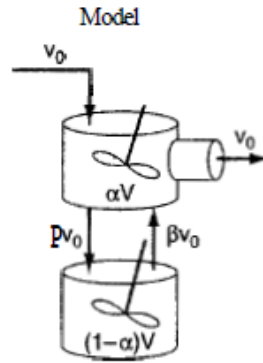
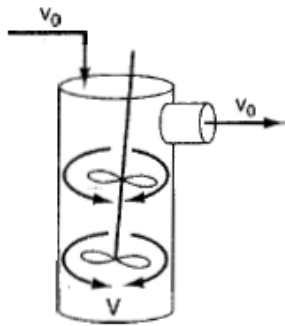
Compartment Models



(b)



Compartment Models



(b)

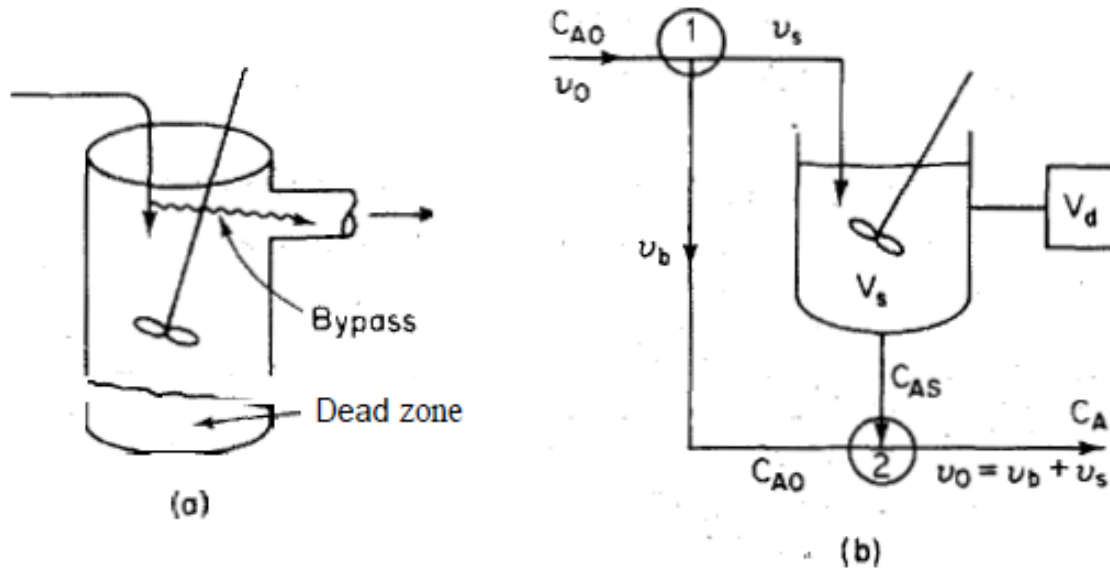
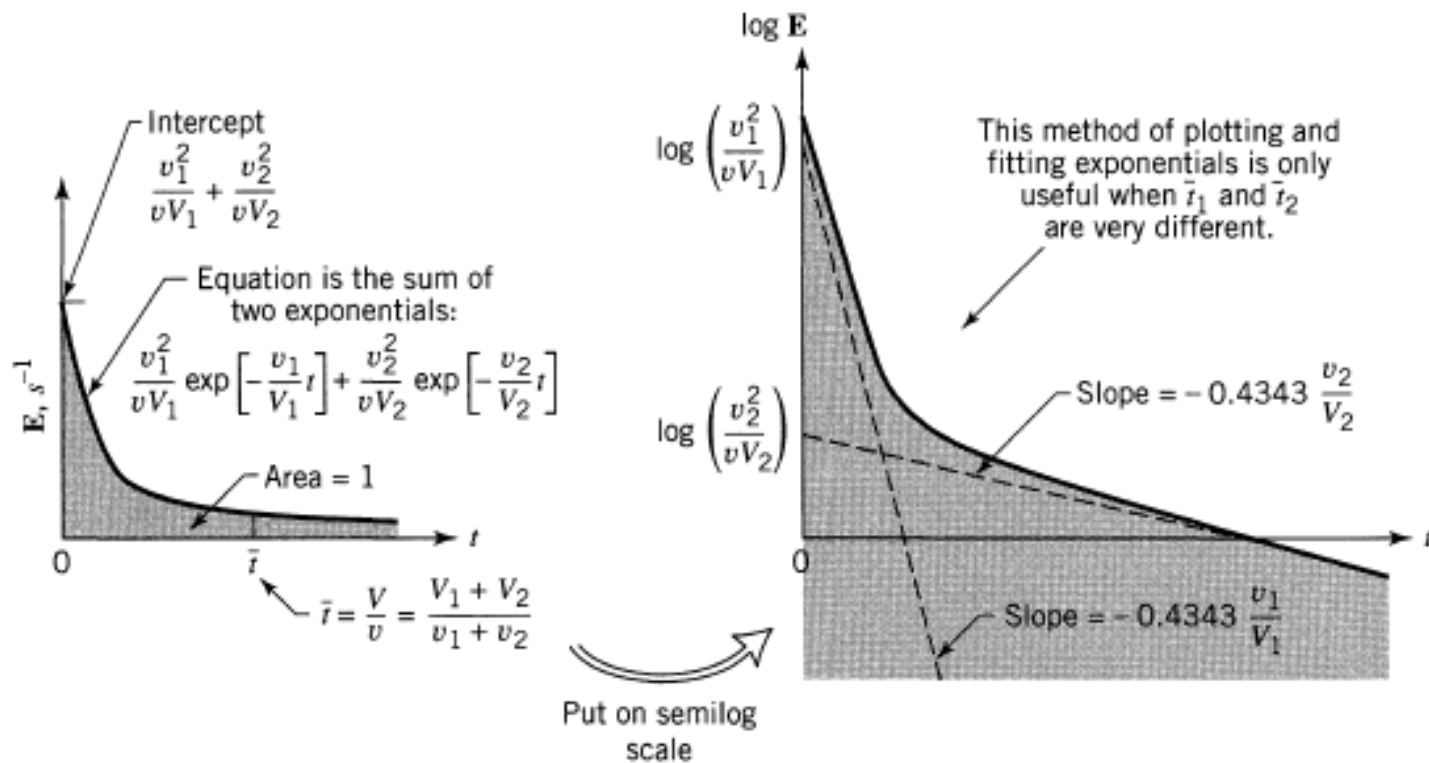
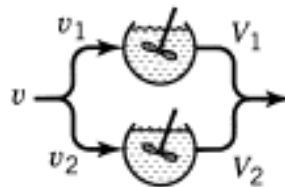


