Apêndice C

Formulários e Tabelas

1) Sendo V_a o valor atual e V_f o valor futuro, e se seu relacionamento for feito por **juros simples,** tem-se que, após **n** períodos à taxa de juros **i** cada um,

$$V_f = V_a\ (1 + ni)$$

2) Sendo V_a o valor atual (ou presente) e V_f o valor futuro, eles estão relacionados pela taxa de **juros composta** $(V_f/V_a;\ i\%;\ n)$

Dado V_a, calcular $V_f \rightarrow (V_f/V_a;\ i\%;\ n) = (1 + i)^n$ e, assim,

$$(V_a/V_f;\ i\%;\ n) = \frac{1}{(1 + i)^n}$$

Observe que $(V_a/V_f;\ i;\ n_1 + n_2) = (V_a/V_f;\ i;\ n_1)\ X\ (V_a/V_f;\ i;\ n_2)$

3) Sendo **A** um valor uniforme ao fim de cada período e os juros compostos, tem-se

$$V_f = A + A\ (1 + i) + A\ (1 + i)^2 + \ldots . + A\ (1 + i)^{n-1}$$

que é a soma de **n** termos de uma progressão geométrica cujo primeiro termo é **A** e a razão é $(1 + i)$, o que resulta em

$$(V_f/A;\ i\%;\ n) = \left[\frac{(1 + i)^n - 1}{i}\right]$$

Para calcular V_a, basta dividir o resultado acima por $(1 + i)^n$, resultando

$$(V_a/A;\ i\%;\ n) = \left[\frac{(1+i)^n - 1}{i\ (1+i)^n}\right]$$

ou

$$\left[\frac{1}{\dfrac{i}{(1+i)^n - 1} + i}\right]$$

Podemos calcular o inverso:

$$(A/V_f;\ i\%;\ n) = \left[\frac{i}{(1+i)^n - 1}\right]$$

$$(A/V_a;\ i\%;\ n) = \left[\frac{i\ (1+i)^n}{(1+i)^n - 1}\right]$$

ou

$$\left[\frac{i}{(1+i)^n - 1} + i\right]$$

onde $(A/V_a;\ i\%;\ n)$ é chamado fator de amortização.

4) Para o gradiente uniforme, resulta

$$(V_a/G;\ i\%;\ n) = \left[\frac{(1+i)^n - 1 - ni}{i^2\ (1+i)^n}\right]$$

$$(A/G;\ i\%;\ n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

5) Se tivermos uma taxa de juros nominal de i% ao ano paga m vezes por ano, isto corresponde a uma taxa efetiva anual

$$i_{cf} = (1 + \frac{i}{m})^m - 1$$

6) **Tabelas**

Com o advento das máquinas de calcular eletrônicas de bolso, as tabelas perderam muito sua importância. Damos, a seguir, apenas algumas delas, que são suficientes para resolver os problemas propostos.

Observe que a tabela I dá $(V_a/V_f;\ i\%;\ n)$, e, se quisermos $(V_f/V_n;\ i\%;\ n)$, basta usar o inverso.

Se quisermos $(V_f/A;\ i\%;\ n)$, podemos utilizar $(V_f/A;\ i\%;\ n) = (V_a/A;\ i\%;\ n)/(V_a/V_f;\ i\%;\ n)$.

Se quisermos $(V_a/G;\ i\%;\ n)$, então utilizamos $(V_a/G;\ i\%;\ n) = (V_a/A;\ i\%;\ n) \times (A/G;\ i\%;\ n)$

Existem tabelas mais completas, que não julgamos necessário reproduzir no presente texto.

