ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Turbulência

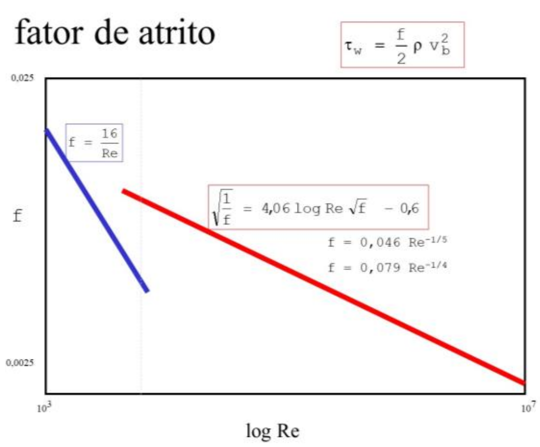
título

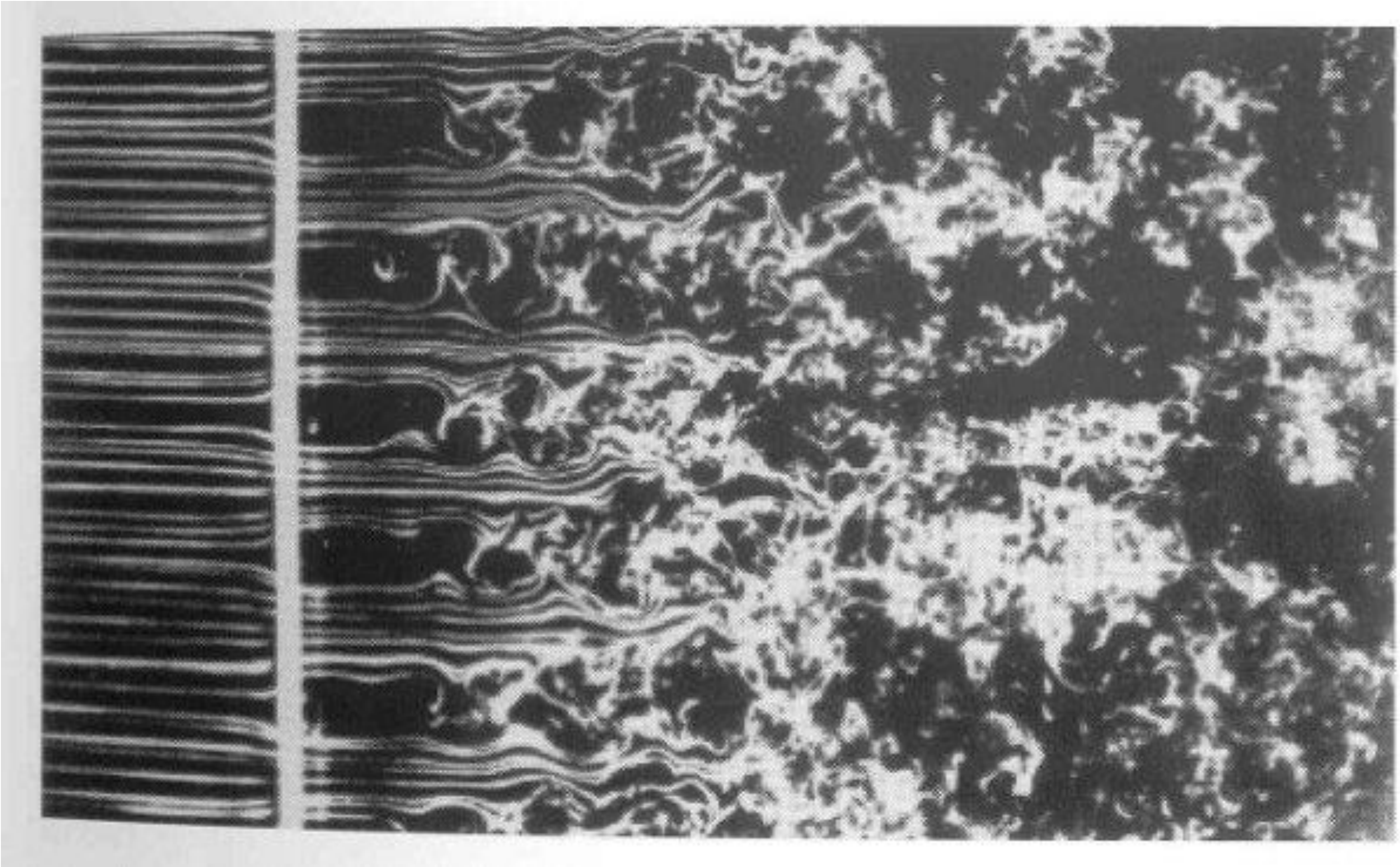
da Vinci

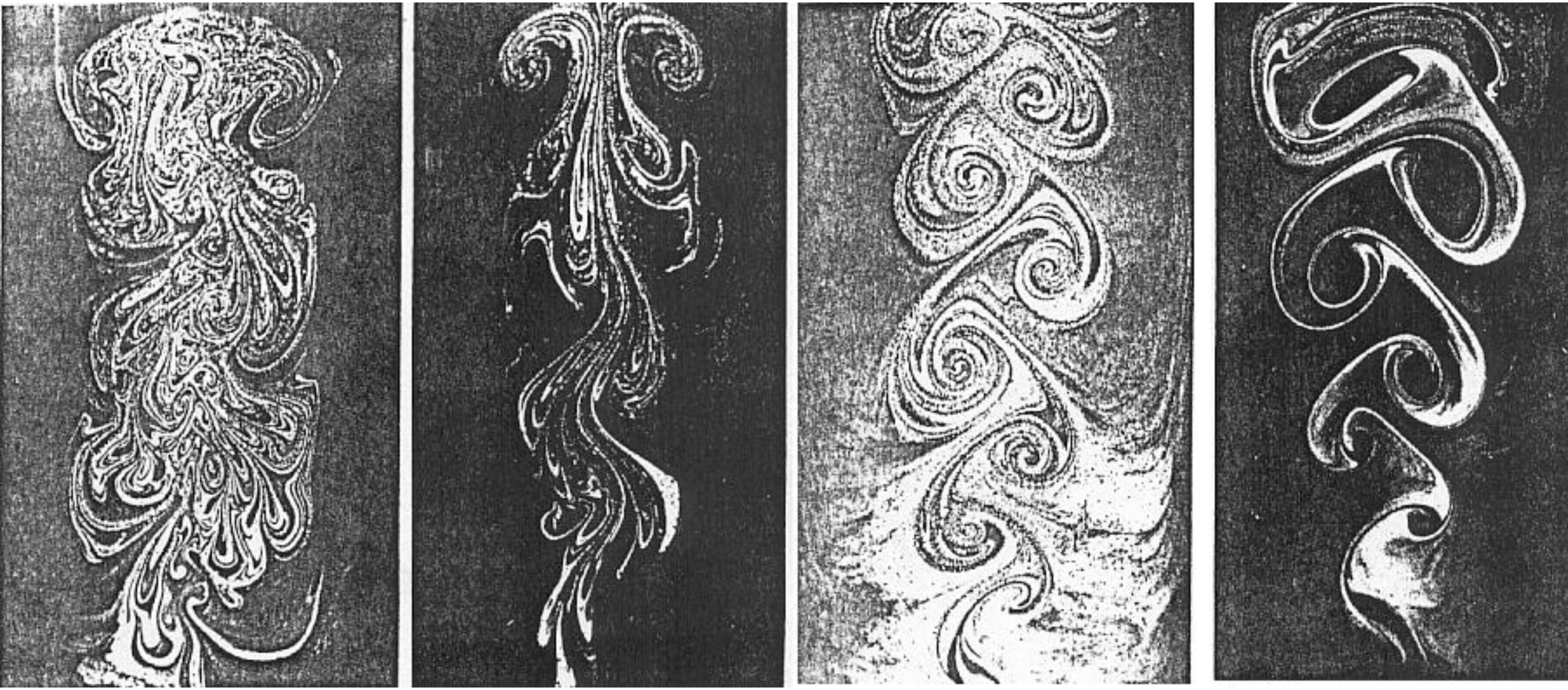
1452-1519

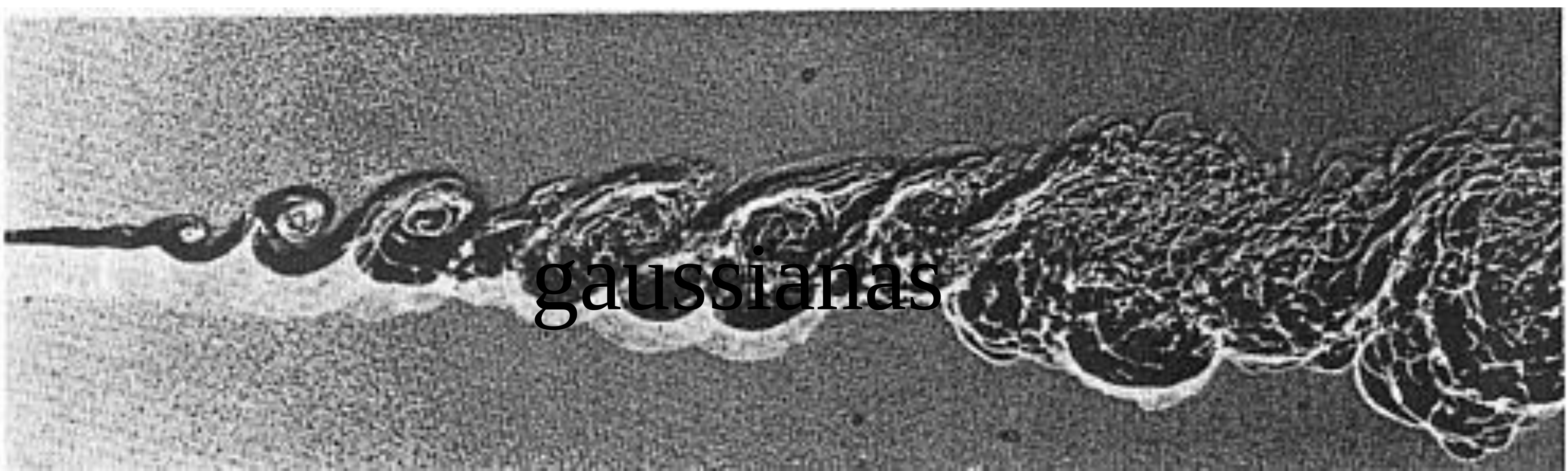
Turbulência

- Maioria dos Escoamentos
- Variação aleatório com o tempo e espaço
- Interpretação rigorosa em andamento
- Modelos de turbulência – CFD
- Transição laminar-turbulento

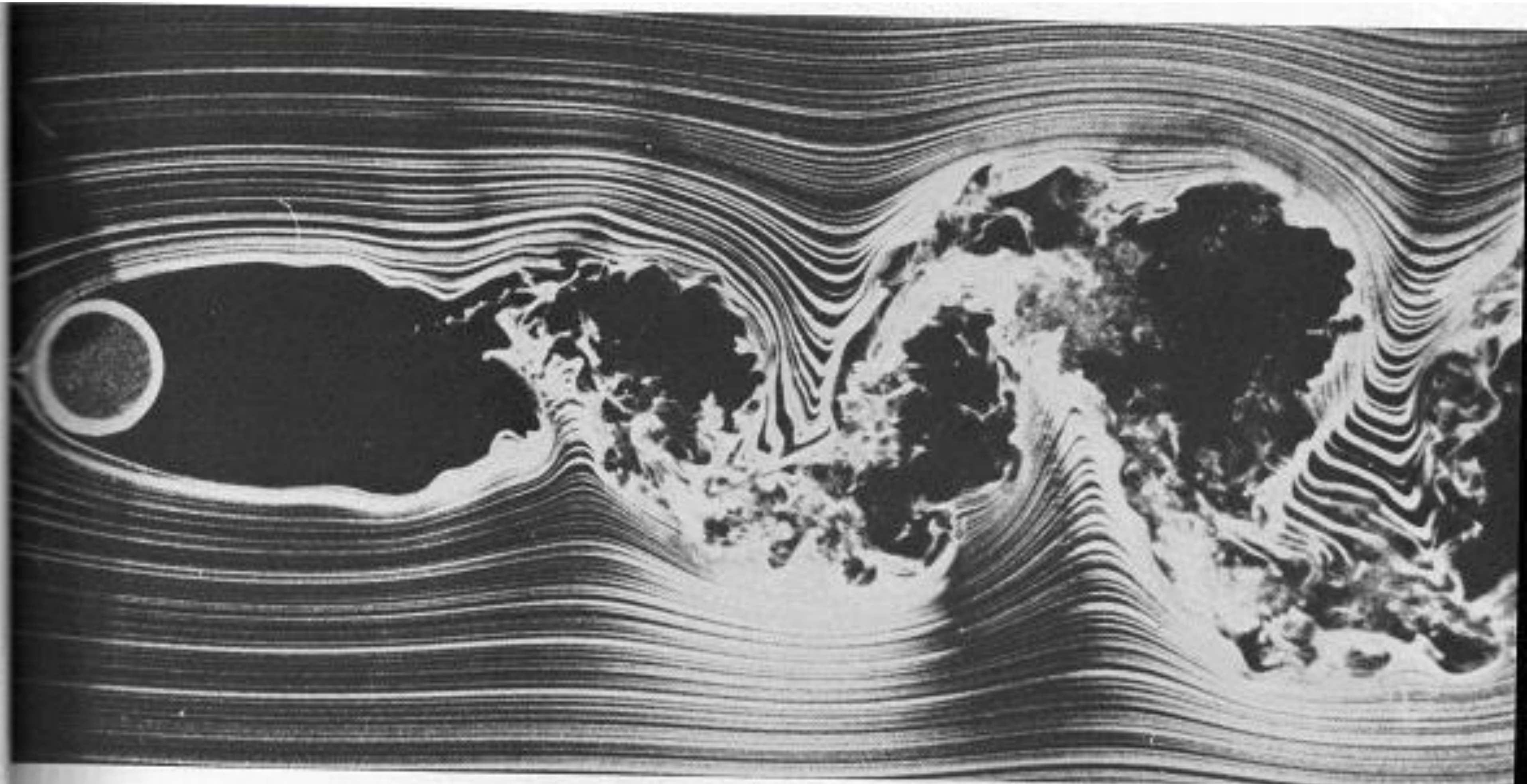






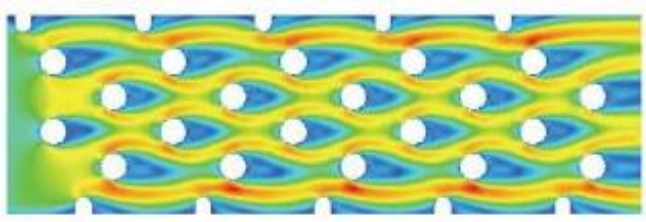

gaussianas

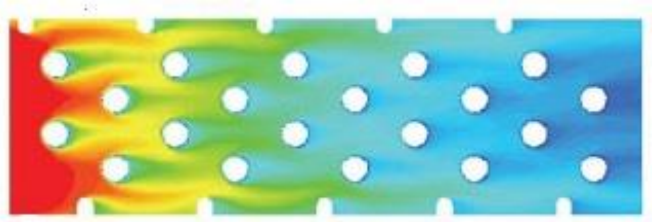
cilindro Re = 10.000

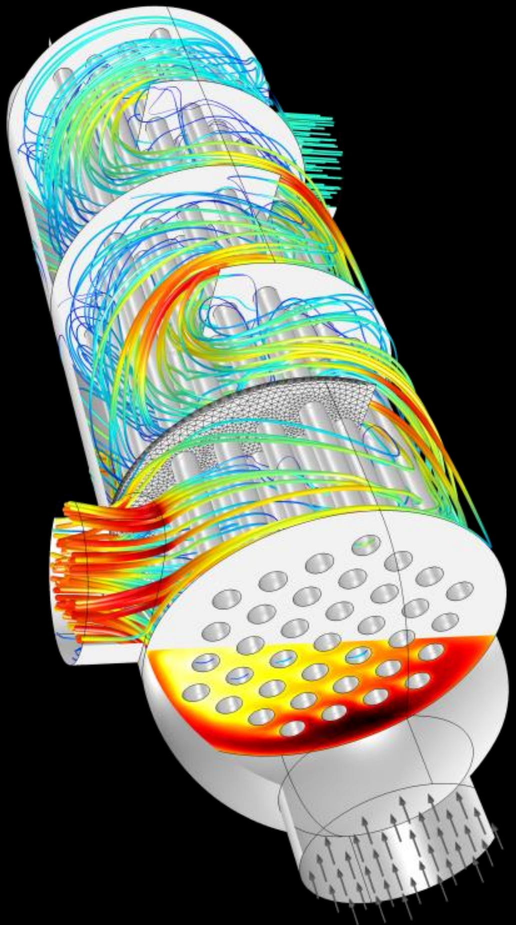


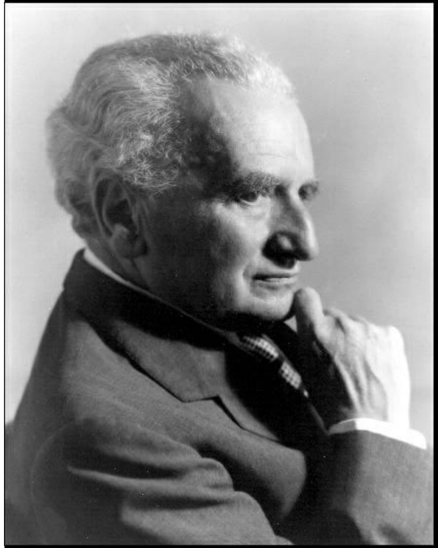
48. Circular cylinder at R**=10,000.** At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

Re 10000

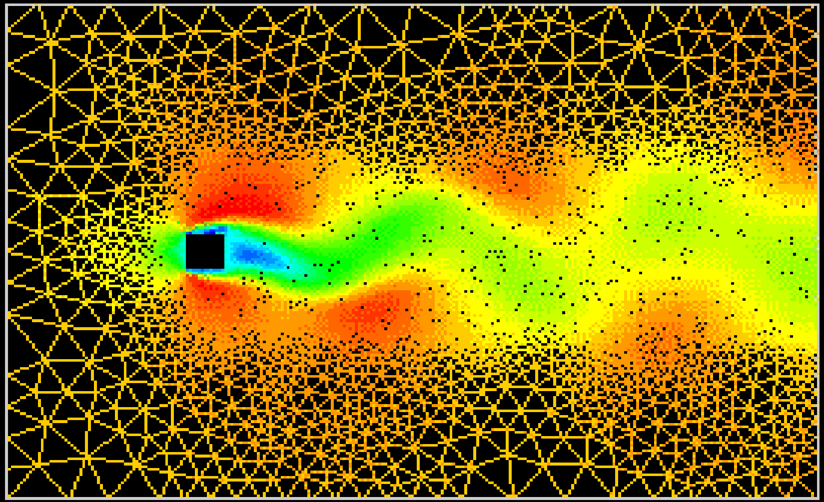






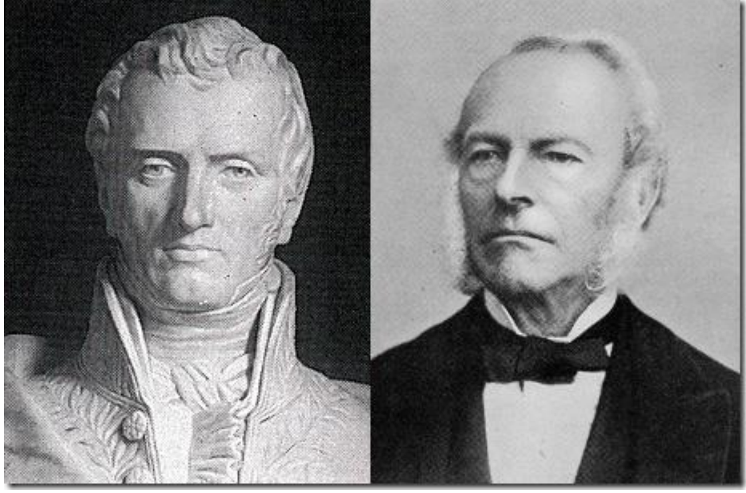


T. von Kármán
1881-1963



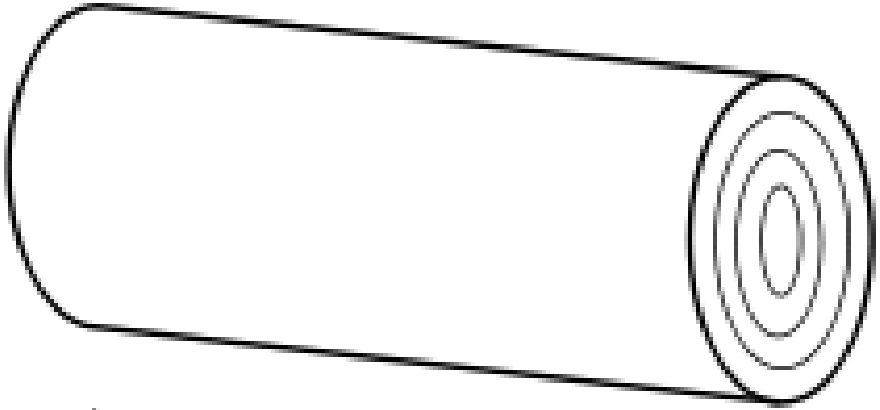
$$\underbrace{\frac{\partial(\rho\vec{v})}{\partial t}}_{\text{transiente}} = \underbrace{-\mathrm{div}(\rho\vec{v}\vec{v})}_{\text{convecção}} - \underbrace{\mathrm{div}\vec{\vec{\tau}}}_{\substack{\text{força}\\ \text{contato}\\ \text{irreversível}}} \underbrace{-\mathrm{gr\vec{a}dp}}_{\substack{\text{força}\\ \text{contato}\\ \text{reversivel}}} + \underbrace{\rho\vec{g}}_{\substack{\text{força}\\ \text{campo}}}$$

