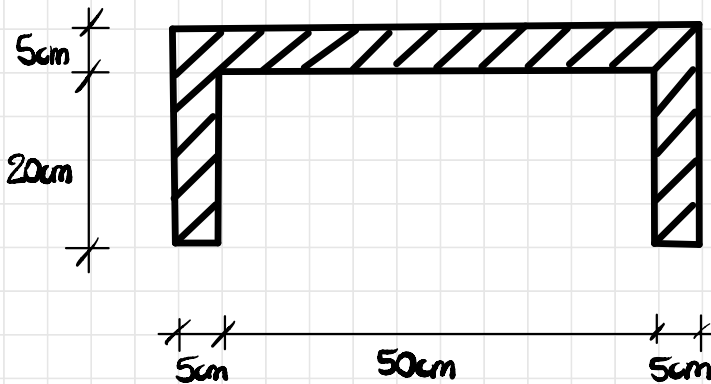
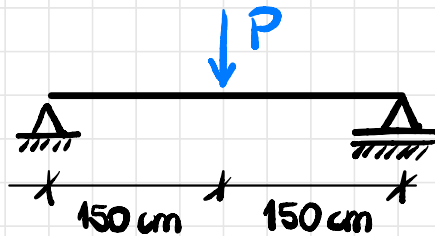
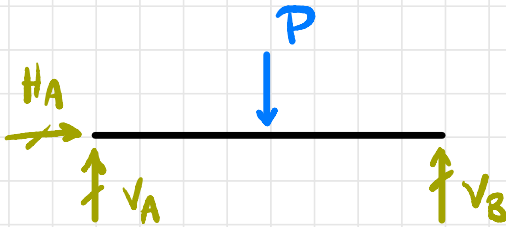


Determinar a carga máxima P que pode ser aplicada na barra considerando que a tensão admissível à tração do material é 1400 kgf/cm^2 e a tensão admissível à compressão é 840 kgf/cm^2 .



Equilibrio:



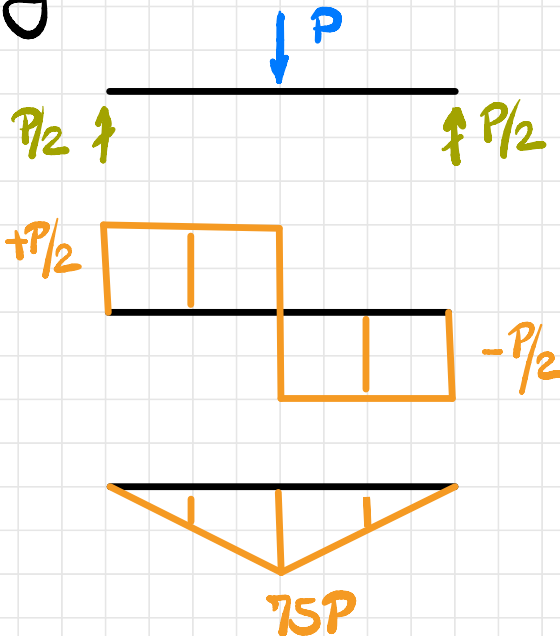
$$\sum F_H = 0: H_A = 0$$

$$\sum F_V = 0: V_A + V_B = P$$

$$\circlearrowleft \sum M_A = 0: -P \cdot 150 + V_B \cdot 300 = 0$$

$$\therefore V_B = P/2 \rightarrow V_A = P/2$$

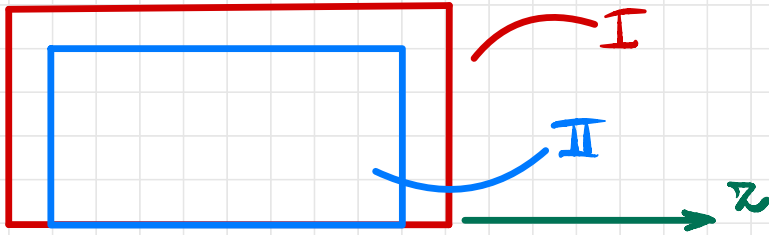
Diagramas:



$V [kgf]$

$M [kgfcm]$

Propriedades da seção transversal:



	y	A	I_{z0}	d
I	12,5	1500	78 125	5
II	10	1000	100 000/3	7,5

$$A = 1500 - 1000 = 500 \text{ cm}^2$$

$$y_G = \frac{12,5 \cdot 1500 - 10 \cdot 1000}{500} \rightarrow y_G = 17,5 \text{ cm}$$

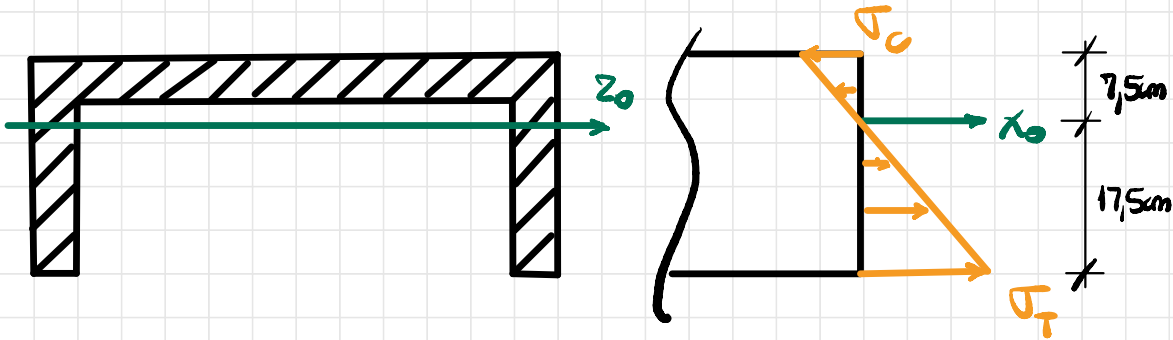
$$I_{z0} = (78 125 + 5^2 \cdot 1500) - (100 000/3 + 7,5^2 \cdot 1000)$$

$$I_{z0} = 115 625 - 268 750/3$$

$$I_{z0} = 78 125/3 \approx 26 041,6 \text{ cm}^4$$

Na seção central, $M = 75P$. Assim:

$$\sigma = \frac{M}{I_{z_0}} y$$



$$\sigma_c = \frac{75 P \cdot 3}{78125} (-7,5) \rightarrow \sigma_c = -\frac{27P}{1250}$$

$$\sigma_T = \frac{75 P \cdot 3}{78125} (17,5) \rightarrow \sigma_T = \frac{63P}{1250}$$

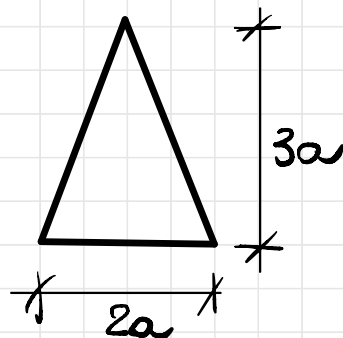
Comparando com os limites:

$$|\sigma_c| \leq \bar{\sigma}_c \rightarrow \frac{27P}{1250} \leq 840 \rightarrow P \leq 38889 \text{ kgf}$$

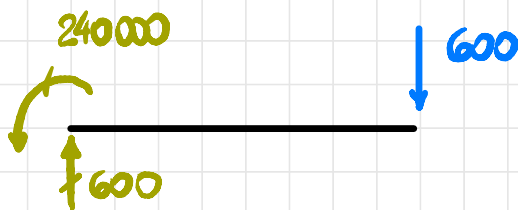
$$\sigma_T \leq \bar{\sigma}_T \rightarrow \frac{63P}{1250} \leq 1400 \rightarrow P \leq 27778 \text{ kgf.}$$

Logo $P_{\max} = 27778 \text{ kgf.}$

Achar a dimensão 'a'. São dados os limites de tensão ($\sigma_r = \sigma_c = \sigma = 40 \text{ kgf/cm}^2$).



