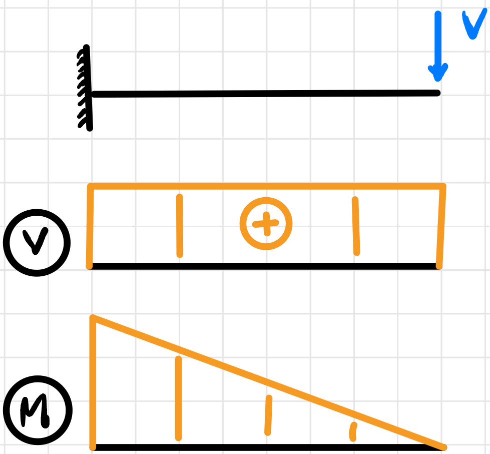
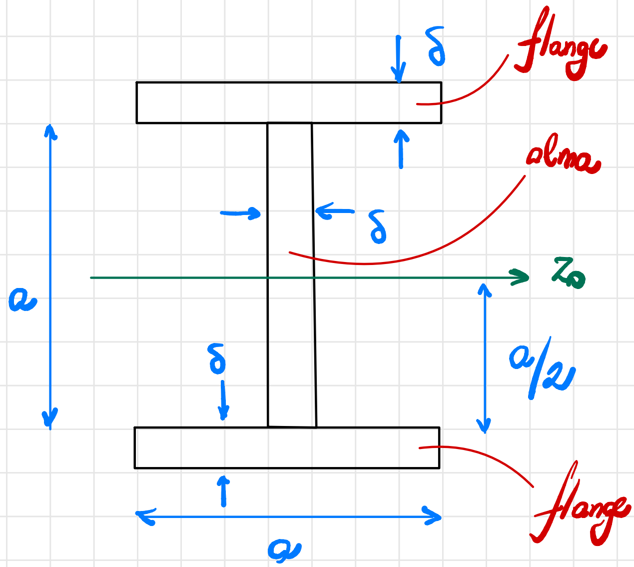
Seções Delgadas Simétricas

Considere uma viga de comprimento l e módulo de rigidez à flexão EI conhecidos. Considere a seção transversal dada em que 'δ' é muito menor que 'a':





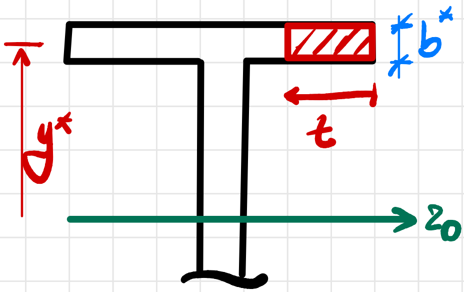
$\delta \ll a$

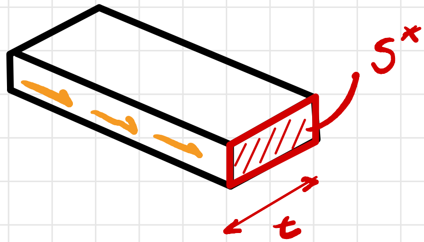
Calculando o momento de inércia:

$$I_{z0} = \frac{\delta a^3}{12} + 2\left[\cancel{\frac{a\delta^3}{12}} + \left(\frac{a}{2}\right)^2 \cdot \delta a\right] \longrightarrow \boxed{I_{z0} = \frac{7}{12}\delta a^3}$$

Fazendo cortes na estrutura (nas flanges e almas):

① Tensões nas flanges





$$S^* = b^* . t \rightarrow S^* = \delta t$$

$$b^* = \delta$$

$$\therefore \; M_s^* = S^* y^* = \frac{\delta a t}{2}$$

$$y^* = a/2$$

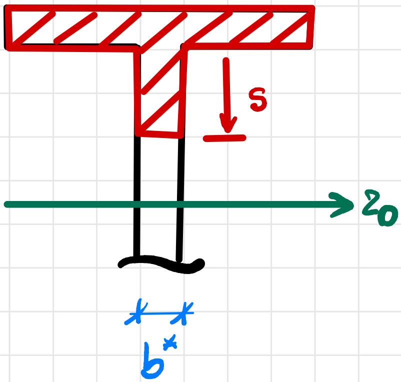
Assim:

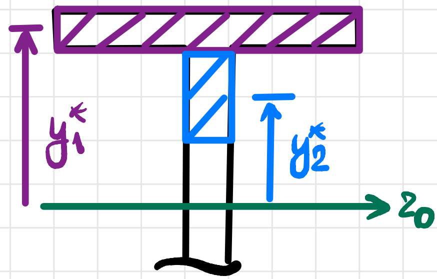
$$\tau = \frac{V . M_s^*}{b^* . I_{z_0}} = V . \frac{\delta a t}{2} . \frac{1}{\delta} . \frac{12}{7 \delta a^3} \rightarrow \boxed{\tau = \frac{6}{7} \frac{V}{\delta a} t}$$