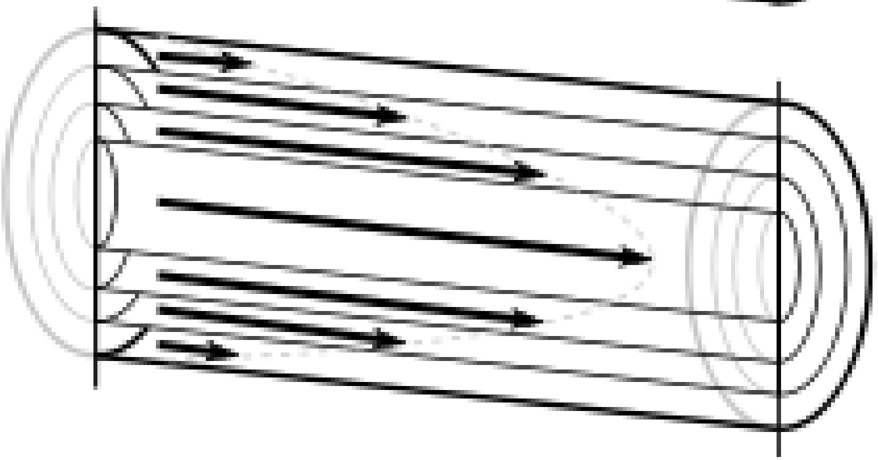
$$\vec{\vec{\tau}} = -\mu\frac{\partial v_z}{\partial r}$$

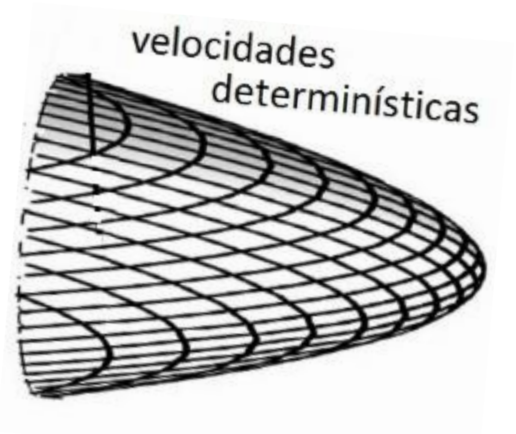


Navier 1785-1836

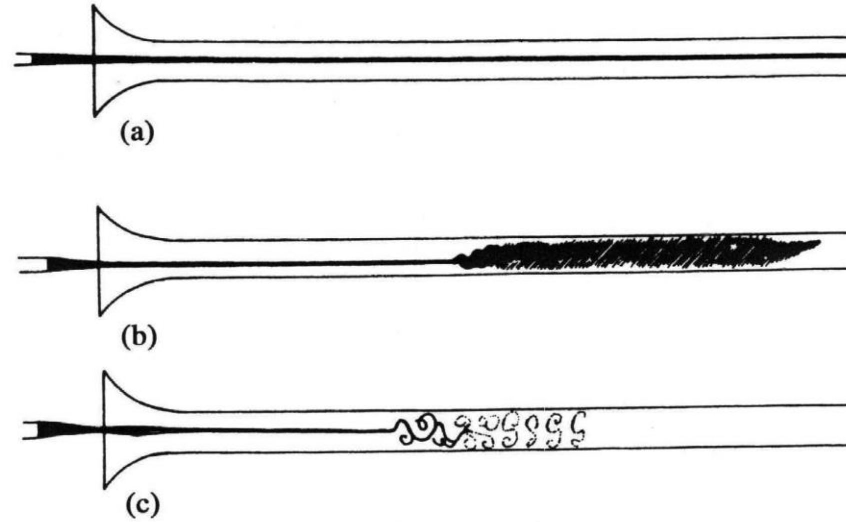
Stokes 1819 -1903



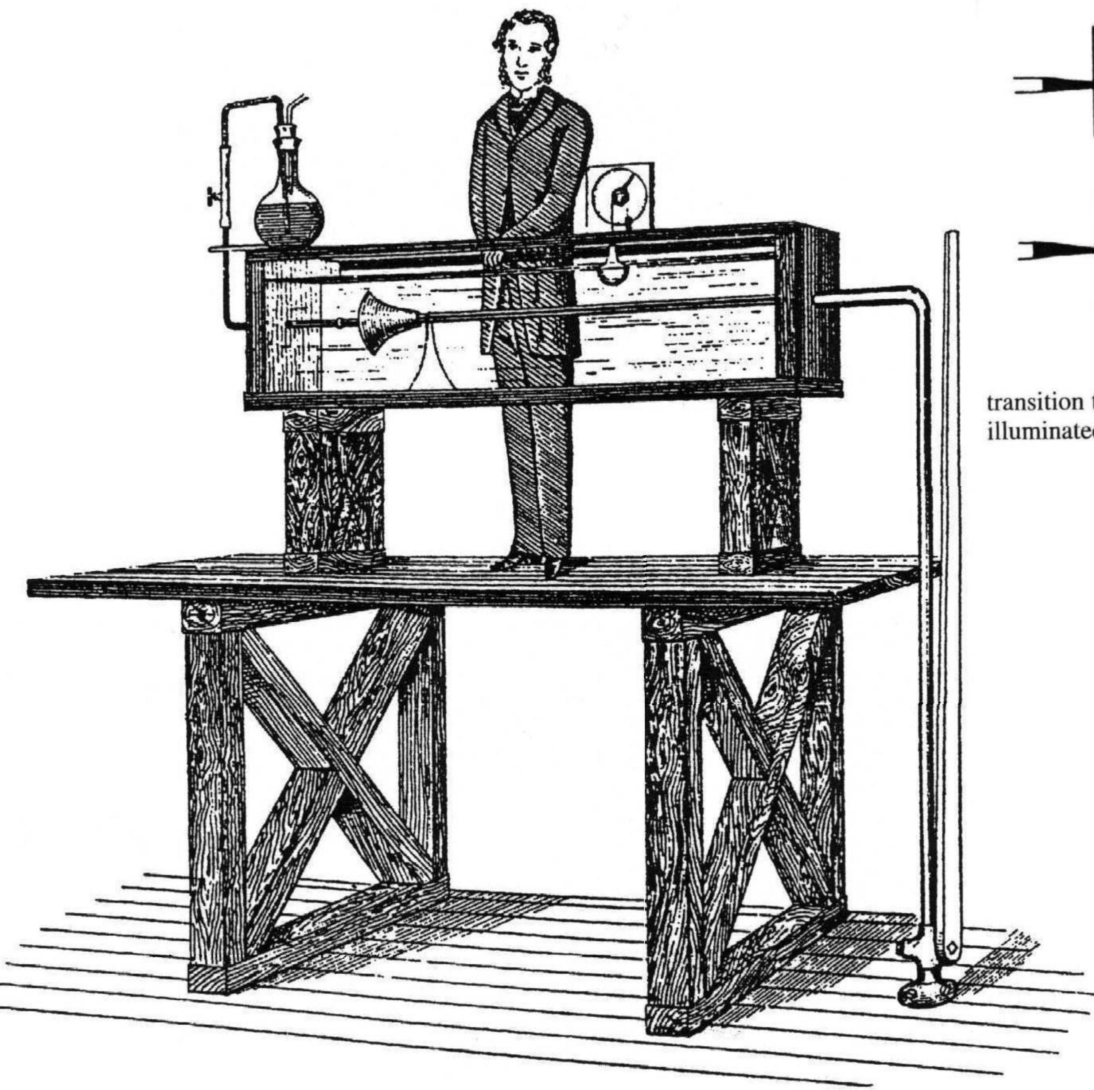




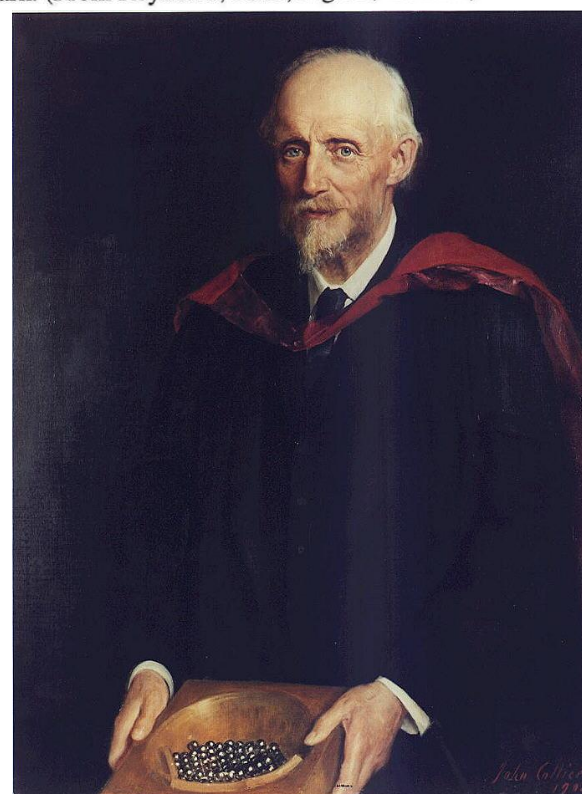
Osborne Reynolds
1842 1912



Sketches of (a) laminar flow in a pipe, indicated by a dye streak; (b) transition to turbulent flow in a pipe; and (c) transition to turbulent flow as seen when illuminated by a spark. (From Reynolds, 1883, Figs. 3, 4 and 5.)



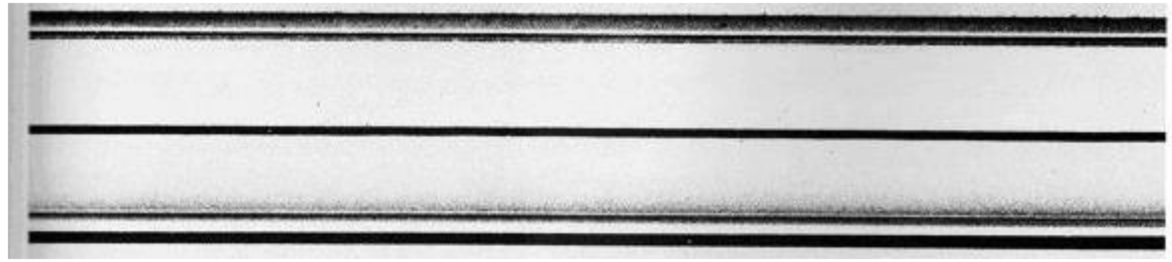
The configuration of Reynolds's experiment on flow along a pipe.



Reynolds

experimento

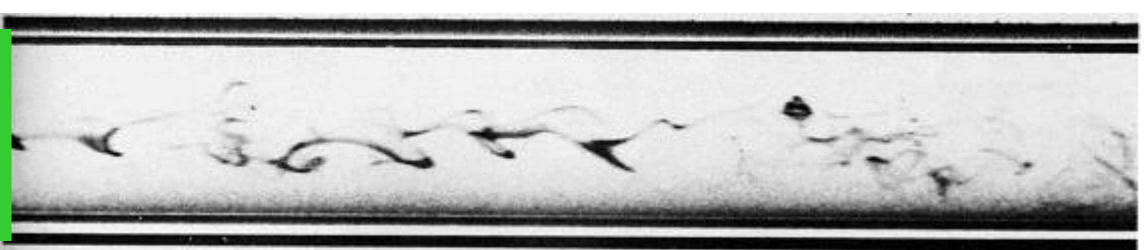
de 1883



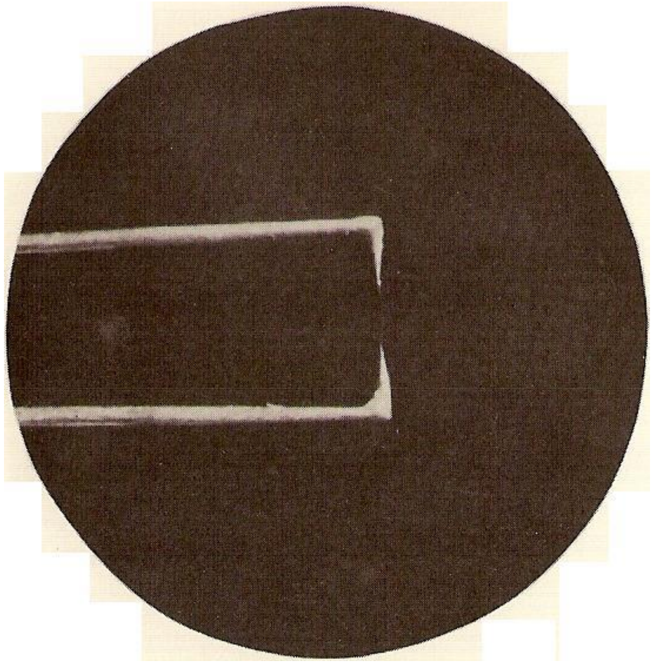
rotacional

3D

irreversível

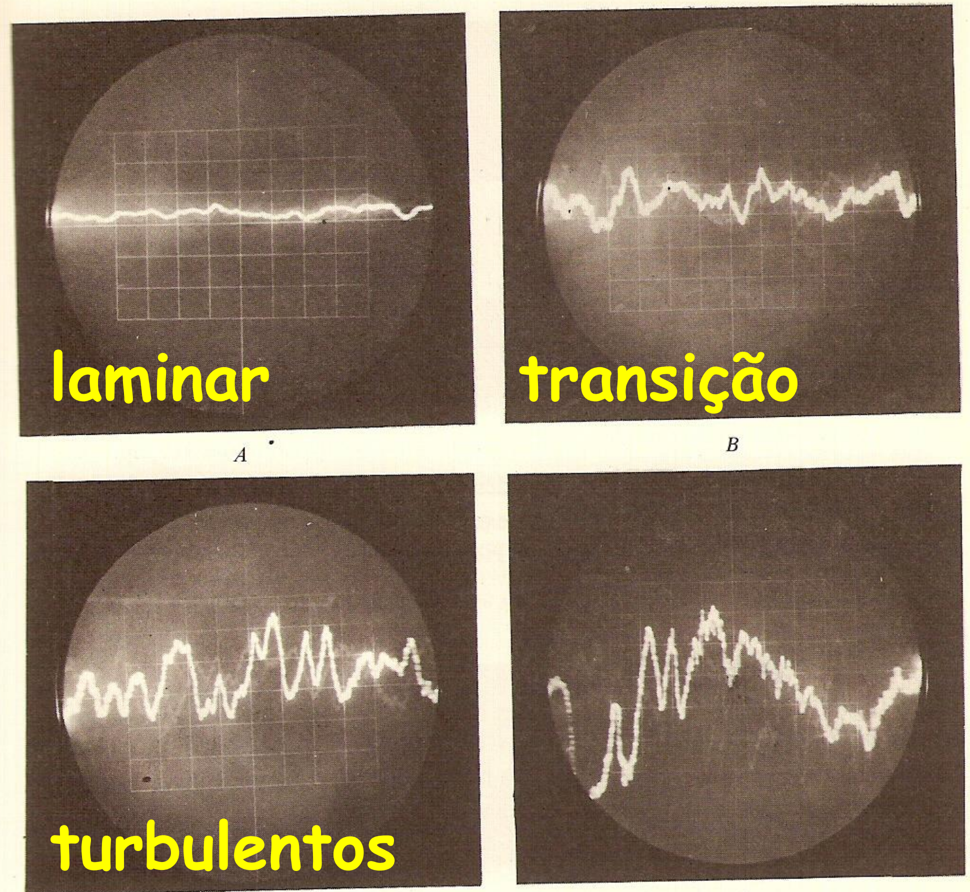


$$\pi = \pi' + \overline{\pi}$$



anemômetro de fio quente

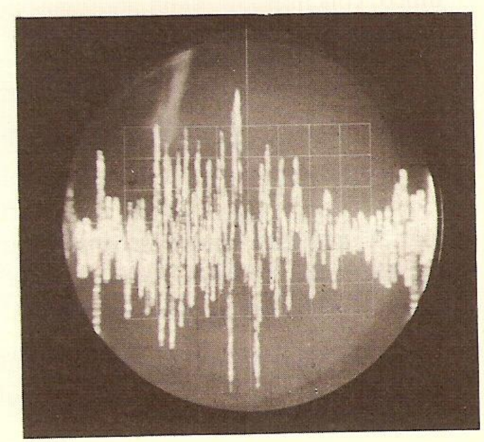
$$\vec{v} = \vec{v}' + \overline{\vec{v}}$$



A B

C D

osciloscópio



E

Fig. 34. Fotografias de flutuações da velocidade de perturbação v' visualizadas na tela de um osciloscópio através da técnica da anemometria de fio quente.

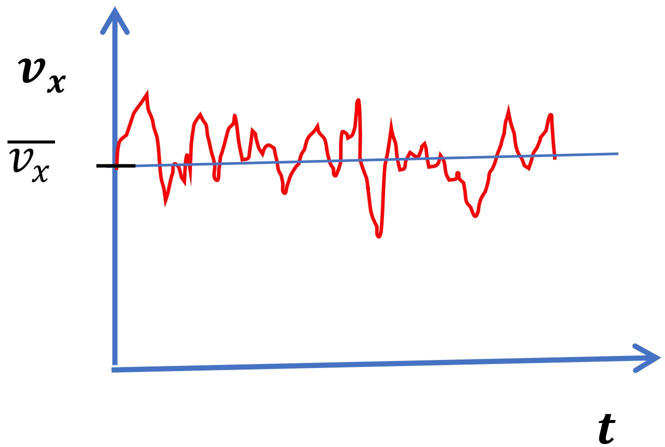
A – regime laminar
B – transição
C, D e E – regime turbulento, com diversas intensidades de turbulência.

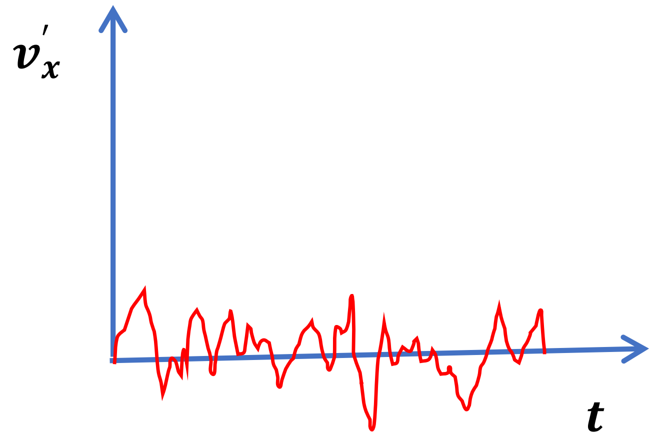
A ordem de grandeza da intensidade de turbulência usual em escoamentos nas aplicações da Engenharia é de 0,05.

Turbulência Estatística – médias temporais

- **Continuidade – incompressível e regime permanente**

$$v_x = \overline{v_x} + v_x'$$





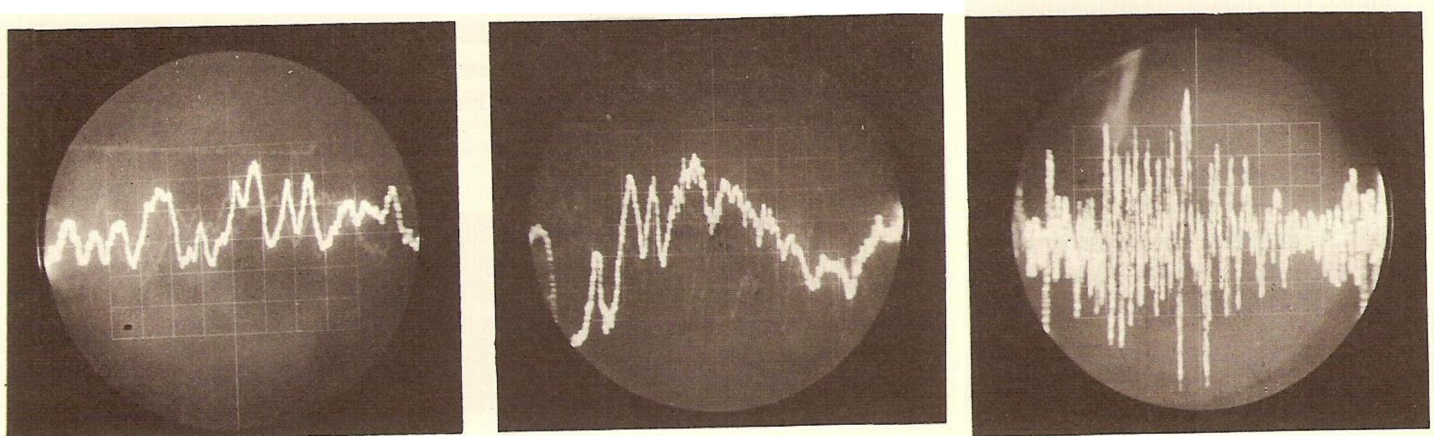


Fig. 34. Fotografias de flutuações da velocidade de perturbação v' visualizadas na tela de um osciloscópio através da técnica da anemometria de fio quente.