7) Valores que não constam da tabela

Para valores que não constam da tabela, podemos obter aproximações por meio de interpolação linear. Assim, por exemplo, se desejarmos calcular $(V_a/A;\ 2{,}3\%;\ 15)$ observamos $(V_a/A;\ 2\%;\ 15) = 12{,}8492$ e $(V_a/A;\ 3\%;\ 15) = 11{,}9379$, de modo que

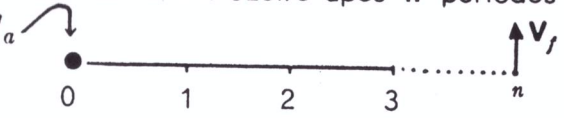
3% − 2% = 1% corresponde a 12,8492 − 11,9379 = 0,9113
2,3% − 2% = 0,3% corresponde a x
x = 0,3 x 0,9113 = 0,27339

e

$(V_a/A;\ 2{,}3\%;\ 15) = 12{,}8492 - 0{,}27339 = 12{,}57581$

TABELA I — Valor Atual de um cruzeiro após n períodos

$(V_a/V_f;\ i\%;\ n)$



n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	n
01	0.99010	0.98039	0.97007	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.88496	0.87719	0.86957	01
02	.98030	.96117	.94260	.92456	.90703	.89000	.87344	.85734	.84168	.82645	.81162	.79719	.78315	.76947	.75614	02
03	.97059	.94232	.91514	.88900	.86384	.83962	.81630	.79383	.77218	.75131	.73119	.71178	.69305	.67497	.65752	03
04	.96098	.92385	.88849	.85480	.82270	.79209	.76290	.73503	.70843	.68301	.65873	.63552	.61332	.59208	.57175	04
05	.95147	.90573	.86261	.82193	.78353	.74726	.71299	.68058	.64993	.62092	.59345	.56743	.54276	.51937	.49718	05
06	.94204	.88797	.83748	.79031	.74622	.70496	.66634	.63017	.59627	.56447	.53464	.50663	.48032	.45559	.43233	06
07	.93272	.87056	.81309	.75992	.71068	.66506	.62275	.58349	.54703	.51316	.48166	.45235	.42506	.39964	.37594	07
08	.92348	.85349	.78941	.73069	.67684	.62741	.58201	.54027	.50187	.46651	.43393	.40388	.37616	.35056	.32690	08
09	.91434	.83675	.76642	.70259	.64461	.59190	.54393	.50025	.46043	.42410	.39092	.36061	.33288	.30751	.28426	09
10	.90529	.82035	.74409	.67556	.61391	.55839	.50835	.46319	.42241	.38554	.35218	.32197	.29459	.26974	.24718	10
11	.89632	.80426	.72242	.64958	.58468	.52679	.47509	.42888	.38753	.35049	.31728	.28748	.26070	.23662	.21494	11
12	.88745	.78849	.70138	.62460	.55684	.49697	.44401	.39711	.35553	.31863	.28584	.25667	.23071	.20755	.18681	12
13	.87866	.77303	.68095	.60057	.53032	.46884	.41496	.36770	.32618	.28966	.25751	.22917	.20416	.18207	.16253	13
14	.86996	.75787	.66112	.57747	.50507	.44230	.38782	.34046	.29925	.26333	.23199	.20462	.18068	.15971	.14133	14
15	.86135	.74301	.64186	.55526	.48102	.41726	.36245	.31524	.27454	.23939	.20900	.18270	.15989	.14010	.12289	15
16	.85282	.72845	.62317	.53391	.45811	.39365	.33873	.29189	.25187	.21763	.18829	.16312	.14150	.12289	.10686	16
17	.84438	.71416	.60502	.51337	.43630	.37136	.31657	.27027	.23107	.19784	.16963	.14564	.12522	.10780	.09293	17
18	.83602	.70016	.58739	.49363	.41552	.35034	.29586	.25025	.21199	.17986	.15282	.13004	.11081	.09456	.08080	18
19	.82774	.68643	.57029	.47464	.39573	.33051	.27651	.23171	.19449	.16351	.13768	.11611	.09806	.08295	.07026	19
20	.81954	.67297	.55367	.45639	.37689	.31180	.25842	.21455	.17843	.14864	.12403	.10367	.08678	.07276	.06110	20
21	.81143	.65978	.53755	.43883	.35894	.29415	.24151	.19866	.16370	.13513	.11174	.09256	.07680	.06383	.05313	21
22	.80340	.64684	.52189	.42195	.34185	.27750	.22571	.18394	.15018	.12285	.10067	.08264	.06796	.05599	.04620	22
23	.79544	.63416	.50669	.40573	.32557	.26180	.21095	.17031	.13778	.11168	.09069	.07379	.06014	.04911	.04017	23
24	.78757	.62172	.49193	.39012	.31007	.24698	.19715	.15770	.12640	.10153	.08170	.06588	.05322	.04308	.03493	24
25	.77977	.60953	.47760	.37512	.29530	.23300	.18425	.14602	.11597	.09230	.07361	.05882	.04710	.03779	.03038	25
30	.74192	.55207	.41199	.30832	.23138	.17411	.13137	.09938	.07537	.05731	.04368	.03338	.02557	.01963	.01510	30