Para alguns pontos de interesse:

$$\tau(t=0) = 0 \quad ; \quad \tau(t = a/2) = 3/7 \; V/\delta a$$

II Tensões na alma





$$M_s^* = M_{s1}^* + M_{s2}^* = S_1^* y_1^* + S_2^* y_2^*$$

$$y_1^* = \frac{a}{2} \quad ; \quad S_1 = \delta a \quad ; \quad M_{s1}^* = \frac{\delta a^2}{2}$$

$$y_2^* = \frac{a}{2} - \frac{s}{2} = \frac{a-s}{2} \quad ; \quad S_2 = s\delta \quad ; \quad M_{s2}^* = \frac{\delta s(a-s)}{2}$$

$$M_s^* = \frac{\delta}{2}(a^2 + sa - s^2)$$

Assim: $$\tau = V \cdot \frac{\delta}{2}(a^2 + sa - s^2) \cdot \frac{1}{\delta} \cdot \frac{12}{7\delta a^3}$$

$$\tau = \frac{6}{7}\frac{V}{\delta a^3}(a^2 + sa - s^2)$$

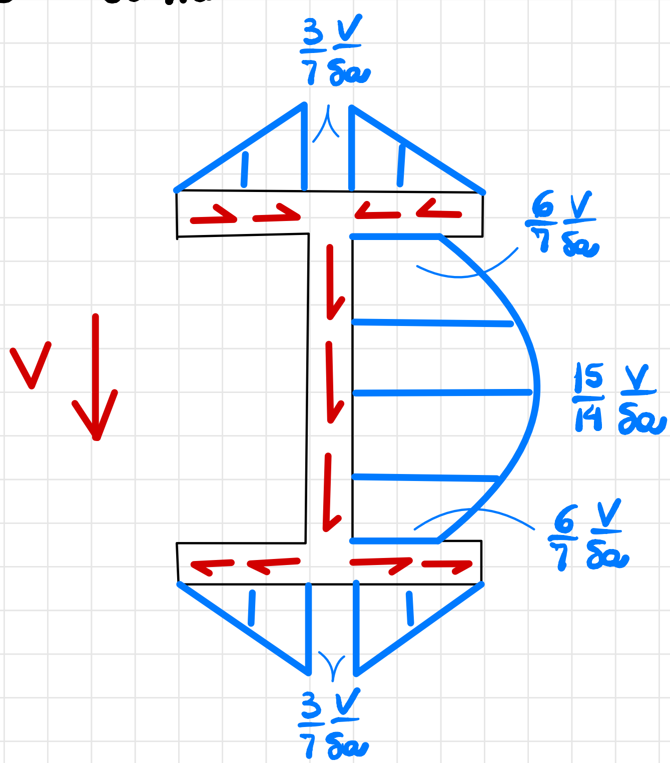
Calculando para alguns pontos de interesse:

$$\tau(s=0) = \frac{6}{7}\frac{V}{\delta a}$$

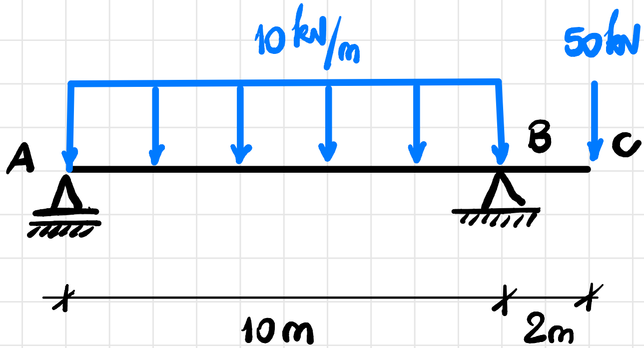
$$\tau(s=a/2) = \frac{15}{14}\frac{V}{\delta a} = \tau_{máximo}$$

$$\tau(s=a) = \frac{6}{7}\frac{V}{\delta a}$$

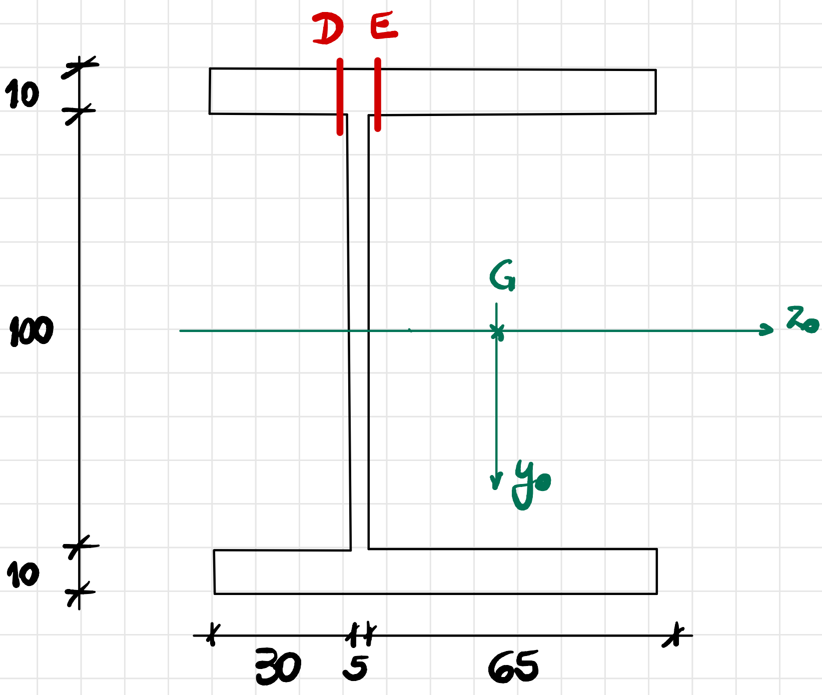
Pode-se então traçar um diagrama de tensões e o fluxo do cisalhamento:



Exemplo: Determine o fluxo de cisalhamento bem como as tensões nos pontos D e E para a seção de maior esforço cortante.

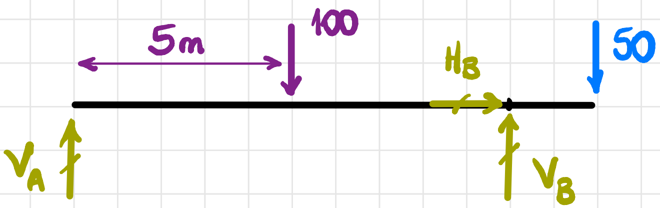


Seção transversal (medidas em mm):



$$I_{z_0} = 6483 \cdot 10^3 \ mm^4$$

Diagramas

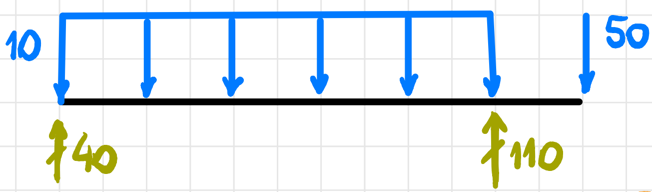


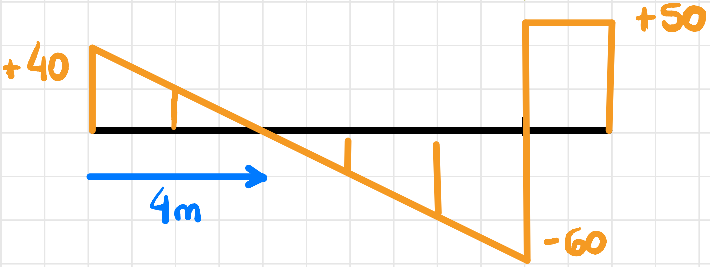
$$\sum F_H = 0 : \quad \underline{H_B = 0}$$

$$\sum F_V = 0 : \quad V_A + V_B = 150$$

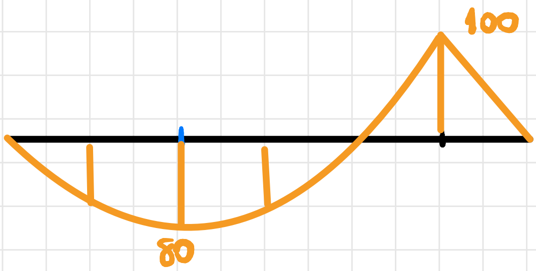
$$\circlearrowleft^{+} \sum M_A = 0 : \quad -100.5 + V_B . 10 - 50 . 12 = 0$$

$$\therefore \quad \underline{V_B = 110 \text{ kN}} \longrightarrow \underline{V_A = 40 \text{ kN}}$$





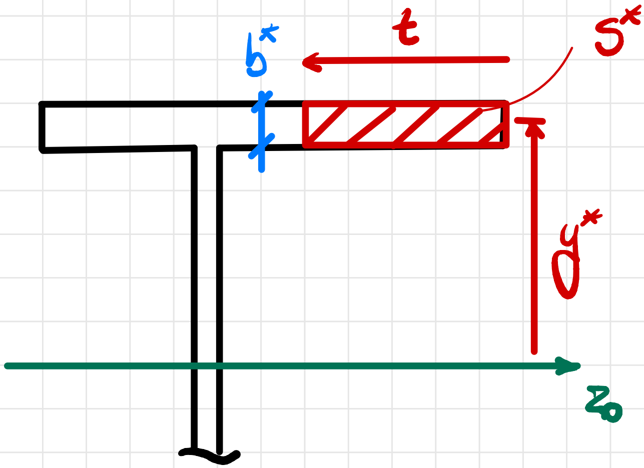
V [kN]



M [kNm]

Assim, o ponto de interesse é o ponto B^- com cortante $V = -60\,kN$.

