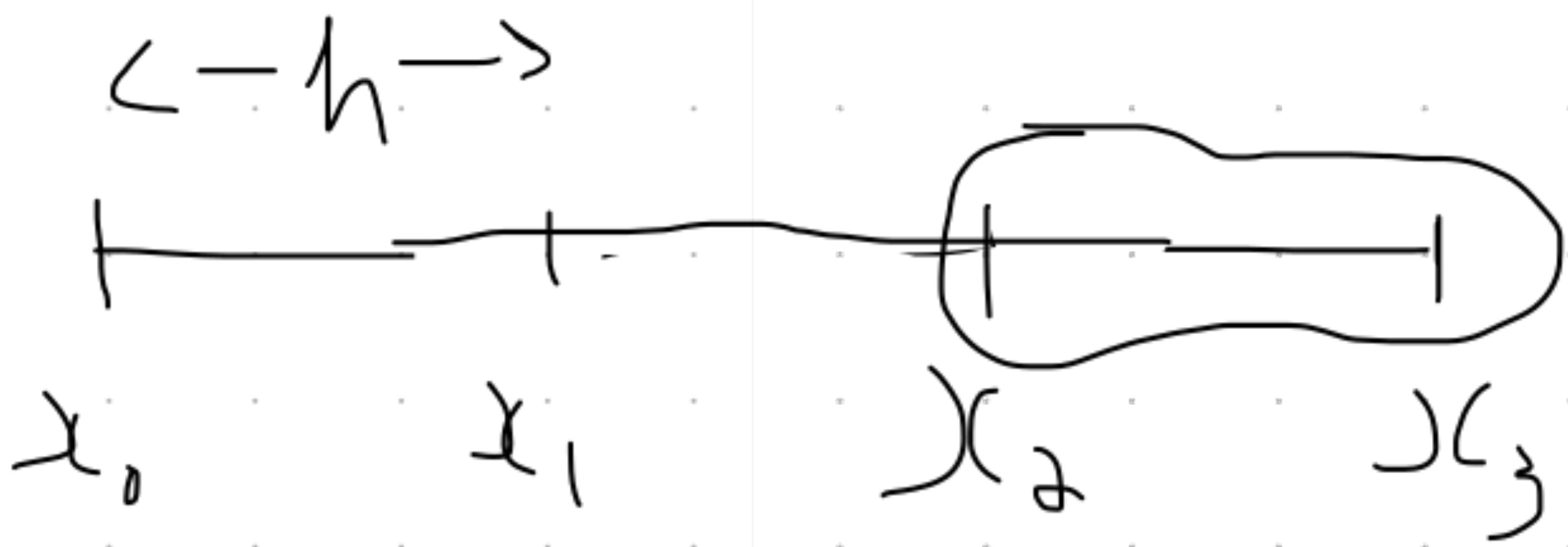
33 x_0, $x_j = x_0 + jh$, $j = 0,1,2,3$



$f \in C^3([x_0, x_3])$

$$\int_{x_2}^{x_3} f(x)\,dx \simeq a f(x_0) + b f(x_1) + c f(x_2)$$

EXATA P/ POLINÔMIOS DE GRAU MENOR OU IGUAL A 2

$$\int_{x_2}^{x_3} f(x)\,dx \simeq \int_{x_2}^{x_3} P_2(x)\,dx,$$

ONDE P_2 É O POLINÔMIO INTERPOLADOR DA TABELA

x	x_0	x_1	x_2
$f(x)$	$f(x_0)$	$f(x_1)$	$f(x_2)$

,

OBTEREMOS O RESULTADO DESEJADO

$$P_2(x) = f(x_0)\,L_0(x) + f(x_1)\,L_1(x) + f(x_2)\,L_2(x)$$

$$\Longrightarrow \left.\begin{aligned} a &= \int_{x_2}^{x_3} L_0(x)\,dx \\ b &= \int_{x_0}^{x_3} L_1(x)\,dx \\ c &= \int_{x_2}^{x_3} L_2(x)\,dx \end{aligned}\right\}$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{\underbrace{(x_0-x_1)}_{-h}\underbrace{(x_0-x_2)}_{-2h}} = \frac{(x-x_1)(x-x_2)}{2h^2}$$