TABELA I

(V_a/V_f; i%; n)

n	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%	n
01	0.86207	0.85470	0.84746	0.84034	0.83333	0.82645	0.81967	0.81301	0.80645	0.80000	0.79365	0.78740	0.78125	0.77519	0.76923	01
02	.74316	.73051	.71818	.70616	.69444	.68301	.67186	.66098	.65036	.64000	.62988	.62000	.61035	.60093	.59172	02
03	.64066	.62437	.60863	.59342	.57870	.56447	.55071	.53738	.52449	.51200	.49991	.48819	.47684	.46583	.45517	03
04	.55229	.53365	.51579	.49867	.48225	.46651	.45140	.43690	.42297	.40960	.39675	.38440	.37253	.36111	.35013	04
05	.47611	.45611	.43711	.41905	.40188	.38554	.37000	.35520	.34111	.32768	.31488	.30268	.29104	.27993	.26933	05
06	.41044	.38984	.37043	.35214	.33490	.31863	.30328	.28878	.27509	.26214	.24991	.23833	.22737	.21700	.20718	06
07	.35383	.33320	.31392	.29592	.27908	.26333	.24859	.23478	.22184	.20972	.19834	.18766	.17764	.16822	.15937	07
08	.30503	.28478	.26604	.24867	.23257	.21763	.20376	.19088	.17891	.16777	.15741	.14776	.13878	.13040	.12259	08
09	.26295	.24340	.22546	.20897	.19381	.17986	.16702	.15519	.14428	.13422	.12493	.11635	.10842	.10109	.09430	09
10	.22668	.20804	.19106	.17560	.16151	.14864	.13690	.12617	.11635	.10737	.09915	.09161	.08470	.07836	.07254	10
11	.19542	.17781	.16192	.14756	.13459	.12285	.11221	.10258	.09383	.08590	.07869	.07214	.06617	.06075	.05580	11
12	.16846	.15197	.13722	.12400	.11216	.10153	.09198	.08339	.07567	.06872	.06245	.05680	.05170	.04709	.04292	12
13	.14523	.12989	.11629	.10420	.09346	.08391	.07539	.06780	.06103	.05498	.04957	.04472	.04039	.03650	.03302	13
14	.12520	.11102	.09855	.08757	.07789	.06934	.06180	.05512	.04921	.04398	.03934	.03522	.03155	.02830	.02540	14
15	.10793	.09489	.08352	.07359	.06491	.05731	.05065	.04481	.03969	.03518	.03122	.02773	.02465	.02194	.01954	15
16	.09304	.08110	.07078	.06184	.05409	.04736	.04152	.03643	.03201	.02815	.02478	.02183	.01926	.01700	.01503	16
17	.08021	.06932	.05998	.05196	.04507	.03914	.03403	.02962	.02581	.02252	.01967	.01719	.01505	.01318	.01156	17
18	.06914	.05925	.05083	.04367	.03756	.03235	.02789	.02408	.02082	.01801	.01561	.01354	.01175	.01022	.00889	18
19	.05961	.05064	.04308	.03669	.03130	.02673	.02286	.01958	.01679	.01441	.01239	.01066	.00918	.00792	.00684	19
20	.05139	.04328	.03651	.03084	.02608	.02209	.01874	.01592	.01354	.01153	.00983	.00839	.00717	.00614	.00526	20
21	.04430	.03699	.03094	.02591	.02174	.01826	.01536	.01294	.01092	.00922	.00780	.00661	.00561	.00476	.00405	21
22	.03819	.03162	.02622	.02178	.01811	.01509	.01259	.01052	.00880	.00738	.00619	.00520	.00438	.00369	.00311	22
23	.03292	.02702	.02222	.01830	.01509	.01247	.01032	.00855	.00710	.00590	.00491	.00410	.00342	.00286	.00239	23
24	.02838	.02310	.01883	.01538	.01258	.01031	.00846	.00695	.00573	.00472	.00390	.00323	.00267	.00222	.00184	24
25	.02447	.01974	.01596	.01292	.01048	.00852	.00693	.00565	.00462	.00378	.00310	.00254	.00209	.00172	.00142	25
30	.01165	.00900	.00697	.00541	.00421	.00328	.00257	0.00201	.00158	.00124	.00097	.00077	.00061	.00048	.00038	30