A – regime laminar
B – transição
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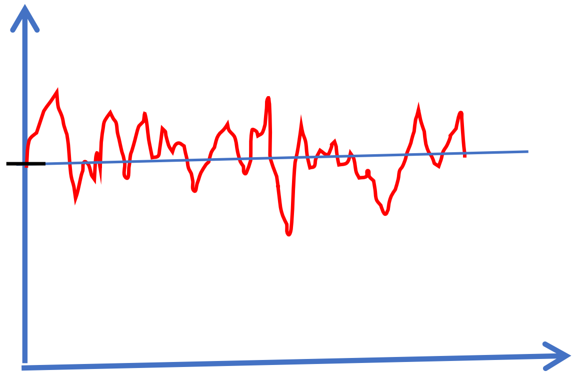
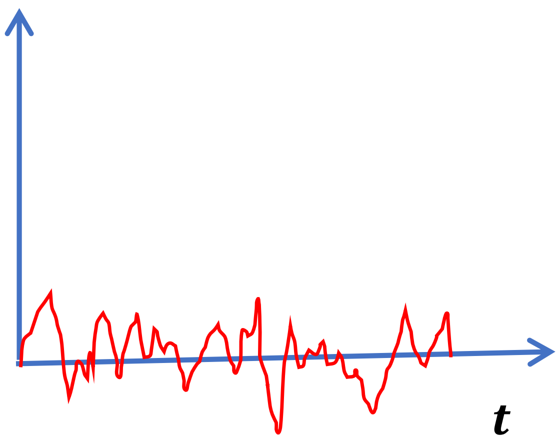
Turbulência Estatística – médias temporais

- **Continuidade – incompressível e regime permanente**

$$v_x = \overline{v_x} + v_x^{'}$$





$$\widetilde{v_x'} = \sqrt{\overline{v_x'^{\,2}}}\,; \qquad \widetilde{v_y'} = \sqrt{\overline{v_y'^{\,2}}}\,; \qquad \widetilde{v_z'} = \sqrt{\overline{v_z'^{\,2}}}\,;$$

$$\widetilde{v'} = \sqrt{\overline{v^2}}$$

Intensidade de turbulência: $I = \widetilde{v'}/\bar{v}\,;$

Energia cinética turbulenta: $k = \dfrac{(v_x'^{\,2} + v_y'^{\,2} + v_z'^{\,2})}{2}$

Turbulência Estatística – médias temporais

- **Continuidade – incompressível e regime permanente**

$$\operatorname{div}\vec{v} = 0 = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

$$v_x = \overline{v_x} + v_x' \;\; ; \quad v_y = \overline{v_y} + v_y' \;\; ; \quad v_z = \overline{v_z} + v_z' \;\; ;$$

$$\overline{v_x'} = \overline{v_y'} = \overline{v_z'} = 0$$

$$\frac{\overline{\partial v_x}}{\partial x} = \frac{\overline{\partial(\overline{v_x} + v_x')}}{\partial x} = \frac{\partial\,\overline{(\overline{v_x} + v_x')}}{\partial x} = \frac{\partial \overline{\overline{v_x}}}{\partial x} + \frac{\partial \overline{v_x'}}{\partial x} = \frac{\partial \overline{v_x}}{\partial x}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \frac{\partial \overline{v_x}}{\partial x} + \frac{\partial v_x'}{\partial x} + \frac{\partial \overline{v_y}}{\partial y} + \frac{\partial v_y'}{\partial y} + \frac{\partial \overline{v_z}}{\partial z} + \frac{\partial v_z'}{\partial z} = 0$$

Turbulência Estatística – médias temporais

- **Continuidade – incompressível e regime permanente**

$$\overline{\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}}=\frac{\partial \overline{v_x}}{\partial x}+\frac{\partial \overline{v_x'}}{\partial x}+\frac{\partial \overline{v_y}}{\partial y}+\frac{\partial \overline{v_y'}}{\partial y}+\frac{\partial \overline{v_z}}{\partial z}+\frac{\partial \overline{v_z'}}{\partial z}=0$$

Efetuando-se a média temporal da continuidade:

$$\overline{\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}}=\frac{\partial \overline{v_x}}{\partial x}+\frac{\partial \overline{v_x'}}{\partial x}+\frac{\partial \overline{v_y}}{\partial y}+\frac{\partial \overline{v_y'}}{\partial y}+\frac{\partial \overline{v_z}}{\partial z}+\frac{\partial \overline{v_z'}}{\partial z}=0$$

$$\boxed{\frac{\partial \overline{v_x}}{\partial x}+\frac{\partial \overline{v_y}}{\partial y}+\frac{\partial \overline{v_z}}{\partial z}=0}$$

Turbulência Estatística – médias temporais

- **Navier-Stokes – direção x – incompressível e regime permanente.**

$$\rho\underbrace{\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)}_{\left(\frac{\partial(v_xv_x)}{\partial x}+\frac{\partial(v_yv_x)}{\partial y}+\frac{\partial(v_zv_x)}{\partial z}\right)}=-\frac{\partial p}{\partial x}+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}+\rho g_x$$

$$\frac{\partial(v_xv_x)}{\partial x}+\frac{\partial(v_yv_x)}{\partial y}+\frac{\partial(v_zv_x)}{\partial z}$$
$$=v_x\frac{\partial v_x}{\partial x}+\underbrace{v_x\frac{\partial v_x}{\partial x}}+v_y\frac{\partial v_x}{\partial y}+\underbrace{v_x\frac{\partial v_y}{\partial y}}+v_z\frac{\partial v_x}{\partial z}+\underbrace{v_x\frac{\partial v_z}{\partial z}}$$

$$\boxed{\frac{\partial(v_xv_x)}{\partial x}+\frac{\partial(v_yv_x)}{\partial y}+\frac{\partial(v_zv_x)}{\partial z}=v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}}$$

Turbulência Estatística – médias temporais

- **Navier-Stokes – direção x – incompressível e regime permanente.**

$$\frac{\partial (v_x v_x)}{\partial x} + \frac{\partial (v_y v_x)}{\partial y} + \frac{\partial (v_z v_x)}{\partial z} = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

Efetuando-se a média temporal do termos:

$$\frac{\partial \overline{(v_x v_x)}}{\partial x} = \frac{\partial \overline{(\overline{v_x} + v_x')^2}}{\partial x} = \frac{\partial \overline{\left(\overline{v_x}^2 + 2\overline{v_x} v_x' + {v_x'}^2\right)}}{\partial x} = \frac{\partial \overline{\overline{v_x}^2}}{\partial x} + 2\frac{\partial \overline{\overline{v_x} v_x'}}{\partial x} + \frac{\partial \overline{{v_x'}^2}}{\partial x} =$$

$$\frac{\partial \overline{\overline{v_x}^2}}{\partial x} + 2\underbrace{\frac{\partial \overline{\overline{v_x} v_x'}}{\partial x}}_{0} + \frac{\partial \overline{{v_x'}^2}}{\partial x} = \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \frac{\partial \overline{{v_x'}^2}}{\partial x}$$

$$\boxed{\frac{\partial \overline{(v_x v_x)}}{\partial x} = \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \frac{\partial \overline{{v_x'}^2}}{\partial x}}$$

Turbulência Estatística – médias temporais

Efetuando-se a média temporal do termos:

$$\frac{\overline{\partial(v_y v_x)}}{\partial y} = \frac{\partial\overline{(\bar{v}_y + v_y')(\bar{v}_x + v_x')}}{\partial y} = \frac{\partial\overline{(\bar{v}_y\,\bar{v}_x + \bar{v}_x v_y' + \bar{v}_y v_x' + v_x' v_y')}}{\partial y}$$

$$\frac{\overline{\partial(v_y v_x)}}{\partial y} = \frac{\partial\overline{(\bar{v}_y\,\bar{v}_x)}}{\partial y} + \underbrace{\frac{\partial\overline{(\bar{v}_x v_y')}}{\partial y}}_{0} + \underbrace{\frac{\partial\overline{\bar{v}_y v_x'}}{\partial y}}_{0} + \frac{\partial\overline{(v_x' v_y')}}{\partial y}$$

$$\boxed{\frac{\overline{\partial(v_y v_x)}}{\partial y} = \bar{v}_y\frac{\partial\bar{v}_x}{\partial y} + \bar{v}_x\frac{\partial\bar{v}_y}{\partial y} + \frac{\partial\overline{v_x' v_y'}}{\partial y}}$$

$$\boxed{\frac{\overline{\partial(v_z v_x)}}{\partial z} = \bar{v}_z\frac{\partial\bar{v}_x}{\partial z} + \bar{v}_x\frac{\partial\bar{v}_z}{\partial z} + \frac{\partial\overline{v_x' v_z'}}{\partial z}}$$

Turbulência Estatística – médias temporais

Substituindo-se no primeiro termo daNavier-Stokes – direção x

$$\boxed{\frac{\partial(v_x v_x)}{\partial x}+\frac{\partial(v_y v_x)}{\partial y}+\frac{\partial(v_z v_x)}{\partial z}=v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}}$$

Efetuando-se a média temporal

$$\boxed{\frac{\partial\overline{(v_x v_x)}}{\partial x}=\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\frac{\partial\overline{v_x'^2}}{\partial x}}$$

$$\boxed{\frac{\partial\overline{(v_z v_x)}}{\partial z}=\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\overline{v_x}\frac{\partial\overline{v_z}}{\partial z}+\frac{\partial\overline{v_x'v_z'}}{\partial z}}$$

$$\boxed{\frac{\partial\overline{(v_y v_x)}}{\partial y}=\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_x}\frac{\partial\overline{v_y}}{\partial y}+\frac{\partial\overline{v_x'v_y'}}{\partial y}}$$

$$\frac{\partial\left(\overline{v_x v_x}\right)}{\partial x}+\frac{\partial\left(\overline{v_y v_x}\right)}{\partial y}+\frac{\partial\left(\overline{v_z v_x}\right)}{\partial z}=$$

$$=\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\frac{\partial\overline{v_x'^2}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_x}\frac{\partial\overline{v_y}}{\partial y}+\frac{\partial\overline{v_x'v_y'}}{\partial z}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\overline{v_x}\frac{\partial\overline{v_z}}{\partial z}+\frac{\partial\overline{v_x'v_z'}}{\partial z}=$$

$$=\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\frac{\partial\overline{v_x'^2}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\frac{\partial\overline{v_x'v_y'}}{\partial z}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\frac{\partial\overline{v_x'v_z'}}{\partial z}=$$

$$\boxed{\frac{\partial\left(\overline{v_x v_x}\right)}{\partial x}+\frac{\partial\left(\overline{v_y v_x}\right)}{\partial y}+\frac{\partial\left(\overline{v_z v_x}\right)}{\partial z}=\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\frac{\partial\overline{v_x'^2}}{\partial x}+\frac{\partial\overline{v_x'v_y'}}{\partial z}+\frac{\partial\overline{v_x'v_z'}}{\partial z}}$$

Turbulência Estatística – médias temporais

$$\boxed{\frac{\partial\left(\overline{v_x v_x}\right)}{\partial x}+\frac{\partial\left(\overline{v_y v_x}\right)}{\partial y}+\frac{\partial\left(\overline{v_z v_x}\right)}{\partial z}=\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\frac{\partial\overline{v_x'^2}}{\partial x}+\frac{\partial\overline{v_x' v_y'}}{\partial z}+\frac{\partial\overline{v_x' v_z'}}{\partial z}}$$

Substituindo-se na Navier-Stokes e efetuando-se a média temporal

$$\rho\left(\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}+\frac{\partial\overline{v_x'^2}}{\partial x}+\frac{\partial\overline{v_x' v_y'}}{\partial z}+\frac{\partial\overline{v_x' v_z'}}{\partial z}\right)=-\frac{\partial\bar{p}}{\partial x}+\frac{\partial\overline{\tau_{xx}}}{\partial x}\frac{\partial\overline{\tau_{xy}}}{\partial y}+\frac{\partial\overline{\tau_{xz}}}{\partial z}+\rho g_x$$

$$\boxed{\begin{aligned}&\rho\left(\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}\right)\\&=-\frac{\partial\bar{p}}{\partial x}+\frac{\partial\left(\overline{\tau_{xx}}-\rho\overline{v_x' v_x'}\right)}{\partial x}+\frac{\partial\left(\overline{\tau_{xy}}-\varrho\overline{v_x' v_y'}\right)}{\partial y}+\frac{\partial\left(\overline{\tau_{xz}}-\rho\overline{v_x' v_z'}\right)}{\partial z}+\rho g_x\end{aligned}}$$

Tensor de Reynolds

$$\boxed{\tau^t=-\rho\begin{bmatrix}\overline{v_x' v_x'} & \overline{v_x' v_y'} & \overline{v_x' v_z'}\\ \overline{v_y' v_x'} & \overline{v_y' v_y'} & \overline{v_y' v_z'}\\ \overline{v_z' v_x'} & \overline{v_z' v_y'} & \overline{v_z' v_z'}\end{bmatrix}}$$

Problema de fechamento - Boussinesq

$$\rho\left(\overline{v_x}\frac{\partial\overline{v_x}}{\partial x}+\overline{v_y}\frac{\partial\overline{v_x}}{\partial y}+\overline{v_z}\frac{\partial\overline{v_x}}{\partial z}\right)$$
$$=-\frac{\partial\overline{p}}{\partial x}+\frac{\partial\left(\overline{\tau_{xx}}-\rho\overline{v_x'v_x'}\right)}{\partial x}+\frac{\partial\left(\overline{\tau_{xy}}-\varrho\overline{v_x'v_y'}\right)}{\partial y}+\frac{\partial\left(\overline{\tau_{xz}}-\rho\overline{v_x'v_z'}\right)}{\partial z}+\rho g_x$$

Tensor de Reynolds

$$\tau^t=-\rho\begin{bmatrix}\overline{v_x'v_x'} & \overline{v_x'v_y'} & \overline{v_x'v_z'}\\ \overline{v_y'v_x'} & \overline{v_y'v_y'} & \overline{v_y'v_z'}\\ \overline{v_z'v_x'} & \overline{v_z'v_y'} & \overline{v_z'v_z'}\end{bmatrix}$$

Boussinesq -

$$\tau^t=\mu_t\frac{d\overline{v_x}}{dy}=\rho\nu_t\frac{d\overline{v_x}}{dy}$$

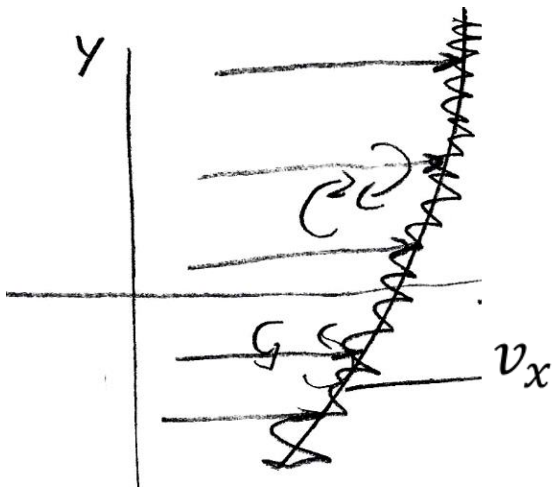
Viscosidade turbulenta (turbilhonar, “eddy”)

Modelos de turbulência

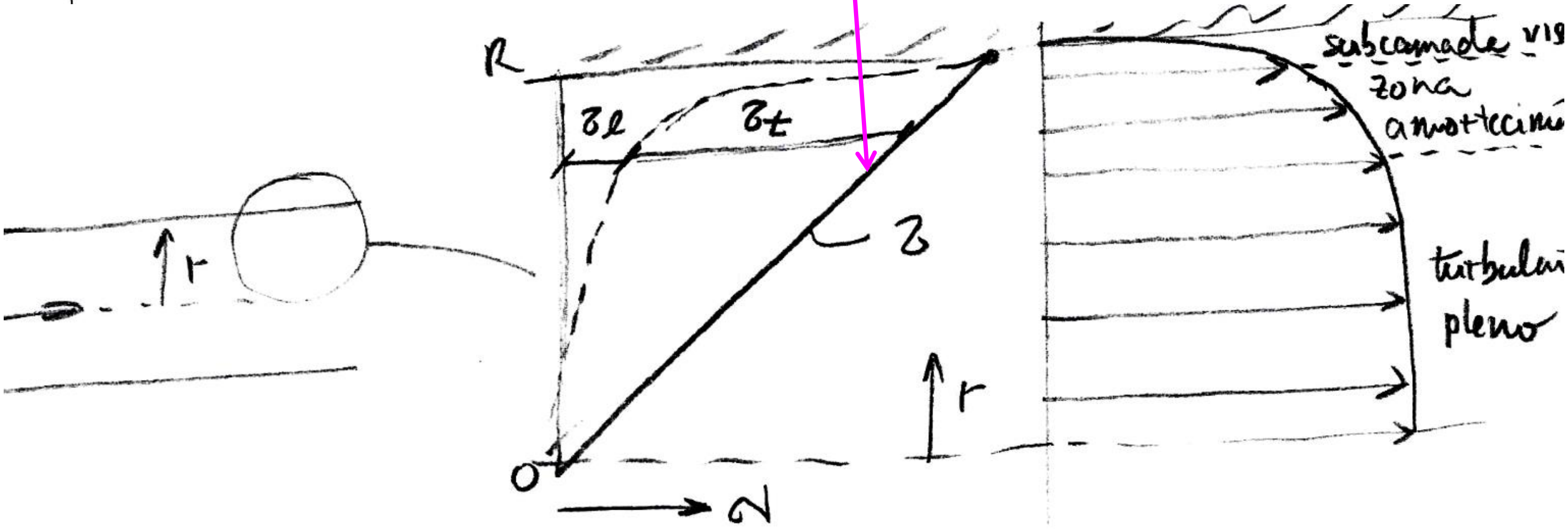
Boussinesq

Tensor tensão

$$\boxed{\tau = \tau^l + \tau^t = \mu_l \frac{d\overline{v_x}}{dy} - \rho\overline{v_x' v_y'}}$$



$$\boxed{\tau = \tau^l + \tau^t = \mu_l \frac{d\overline{v_x}}{dy} - \rho\overline{v_x' v_y'}}$$



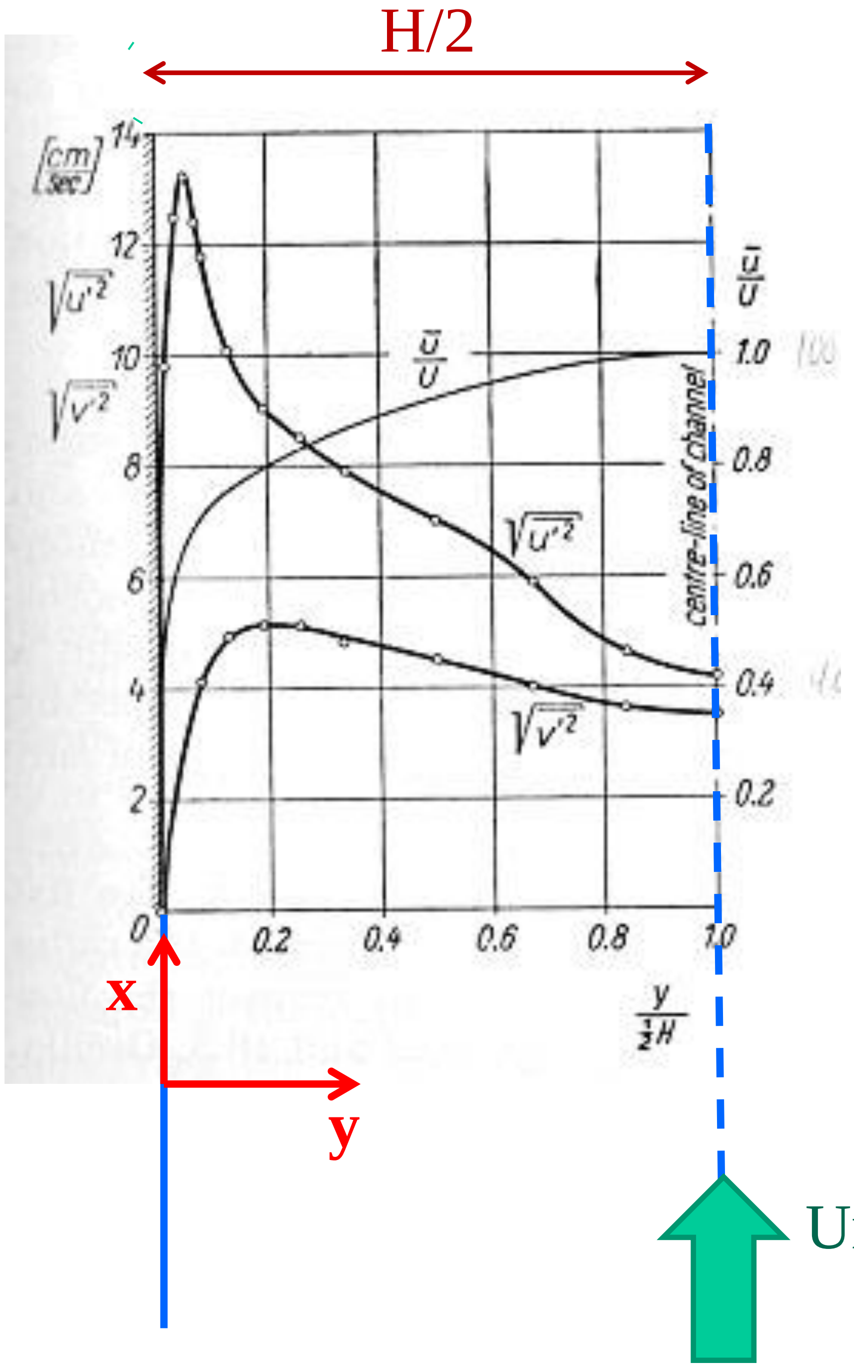


Fig. 18.3. Measurement of fluctuating turbulent components in a wind tunnel, at maximum velocity $U = 100$ cm/sec after Reichardt [41]
Root-mean-square of longitudinal fluctuation $\sqrt{\overline{u'^2}}$, transverse fluctuation $\sqrt{\overline{v'^2}}$, mean velocity $\bar{u}$

$$u = v_x$$
$$u' = v_x'$$
$$v = v_y$$
$$v' = v_y'$$

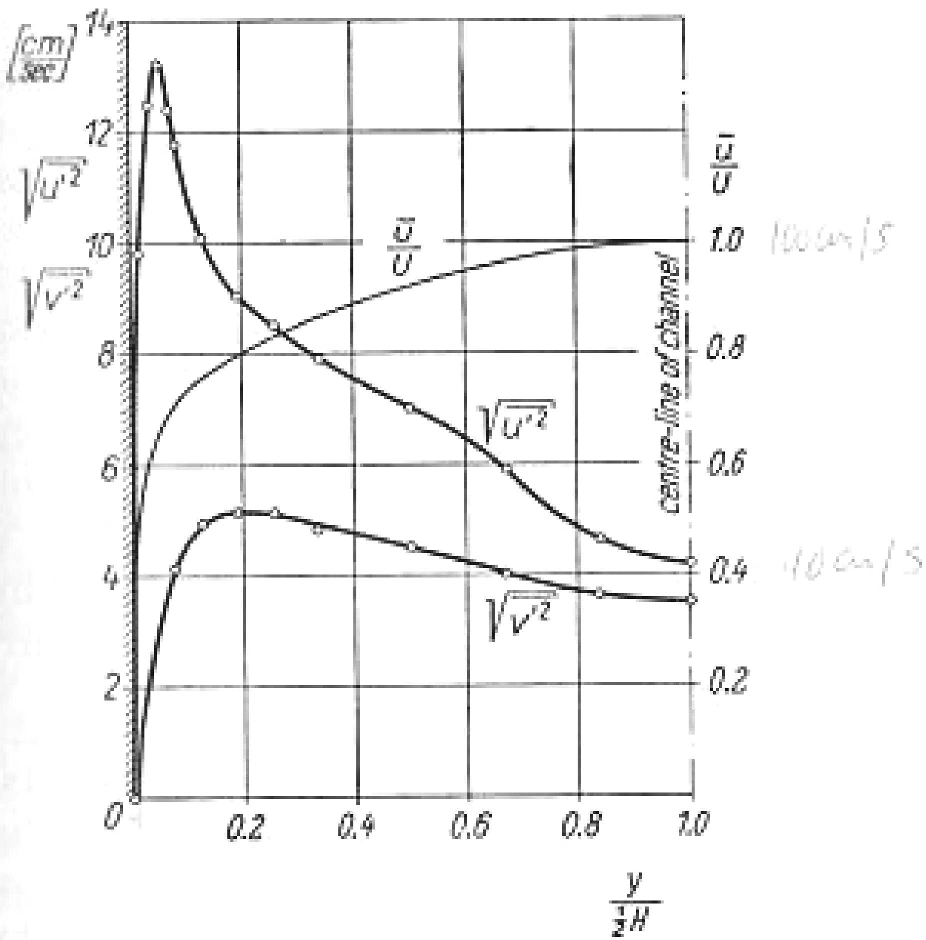


Fig. 18.3. Measurement of fluctuating turbulent components in a wind tunnel, at maximum velocity $U = 100$ cm/sec after Reichardt [41]