$$a = \frac{1}{2h^2}\int_{x_2}^{x_3}(x-x_1)(x-x_2)\,dx$$

$$x = x_2 + y,$$

$$dx = dy,\quad x - x_1 = x_2 + y - x_1 = y + h$$

$$x - x_2 = y$$

$$x = x_2 \Rightarrow y = 0,\quad x = x_3 \Rightarrow y = h$$

$$a = \frac{1}{2h^2}\int_0^h (y+h)\cdot y\,dy = \frac{1}{2h^2}\int_0^h (y^2 + h\,y)\,dy$$

$$= \frac{1}{2h^2}\left(\frac{h^3}{3}+\frac{h^3}{2}\right) = \frac{5}{12}h \quad (= a)$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{\underbrace{(x_1-x_0)}_{h}\underbrace{(x_1-x_2)}_{-h}} = -\frac{(x-x_0)(x-x_2)}{h^2}$$

$$b = -\frac{1}{h^2}\int_{x_2}^{x_3}(x-x_0)(x-x_2)\,dx \quad \left(y = x - x_2\right)$$

$$= -\frac{1}{h^2}\int_{0}^{h}(y+2h)\,y\,dy$$

$$= -\frac{1}{h^2}\left(\frac{h^3}{3}+h^3\right) = -\frac{4}{3}h$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{\underbrace{(x_2-x_0)}_{2h}\underbrace{(x_2-x_1)}_{h}} = \frac{(x-x_0)(x-x_1)}{2h^2}$$

$$C = \frac{1}{2h^2}\int_{x_2}^{x_3} (x-x_0)(x-x_1)\,dx \quad (y = x - x_2)$$

$$= \frac{1}{2h^2}\int_0^h (y+2h)(y+h)\,dy$$

$$= \frac{1}{2h^2}\int_0^h (y^2+3hy+2h^2)\,dy$$

$$= \frac{1}{2h^2}\left(\frac{h^3}{3} + \frac{3h^3}{2} + 2h^3\right) = \frac{23}{12}h$$

$$a = \frac{5}{12}h, \quad b = -\frac{4}{3}h, \quad c = \frac{13}{12}h$$

$$\underbrace{\int_{x_2}^{x_3} f(x)\,dx}_{I} \simeq \underbrace{h\left[\frac{5}{12}f(x_0) - \frac{4}{3}f(x_1) + \frac{13}{12}f(x_2)\right]}_{Q}$$

ERRO

$$E = I - Q = \int_{x_2}^{x_3} f(x)\,dx - \int_{x_2}^{x_3} P_2(x)\,dx$$

$$= \int_{x_2}^{x_3} \underbrace{\left[f(x) - P_2(x)\right]}\,dx$$

ERRO P/ INTERP POLINOMIAL

$$f(x) - P_2(x) = \frac{f'''(t_x)}{3!}(x-x_0)(x-x_1)(x-x_2)$$

ONDE

$$t_x \in [x_0, x_3] \text{ SE } x \in \underbrace{[x_1, x_3]}$$

<u>OBS</u>.

$$x \in [x_2, x_3] \Rightarrow (x-x_0)(x-x_1)(x-x_2) \geq 0$$

NESTE INTERVALO

$$|f(x) - P_2(x)| = \frac{|f'''(t_x)|}{6}(x-x_0)(x-x_1)(x-x_2)$$

$$\Longrightarrow$$

$$|f(x) - P_2(x)| \leq \frac{M_3}{6}(x-x_0)(x-x_1)(x-x_2)$$

ONDE

$$M_3 = \max_{x \in [x_2, x_3]} |f'''(x)|$$

$$|E| = \left| \int_{x_2}^{x_3} [f(x) - P_2(x)]\, dx \right|$$

$$\leqslant \int_{x_2}^{x_3} |f(x) - P_2(x)|\, dx$$

$$\leqslant \frac{M_3}{6} \int_{x_2}^{x_3} (x-x_0)(x-x_1)(x-x_2)\, dx$$

$$= \frac{M_3}{6} \int_{0}^{h} (y+2h)(y+h)y\, dy$$

$$= \frac{M_3}{6}\int_0^h (y^2+3hy+2h^2)\,y\,dy$$

$$= \frac{M_3}{6}\int_0^h (y^3+3hy^2+2h^2y)\,dy$$

$$= \frac{M_3}{6}\left(\frac{h^4}{4}+h^4+h^4\right) = \frac{3}{8}M_3h^4$$