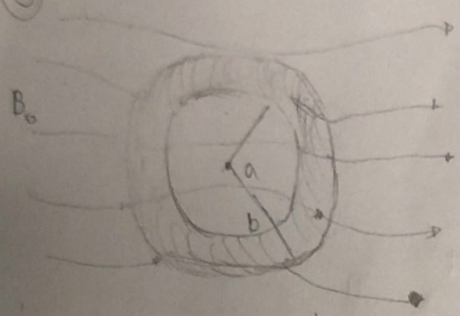
5



Sol geral:
$$\boxed{\Psi_M = \sum_{m=1}^{\infty} \left(a_m \rho^{m} + b_m \rho^{-m}\right)\left(A_m e^{im\phi} + B_m e^{-im\phi}\right)}$$

Escrevemos o potencial nas três regiões:

Sabemos: $\nabla^2 \Psi_M = 0 \rightsquigarrow B = -\nabla \Psi_M$, e o potencial para $\rho \to \infty$ é: $\Psi_M = -B_0 \rho \cos\phi$

Para $\rho > b$:

$$\boxed{\Psi_{out} = -B_0 \rho \cos\phi + \sum_{m=1}^{\infty} \rho^{-m}\left(A_m e^{im\phi} + B_m e^{-im\phi}\right)}$$

Na metade do cilindro; $a < \rho < b$:

$$\boxed{\Psi_{Med} = \sum_{m=1}^{\infty} \left(C_m \rho^{m} + \rho^{-m}\right)\left(G_m e^{im\phi} + D_m e^{-im\phi}\right)}$$

Dentro do cilindro: $\rho < a$:

$$\boxed{\Psi_{Int} = \sum_{m=1}^{\infty} \rho^{-m}\left(F_m e^{im\phi} + G_m e^{-im\phi}\right)}$$

...a corrente livre é zero no tudo o material.

...ndo as condições de contorno:

$$(B_{out} - B_{med}) = 0 \qquad \frac{B_{out}}{\mu_0} - \frac{B_{mid}}{\mu} = 0 \rightsquigarrow \left(B_{out} - \frac{1}{\mu_r} B_{mid}\right) = 0 \;, \rho = b$$

$$(B_{med} - B_{int}) = 0 \qquad \frac{B_{med}}{\mu} - \frac{B_{int}}{\mu_0} = 0 \rightsquigarrow \left(\frac{B_{mid}}{\mu_r} - B_{INT}\right) = 0 \;, \rho = a$$

Obtendo: , $\mu_r = \mu/\mu_0$

1

$$\frac{\partial \Psi_{out}}{\partial \rho} = \frac{\partial \Psi_{mid}}{\partial \rho} \quad \dots (1)$$

$$\frac{\partial \Psi_{mid}}{\partial \rho} = \frac{\partial \Psi_{INT}}{\partial \rho} \quad \dots (2)$$

2

$$\frac{\partial \Psi_{out}}{\partial \phi} = \frac{1}{\mu_r}\frac{\partial \Psi_{mid}}{\partial \psi} \;, \quad \rho = b \quad \dots (3)$$

$$\frac{1}{\mu_r}\frac{\partial \Psi_{mid}}{\partial \phi} = \frac{\partial \Psi_{int}}{\partial \psi} \;, \quad \rho = a \quad \dots (4)$$

de ..(1), obtemos:

$$\boxed{\frac{A_1}{b^2} + (c_1 - b^{-2})\, C_1 = -B_0/2} \quad \dots (5) \quad \text{e} \quad \boxed{B_1 = A_1} \;, \quad \boxed{D_1 = C_1}$$

de ..(2)

$$\boxed{\frac{A_1}{b^2} - \frac{1}{\mu_r}(c_1 + b^{-2})\, C_1 = B_0/2} \quad \dots (6)$$

de ..(3)

$$\boxed{(c_1 - a^{-2})\, C_1 = F_1} \quad \dots (7) \;, \quad \boxed{F_1 = G_1}$$

de ..(4)

$$\boxed{\frac{1}{\mu_r}(c_1 + a^{-2})\, C_1 = F_1} \quad \dots (8)$$

de .. (7) e .. (8) $(c_m - a^{-2m})$

$$(c_1 - a^{-2})\, \cancel{C_1'} = \frac{1}{\mu_r}\,(c_1 + a^{-2})\, \cancel{C_1'}$$

