

Gabarito da prova de Cálculo II (P3)

1ª Questão

$$\gamma(t) = (t^2, 3t-1, z(t)), t \in \mathbb{R}$$

$$\gamma(1) = (1, 2, z(1)) = (1, 2, f(1, 2)) = (1, 2, -2)$$

$$\gamma'(t) = \left(2t, 3, \frac{dz}{dt} \right)$$

$$z = f(t^2, 3t-1) \Rightarrow \frac{dz}{dt} = \frac{\partial f}{\partial x}(t^2, 3t-1) \frac{dx}{dt} + \frac{\partial f}{\partial y}(t^2, 3t-1) \frac{dy}{dt}$$

$$\frac{dz}{dt} = 2t \frac{\partial f}{\partial x}(t^2, 3t-1) + 3 \frac{\partial f}{\partial y}(t^2, 3t-1)$$

$$\left. \frac{dz}{dt} \right|_{t=1} = 18$$

$$\gamma'(1) = (2, 3, 18)$$

$$(x, y, z) = (1, 2, -2) + \lambda (2, 3, 18), \lambda \in \mathbb{R}$$

$$z = f(x, y)$$

$$f(1, 2) = -2, \quad \frac{\partial f}{\partial x}(1, 2) = 3 \quad \text{e} \quad \frac{\partial f}{\partial y}(1, 2) = 4$$

$$z(t) = f(t^2, 3t-1)$$

3ª Questão

$$g(x, y, z) = x^2 + y^2 + z^2$$

$$\nabla g(x, y, z) = (2x, 2y, 2z)$$

$$f(x, y, z) = 2x^2 + y^2 + z$$

$$\nabla f(x, y, z) = (4x, 2y, 1)$$

$$\begin{cases} \nabla f(x, y, z) = \lambda \nabla g(x, y, z) \\ x^2 + y^2 + z^2 = 1 \end{cases} \Rightarrow \begin{cases} 4x = \lambda 2x & (1) \\ 2y = \lambda 2y \\ 1 = \lambda 2z \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$4x = \lambda 2x$$

$$\lambda = 2$$

$$\text{Se } x = 0$$

$$\begin{cases} y = \lambda y & (2) \end{cases}$$

$$\begin{cases} 1 = \lambda 2z & (3) \end{cases}$$

$$\begin{cases} y^2 + z^2 = 1 & (4) \end{cases}$$

$$\text{De (2) } y = 0 \text{ e } \lambda = 1$$

$$\text{Se } y = 0 \text{ em (4) } z = \pm 1 \text{ logo obtemos os pontos } (0, 0, \pm 1) \text{ e } f(0, 0, \pm 1) = \pm 1$$

$$\text{Se } \lambda = 1$$

$$z = \frac{1}{2}$$

$$y^2 + \frac{1}{4} = 1 \Rightarrow y = \pm \frac{\sqrt{3}}{2}$$

$$\text{Logo obtemos os pontos } (0, \pm \frac{\sqrt{3}}{2}, \frac{1}{2}) \text{ e } f(0, \pm \frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{5}{4}$$

$$\text{Se } \lambda = 2 \text{ então do sistema original tem-se:}$$

$$\begin{cases} 2y = 4y \\ 1 = 4z \end{cases}$$

$$y = 0$$

$$z = \frac{1}{4}$$

$$\begin{cases} x^2 + y^2 + z^2 = 1 \end{cases}$$

$$x^2 + \frac{1}{16} = 1 \Rightarrow x = \pm \frac{\sqrt{15}}{4}$$

$$\text{Logo obtemos os pontos } (\pm \frac{\sqrt{15}}{4}, 0, \frac{1}{4}) \text{ e } f(\pm \frac{\sqrt{15}}{4}, 0, \frac{1}{4}) = \frac{17}{8}$$