Figure 14-11 (a) Real system; (b) model system.

$$E(t) = \left(\frac{v_b}{v_0} \right) \delta(t) + \left(\frac{v_a^2}{V_s v_0} \right) \exp\left(-\frac{v_a}{V_s} t \right) = \beta \cdot \delta(t) + \frac{(1-\beta)^2}{\alpha \tau} \exp\left(-\frac{(1-\beta)}{\alpha \tau} t \right)$$

$$F(t) = 1 - \left(\frac{v_a}{v_0} \right) \exp\left(-\frac{v_a}{V_s} t \right) = 1 - (1-\beta) \exp\left(-\frac{(1-\beta)}{\alpha \tau} t \right)$$

Compartment Models

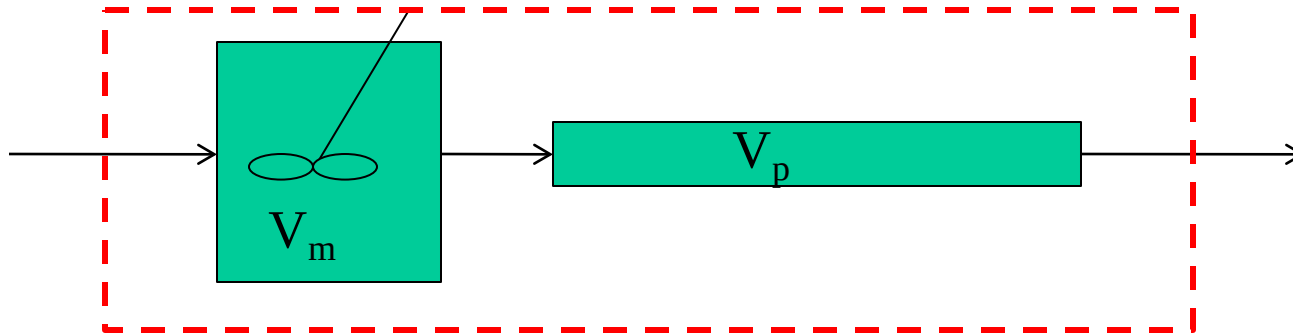


Assignment #2:

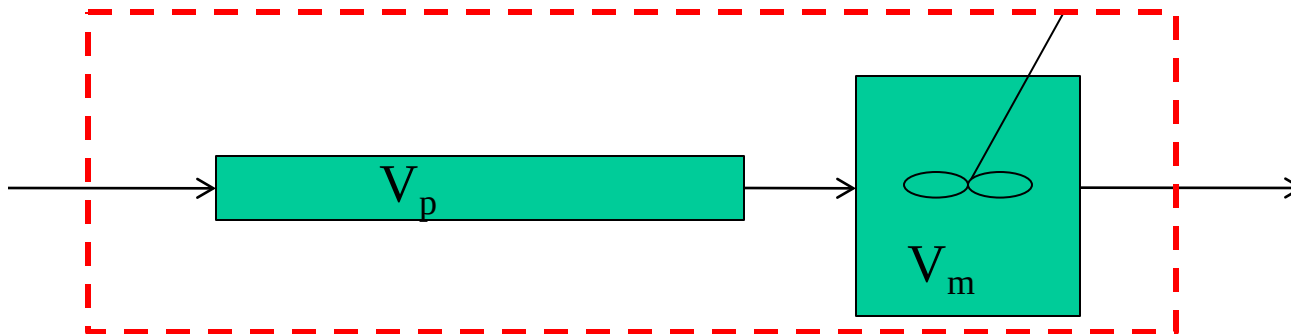
Draw the RTD curves $E(t)$ for the systems shown bellow.

Explain your results (via mass balances and via “thought experiment”)

- (a) a system formed by a CSTR followed by a PFR.



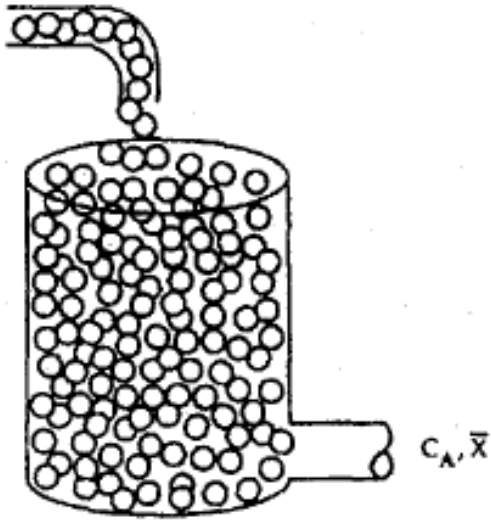
- (b) a system formed by a PFR followed by a CSTR.





Conversion directly from the RTD for Segregated Flow

Segregated Flow



MACROFLUID:

- mixing at the level of macroscopic portions of fluid
- molecules flow together in groups (globules) and they are not mixed until they exit the reactor
- different from a microfluid (where mixing occurs at the molecular level)
- each globule is a closed system (batch reactor)

$$\left(\begin{array}{c} \text{mean conversion} \\ \text{of all globules} \\ \text{exiting} \\ \text{the reactor} \end{array} \right) = \sum_{\substack{\text{all} \\ \text{globules} \\ \text{exiting} \\ \text{the reactor}}} \left(\begin{array}{c} \text{conversion} \\ \text{achieved in} \\ \text{a globule} \\ \text{that spend} \\ \text{a time } t \\ \text{in the reactor} \end{array} \right) \times \left(\begin{array}{c} \text{fraction of} \\ \text{globules that} \\ \text{spend between} \\ \text{time} \\ t \text{ and } t + dt \\ \text{in the reactor} \end{array} \right)$$

$$\bar{X} = \int_0^{\infty} \underbrace{X_{\text{batch}}(t)}_{\text{batch reactor}} E(t) dt$$



Assignments RTD

(laboratory activity, gathering real data)

Assignment #3: Given the experimental data (absorbance versus time) of a pulse RTD experiment (fluid = water) in a real tubular reactor (L=90 cm, D=4 cm) packed with 0,8 cm Raschig rings (void fraction of the packed bed $\phi=0,70$).