Root-mean-square of longitudinal fluctuation $\sqrt{\overline{u'^2}}$, transverse fluctuation $\sqrt{\overline{v'^2}}$, mean velocity $\bar{u}$

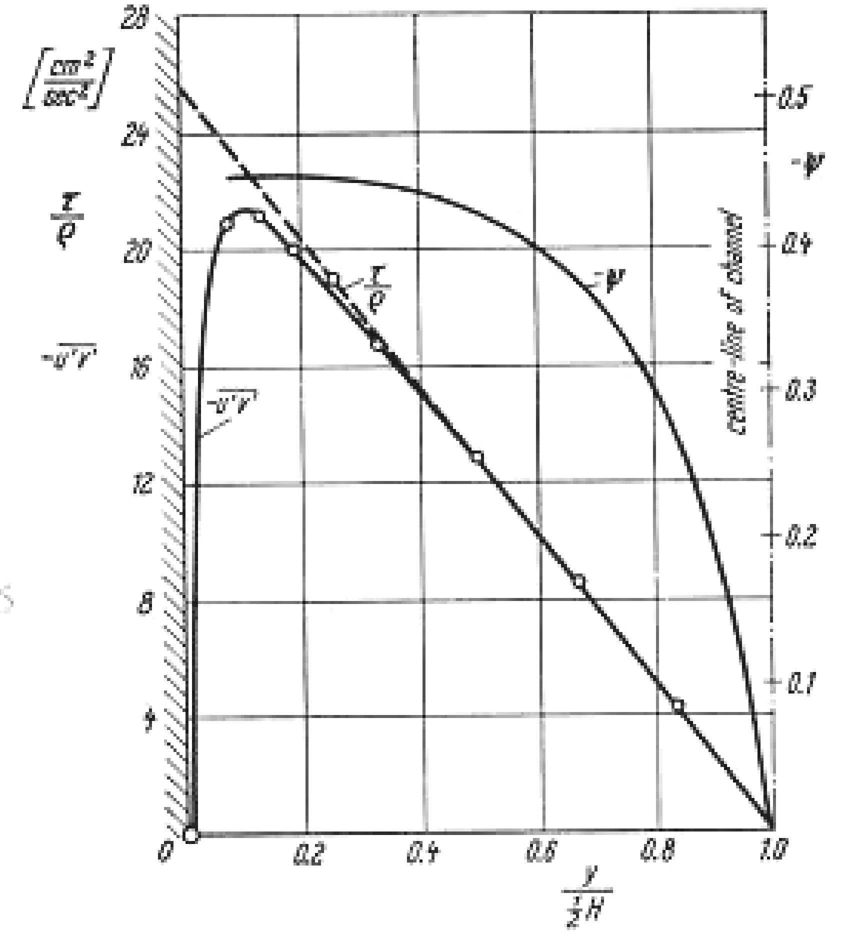


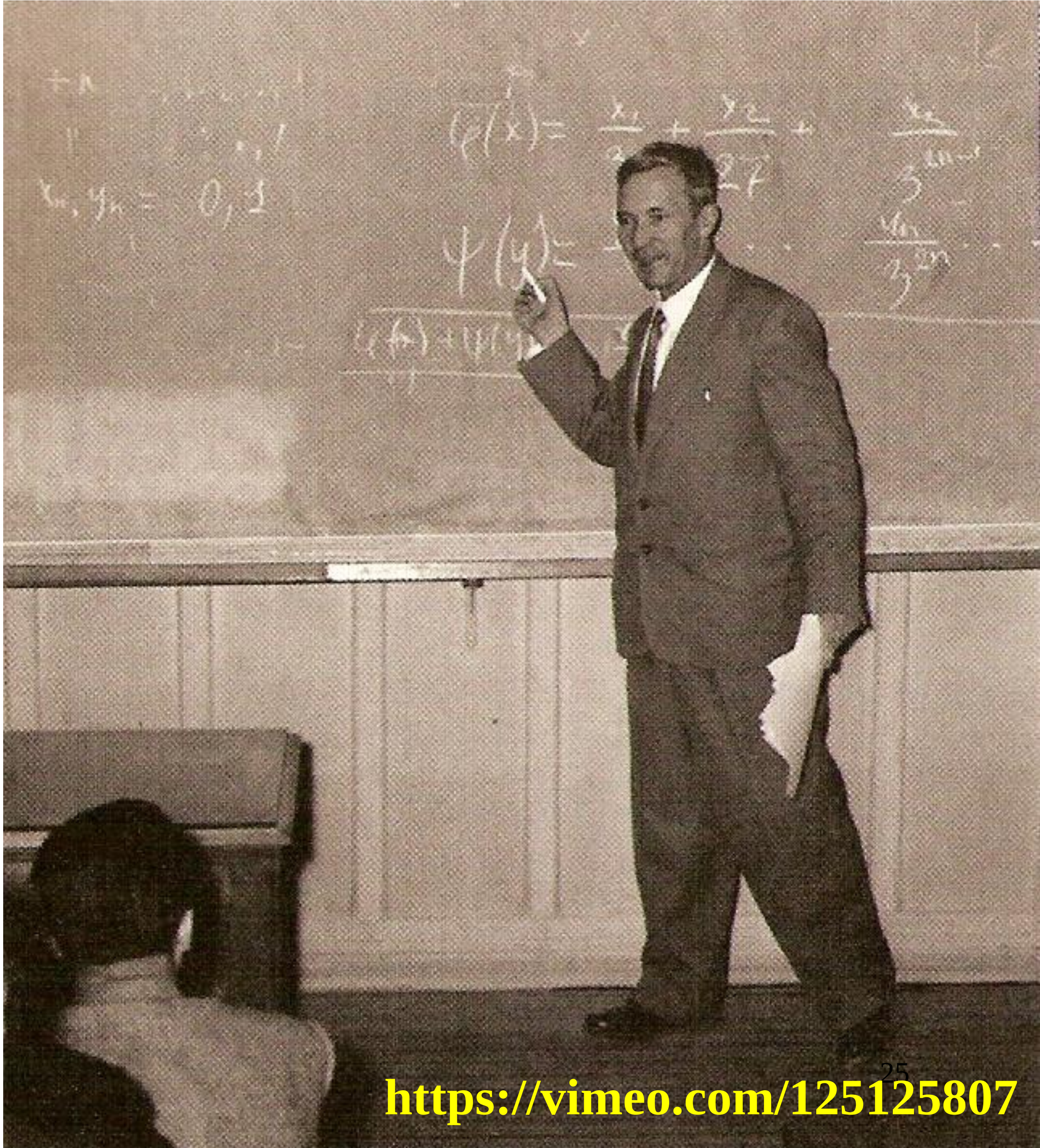
Fig. 18.4. Measurement of fluctuating components in a channel, after Reichardt [41]

The product $\overline{u'v'}$, the shearing stress τ/ϱ, and the correlation coefficient ψ

Intensidade de Turbulência

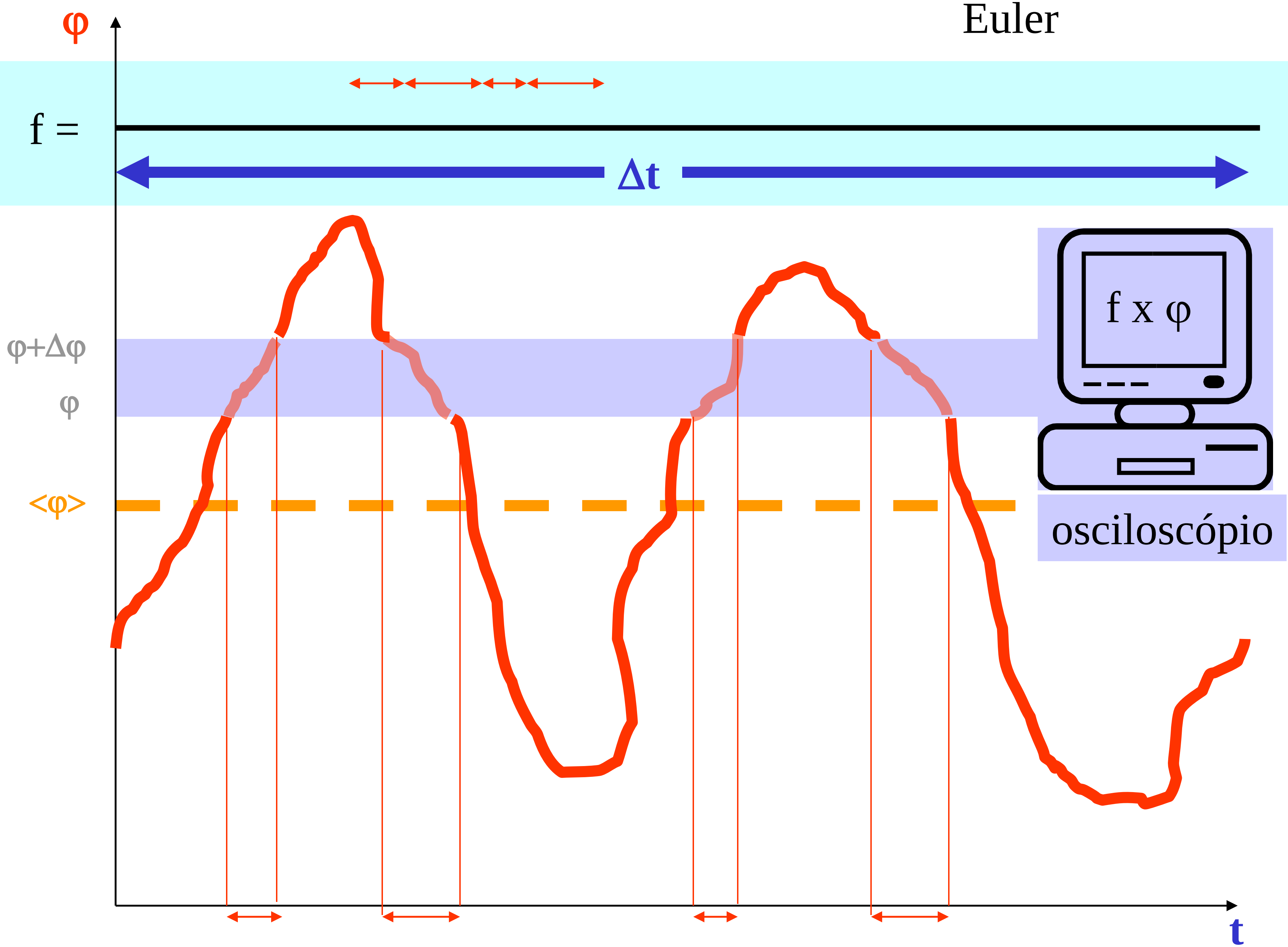
Kolmogorov
1903 -1987

“meu interesse pelo estudo dos escoamentos turbulentos surgiu no fim dos anos 30. Pareceu-me evidente que a técnica matemática principal deveria ser a teoria das **funções aleatórias de diversas variáveis**, que estava então nascendo.”



https://vimeo.com/125125807

Euler



medidas

$$v = \langle v \rangle + v'$$

Δv

v'

$\langle v \rangle$

Δv

v'

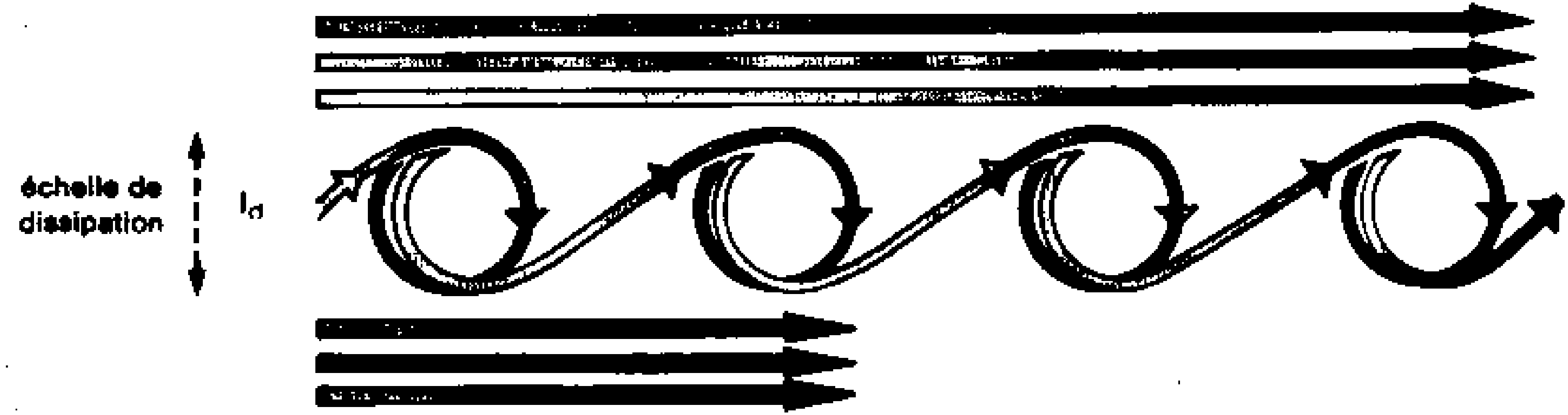
$\langle v \rangle$

$$f_v = f_v(t, r)$$

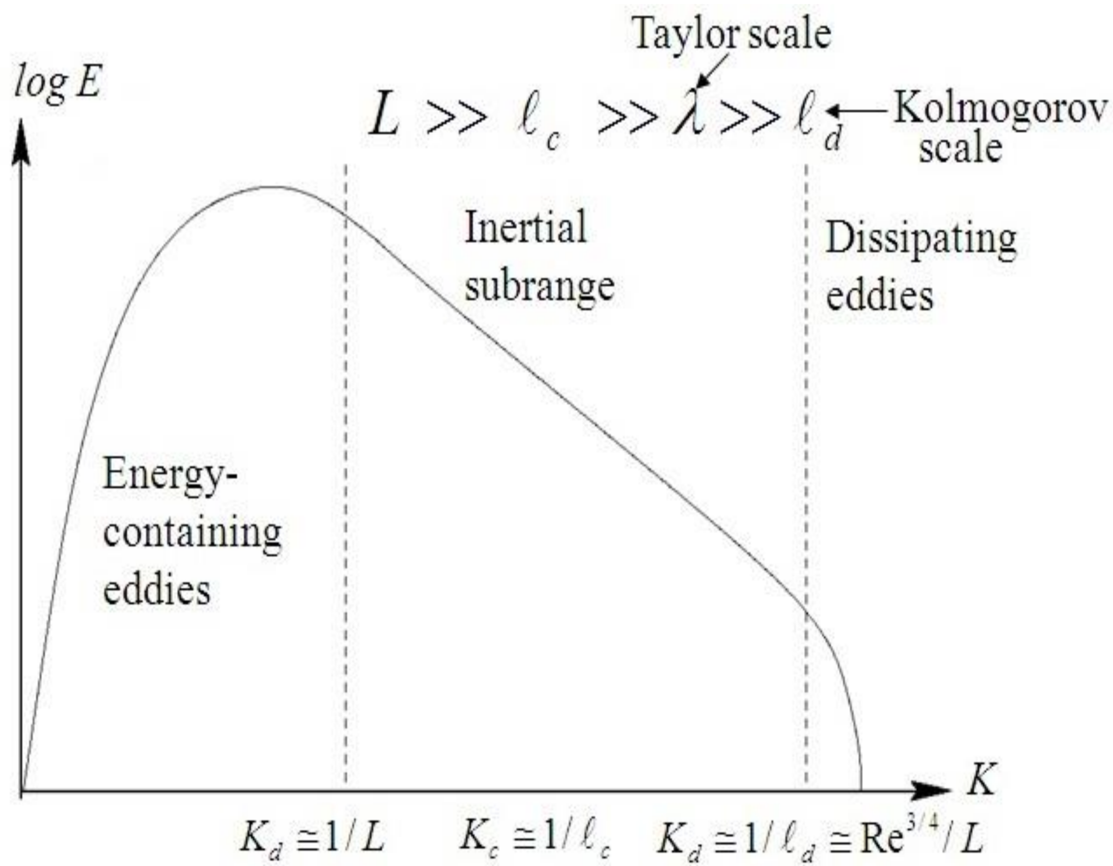
$$f_v = f_v[\, v \,,\, v + \Delta v \,]$$