TABELA I

$(V_a/V_f;\ i\%;\ n)$

n	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%	50%	60%	70%	80%	90%	n
01	0.76336	0.75758	0.75188	0.7463	0.74074	0.73529	0.72993	0.72464	0.71942	0.71429	.667	.625	.558	.556	.526	01
02	.58272	.57392	.56532	.55692	.54870	.54066	.53279	.52510	.51757	.51020	.444	.391	.346	.309	.277	02
03	.44482	.43479	.42505	.41561	.40644	.39754	.38890	.38051	.37235	.36443	.296	.244	.204	.171	.146	03
04	.33956	.32939	.31959	.31016	.30107	.29231	.28387	.27573	.26788	.26031	.198	.153	.120	.095	.077	04
05	.25920	.24953	.24029	.23146	.22301	.21493	.20720	.19980	.19272	.18593	.132	.095	.070	.053	.040	05
06	.19787	.18904	.18067	.17273	.16520	.15804	.15124	.14479	.13865	.13281	.088	.060	.041	.029	.021	06
07	.15104	.14321	.13584	.12890	.12237	.11621	.11040	.10492	.09975	.09486	.059	.037	.024	.016	.011	07
08	.11530	.10849	.10214	.09620	.09064	.08545	.08058	.07603	.07176	.06776	.039	.023	.014	.009	.006	08
09	.08802	.08219	.07680	.07179	.06714	.06283	.05882	.05509	.05163	.04840	.026	.015	.008	.005	.003	09
10	.06719	.06227	.05774	.05357	.04973	.04620	.04293	.03992	.03714	.03457	.017	.009	.005	.003	.002	10
11	.05129	.04717	.04341	.03998	.03684	.03397	.03134	.02893	.02672	.02469	.012	.006	.003	.002	.001	11
12	.03915	.03574	.03264	.02984	.02729	.02498	.02287	.02096	.01922	.01764	.008	.004	.002	.001	.001	12
13	.02989	.02707	.02454	.02227	.02021	.01837	.01670	.01519	.01383	.01260	.005	.002	.001	.001	.000	13
14	.02281	.02051	.01845	.01662	.01497	.01350	.01219	.01101	.00995	.00900	.003	.001	.001	.000	.000	14
15	.01742	.01554	.01387	.01240	.01109	.00993	.00890	.00798	.00716	.00643	.002	.001	.000	.000	.000	15
16	.01329	.01177	.01043	.00925	.00822	.00730	.00649	.00578	.00515	.00459						
17	.01015	.00892	.00784	.00691	.00609	.00537	.00474	.00419	.00370	.00328						
18	.00775	.00676	.00590	.00515	.00451	.00395	.00346	.00304	.00267	.00234						
19	.00591	.00512	.00443	.00385	.00334	.00290	.00253	.00220	.00192	.00167						
20	.00451	.00388	.00333	.00287	.00247	.00213	.00184	.00159	.00138	.00120						
21	.00345	.00294	.00251	.00214	.00183	.00157	.00135	.00115	.00099	.00085						
22	.00263	.00223	.00188	.00160	.00136	.00115	.00098	.00084	.00071	.00061						
23	.00201	.00169	.00142	.00119	.00101	.00085	.00072	.00061	.00051	.00044						
24	.00153	.00128	.00107	.00089	.00074	.00062	.00052	.00044	.00037	.00031						
25	.00117	.00097	.00080	.00066	.00055	.00046	.00038	.00032	.00027	.00022						