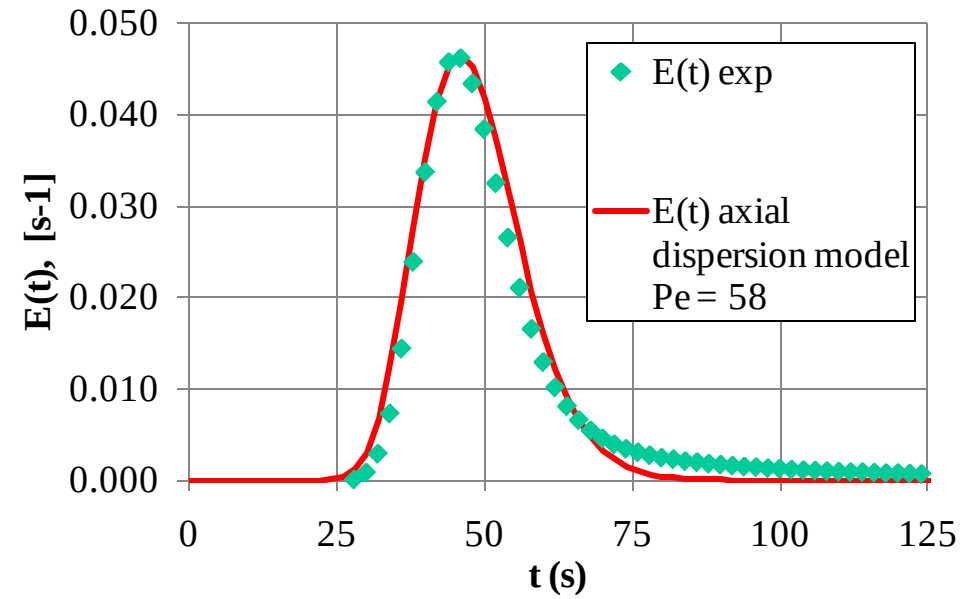
- (a) Fit the axial dispersion model to the data and determine the value of Peclet number, $Pe = u.L/D_A$ and τ . Compare the obtained Peclet value with literature values (see slide 37). Use the approximated solution for the RTD of the axial dispersion model.

$$E(t) \cong \frac{1}{\tau} \left(\frac{Pe + 1}{4\pi(t/\tau)^3} \right)^{1/2} \exp \left(\frac{-(Pe + 1)[1 - (t/\tau)]^2}{4(t/\tau)} \right)$$

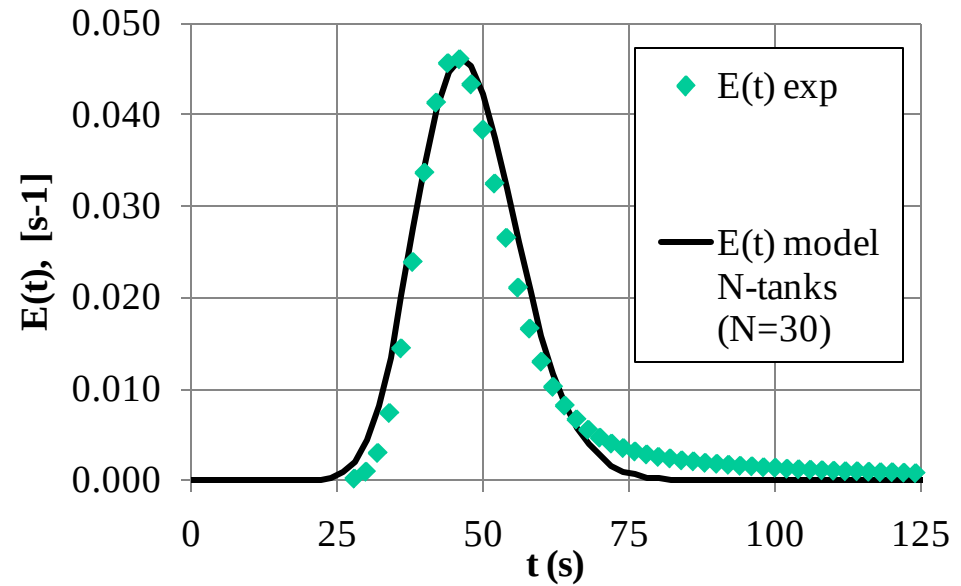
- (a) Fit the N-tanks-in-series model to the exp. data and determine N and τ