f_v = Probality Density Function = PDF

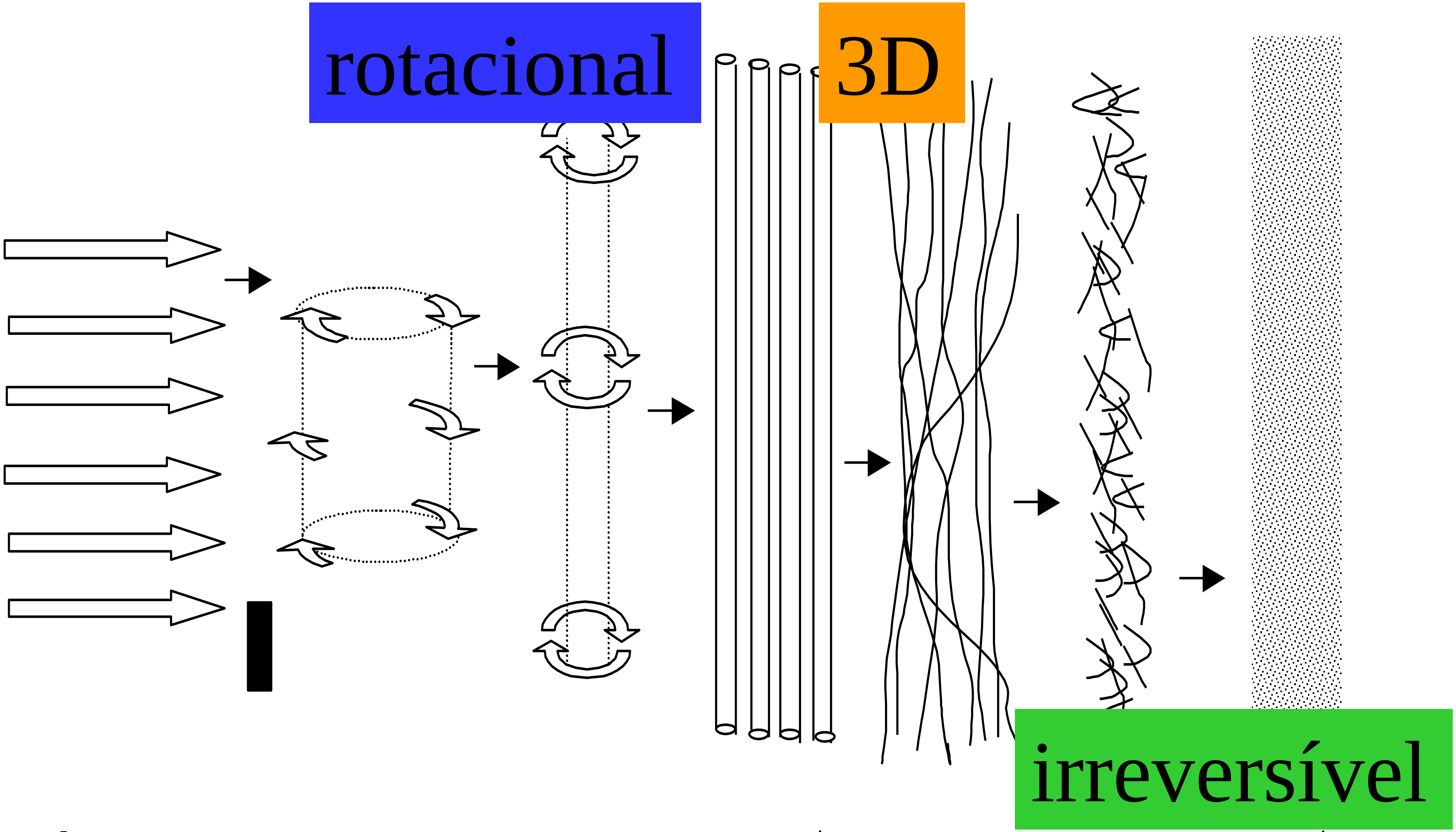
foco



échelle de dissipation
l_d

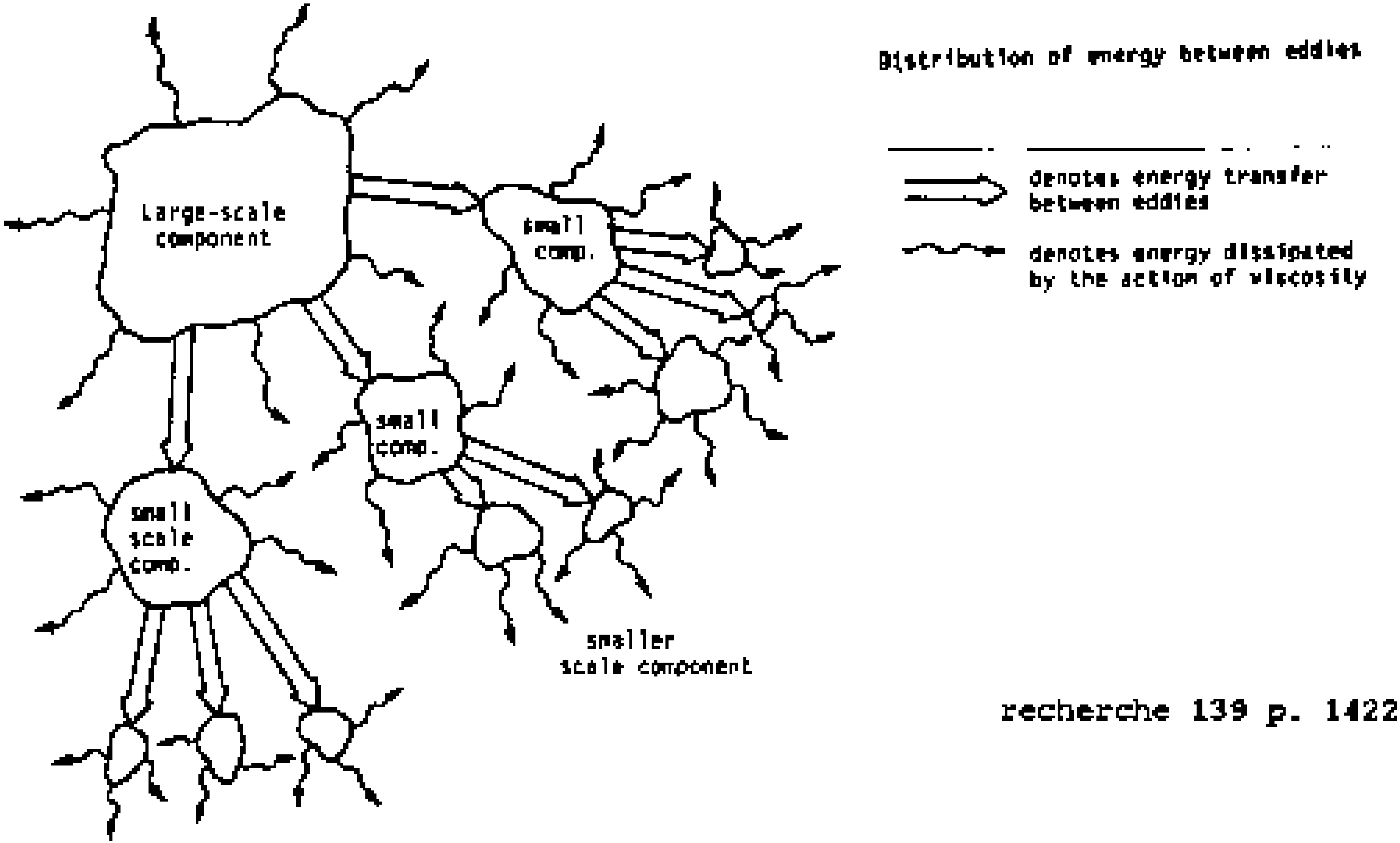


Taylor scale
$log\,E$
$L >> \ell_c >> \lambda >> \ell_d$
Kolmogorov scale
Inertial subrange
Dissipating eddies
Energy-containing eddies
K
$K_d \cong 1/L$
$K_c \cong 1/\ell_c$
$K_d \cong 1/\ell_d \cong \mathrm{Re}^{3/4}/L$



escala:	macro	transição	micro
energia cinética	macro	turbulenta	micro
proporcional:	**v**	←--------------- k -----------------→	T

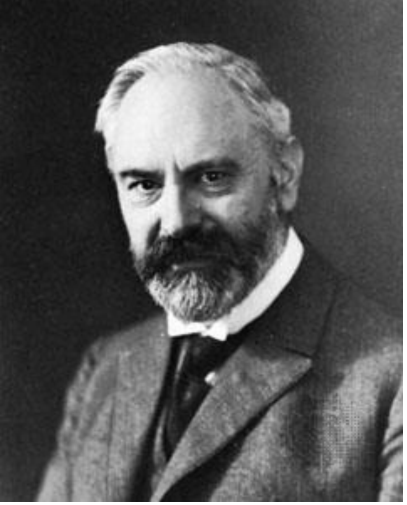
escalas de turbulências


Distribution of energy between eddies
denotes energy transfer between eddies
denotes energy dissipated by the action of viscosity
Large-scale component
small comp.
small comp.
small scale comp.
smaller scale component
recherche 139 p. 1422

Kolmogorov

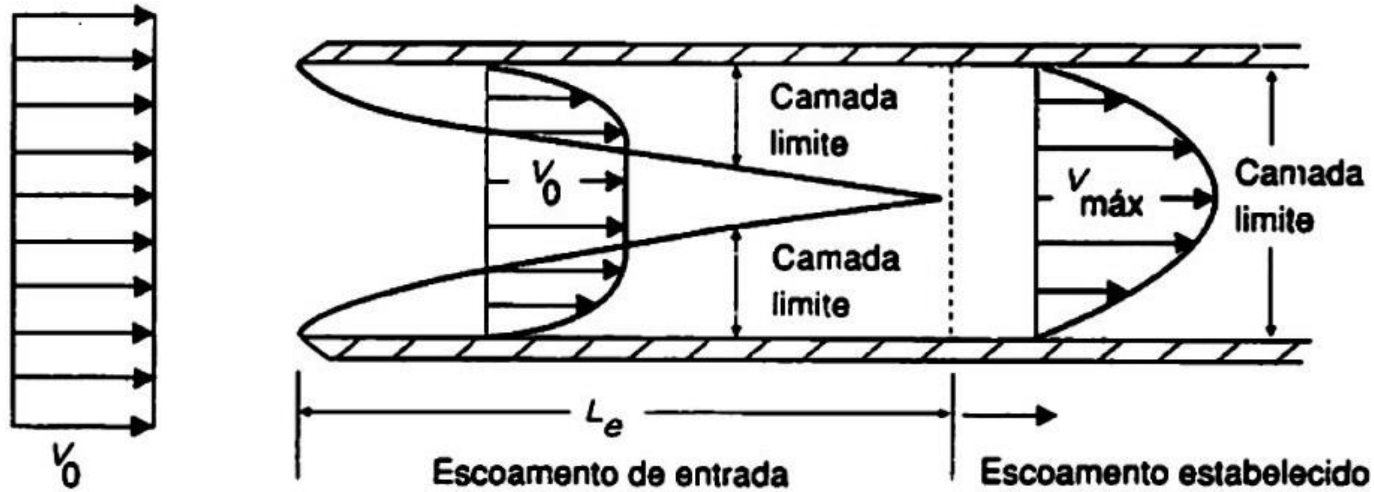
Escalas de Kolmogorov – menores escalas de turbulência

- Comprimento $$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \qquad \frac{\eta}{l_0} = \mathrm{Re}^{-3/4}$$
- Velocidade $$u_\eta = (\nu\varepsilon)^{1/4} \qquad \frac{u_\eta}{u_0} = \mathrm{Re}^{-1/4}$$
- Tempo $$\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2} \qquad \frac{\tau_\eta}{\tau_0} = \mathrm{Re}^{-1/2}$$
- Reynolds $$\mathrm{Re}_\eta = \frac{u_\eta \eta}{\nu} = 1$$



Prandtl
1875 –1953

$$\frac{\partial \vec{\bar{v}}}{\partial t} + \operatorname{div} \vec{\bar{v}}\,\vec{\bar{v}} = \operatorname{div}\left[\mu + \mu_T\right] \operatorname{gr\vec{a}d} \vec{\bar{v}} + \vec{\bar{g}}$$



$$\mu_T = {l_m}^2 gr\vec{a}d\vec{\bar{v}}$$

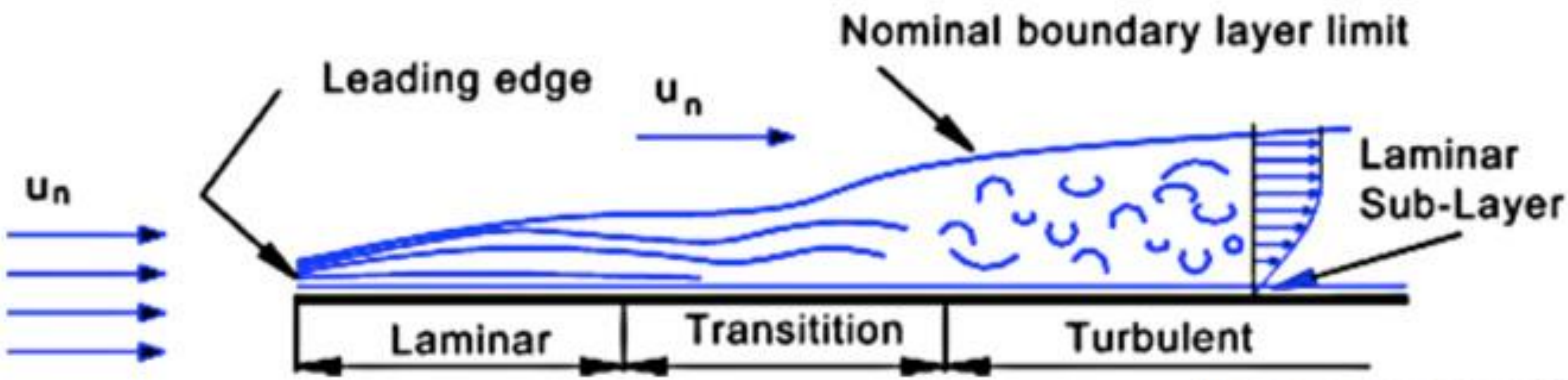
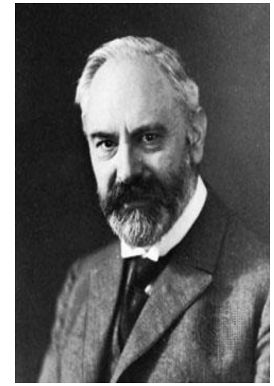


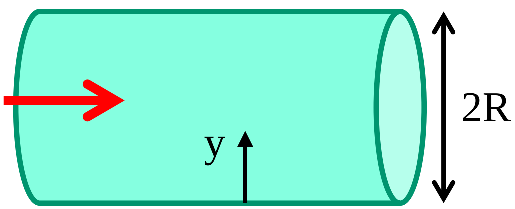
Image: www.roymech.co.uk/

Modelo – Comprimento de mistura de Prandtl



Prandtl
1875 –
1953

$$\mu_t = \rho {l_m}^2 \left| \frac{d\overline{v_x}}{dy} \right|$$



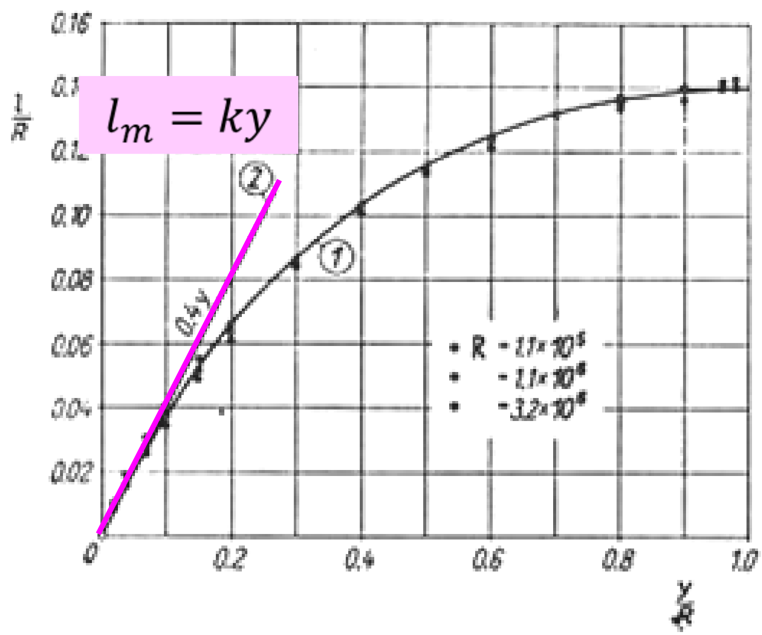
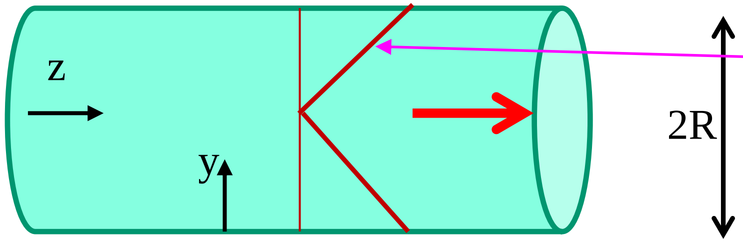


Fig. 20.5. Variation of mixing length over pipe diameter for smooth pipes at different Reynolds numbers

hipótese

Modelo – Comprimento de mistura de Prandtl

Distribuição Universal de Velocidades



$$\tau = \tau_w \left(1 - \frac{y}{R}\right)$$

$$\tau = \frac{r}{2}\left(\frac{dP}{dz}\right)$$

Velocidade de atrito, "drift velocity"

$$\boxed{v^* = \sqrt{\tau_w / \rho}}$$

Subcamada viscosa

$$\tau \cong \tau_w = \mu \frac{d\overline{v_z}}{dy} \rightarrow \overline{v_x} = \frac{\tau_w}{\mu} y = \frac{\tau_w}{\rho}\frac{y}{\nu} \rightarrow \overline{v_x} = v^{*2}\frac{y}{\nu}$$

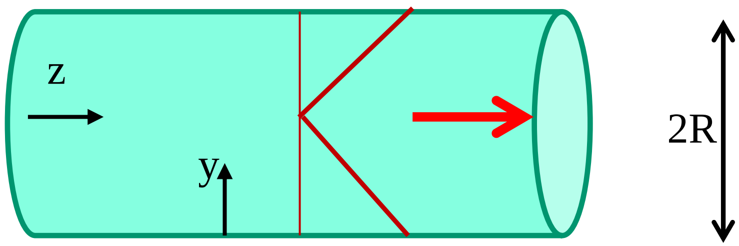
$$\frac{\overline{v_x}}{v^*} = v^* \frac{y}{\nu} \rightarrow \bar{v}^+ = y^+$$

$$\boxed{\bar{v}^+ = \frac{\overline{v_z}}{v^*}} \qquad \boxed{y^+ = \frac{y v^*}{\upsilon}}$$

hipótese

Modelo – Comprimento de mistura de Prandtl

Distribuição Universal de Velocidades

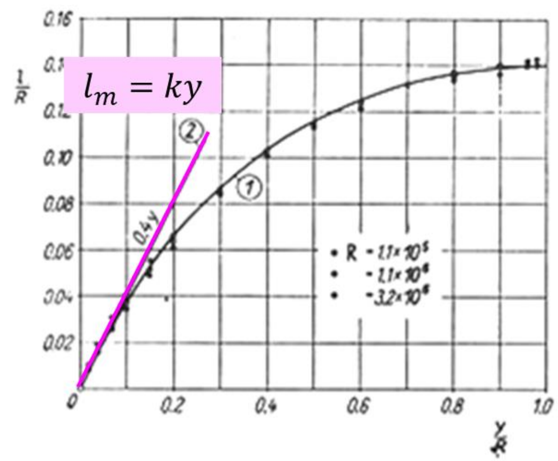


Fora da subcamada

$$\tau \cong \tau_w \quad \text{e} \quad l_m = ky$$

$$\tau_w = \mu_t \frac{d\overline{v_z}}{dy} = \rho \nu_t \frac{d\overline{v_z}}{dy} \qquad \mu_t = \rho {l_m}^2 \left| \frac{d\overline{v_z}}{dy} \right|$$

$$\tau_w = \rho {l_m}^2 \left(\frac{d\overline{v_z}}{dy} \right)^2 = \quad \tau_w = \rho (ky)^2 \left(\frac{d\overline{v_z}}{dy} \right)^2$$



$$\sqrt{\frac{\tau_w}{\rho}} = v^* = ky \left(\frac{d\overline{v_z}}{dy} \right) \rightarrow \quad v^* \frac{dy}{y} = k d\overline{v_z}$$

$$\overline{v_z} = v^* \frac{1}{k} ln y + Cte \quad \rightarrow \quad \frac{\overline{v_z}}{v^*} = \quad \overline{v}^+ = \frac{1}{k} ln y + C$$

ótese

Turbulento

$$\tau = \tau^l + \tau^t = \mu_l \frac{d\bar{v}_z}{dy} - \rho\overline{v_r' v_z'}$$

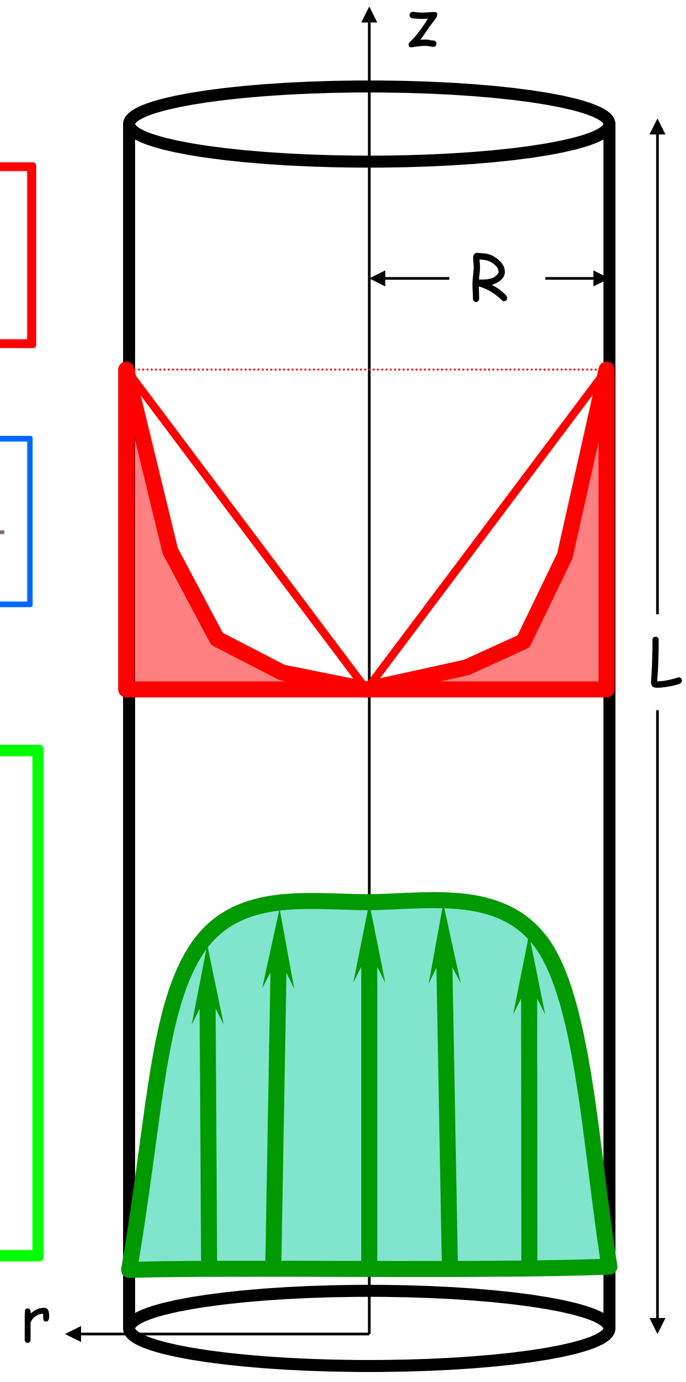
Velocidade de atrito,
"drift velocity"

$$v^* = \sqrt{\tau_w/\rho}$$

$$\bar{v}^+ = \frac{\bar{v}_z}{v^*}$$

$$y^+ = \frac{y v^*}{\upsilon}$$

$0 > y^+ > 5$	$\bar{v}^+ = y^+$
$5 > y^+ > 30$	$\bar{v}^+ = 5\, ln\, y^+ - 3{,}05$
$y^+ > 30$	$\bar{v}^+ = 2{,}5\, ln\, y^+ + 5{,}5$



Comprimento de Mistura - Prandtl

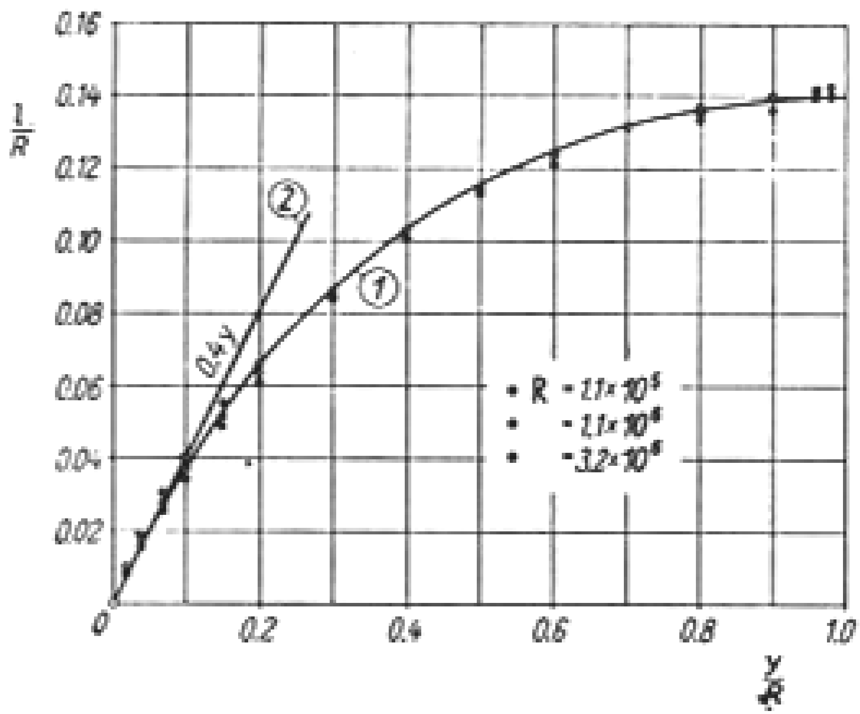


Fig. 20.5. Variation of mixing length over pipe diameter for smooth pipes at different Reynolds numbers