Diagramas

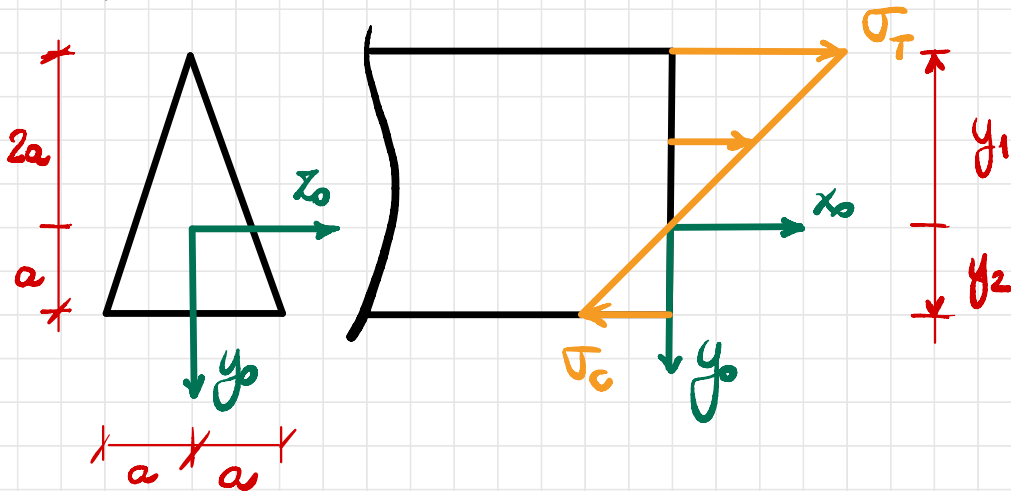


$V [\text{kgf}]$



$M [\text{kgf.cm}]$

Tensão:



$$I_{z_0} = \frac{2a(3a)^3}{36} = \frac{3}{2}a^4, \quad M = -M_A$$

$$M_A = 240000$$

$$\sigma_T = \frac{M}{I_{z_0}} \cdot y_1 = \frac{-M_A}{\frac{3}{2}a^4} \cdot (-2a) = \frac{4}{3} \frac{M_A}{a^3}$$

$$\sigma_C = \frac{M}{I_{z_0}} \cdot y_2 = \frac{-M_A}{\frac{3}{2}a^4} \cdot a = -\frac{2}{3} \frac{M_A}{a^3}$$

Como $\bar{\sigma}_T = \bar{\sigma}_C = \bar{\sigma}$, então:

$$\max(\sigma_T, |\sigma_C|) \leq \bar{\sigma}$$

$$\sigma_T \leq \bar{\sigma}$$

$$\frac{4}{3} \frac{M_A}{a^3} \leq \bar{\sigma}$$

$$\therefore a \geq \sqrt[3]{\frac{4M_A}{3\bar{\sigma}}}$$

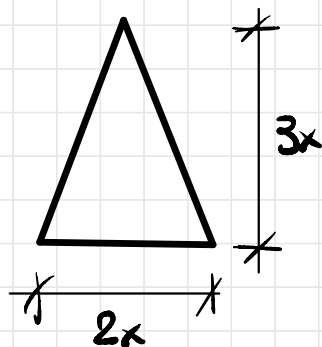
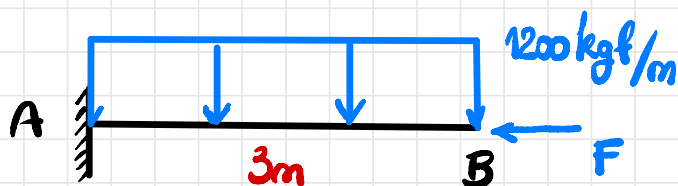
$$a \geq \sqrt[3]{\frac{4.240000}{3 \cdot 40}}$$

$$a \geq 20 \text{ cm}$$

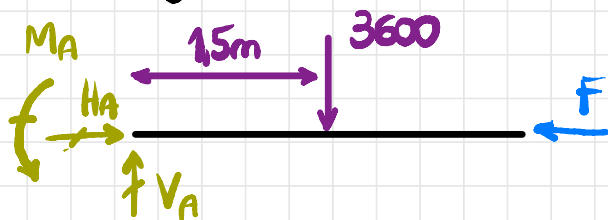
Logo $a_{\min} = 20 \text{ cm}$.

Obter F para minimizar x . Qual o valor mínimo de x ? São dados $\bar{\sigma}_T = 90 \text{ kgf/cm}^2$ e $\bar{\sigma}_c = 230 \text{ kgf/cm}^2$.

Para $F=0$, quanto vale x ?



Diagramas:

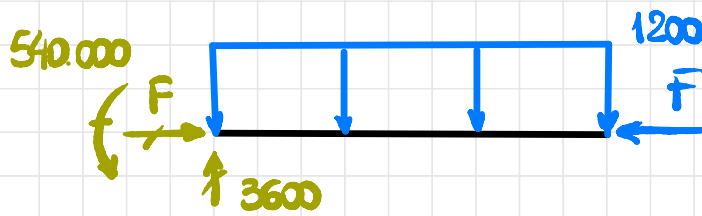


$$\sum F_H = 0: H_A = F$$

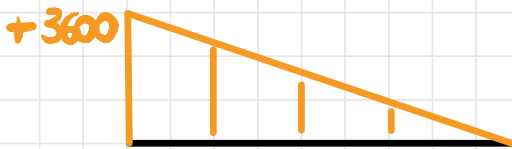
$$\sum F_V = 0: V_A = 3600 \text{ kgf}$$

$$\odot \sum M_A = 0: M_A - 3600 \cdot 1.50 = 0$$

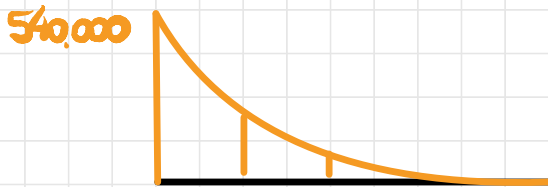
$$M_A = 540.000 \text{ kgfcm}$$



$N[\text{kgf}]$



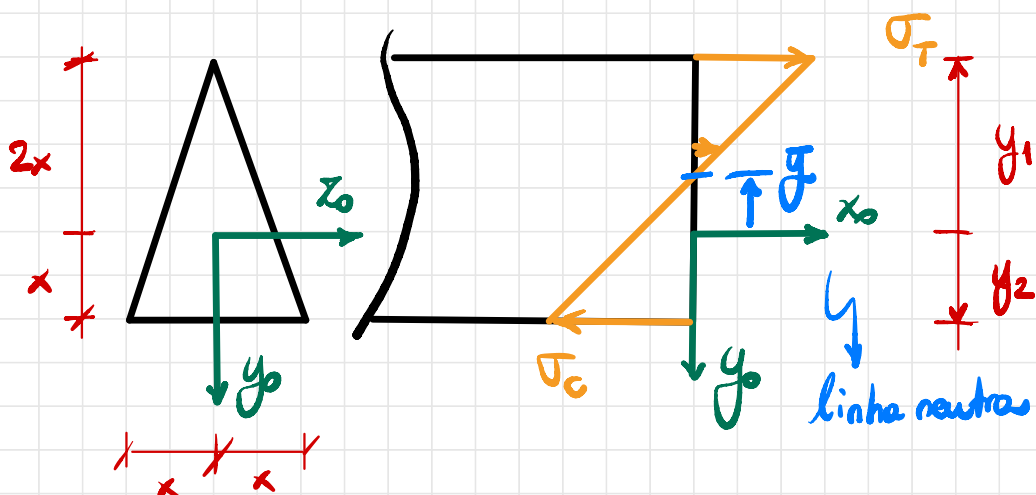
$V[\text{kgf}]$



$M[\text{kgf.cm}]$

Sego crítica A ($M_A = -540000 \text{ kgfcm}$,
 $N = -F$)

Chamando $M_A = -M$, então:



$$I_{z_0} = \frac{2x \cdot (3x)^3}{36} = \frac{3}{2}x^4 \quad ; \quad A = 3x^2$$

$$\sigma_T = \frac{N}{A} + \frac{M_A}{I_{z_0}} \cdot y_1 = -\frac{F}{3x^2} - \frac{M}{\frac{3}{2}x^4} \cdot (-2x)$$

$$\sigma_T = -\frac{F}{3x^2} + \frac{4M}{3x^3} = \frac{4M - Fx}{3x^3}$$

$$\sigma_c = \frac{N}{A} + \frac{M_A}{I_{z_0}} \cdot y_2 = -\frac{F}{3x^2} - \frac{M}{\frac{3}{2}x^4} \cdot x$$

$$\sigma_c = -\frac{F}{3x^2} - \frac{2M}{3x^3} = -\left(\frac{2M + Fx}{3x^3}\right)$$