$$E(t) = \frac{N \left(N \frac{t}{\tau} \right)^{N-1}}{\tau (N-1)!} \exp \left(-N \frac{t}{\tau} \right) = \frac{t^{N-1}}{(N-1)! (\tau/N)^N} \exp \left(-N \frac{t}{\tau} \right)$$

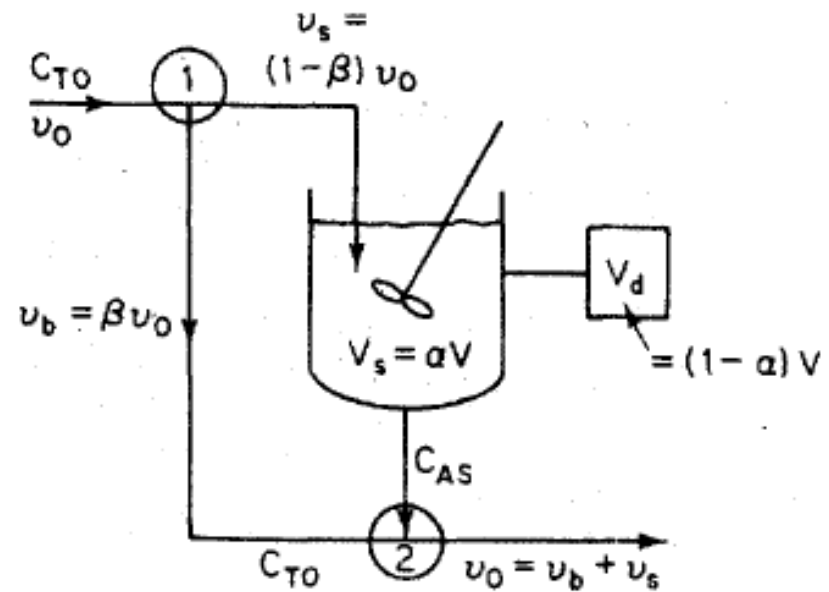
$$E(t) \cong \frac{1}{\tau} \left(\frac{Pe+1}{4\pi \left(\frac{t}{\tau}\right)^3} \right)^{1/2} \exp \left(\frac{-(Pe+1) \left[1 - \left(\frac{t}{\tau}\right) \right]^2}{4 \left(\frac{t}{\tau}\right)} \right)$$



$$E(t) = \frac{t^{N-1}}{(N-1)! (\tau/N)^N} \exp \left(-N \frac{t}{\tau} \right)$$



Assignment #4: Given the experimental data (absorbance versus time) of a pulse RTD experiment in a real tank reactor (fluid = water), fit the compartment model considering a dead volume and a by-pass, and determine the fraction of by-pass $\beta = v_b/v$ and the fraction of active volume $\alpha = V_s/V$



$$E(t) = \left(\frac{v_b}{v_0} \right) \delta(t) + \left(\frac{v_a^2}{V_s v_0} \right) \exp\left(-\frac{v_a}{V_s} t \right) = \beta \cdot \delta(t) + \frac{(1-\beta)^2}{\alpha \tau} \exp\left(-\frac{(1-\beta)}{\alpha \tau} t \right)$$

$$F(t) = 1 - \left(\frac{v_a}{v_0} \right) \exp\left(-\frac{v_a}{V_s} t \right) = 1 - (1-\beta) \exp\left(-\frac{(1-\beta)}{\alpha \tau} t \right)$$