Curve (1) from eqn. (20.18)

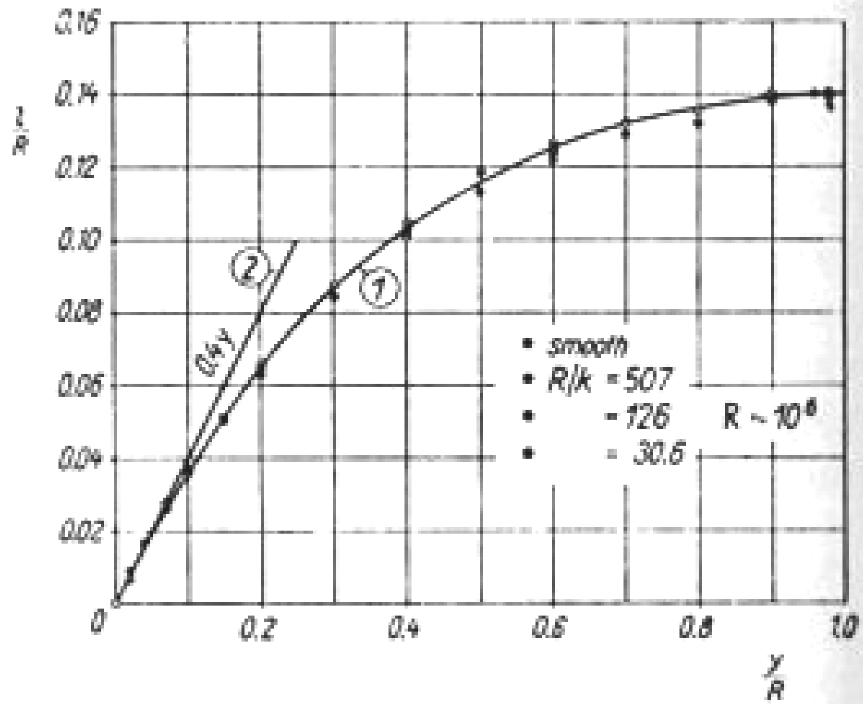


Fig. 20.6. Variation of mixing length over pipe diameter for rough pipes

Curve (1) from eqn. (20.18)

Comprimento

Comprimento de Mistura - Prandtl

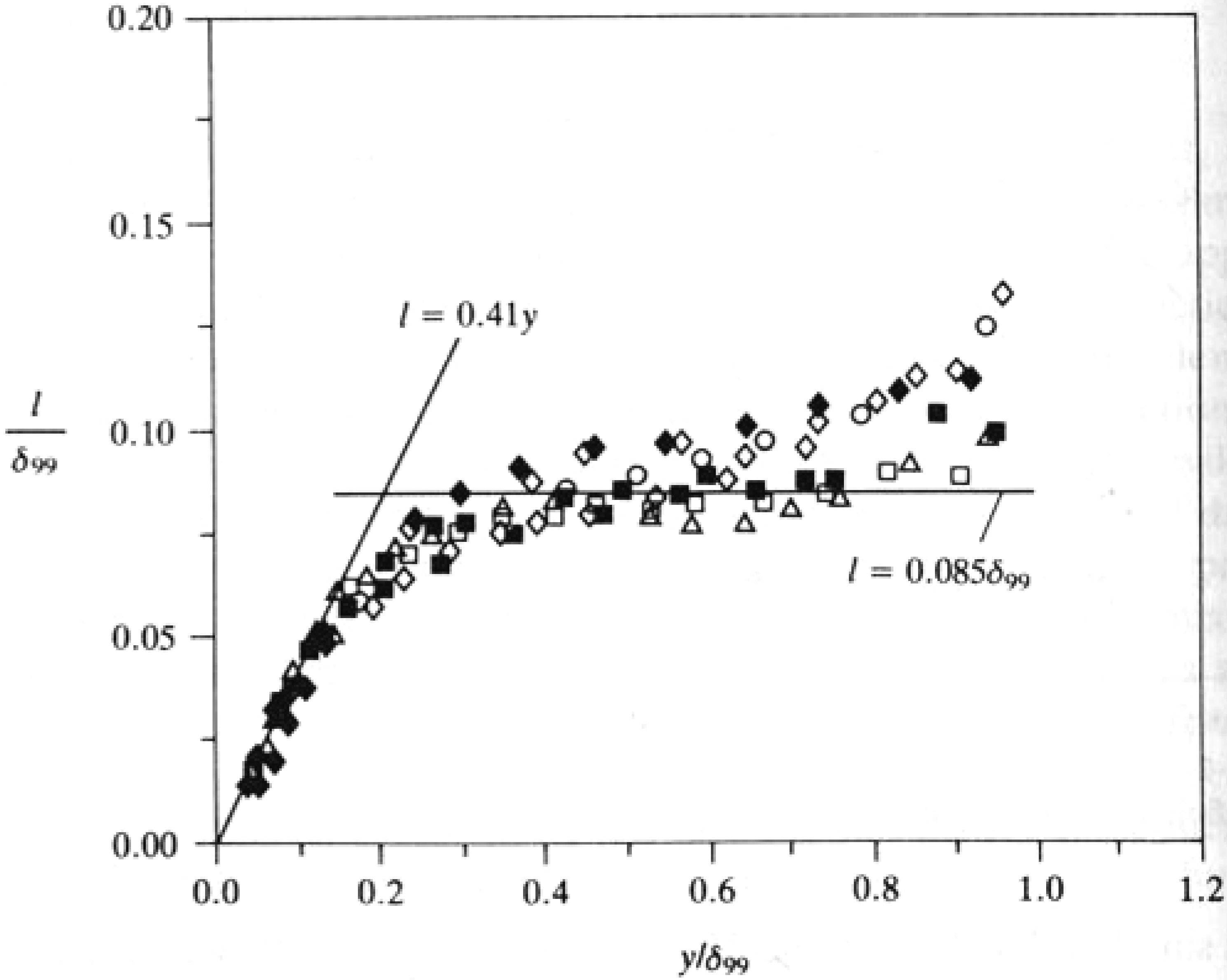


FIGURE 11-2
Mixing-length measurements of Andersen[1] for no pressure gradient, advers gradient, blowing, and suction.

$$\boxed{v_z = \frac{\Delta p R^2}{2\mu L}\left[1-\left(\frac{r}{R}\right)^2\right]}$$

$$\boxed{\frac{v_z}{v_o} = 1-\left(\frac{r}{R}\right)^2}$$

v_{bulk} = $\dot{m} v_b = \rho v_b \pi R^2 = \int_0^R \rho v_z 2\pi r\, dr$

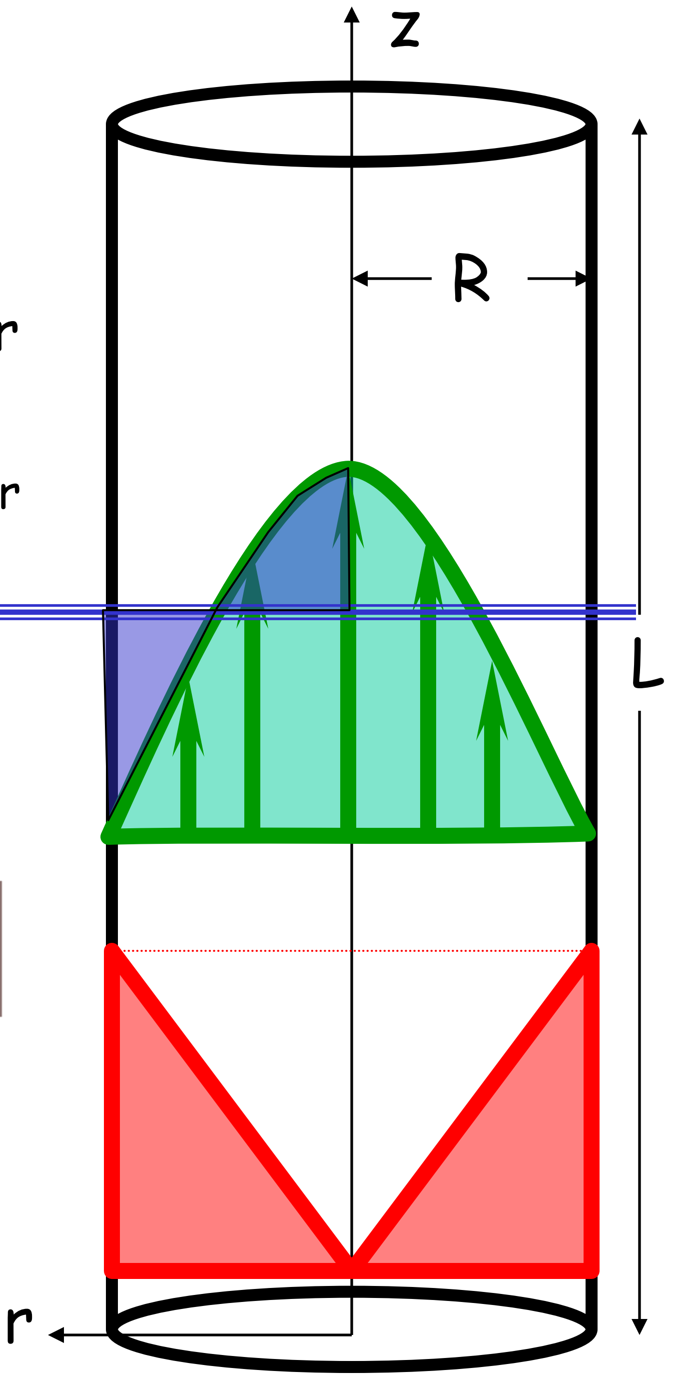
$$v_b = \frac{\rho 2\pi}{\rho \pi R^2}\int_0^R v_z\, r\, dr = \frac{2}{R^2}\frac{\Delta p R^2}{2\mu L}\int_0^R \left(1-\left(\frac{r}{R}\right)^2\right) r\, dr$$

$$\boxed{v_b = \frac{R^2 \Delta p}{4\mu L}}$$

tensão

$$\tau_{rz} = -\mu\frac{\partial v_z}{\partial r} = -\mu\frac{\partial}{\partial r}\left[\frac{\Delta p}{2\mu L}(R^2 - r^2)\right]$$

$$\tau_{rz} = \frac{\Delta p}{L} r \qquad \tau_w = \frac{\Delta p}{L} R$$



Perfil Universal de Velocidades

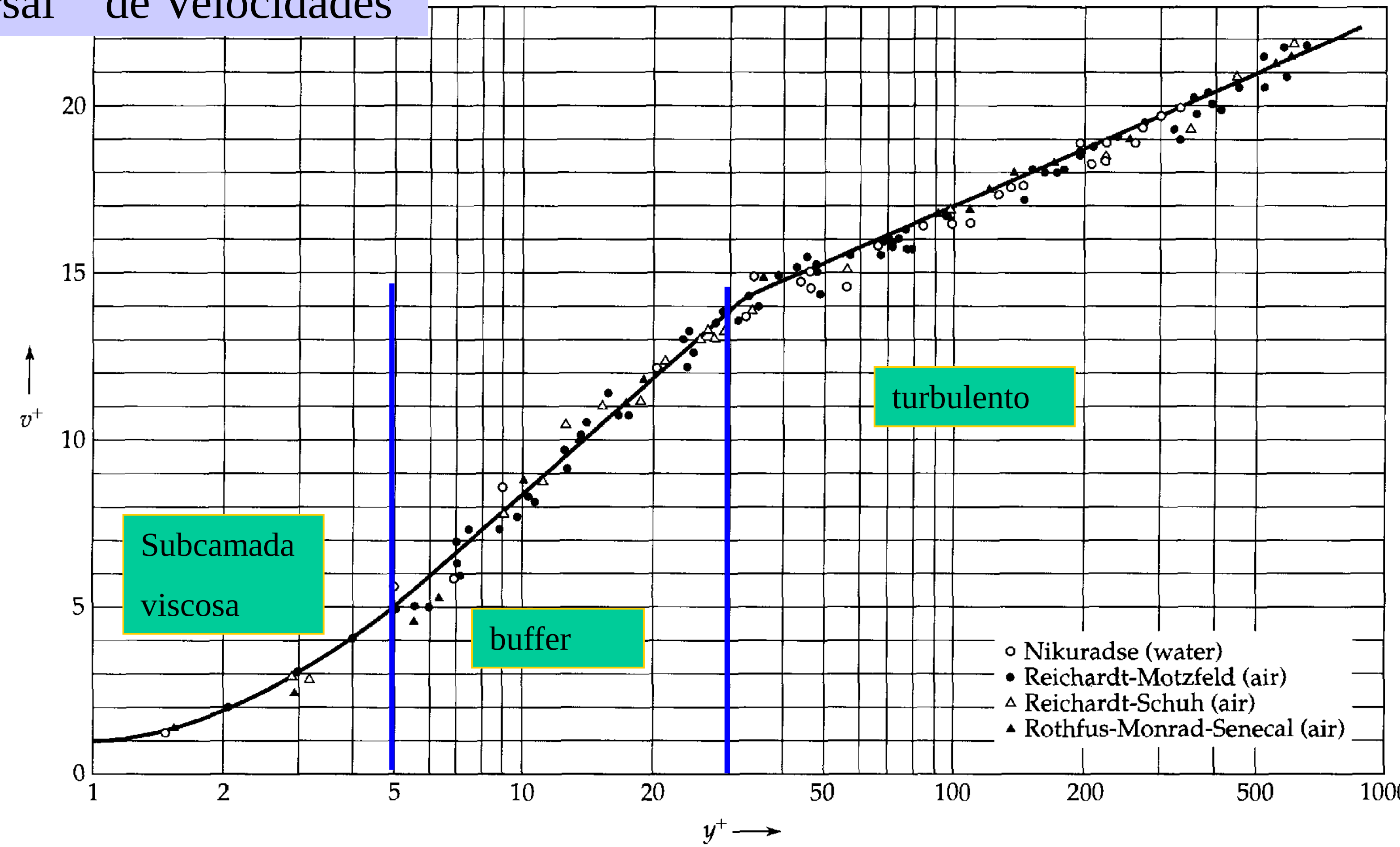


Fig. 5.5-3. Dimensionless velocity distribution for turbulent flow in circular tubes, presented as $v^+ = \bar{v}_z/v_*$ vs. $y^+ = yv_*\rho/\mu$, where $v_* = \sqrt{\tau_0/\rho}$ and τ_0 is the wall shear stress. The solid curves are those suggested by Lin, Moulton, and Putnam [*Ind. Eng. Chem.*, **45**, 636–640 (1953)]:

$$0 < y^+ < 5: \qquad v^+ = y^+[1 - \tfrac{1}{4}(y^+/14.5)^3]$$

$$5 < y^+ < 30: \qquad v^+ = 5\ln(y^+ + 0.205) - 3.27$$

$$30 < y^+: \qquad v^+ = 2.5\ln y^+ + 5.5$$

The experimental data are those of J. Nikuradse for water (○) [*VDI Forschungsheft*, **H356** (1932)]; Reichardt and Motzfeld for air (●); Reichardt and Schuh (△) for air [H. Reichardt, NACA Tech. Mem. 1047 (1943)]; and R. R. Rothfus, C. C. Monrad, and V. E. Senecal for air (■) [*Ind. Eng. Chem.*, **42**, 2511–2520 (1950)}.

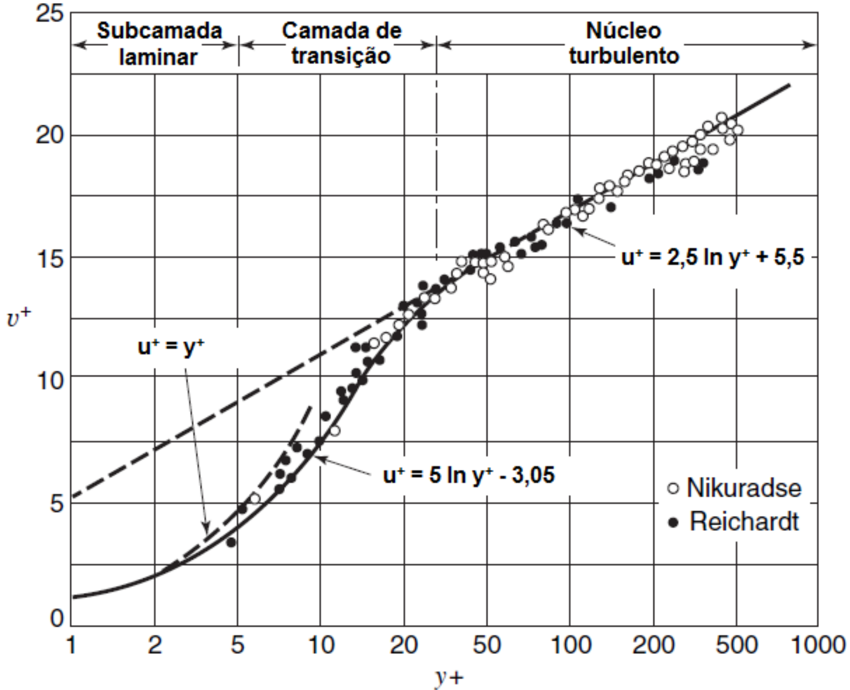
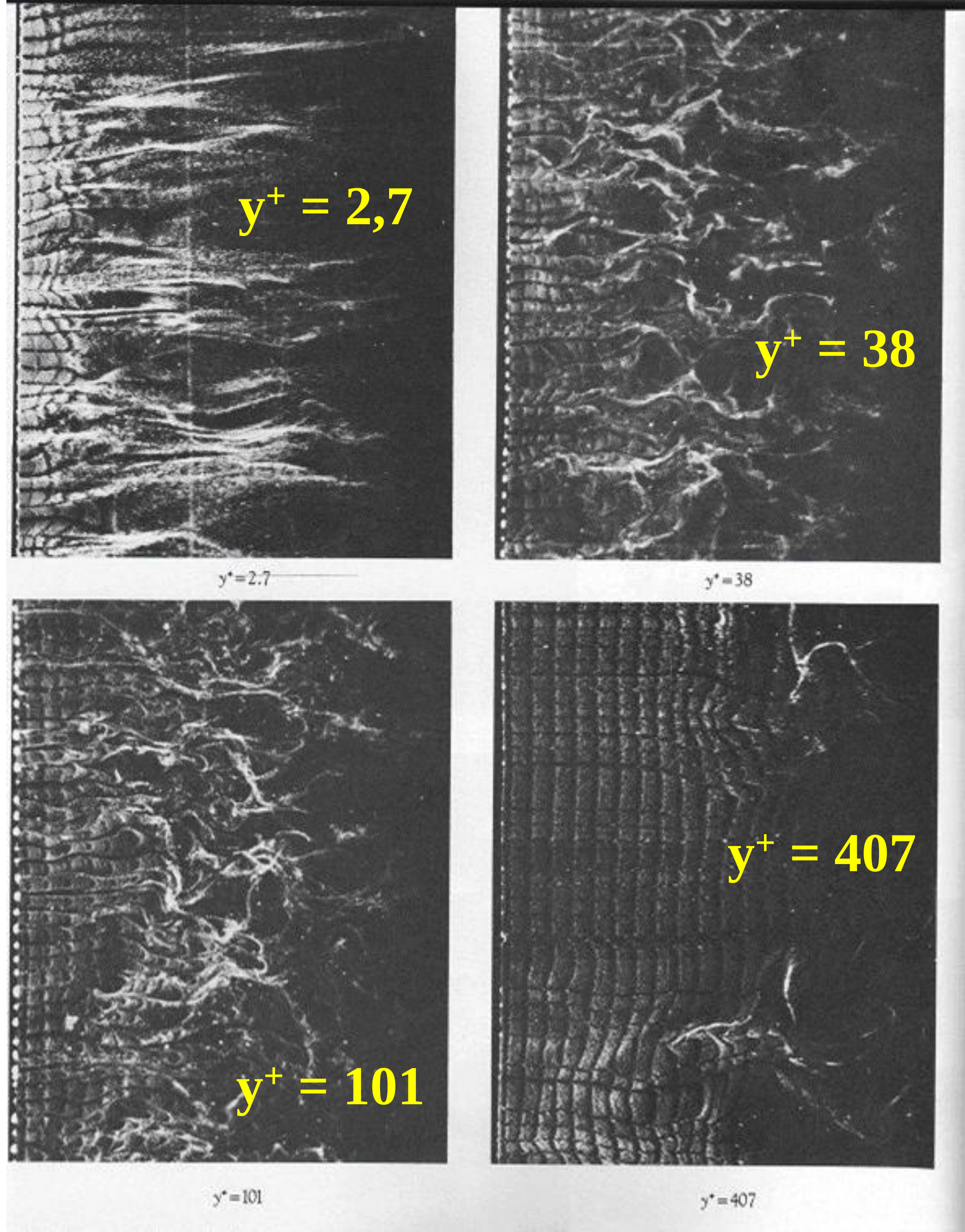