Aplicando as condições limites:

$$\sigma_T \leq \bar{\sigma}_T$$

$$|\sigma_C| \leq \bar{\sigma}_C$$

$$\frac{4M - Fx}{3x^3} \leq \bar{\sigma}_T$$

$$\frac{2M + Fx}{3x^3} \leq \bar{\sigma}_C$$

Se x é tal que ambas as tensões estão nos seus respectivos limites, então:

$$\begin{cases} 4M - \cancel{Fx} = 3\bar{\sigma}_T x^3 \\ 2M + \cancel{Fx} = 3\bar{\sigma}_C x^3 \end{cases}$$

$$6M = 3(\bar{\sigma}_T + \bar{\sigma}_C)x^3$$

$$\therefore x^3 = \frac{2M}{\bar{\sigma}_T + \bar{\sigma}_C} \rightarrow x = \sqrt[3]{\frac{2M}{\bar{\sigma}_T + \bar{\sigma}_C}}$$

$$x = \sqrt[3]{\frac{2.540000}{90 + 230}} \rightarrow x = 15\text{cm}$$

Assim:

$$F_x = 3\bar{\sigma}_c x^3 - 2M$$

$$F = 3\bar{\sigma}_c x^2 - \frac{2M}{x}$$

$$F = 3 \cdot 230 \cdot 15^2 - \frac{2 \cdot 540000}{15}$$

$$F = 155250 - 72000 \rightarrow F = 83250 \text{ kgf}$$

Se $F = 0$, os limites ficam:

$$\frac{4M}{3x^3} \leq \bar{\sigma}_T$$

$$\frac{2M}{3x^3} \leq \bar{\sigma}_c$$

$$x \geq \sqrt[3]{\frac{4M}{3\bar{\sigma}_T}}$$

$$x \geq \sqrt[3]{\frac{2M}{3\bar{\sigma}_c}}$$

$$x \geq \sqrt[3]{\frac{4 \cdot 540000}{3 \cdot 90}}$$

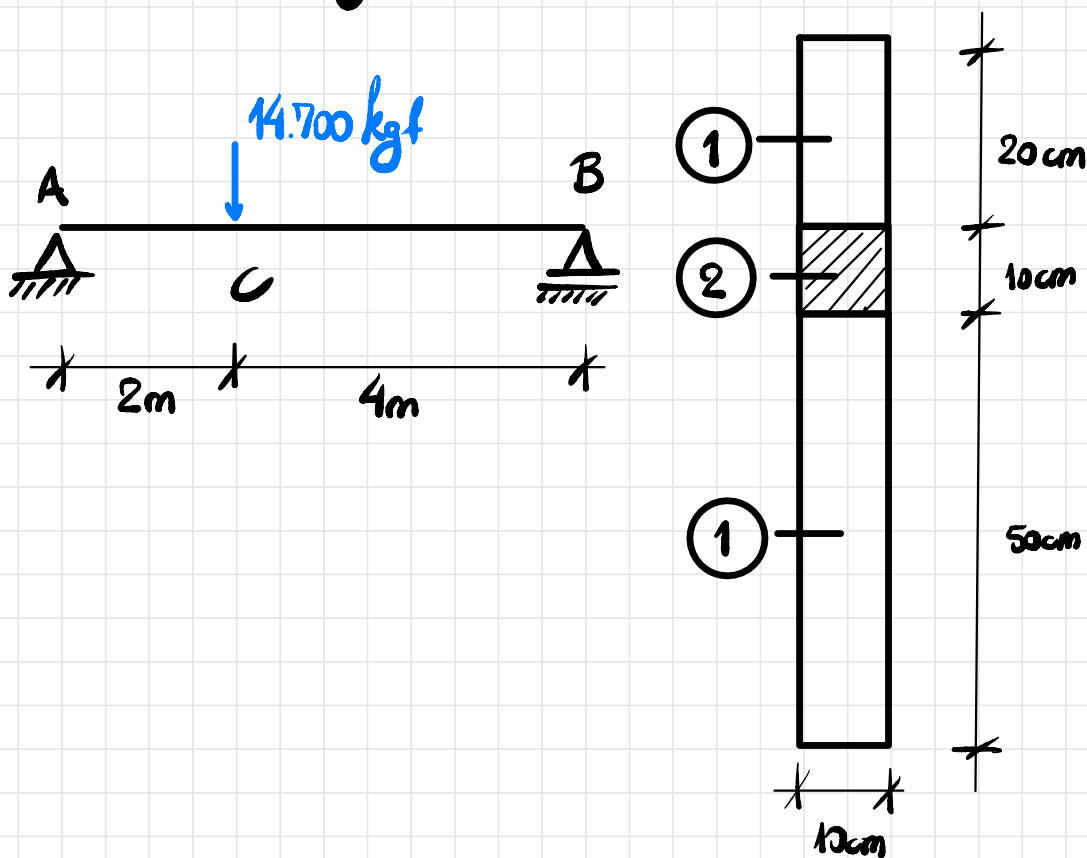
$$x \geq \sqrt[3]{\frac{2 \cdot 540000}{3 \cdot 230}}$$

$$x \geq 20 \text{ cm}$$

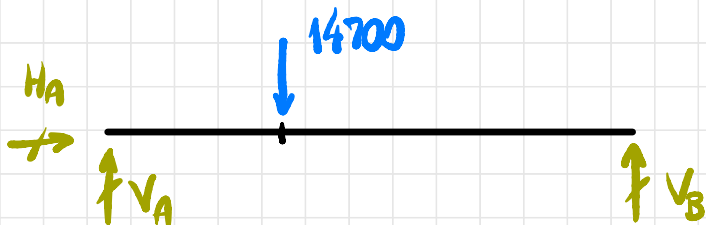
$$x \geq 11,61 \text{ cm}$$

Logo para $F = 0$, $x_{\min} = 20 \text{ cm}$.

Para a viga indicada na figura, formada por dois materiais diferentes, determinar o diagrama de tensões normais na seção transversal crítica, ou seja, a variação de σ ao longo da altura da seção para a seção mais solicitada pelo momento fletor. Dados $E_1 = 150.000 \text{ kgf/cm}^2$ e $E_2 = 750.000 \text{ kgf/cm}^2$.



Diagramas:

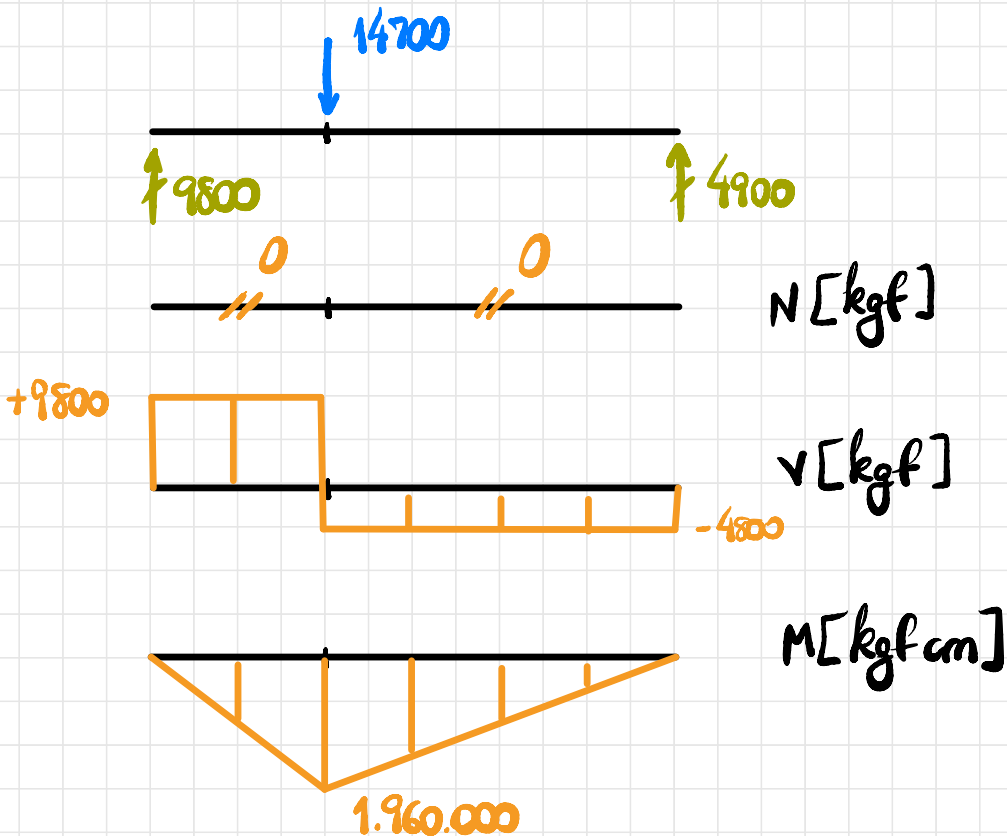


$$\sum F_H = 0: H_A = 0$$

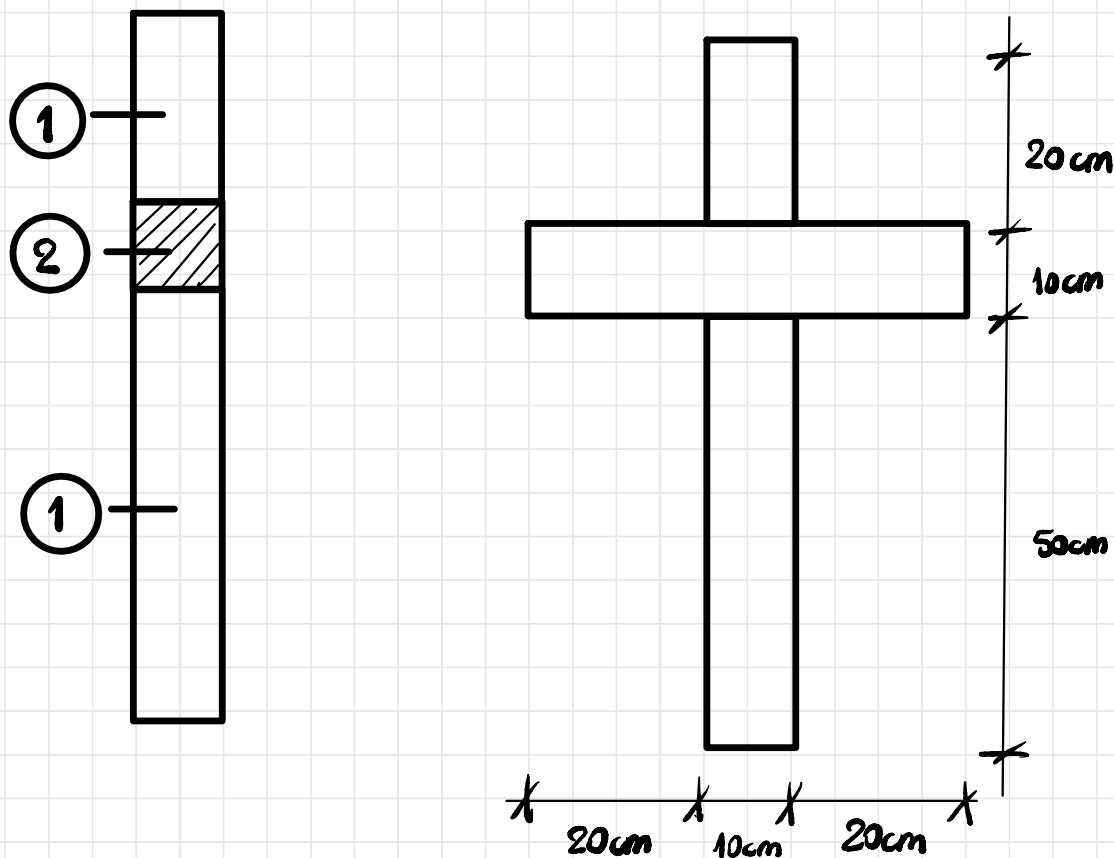
$$\sum F_V = 0: V_A + V_B = 14700$$

$$\oplus \sum M_A = 0: -14700 \cdot 200 + V_B \cdot 600 = 0$$

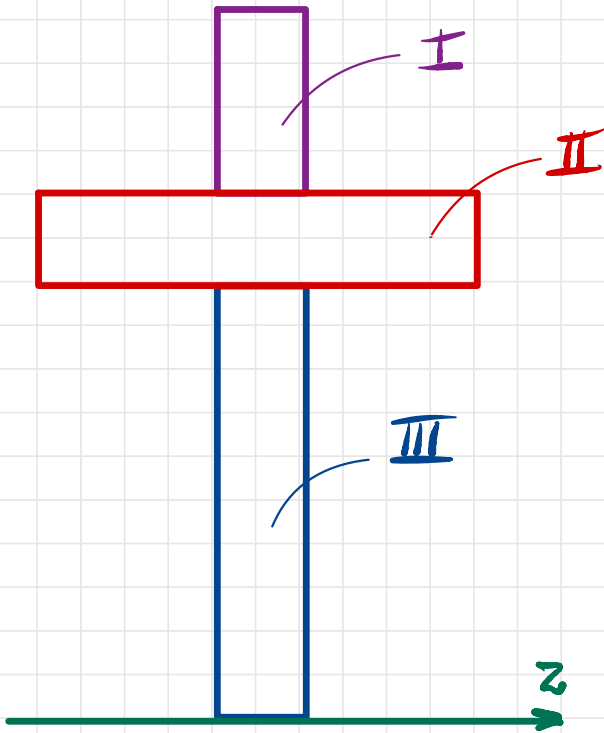
$$\therefore V_B = 4900 \text{ kgf} \rightarrow V_A = 9800 \text{ kgf}$$



Seção transversal equivalente. Escolhendo ① como base:



$$b_{eq} = \frac{E_2}{E_1} \cdot b = \frac{750\,000}{150\,000} \cdot 10 = 50\text{ cm}$$



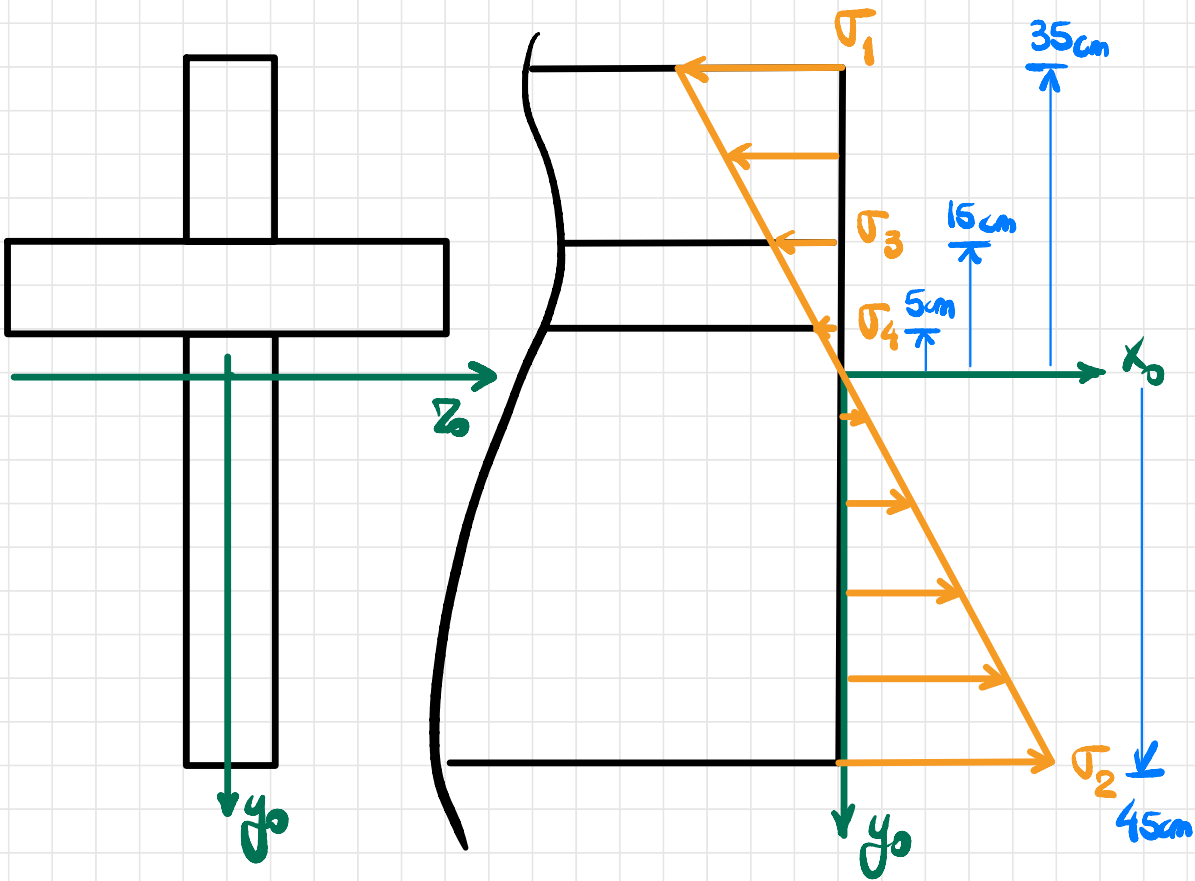
$$y_G = \frac{70 \cdot 200 + 55 \cdot 500 + 25 \cdot 500}{200 + 500 + 500}$$