Calculando a tensão de cisalhamento na flange de maneira paramétrica:



$y^* = 55\,mm$; $b^* = 10\,mm$; $S^* = 10t \longrightarrow M_S^* = 550t$

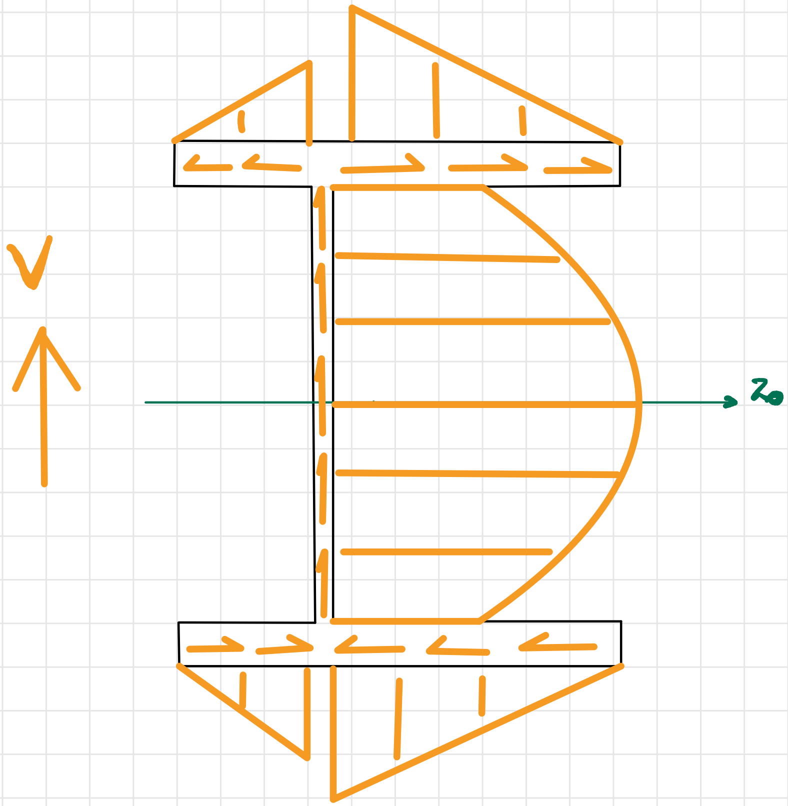
$$\tau = \frac{V.M_S^*}{b^* I_{z_0}} = \frac{60.\cancel{10}^3.550t}{10.6483.\cancel{10}^3} = 0{,}509t \qquad \left[\frac{N}{mm^2}\right] = [MPa]$$

em mm

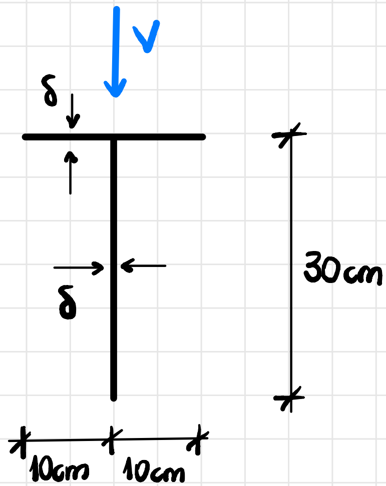
$\tau_D = \tau(t=30) \longrightarrow \tau_D = 15{,}27\,MPa$

$\tau_E = \tau(t=65) \longrightarrow \tau_E = 33{,}09\,MPa$

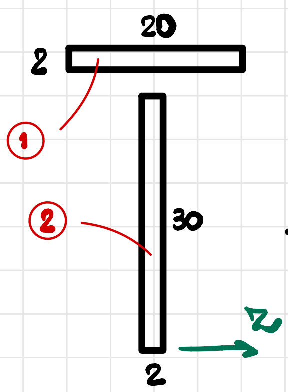
Esboço do fluxo de cisalhamento:



Exemplo: Sendo V = 79200 kgf, achar a distribuição do cisalhamento. Considere $\delta = 2$cm.



Determinando as propriedades da seção:



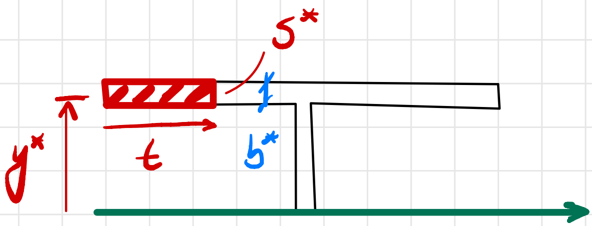
	y_G	A	I_{zo}	d
1	30	40	40/3	9
2	15	60	4500	6

$$y_G = \frac{30 \cdot 40 + 15 \cdot 60}{40 + 60} = \frac{1200 + 900}{100} = 21\text{cm}$$

$$I_{z_0} = \left[\cancel{40/3} + 9^2 \cdot 40\right] + \left[4500 + 6^2 \cdot 60\right]$$

$$I_{z_0} = 3240 + 4500 + 2160 \longrightarrow I_{z_0} = 9900\ cm^4.$$

Encontrando a distribuição de tensões de cisalhamento:



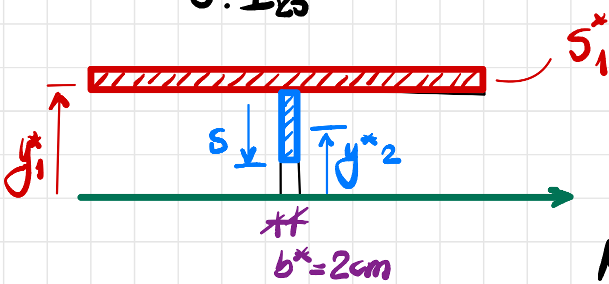
$y^* = 30 - 21 = 9cm$

$S^* = t.\delta = 2t$

$b^* = \delta = 2$

$M_S^* = 18t$

$$\tau = \frac{V.M_S^*}{b^*.I_{z_0}} = \frac{79.200.18t}{2.9900} = 72t \qquad \begin{cases} \tau(0) = 0 \\ \tau(10) = 720\ kgf/cm^2 \end{cases}$$



$S_1^* = 40cm$ $\quad S_2^* = 2s$

$y_1^* = 9cm$ $\quad y_2^* = 9 - s/2$

$M_S^* = 40.9 + 2s(9 - s/2)$

$M_S^* = 360 + 18s - s^2$

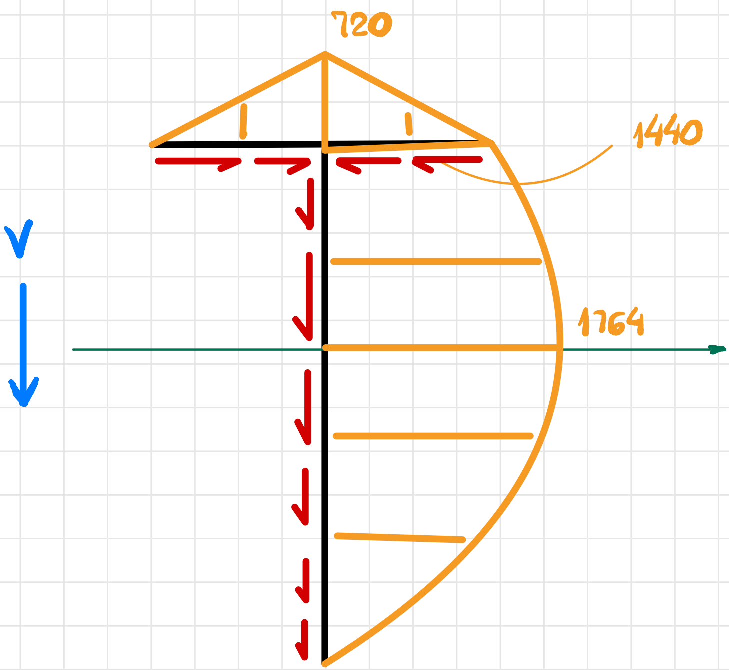
$$\therefore \tau = \frac{79200.(360 + 18s - s^2)}{2.9900} = 4(360 + 18s - s^2)$$

Calculando o máximo:

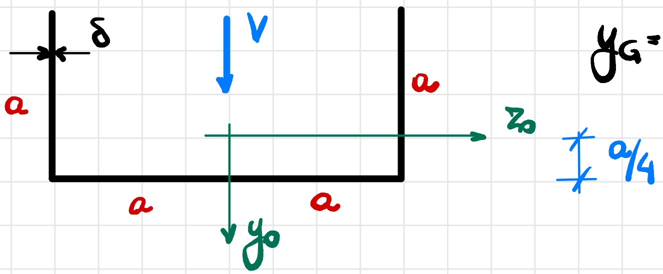
$$\tau_{máx} \longrightarrow \frac{\partial \tau}{\partial s} = 0 \longrightarrow 18 - 2s = 0 \quad s = 9cm$$

$$\tau(0) = 1440 \ kgf/cm^2$$
$$\tau(9) = 1764 \ kgf/cm^2$$
$$\tau(30) = 0$$

Desenhando o fluxo e as tensões de cisalhamento:

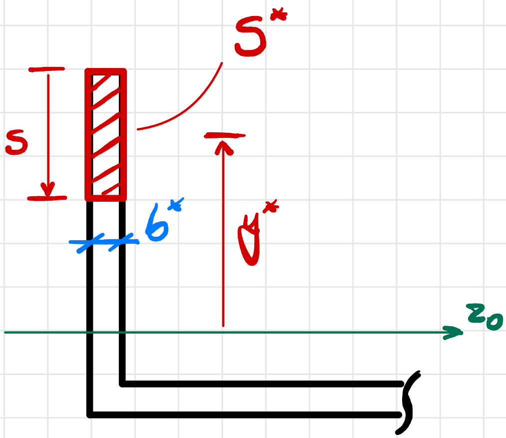


Exemplo: Obtenha o diagrama e fluxos de cisalhamento para a seção a seguir (considere $\delta << a$):



$$y_G = \frac{a}{4} \; ; \; I_{z_0} = \frac{5\delta a^3}{12}$$

Calculando a tensão nas almas:



$$y^* = \frac{3}{4}a - \frac{s}{2} = \frac{(3a-2s)}{4}$$

$$S^* = \delta s$$

$$\therefore M_S^* = \frac{\delta s(3a-2s)}{4}$$

$$b^* = \delta$$

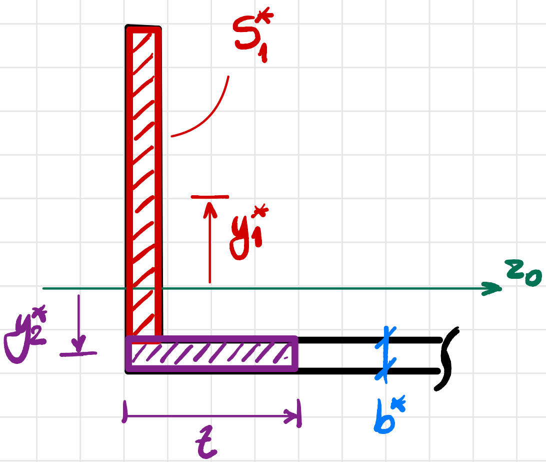
$$\tau = \frac{V M_S^*}{b^* I_{z_0}} = V \cdot \frac{\delta s(3a-2s)}{4} \cdot \frac{1}{\delta} \cdot \frac{12}{5\delta a^3} = \frac{3}{5} \frac{V}{\delta a^3}(3as-2s^2)$$

Calculando o máximo:

$$\frac{\partial \tau}{\partial s} = 0 \rightarrow 3a - 4s = 0 \rightarrow s = \frac{3}{4}a$$

$$\begin{cases} \tau(0) = 0 \\ \tau(3/4\, a) = 27/40\; V/\delta a \\ \tau(a) = 3/5\; V/\delta a \end{cases}$$

Calculando as tensões na flange:



$y_1^* = a/4 \;;\; S_1^* = \delta a$

$y_2^* = -a/4 \;;\; S_2^* = \delta t$

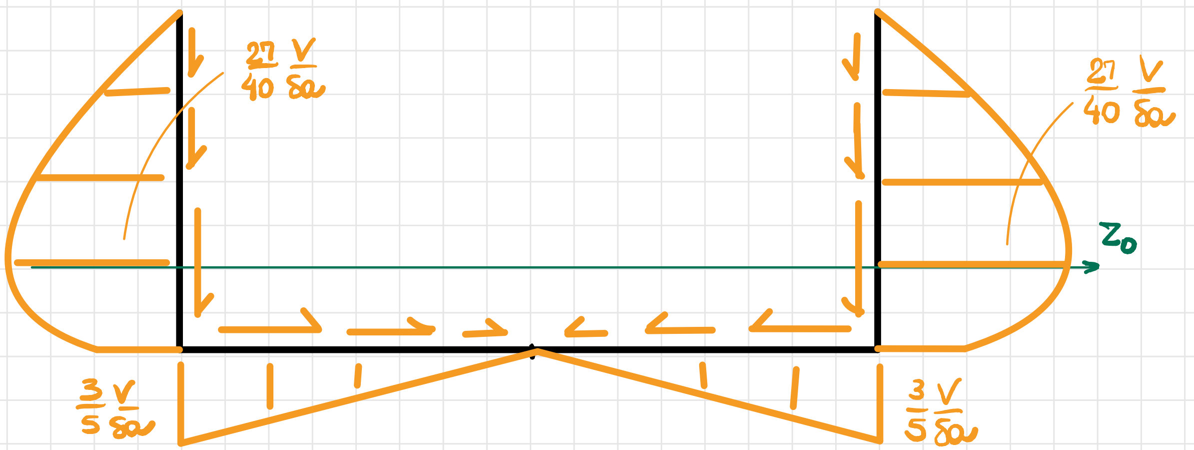
importante! é necessário que a diferença de posição (cruzar o eixo) seja levada em conta!

$$M_s^* = \frac{a}{4} \cdot \delta a - \frac{a}{4} \cdot \delta t = \frac{\delta a}{4}(a - t)$$

$$\tau = \frac{V \cdot M_s^*}{b^* \cdot I_{z_0}} = V \cdot \frac{\delta a}{4}(a-t) \cdot \frac{1}{\delta} \cdot \frac{12}{5\delta a^3} = \frac{3}{5}\frac{V}{\delta a^2}(a-t)$$