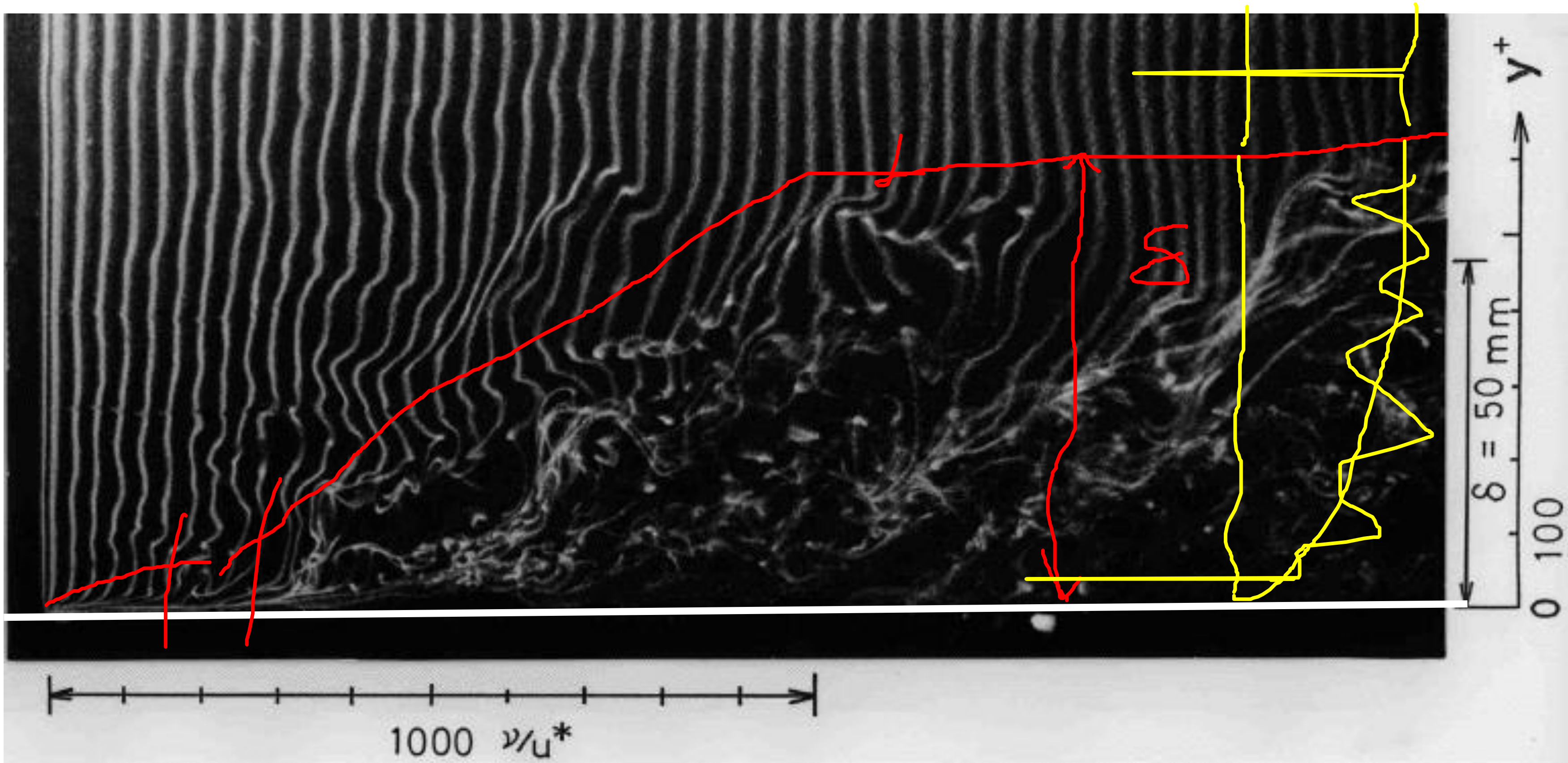
Figura 53. Perfil universal de velocidade para escoamento em um tubo circular liso.

Referência: WELTY, J. R. et al. Fundamentals of Momentum, Heat and Mass Transfer. 5ª edição, Fig. 12.15, pg. 162, John Wiley and Sons, 2008, adaptado.

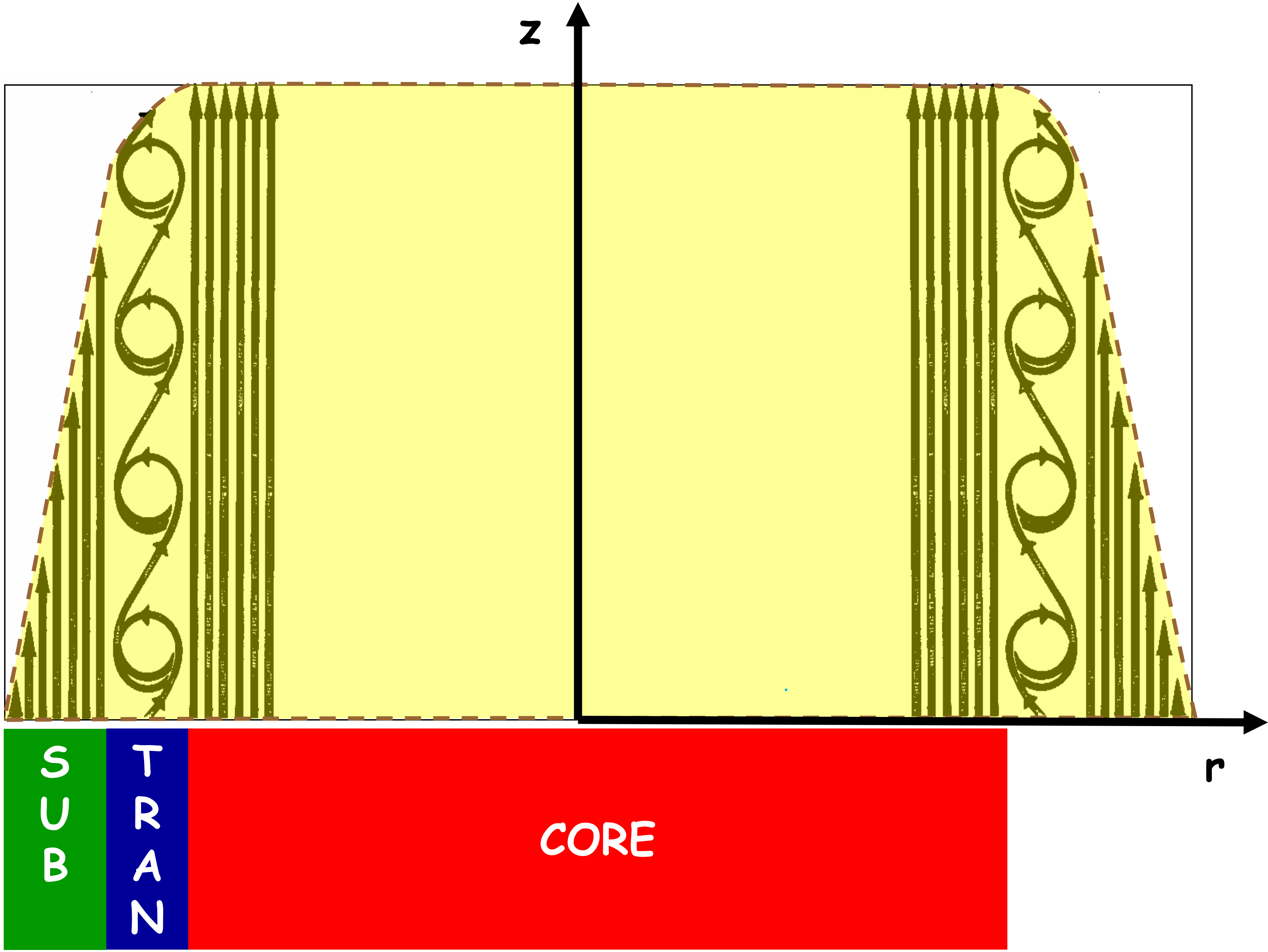
161. Structure of a turbulent boundary layer. Successive layers of the flow near a flat plate in a water channel are shown by tiny hydrogen bubbles released periodically from a thin platinum wire seen at the left. The height $y^{+} = y\,u_{\tau}/\nu$ of the wire above the plate is shown in wall variables, where $u_{\tau} = (\tau_w/\varrho)^{1/2}$ is the friction velocity. The characteristic low- and high-speed streaks shown in the viscous sublayer at $y^{+} = 2.7$ become less noticeable farther away, and have disappeared in the logarithmic region at $y^{+} = 101$. In the wake region at $y^{+} = 407$ the turbulence is seen to be intermittent and of larger scale. *Kline, Reynolds, Schraub & Runstadler 1967*



y+



$\frac{\delta_t}{\delta_s}$


z
r
SUB
TRAN
CORE

Modelos de Turbulência

Reynolds Stress $\vec{\vec{R}} = \overline{\vec{v}'.\vec{v}'}$

sete equações diferenciais	$\frac{D\vec{\vec{R}}}{Dt} = -\,\vec{\mathrm{div}}\left[\vec{\vec{\vec{j}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\pi}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\Omega}}}_{\vec{\vec{R}}}\right] + \overline{\dot{\vec{\vec{\sigma}}}_{M\vec{\vec{R}}}} - \overline{\vec{\vec{\varepsilon}}_{M\vec{\vec{R}}}}$
algébrico	$\vec{\vec{R}} = \frac{2}{3}K\,\vec{\vec{\delta}} + \left[\frac{C_D}{C_1 - 1 + p/\varepsilon}\right]\left(\overline{\dot{\vec{\vec{\sigma}}}_{M\vec{\vec{R}}}} - \frac{2}{3}p\,\vec{\vec{\delta}}\right)\frac{K}{\varepsilon}$
2 eq. K ε	$\nu_T = C_\mu\,K^2/\varepsilon$
0 eq. Prandtl mixing lenght	$\nu_T = \ell_m^2\left|\overrightarrow{\mathrm{grad}}\;\overline{\vec{v}}\right|$
large eddy simulation LES	por hora apenas fornecem parâmetros

tabela

Rayleight

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2$$

Dissipação da energia cinética de turbulência - ε

$$\varepsilon = \nu\left[2\left\langle\left(\overline{\frac{\partial v'_x}{\partial x}}\right)^2\right\rangle + 2\left\langle\left(\overline{\frac{\partial v'_y}{\partial y}}\right)^2\right\rangle + 2\left\langle\left(\overline{\frac{\partial v'_z}{\partial z}}\right)^2\right\rangle + \left\langle\left(\overline{\frac{\partial v'_y}{\partial x}} + \overline{\frac{\partial v'_x}{\partial y}}\right)^2\right\rangle + \left\langle\left(\overline{\frac{\partial v'_z}{\partial y}} + \overline{\frac{\partial v'_y}{\partial z}}\right)^2\right\rangle + \left\langle\left(\overline{\frac{\partial v'_z}{\partial x}} + \overline{\frac{\partial v'_x}{\partial z}}\right)^2\right\rangle\right]$$

Energia cinética de turbulência - k

$$k = \frac{1}{2}\left({v'_x}^2 + {v'_y}^2 + {v'_z}^2\right)$$

$$\frac{\partial \bar{k}}{\partial t} + div\,\bar{\vec{v}}\,\bar{k} = div\,\frac{\nu_T}{\sigma_k}\,gr\vec{a}d\,\bar{k} + P_k - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} + div\,\bar{\vec{v}}\,\varepsilon = div\,\frac{\nu_T}{\sigma_\varepsilon}\,gr\vec{a}d\,\varepsilon + C_{\varepsilon 1}\frac{\varepsilon}{\bar{k}}P_k - C_{\varepsilon 2}\frac{\varepsilon^2}{\bar{k}}$$

$$P_k = -\bar{\rho}\,\nu_T\left(gr\vec{a}d\,\bar{\vec{v}} : gr\vec{a}d\,\bar{\vec{v}}\right)$$

$$\nu_T = C_\mu \frac{\bar{k}^2}{\varepsilon}$$

σ_k = 1,0 ; σ_ε = 1,217 ;
$C_{\varepsilon 1}$ = 1,44 ; $C_{\varepsilon 2}$ = 1,92 ; C_μ = 0.09

constantes experimentais

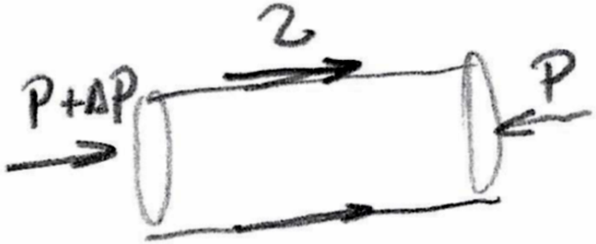
Água (20 ºC) escoa em um tubo de 0,1 m de diâmetro. A velocidade média é de 5 m/s. O tubo é hidraulicamente liso.

$$Re = \frac{VD}{\nu} = \frac{5 \times 0{,}1}{10^{-6}} = 5.10^5$$

Tubo liso $f = 0{,}0032$

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2} \Rightarrow \tau_w = \frac{1}{2} \cdot 10^3 \cdot 5^2 \cdot 0{,}0032$$

$$\tau_w = 40 \ N/m^2$$



$$\tau \cdot 2\pi r L = \Delta P \cdot \pi r^2$$

$$\tau = \frac{\Delta P}{L} \cdot \frac{r}{2}$$

$$v^* = \sqrt{\frac{\tau_w}{\rho}} = 0{,}2\ m/s$$

Subcamada $\longrightarrow$ $y^+ = v^+ \quad 0 \leq y^+ \leq 5$

$$\frac{y\, v^*}{\nu} = \frac{\bar{v}_z}{v^*} \longrightarrow \bar{v}_z = \frac{v^{*2}\, y}{\nu}$$

$$y = \delta \longrightarrow y^+ = 5 \longrightarrow v^+ = 5$$

$$\bar{v}_z = 5 \times 0{,}2 = 1\ m/s$$

$$y^+ = 5 = \frac{y\, v^*}{\nu} = \frac{y \cdot 0{,}2}{10^{-6}} \longrightarrow \delta = 2{,}5 \cdot 10^{-5}\ m$$

$$y^+ = 30 \longrightarrow y = 1{,}5 \cdot 10^{-4}\ m$$

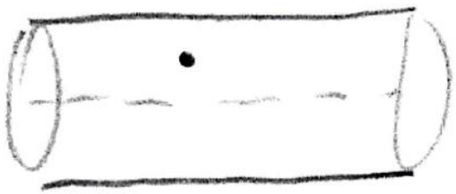
$$v^+ = 5 \ln y^+ - 3{,}05 = 5 \ln 30 - 3{,}05$$

$$v^+ = 13{,}96 \longrightarrow \bar{v}_z = 13{,}96 \times 0{,}2 = 2{,}8\ m/s$$

$p/t = 0{,}025\ m \longrightarrow y = 0{,}025\,m \longrightarrow y^+ = 5.10^3$

$v^+ = 2{,}5 \ln y^+ + 5{,}5$

$v^+ = 26{,}8 \longrightarrow \bar{v}_z = 5{,}36\ m/s$



$$\tau = \tau_w \left(1 - \frac{y}{R}\right) = 40\left(1 - \frac{0{,}025}{0{,}05}\right) = 20\ \frac{N}{m^2}$$

$$\tau = \mu_T \frac{d\bar{v}_z}{dr}$$

$$\frac{\quad}{dr}$$

$$v^+ = 2{,}5 \ln y^+ + 5{,}5 \longrightarrow \frac{dv^+}{dy^+} = \frac{2{,}5}{y^+}$$

$$\frac{dv^+}{dy^+} = \frac{d(\bar{v}/v^*)}{d\left(\frac{y v^*}{\nu}\right)} = \frac{\nu}{v^{*2}} \frac{d\bar{v}}{dy} \leftarrow -\frac{d\bar{v}}{dr}$$

$$\frac{dv^+}{dy^+} = \frac{2{,}5}{y^+} = \frac{2{,}5}{\frac{y v^*}{\nu}} = \frac{\nu}{v^{*2}} \frac{d\bar{v}}{dy}$$

$$\rightarrow \frac{d\bar{v}}{dy} = \frac{2{,}5}{y} v^* = \frac{2{,}5 \cdot 0{,}2}{0{,}025} = 20\ s^{-1}$$

$$\tau = \mu_T \frac{d\bar{v}}{dy} = 20 = \mu_T \cdot 20 \rightarrow \mu = 1\ \text{Pa.s}$$

$$\tau_{laminar} = \mu \frac{d\bar{v}}{dy} = 10^{-3} \cdot 20 = 2 \cdot 10^{-2}\ \text{Pa}$$

$$\tau_l / \tau_T = 2 \cdot 10^{-2} / 20 = 10^{-3}$$

$$\mu_T / \mu_l = 1 / 10^{-3} = 1000$$

p/ $y^+ = 30$

$$\tau \simeq \tau_w = 40 \ N/m^2$$

$$\frac{d\bar{v}}{dy} = 3333 \ s^{-1}$$

$$\mu_T = \frac{\tau}{d\bar{v}/dy} = 0{,}012 \ Pa.s = \rho l_m^2 \frac{d\bar{v}}{dy}$$

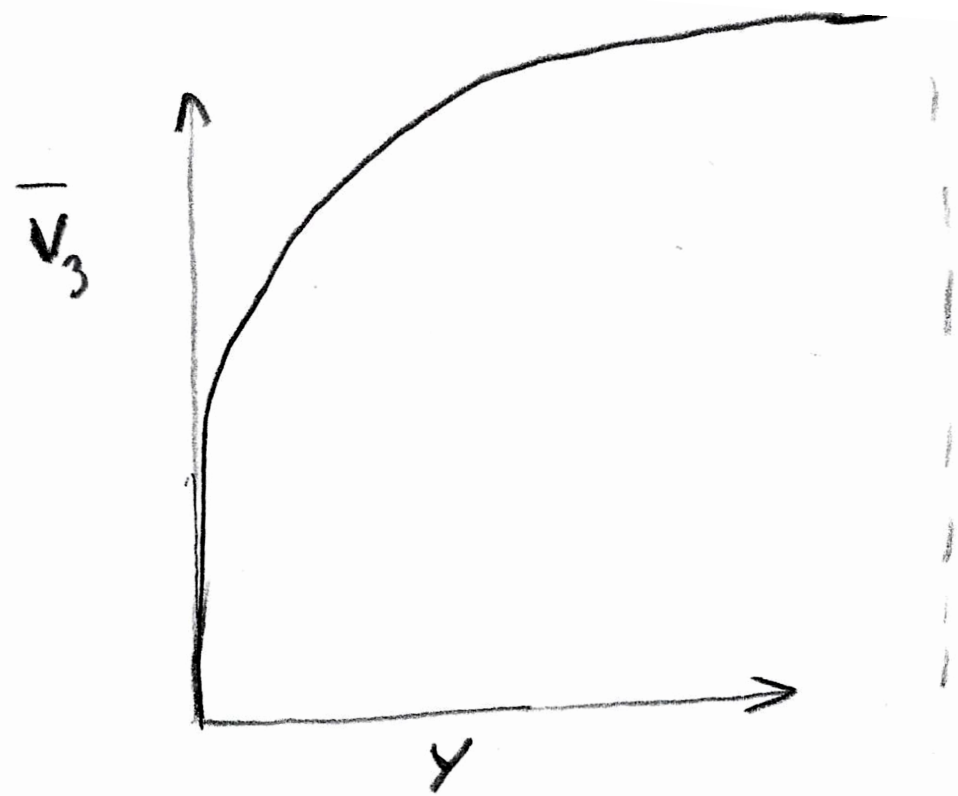
$$\tau_l = \mu \frac{d\bar{v}}{dy} = 3{,}33 \ Pa$$

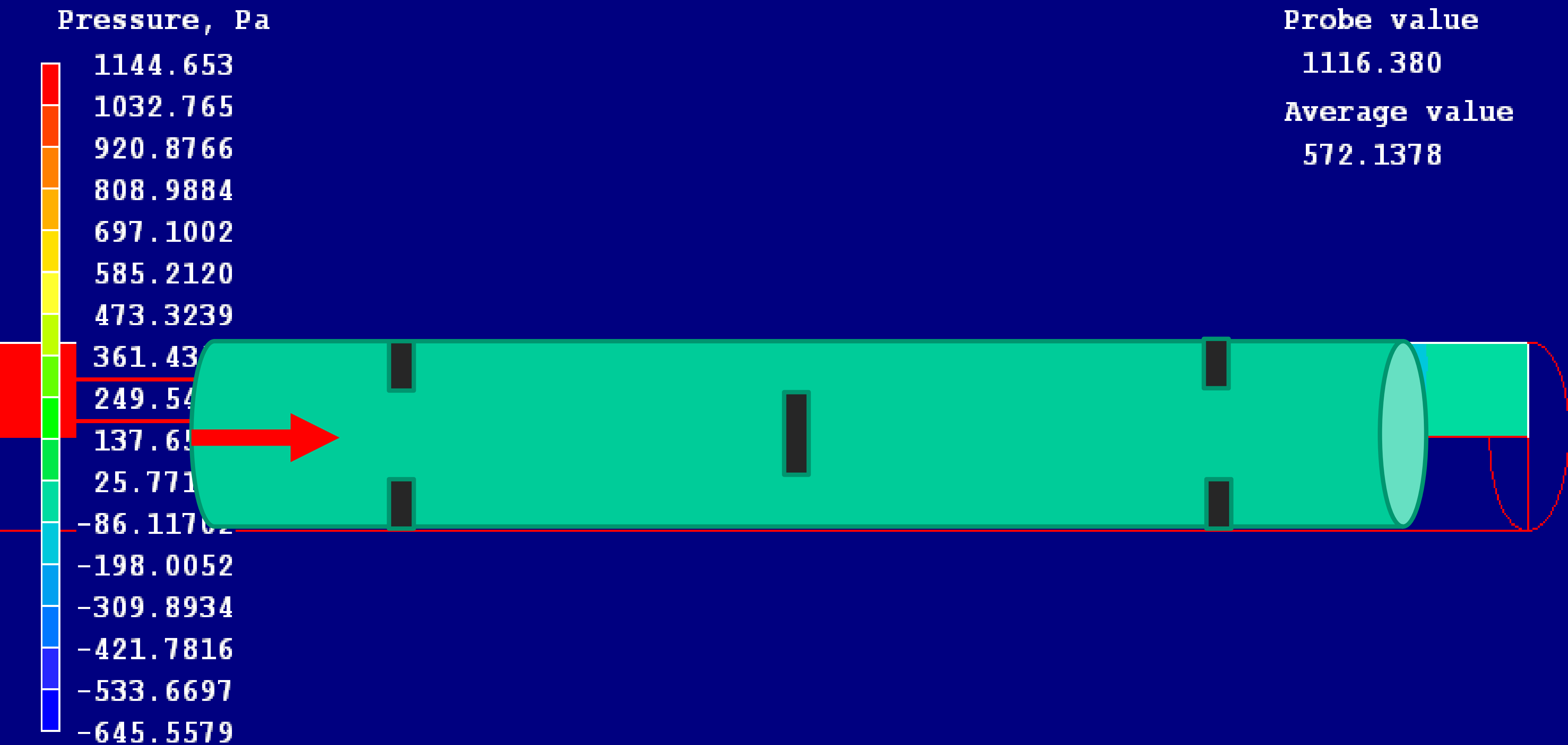
$$\tau_T = 40 - 3{,}33 = 36{,}67 \ Pa.s$$

P/ $y^+ = 0$

$$\tau = \tau_w = 40\ N/m^2$$

$$\frac{d\bar{v}}{dy} = \frac{v_*^2}{\nu} = \frac{0{,}2^2}{10^{-6}} = 4.10^4\ s^{-1}$$

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808.9884
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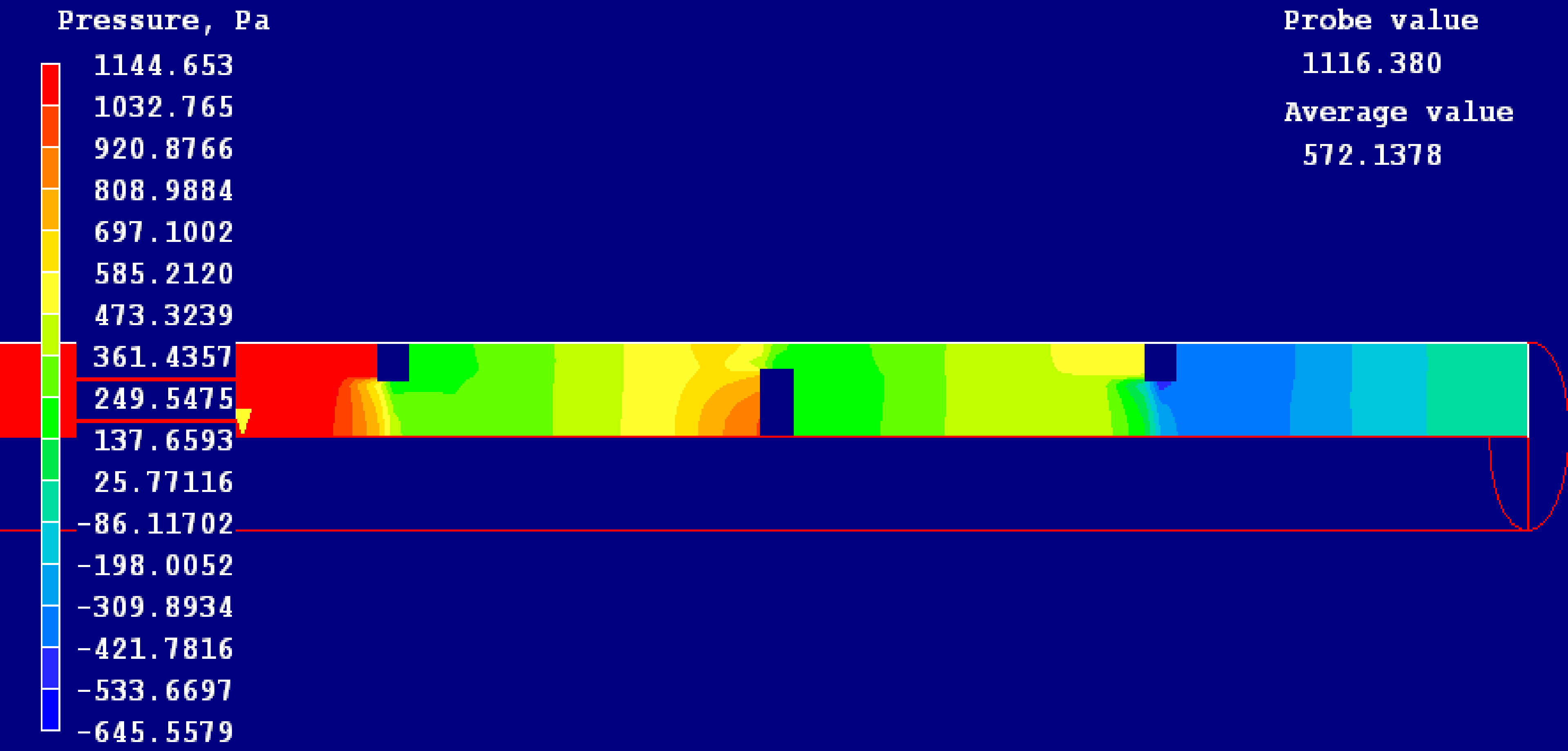
-533.6697
-645.5579
Probe value
1116.380
Average value
572.1378

pressão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Probe value
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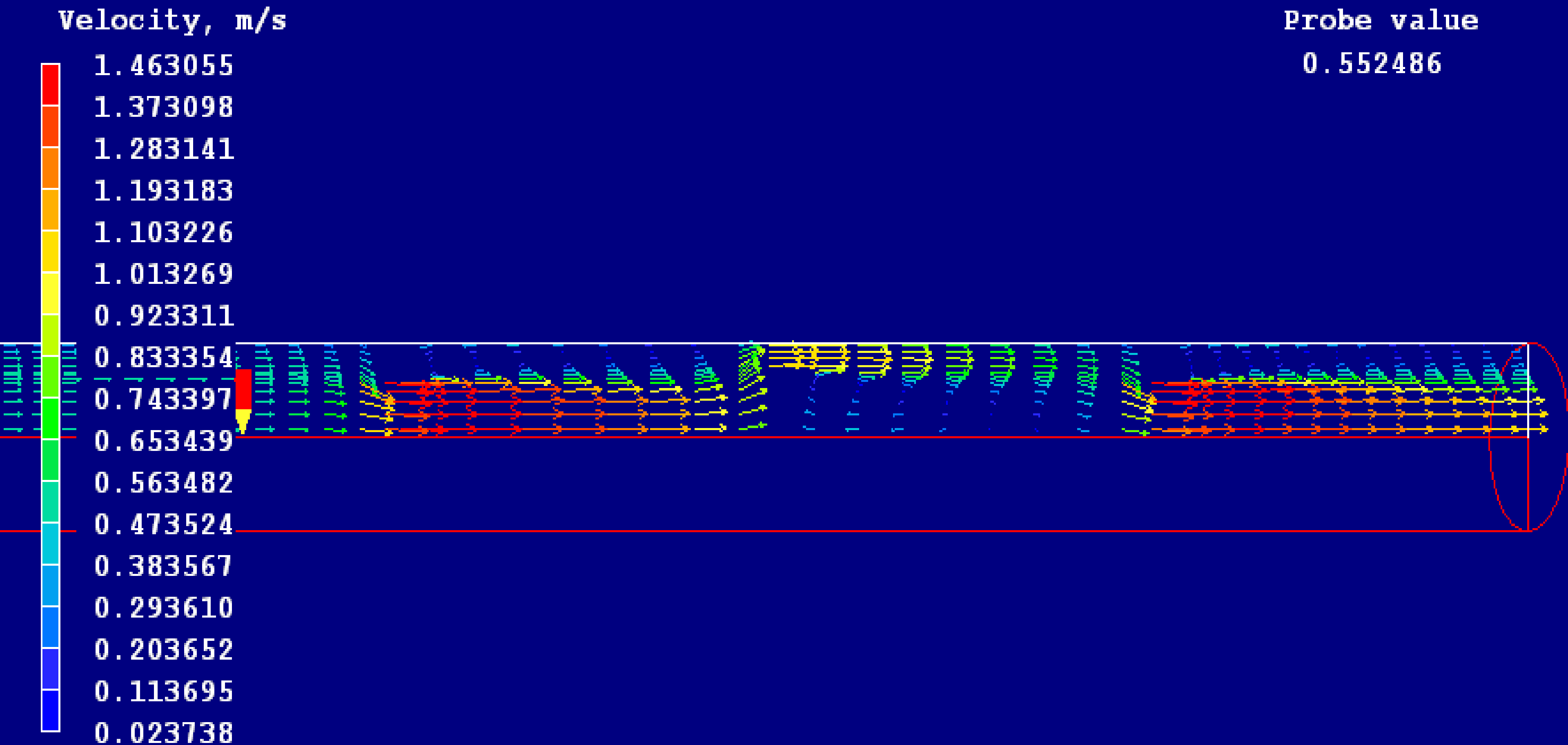
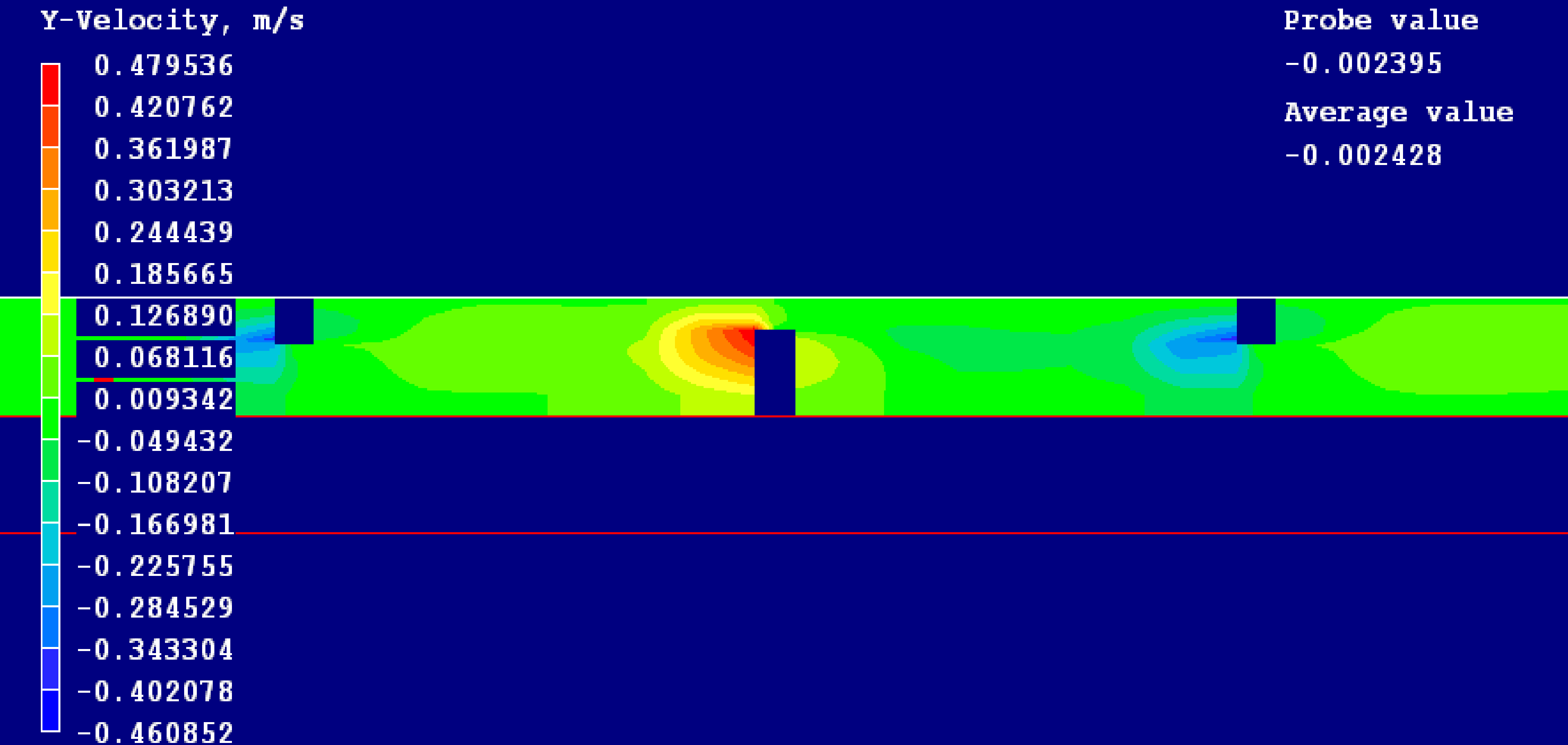
Average value
572.1378

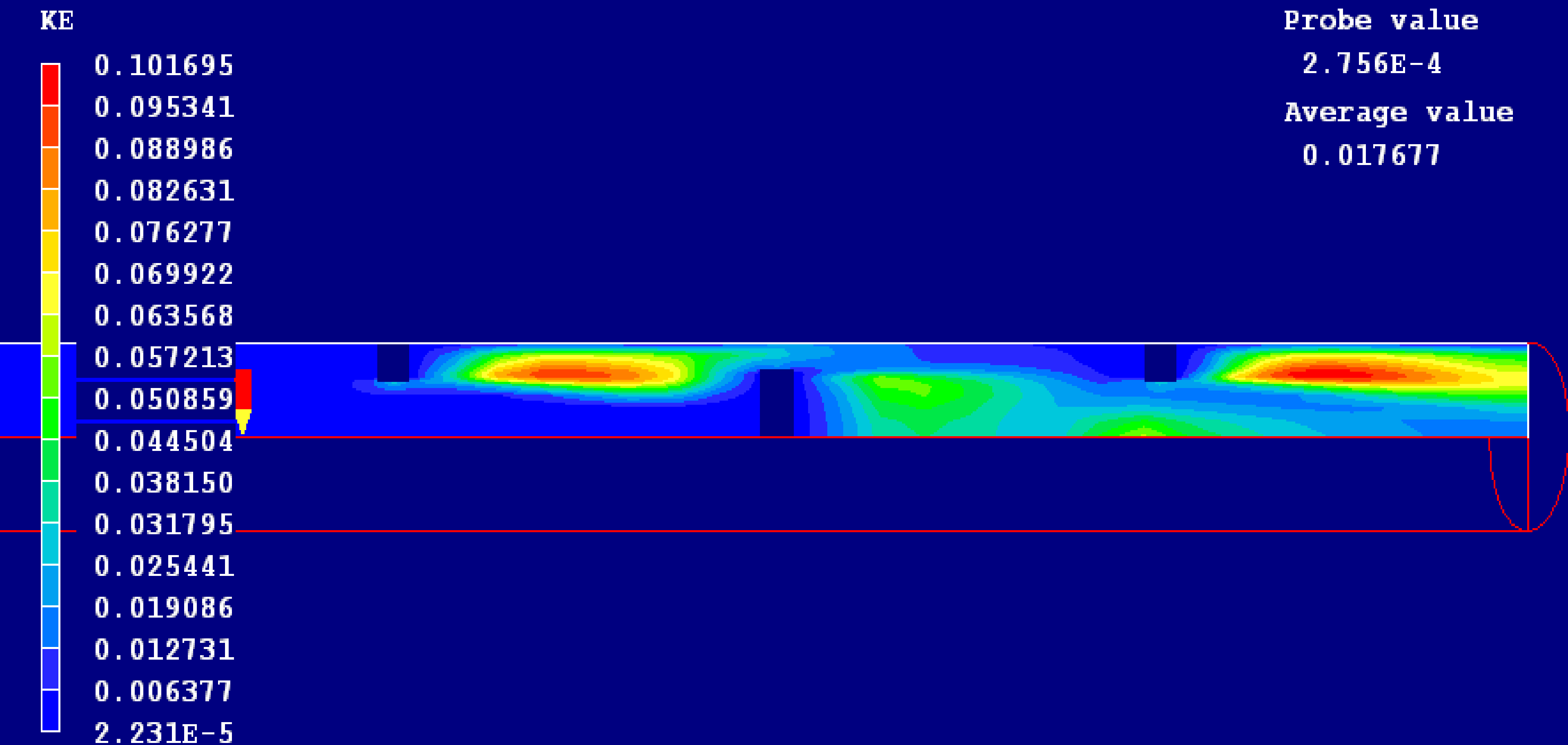
pressão



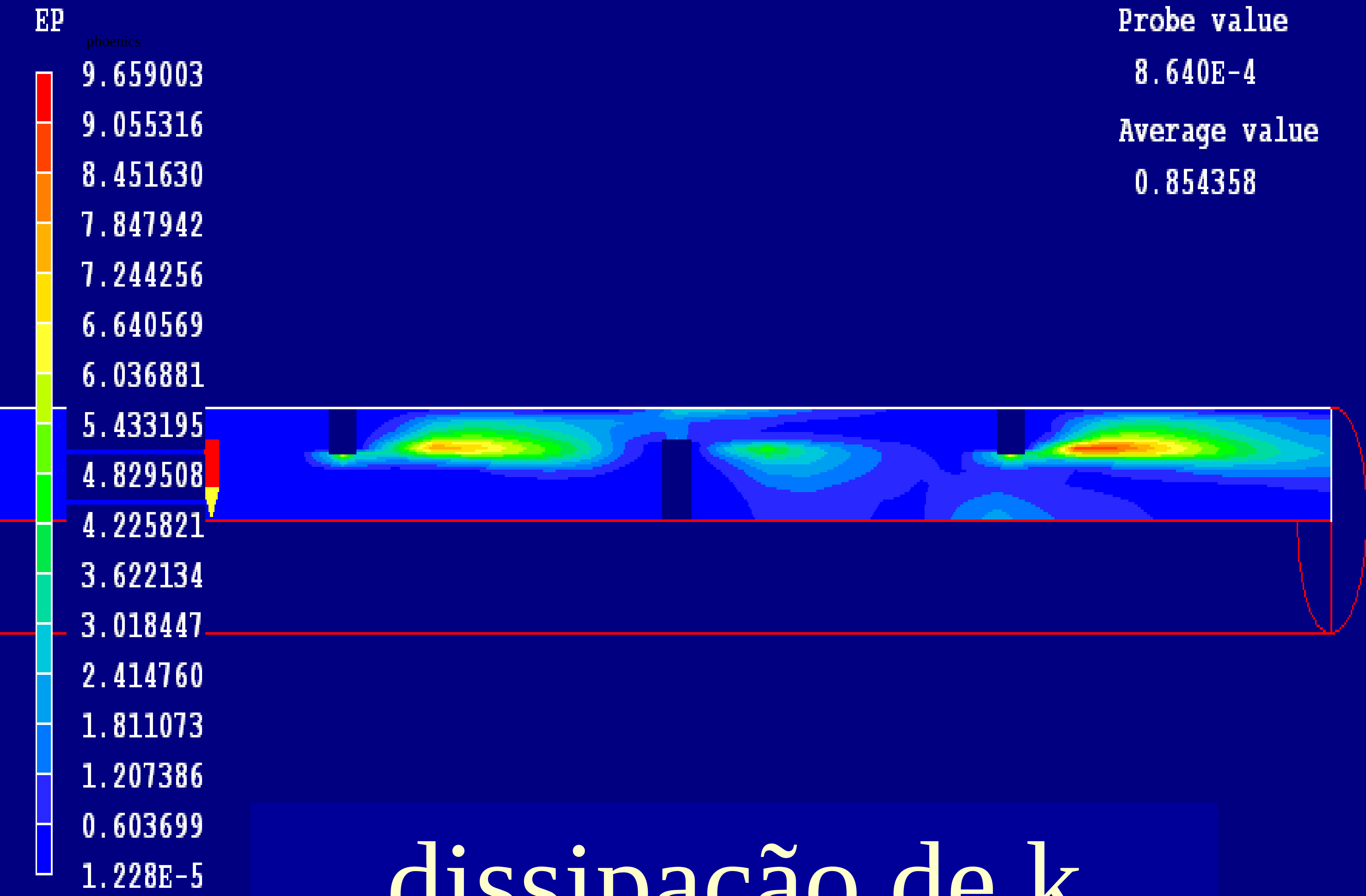
velocidades



velocidade radial



energia cinética de turbulência k



dissipação de k

12) Qual a relação entre a espessura da subcamada viscosa e a camada limite turbulenta em placa plana? Depende do fluido?

turbulenta $\quad \frac{\delta}{x} = \frac{0{,}376}{Re_x^{1/5}} \qquad C_D = \frac{0{,}0576}{Re_x^{1/5}}$

subcamada: $y^+ = 5$

$$y^+ = 5 = \frac{v^* y}{\nu} \longrightarrow \boxed{y = \frac{5\nu}{v^*}}$$

$$v^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{f \cdot \frac{1}{2}\rho U^2}{\rho}} = U\sqrt{\frac{f}{2}}$$

$$v^* = U\sqrt{\frac{0{,}0576}{2}} \cdot \sqrt{\frac{1}{Re_x^{1/5}}} = \frac{0{,}17\,U}{Re_x^{1/10}}$$

$$y = \frac{5\nu}{v^*} = \frac{5\nu\, Re_x^{1/10}}{0{,}17\,U} = 29{,}5\,\frac{\nu}{U} \cdot Re_x^{1/10}$$

$$\frac{y}{x} = 29{,}5 \frac{\nu}{Ux} Re_x^{1/10} = 29{,}5\, Re_x^{-9/10}$$

Camada turbulenta $\frac{\delta}{x} = 0{,}376\, Re^{-1/5}$

Relação: subcamada / camada turb.

$$\frac{y}{\delta} = \frac{29{,}5}{0{,}376} \cdot \frac{Re_x^{-9/10}}{Re_x^{-1/5}} = 78\, Re_x^{-7/10}$$

$Re_x = 10^6 \Rightarrow$ relação $= 4{,}9 \cdot 10^{-3}$

$Re_x = 10^7 \Rightarrow$ relação $= 9{,}8 \cdot 10^{-4}$