$$y_G = 45 \text{ cm}$$

$$I_{z0} = 490.000 \text{ cm}^4$$

	y	A	I_{z0}	d	$d^2 A$
I	70	200	$\frac{20.000}{3}$	25	125.000
II	55	500	$\frac{12.500}{3}$	10	50.000
III	25	500	$\frac{312.500}{3}$	20	200.000

Calculando as tensões:

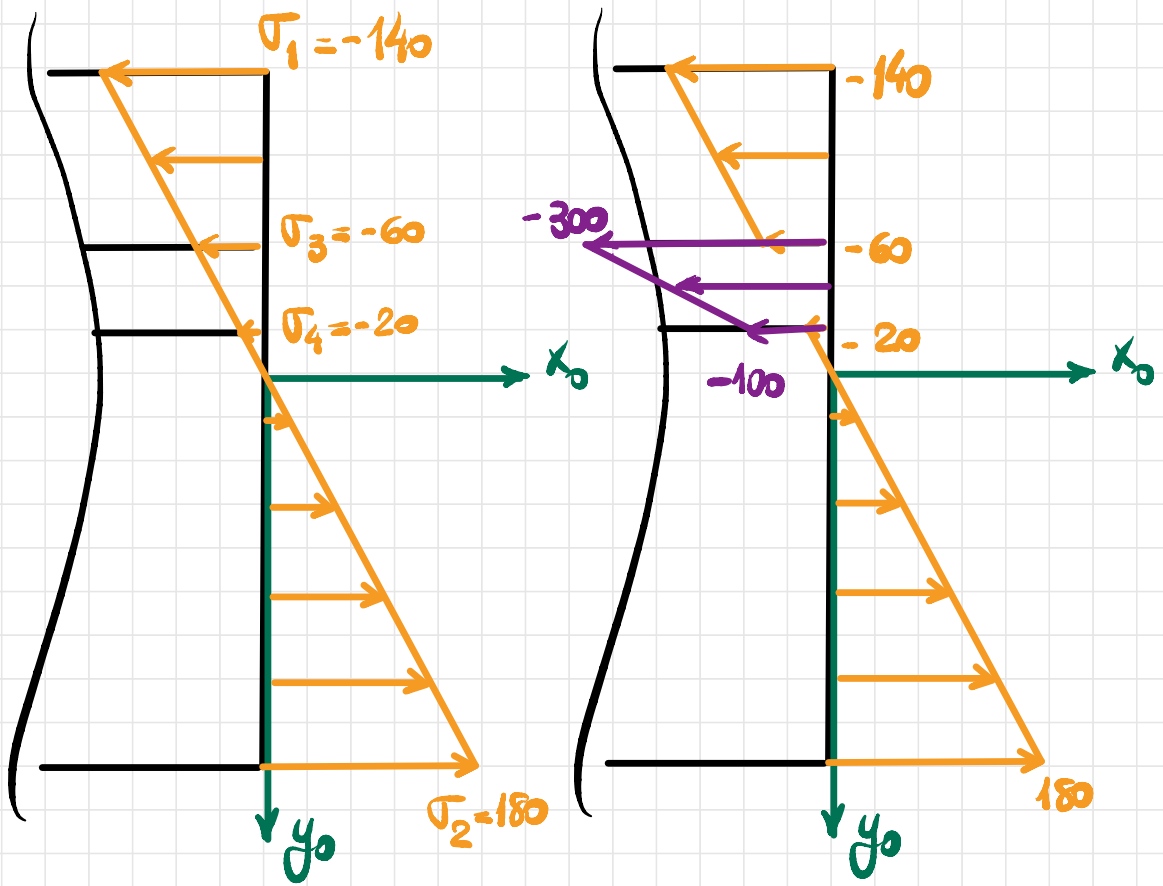


$$\sigma_1 = \frac{M}{I_{z_0}} \cdot h_1 = - \frac{1960000}{490000} \cdot 35 = -4 \cdot 35 \rightarrow \sigma_1 = -140 \text{ kgf/cm}^2$$

$$\sigma_2 = \frac{M}{I_{z_0}} \cdot h_2 = 4 \cdot 45 = 180 \text{ kgf/cm}^2$$

$$\sigma_3 = \frac{M}{I_{z_0}} \cdot h_3 = -4 \cdot 15 = -60 \text{ kgf/cm}^2$$

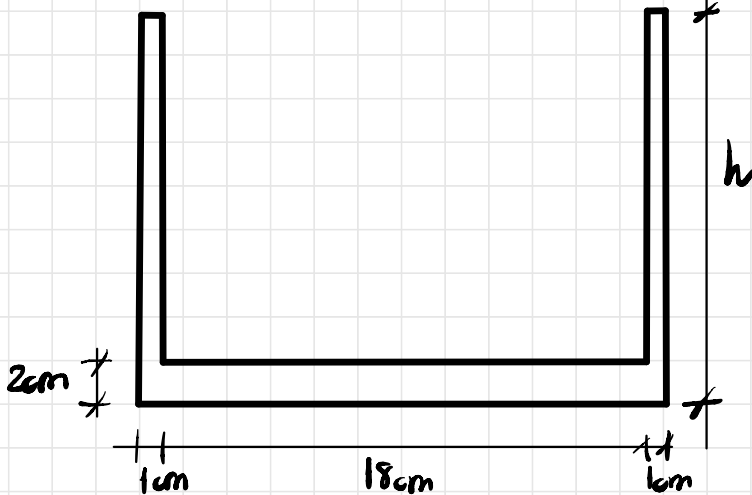
$$\sigma_4 = \frac{M}{I_{z_0}} \cdot h_4 = -4 \cdot 5 = -20 \text{ kgf/cm}^2$$



$$\sigma_3^{(2)} = \frac{E_2}{E_1} \sigma_3^{(1)} = 5 \cdot -60 = -300 \text{ kgf/cm}^2$$

$$\sigma_4^{(2)} = \frac{E_2}{E_1} \sigma_4^{(1)} = 5 \cdot -20 = -100 \text{ kgf/cm}^2$$

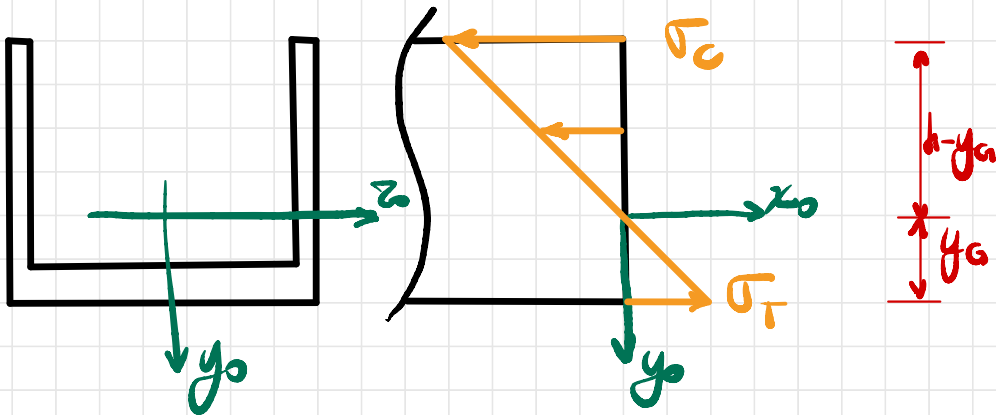
Achar a altura racional da seção:



Dado:

$$\frac{\sigma_T}{\sigma_C} = \frac{1}{3}$$

Assumindo um momento positivo:



$$\sigma_T = \frac{M}{I_{z0}} \cdot y_G$$

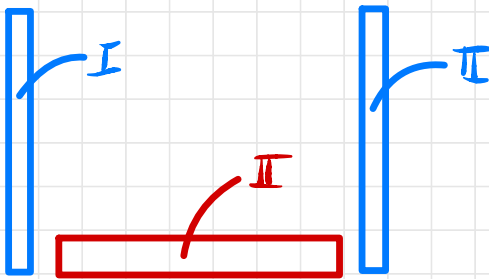
$$|\sigma_C| = \frac{M}{I_{z0}} (h - y_G)$$

$$\frac{\sigma_T}{|\sigma_C|} = \frac{M/I_{z0} \cdot y_G}{M/I_{z0} \cdot (h - y_G)}$$

$$\frac{y_G}{h - y_G} = \frac{1}{3} \rightarrow 3y_G = h - y_G$$

$$\therefore y_G = h/4$$

Propriedades da seção:



	y	A
<u>I</u>	$h/2$	b
<u>II</u>	1	$3b$
<u>III</u>	$h/2$	b

$$y_G = \frac{h/2 \cdot b + 1 \cdot 3b + h/2 \cdot b}{3b + 2b} = h/4$$

$$\frac{3b + h^2}{3b + 2h} = \frac{h}{4} \rightarrow 4(3b + h^2) = h(3b + 2h)$$

$$4h^2 + 144 = 3bh + 2h^2$$

$$2h^2 - 36h + 144 = 0$$

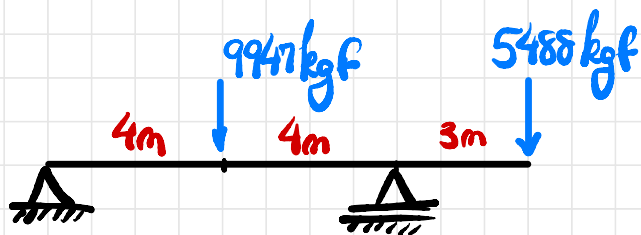
$$h^2 - 18h + 72 = 0$$

$$\therefore h = \frac{18 \pm \sqrt{18^2 - 4 \cdot 1 \cdot 72}}{2} = \frac{18 \pm \sqrt{324 - 288}}{2}$$

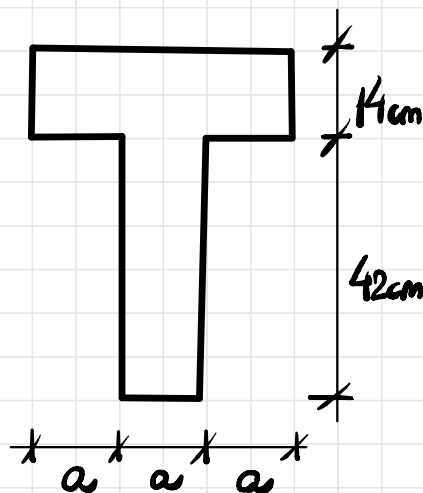
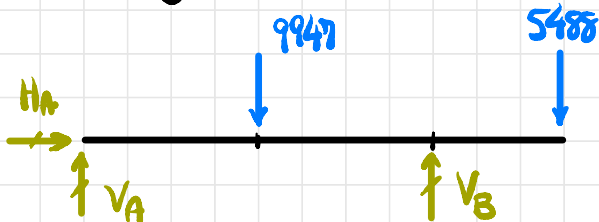
$$h = \frac{18 \pm \sqrt{36}}{2} = \frac{18 \pm 6}{2}$$

$$\therefore h = 12 \text{ cm or } h = 6 \text{ cm.}$$

Achar o valor da dimensão 'a', dados
 $\bar{\sigma}_t = 125 \text{ kgf/cm}^2$ e $\bar{\sigma}_c = 200 \text{ kgf/cm}^2$.



Diagramas:



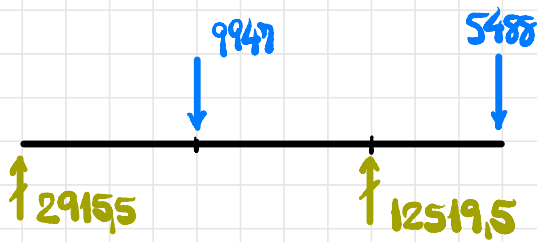
$$\sum F_H = 0: H_A = 0$$

$$\sum F_V = 0: V_A + V_B = 15435$$

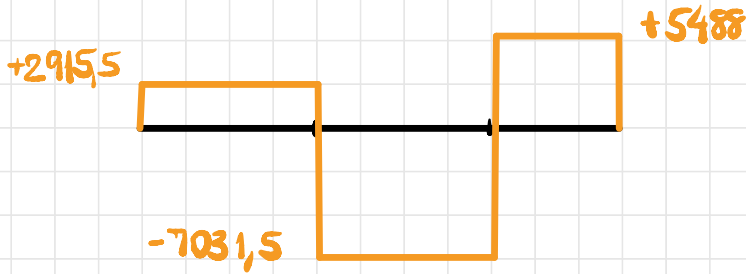
$$\curvearrowright \sum M_A = 0: -9947 \cdot 400 + V_B \cdot 800 - 5488 \cdot 1100 = 0$$

$$8V_B = 159842 \rightarrow V_B = 12.519,5 \text{ kgf}$$

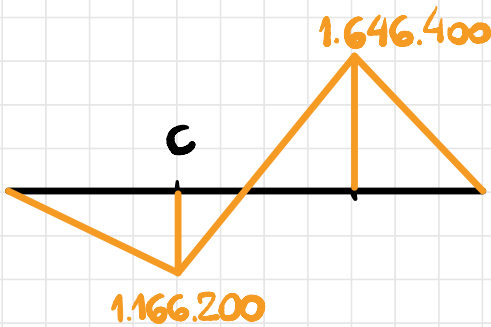
$$\therefore V_A = 2915,5 \text{ kgf}$$



$N[kgf]$

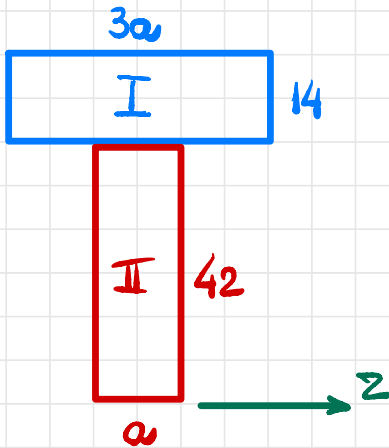


$v[kgf]$



$M[kgfcm]$

Propriedades da seção:



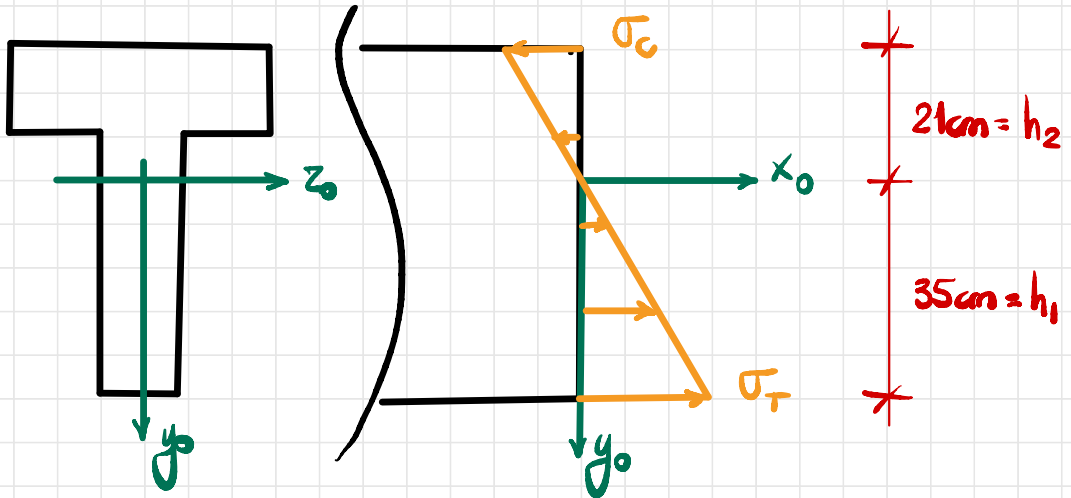
	y	A	I_{z0}	d	$d^2 A$
I	49	$42a$	$686a$	14	$8232a$
II	21	$42a$	$6174a$	14	$8232a$

$$y_G = \frac{49 \cdot 42a + 21 \cdot 42a}{84a} = 35 \text{ cm}$$

$$I_{z0} = (686a + 8232a) + (6174a + 8232a)$$

$$I_{z0} = 23324a$$

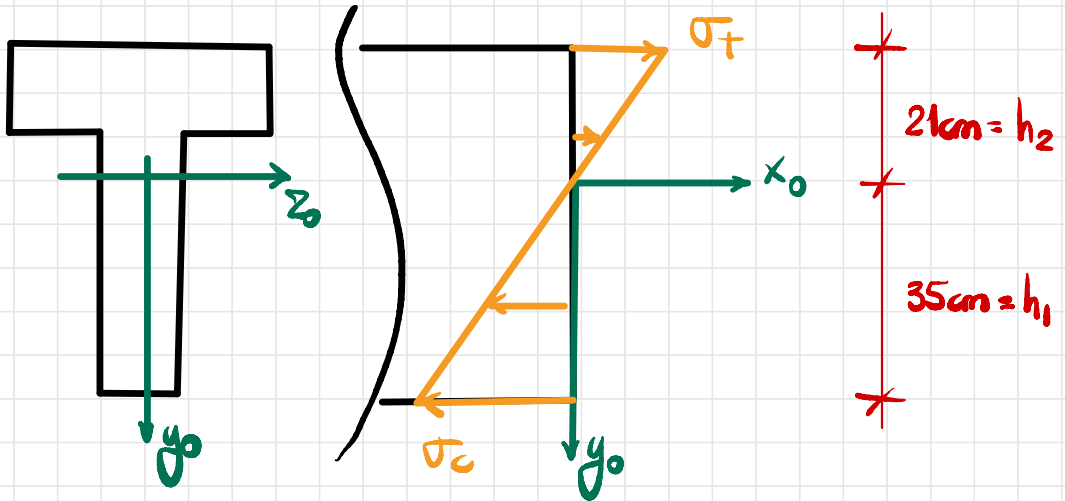
Seção C ($M = 1.166.200 \text{ kgf.cm}$):



$$\sigma_t = \frac{M}{I_{z_0}} \cdot h_1 = \frac{1750}{\omega}$$

$$\sigma_c = \frac{M}{I_{z_0}} \cdot -h_2 = -\frac{1050}{\omega}$$

Sección C ($M = -1.646.400 \text{ kgf.cm}$):



$$\sigma_t = \frac{M}{I_{z_0}} \cdot -h_2 = \frac{25200}{1700} \left(\frac{14823}{0} \right)$$

$$\sigma_c = \frac{M}{I_{z_0}} \cdot h_1 = -\frac{42000}{1700} \left(-\frac{24706}{0} \right)$$

Comparando com os limites:

$$\max(\sigma_T) \leq \bar{\sigma}_T$$

$$\frac{1750}{a} \leq 125$$

$$a \geq 14 \text{ cm}$$

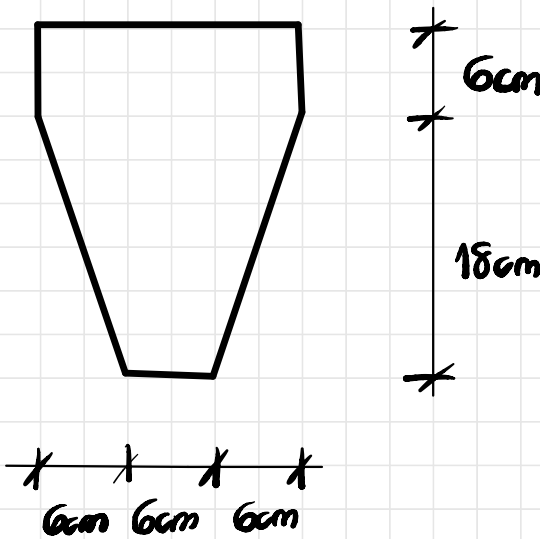
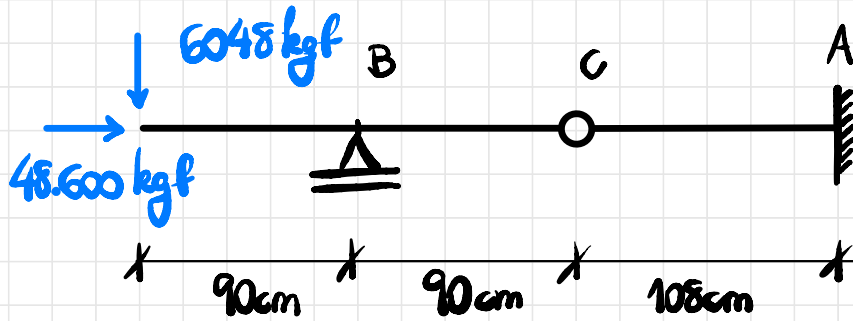
$$\max(|\sigma_c|) \leq \bar{\sigma}_c$$

$$\frac{42000}{17a} \leq 200$$

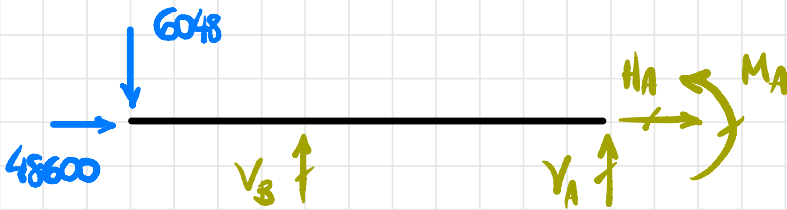
$$a \geq 12,35 \text{ cm}$$

Logo $a_{\min} = 14 \text{ cm}$.

Achar as tensões normais extremas ($\sigma_{\max}$ e $\sigma_{\min}$)
na seção transversal:



Diagramas:

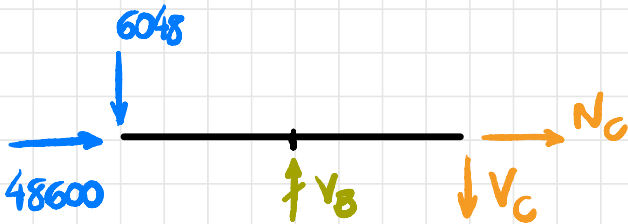


$$\sum F_H = 0: 48600 + H_A = 0 \Rightarrow H_A = -48600 \text{ kgf}$$

$$\sum F_V = 0: V_A + V_B = 6048$$

$$\curvearrowright \sum M_A = 0: M_A - V_B \cdot 198 + 6048 \cdot 288 = 0$$

Fazendo um corte em C:

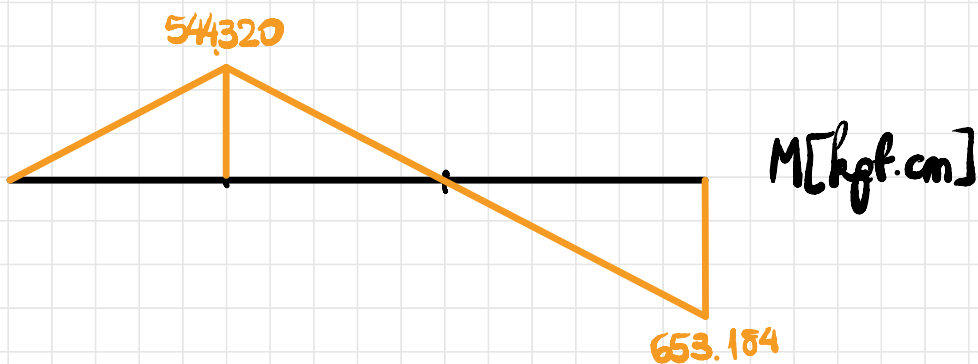
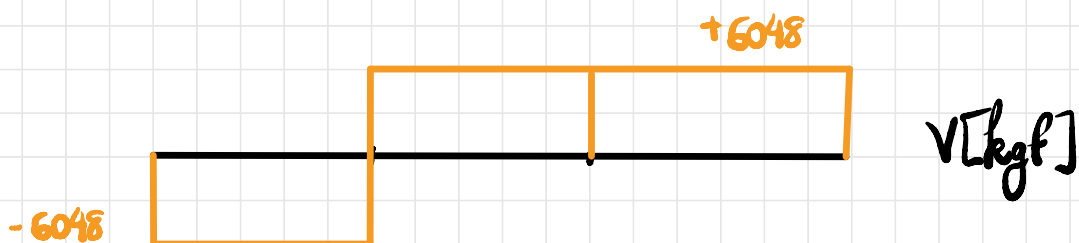
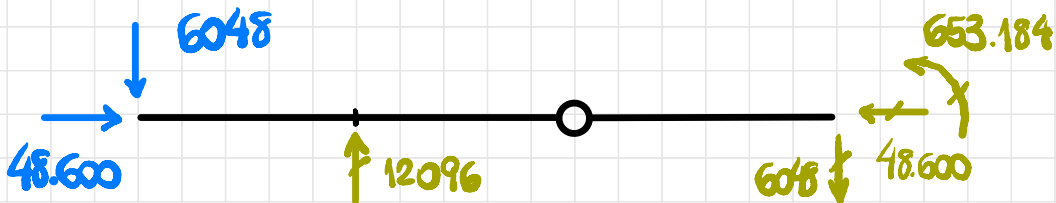


$$\curvearrowright \sum M_C = 0: 6048 \cdot 180 - V_B \cdot 90 = 0$$

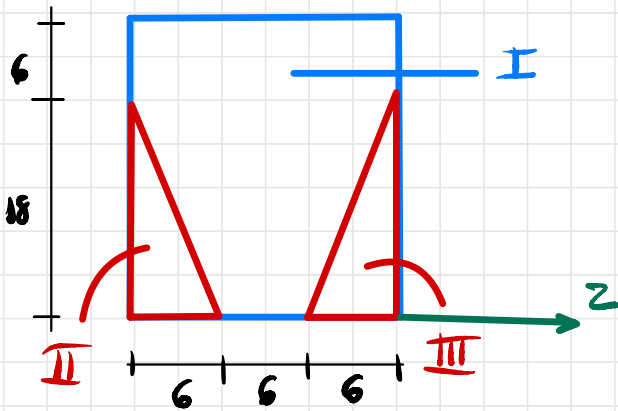
$$\therefore V_B = 12096 \text{ kgf}$$

$$M_A = 653.184 \text{ kgfcm}$$

$$V_A = -6048 \text{ kgf}$$



Propriedades da seção transversal:



I - II - III

	y	A	I_z	d	δA
I	12	432	20736	2	1728
II	6	54	972	8	3456
III	6	54	972	8	3456

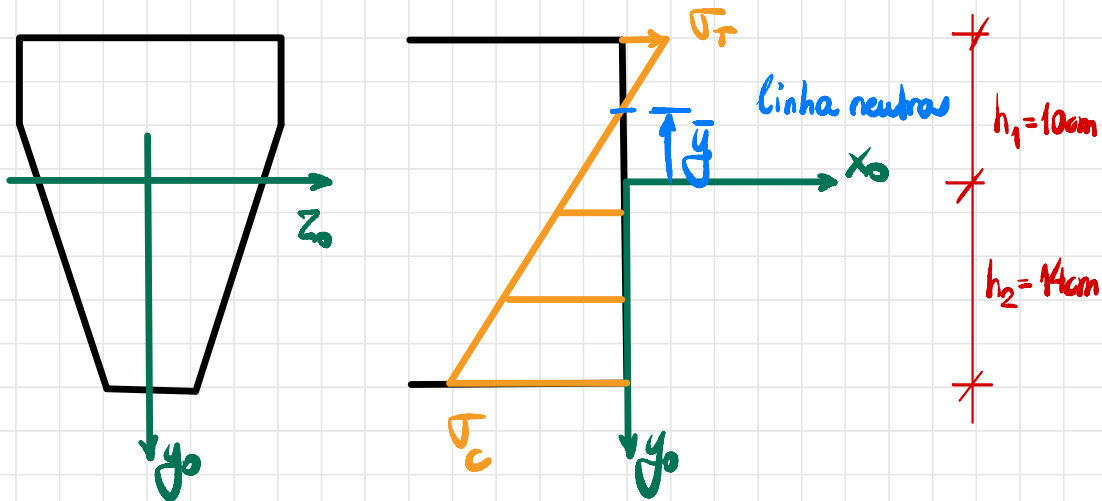
$$A = 432 - 54 - 54 = \underline{324 \text{ cm}^2}$$

$$y_G = \frac{12 \cdot 432 - 6 \cdot 54 - 6 \cdot 54}{324} = \underline{14 \text{ cm}}$$

$$I_{z0} = (20736 + 1728) - 2 \cdot (972 + 3456)$$

$$\underline{I_{z0} = 13608 \text{ cm}^4}$$

Tensões na seção A ($N = -48600 \text{ kgf}$; $M = -544320 \text{ kgfcm}$)



$$\sigma_T = \frac{N}{A} + \frac{M}{I_{z_0}} \cdot (-h_1) = -\frac{48600}{324} - \frac{544.320}{13.608} \cdot (-10)$$

$$\sigma_T = -150 + 40 \cdot 10 \rightarrow \sigma_T = 250 \text{ kgf/cm}^2$$

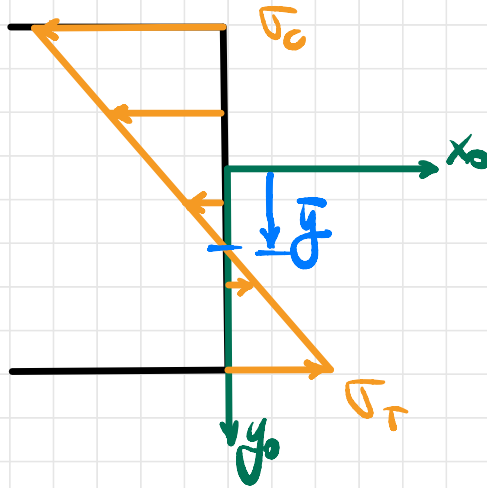
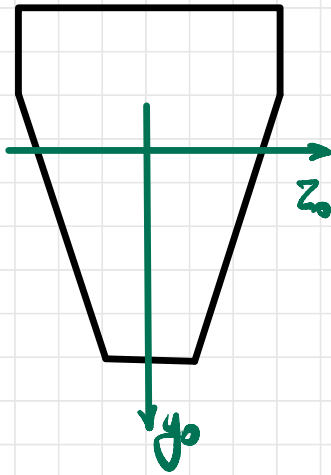
$$\sigma_C = \frac{N}{A} + \frac{M}{I_{z_0}} \cdot h_2 = -150 - 40 \cdot 14$$

$$\sigma_C = -710 \text{ kgf/cm}^2$$

~~X~~

Linha neutra: $\sigma = 0 \rightarrow -150 - 40\bar{y} = 0 \rightarrow \bar{y} = -3,75 \text{ cm}$

Tensões na seção A ($N = -48600 \text{ kgf}$; $M = 653.184 \text{ kgfcm}$)



$$\sigma_c = \frac{N}{A} + \frac{M}{I_{z_0}} \cdot (-h_1) = -\frac{48600}{432} + \frac{653184}{13608} \cdot (-10)$$

$$\sigma_c = -150 + 48 \cdot (-10) \rightarrow \sigma_c = -630 \text{ kgf/cm}^2$$

$$\sigma_t = \frac{N}{A} + \frac{M}{I_{z_0}} \cdot h_2 = -150 + 48 \cdot 14$$

$$\sigma_t = 522 \text{ kgf/cm}^2$$

~~X~~

Linha neutra: $\sigma = 0 \rightarrow -150 + 48\bar{y} = 0 \rightarrow \bar{y} = 3,125 \text{ cm}$

Assim, as tensões máximas que ocorrem são:

$$\sigma_{\max} = 522 \text{ kgf/cm}^2, \text{ que ocorre no engaste.}$$

$$\sigma_{\min} = -710 \text{ kgf/cm}^2, \text{ que ocorre no apoio simples.}$$