$$c_1 \mu_r - a^{-2}\mu_r = c_1 + a^{-2}$$

$$c_1 \mu_r - c_1 = a^{-2}\mu_r + a^{-2}$$

$$c_1(\mu_r - 1) = a^{-2}(\mu_r + 1)$$

$$\boxed{c_1 = -\frac{1}{a^2}\,\frac{(1+\mu_r)}{(1-\mu_r)}}$$

de .. (5) e .. (6)

$$-(c_1 - b^{-2})\, C_1' - \frac{1}{\mu_r}\,(c_1 + b^{-2})\, C_1' = B_0$$

$$-\mu_r (c_1 - b^{-2})\, C_1' - (c_1 + b^{-2})\, C_1' = \mu_r B_0$$

$$\boxed{C_1' = \frac{\mu_r \; B_0 \; (1-\mu_r)\, b^2}{\frac{b^2}{a^2}\,(1+\mu_r)^2 - (1-\mu_r)^2}}$$

em .. (7) $\qquad (c_1 - a^{-2})\, C_1' = F_1$

$$\left(-\frac{1}{a^2}\,\frac{(1+\mu_r)}{(1-\mu_r)} - a^2\right) \frac{\mu_r \; B_0 \,(1-\mu_r)\, b^2}{\frac{b^2}{a^2}\,(1+\mu_r) - (1-\mu_r)^2} = F_1$$

$$\sim \; F_1 = -2 B_0 \frac{b^2}{a^2}\, \frac{\mu_r}{\frac{b^2}{a^2}\,(1+\mu_r)^2 - (1-\mu_r)^2}$$

...nalmente:

$$A_1 = -\frac{B_0}{2b^2(1-\mu_r^2)} \frac{\frac{b^2}{a^2}-1}{\frac{b^2}{a^2}(1+\mu_r)^2-(1-\mu_r)^2}$$

Portanto:

$$\Psi_{out} = -B_0 \rho \cos\phi + \frac{(A_1 e^{i\phi} + B_1 e^{-i\phi})}{\rho}$$

$$\Psi_{out} = -B_0 \rho \cos\phi + \frac{(1-\mu_r^2)(b^2-a^2)}{2\mu_r ab}\left(\frac{2\mu_r ab}{b^2(1+\mu_r)^2-a^2(1-\mu_r)^2}\right)\frac{b}{\rho} B_0 b \cos\phi$$

$$\Psi_{Mid} = \frac{-2\mu_r ab}{b^2(1+\mu_r)-a^2(1-\mu_r)^2}\left\{(1+\mu_r)\frac{\rho}{a} - (1-\mu_r)\frac{a}{\rho}\right\} B_0 b \cos\phi$$

$$\Psi_{Int} = -\frac{b}{a}\, 2\left(\frac{2\mu_r ab}{b^2(1+\mu_r)^2-a^2(1-\mu_r)^2}\right) B_0 \rho \cos\phi$$

usando $\quad B = -\nabla \Psi_M$ $\qquad\qquad \mu_r = \mu_0/\mu$

$$\rightsquigarrow \quad B_{int} = \frac{b}{a}\, 2 \left(\frac{2\mu_r ab\, B_0}{b^2(1+\mu_r)^2 - a^2(1-\mu_r)^2} \right) B_0\, \hat{x}$$

$$B_{int} = \frac{4\mu_r b^2 B_0}{b^2(1+\mu_r)^2 - a^2(1-\mu_r)^2}\,\hat{x} \qquad s<a$$

$a<s<b$

$$B_{mid} = \frac{2\mu_r ab}{b^2(1+\mu_r)^2 - a^2(1-\mu_r)^2} \quad B_0 \frac{b}{a}(1+\mu_r)\,\hat{x} + \frac{2\mu_r ab\, B_0(1-\mu_r)}{b^2(1+\mu_r)^2 - a^2(1-\mu_r)^2}\frac{ab}{\rho^2}\left[\hat{x} + 2\hat{\phi}\sin\phi\right]$$

$s>b$

$$B_{out} = B_0\,\hat{x} - \frac{(1-\mu_r^2)(b^2-a^2)}{2\mu_r ab}\cdot\frac{2\mu_r ab}{b^2(1+\mu_r)^2 - a^2(1-\mu_r)^2}\left(\frac{b^2}{\rho^2}\right) B_0\left[\hat{x} + 2\hat{\phi}\sin\phi\right]$$