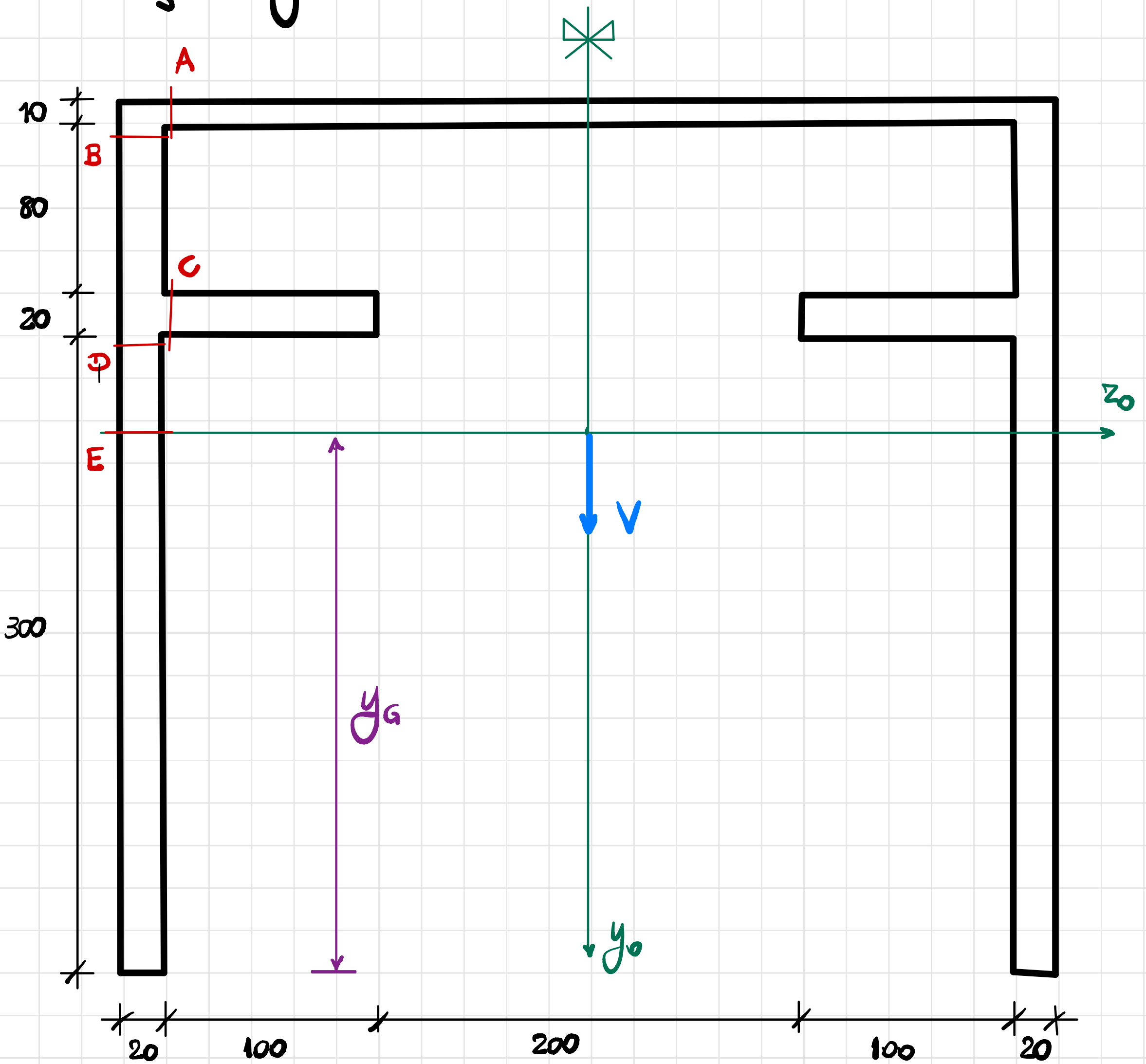
$$\begin{cases} \tau(0) = 3/5 \; V/\delta a \\ \tau(a) = 0 \\ \tau(2a) = -3/5 \; V/\delta a \end{cases}$$

Traçando os diagramas e o fluxo de cisalhamento:

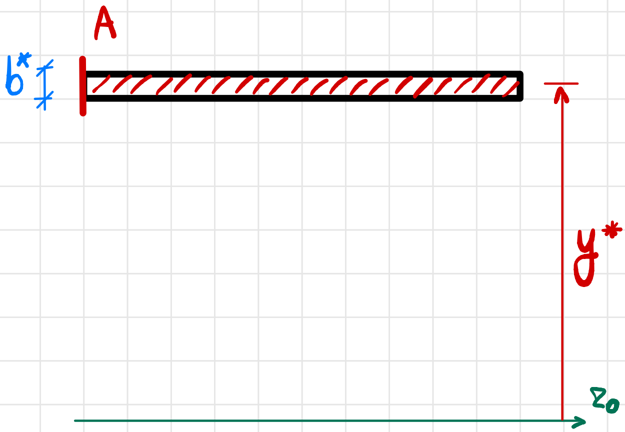


Considerando $V = 225\,kN$, calcule as tensões de cisalhamento nos pontos A, B, C, D e E. São dados $y_G = 255\,mm$ e $I_{z0} = 3{,}74 \cdot 10^8\,mm^4$.

Esboce o diagrama de tensões de cisalhamento e o fluxo das tensões.



Tensões em A:



$y^* = 405 - 255 = 150\ mm$

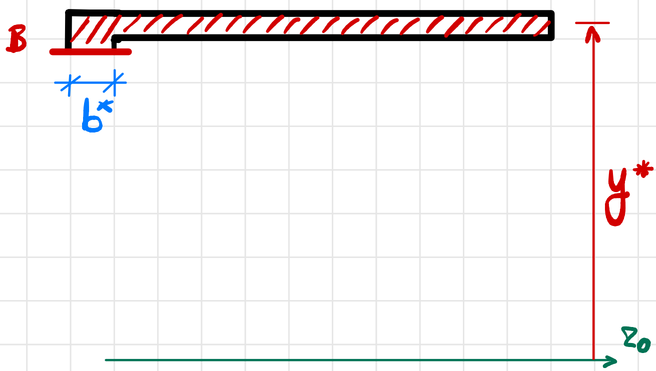
$S^* = 10.200 = 2000\ mm^2$

$M_S^* = 300\,000\ mm^3$

$b^* = 10\ mm$

$$\tau_A = \frac{225.10^3 . 3.10^5}{10 . 3{,}74 . 10^8} = 18{,}05\ N/mm^2 \longrightarrow \tau_A = 18{,}05\,MPa$$

Tensões em B:



$y^* = 150\ mm$

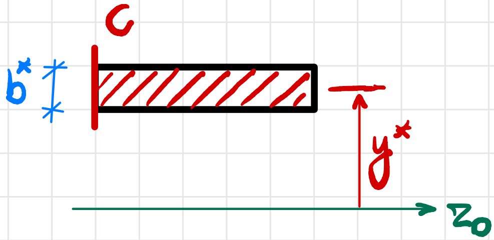
$S^* = 10.220 = 2200\ mm^2$

$M_S^* = 330\,000\ mm^3$

$b^* = 20\ cm$

$$\tau_B = \frac{225.10^3 . 3{,}3.10^5}{20 . 3{,}74 . 10^8} = 9{,}93\ N/mm^2 \longrightarrow \tau_B = 9{,}93\ MPa$$

Tensões em C:



$y^* = 310 - 255 = 55\,mm$

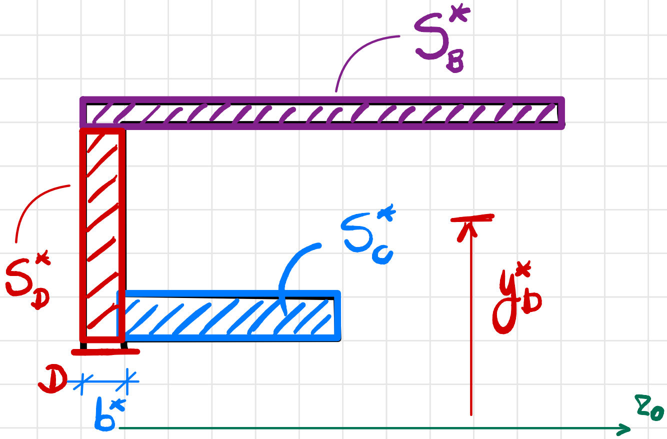
$S^* = 20.100 = 2000\,mm^2$

$M_S^* = 110\,000\,mm^3$

$b^* = 20\,mm$

$$\tau_C = \frac{225.10^3 . 1{,}1.10^5}{20.\ 3{,}74.\ 10^8} = 3{,}31\ N/mm^2 \longrightarrow \tau_C = 3{,}31\,MPa$$

Tensões em D:



$y_D^* = 350 - 255 = 95\,mm$

$S_D^* = 100.20 = 2000\,mm^2$

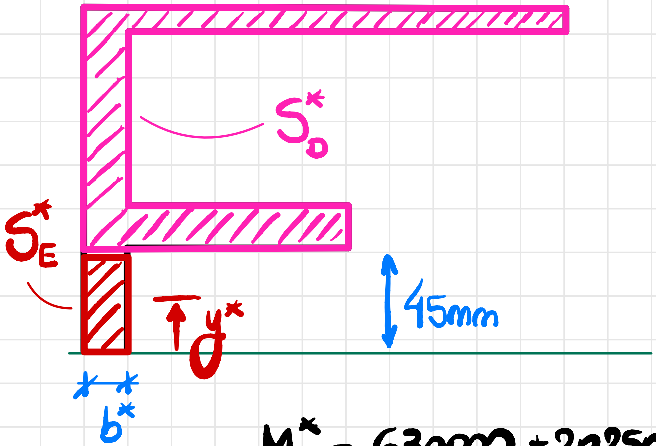
$M_{SD}^* = 190\,000\,mm^3$

$b^* = 20\,mm$

$$M_S^* = M_{SB}^* + M_{SC}^* + M_{SD}^* = 630\,000\,mm^3$$

$$\tau_D = \frac{225.10^3 . 6{,}3.10^5}{20.\ 3{,}74.10^8} = 18{,}95\,N/mm^2 \longrightarrow \tau_D = 18{,}95\,MPa$$

Tensões em E:



$y^* = 22{,}5\,mm$

$S_E^* = 20 \cdot 45 = 900\,mm^2$

$M_{S_E}^* = 20250\,mm^3$

$$M_S^* = 630000 + 20250 = 650250\,mm^3$$

$$\tau_E = \frac{225 \cdot 10^3 \cdot 6{,}5025 \cdot 10^5}{20 \cdot 3{,}74 \cdot 10^8} = 19{,}56\ N/mm^2$$

$$\tau_E = 19{,}56\,MPa$$

Esboçando as tensões de cisalhamento e dos fluxos de tensões: