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Microeconomia II
Programa de Pós-Graduação em Economia
FEARP - USP

Seção 23.D e 23.E - Bayesian Implementation and Participation Constraints

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Exercise 1. Considere a Função de Escolha Social (FES) $f : \Theta \mapsto \mathbb{X}$, ou seja, para cada $\theta \in \Theta$

$$f(\theta) = [k(\theta), t_1(\theta), \cdots, t_I(\theta)],$$

$k(\theta) \in \mathbb{K}$ e $\sum_{i=1}^{I} t_i(\theta) \leqslant 0$. Mostre que

(a) Se $f(\cdot)$ é *ex post* eficiente, então $\forall\theta \in \Theta$,

  (i) $k(\theta)$ satisfaz
$$\sum_{i=1}^{I} v_i(k(\theta), \theta_i) \geqslant \sum_{i=1}^{I} v_i(k, \theta_i), \quad \forall k \in \mathbb{K} \tag{23.C.7}$$

  (ii) $t_i(\theta)$ satisfaz
$$\sum_{i=1}^{I} t_i(\theta) = 0. \tag{23.C.12}$$

(b) Se $f(\cdot)$ satisfaz (23.C.7) e (23.C.12), então $f(\cdot)$ é *ex post* eficiente.

Exercise 2. Consider a sealed bid first price auction with *iid* valuations uniformily distributed on $[0, 1]$. Compute the optimal reservation price and show that it is independent of the number of bidders. What is the optimal reservation price for a second-price auction in the same environment?

Exercise 3. Assuming *iid* uniformily distributed valuations, consider an auction in which the highest bidder pays 50% of his bid plus 50% of the bid of the second highest bidder. Solve for a symmetric BNE.

Exercise 4 (MWG 23.D.2). . Consider a bilateral trade setting in which each $\theta_i (i = 1, 2)$ is independently drawn from a uniform distribution on $[0, 1]$.

(a) Compute the transfer functions in the expected externality mechanism.

(b) Verify that truth telling is a Bayesian Nash equilibrium.

Exercise 5 (**MWG 23.D.5**). . For the same assumptions as in Exercise 23.D.4, consider a sealed-bid all-pay auction in which every buyer submits a bid, the highest bidder receives the good, and every buyer pays the seller the amount of his bid regardless of whether he wins. Argue that any symmetric equilibrium of his auction also yields the seller the same expected revenue as the sealed-bid second-price auction. [Hint: Follow similar steps as in Exercise 23.D.]

Exercise 6 (**MWG 23.D.6**). . Suppose that I symmetric individuals wish to acquire the single remaining ticket to a concert. The ticket office opens at 9 a.m on Monday. Each individual must decide what time to go to get on line: the first individual to get on line will get the ticket. An individual who waits t hours incurs a (monetary equivalent) disutility of βt. Suppose also that an individual showing up after the first one can go home immediately and do incurs no waiting cost. Individual i's value of receiving the ticket is θ_i, and each individual's θ_i is independently drawn from a uniform distribution on [0, 1]. What is the expected value of the number of hours that the first individual in line will wait? [Hint: Note the analogy to a first-price sealed-bid auction and use the revenue equivalence theorem.] How does this vary when β doubles? When I doubles?

Exercise 7. The federal government wants to build a new hospital and sets up an auction. The specific format chosen is a first-price closed-bid procurement auction, meaning that each bidder submits a price proposal for committing to construct the hospital and the contract is awarded to any of the bidders submitting a lowest price.

To be more concrete, suppose there are $N \geqslant 2$ companies bidding for the contract. Each bidder is characterized by a number θ which measures the quality of the hospital she can construct and is private information at the time of the auction. The types are independently and uniformily distributed over $[0, 1]$. There is a common cost function $c(\theta) = \theta^2$ for constructing a hospital of quality θ.

(a) Derive a symmetric Bayesian Nash equilibrium (be careful while writing the equilibrium probability of winning for a given type).

(b) Determine the equilibrium expected cost for the government and the equilibrium expected quality of the hospital.

Exercise 8 (**MWG 23.E.2**). . Argue that when the assumptions of Proposition 23.E.1 hold in the bilateral trade setting:

(a) There is no social choice function $f(.)$ that is dominant strategy incentive compatible and interim individually rational (i.e., that gives each agent i nonnegative gains from participation conditional on his type θ_i, for all θ_i).

(b) There is no social choice function $f(.)$ that is Bayesian incentive compatible and ex post individually rational [i.e., that gives each agent nonnegative gains from participation for every pair of types (θ_1, θ_2)].

Exercise 9 (**MWG 23.E.6**). . Consider a bilateral trading setting in which both agents initially own one unit of a good. Each agent i's ($i = 1, 2$) valuation per unit consumed of the good is θ_i. Assume that θ_i is independently drawn from a uniform distribution on [0, 1].

(a) Characterize the trading rule in an ex post efficient social choice function.

(b) Consider the following mechanism: Each agent submits a bid; the highest bidder buys the other agent's unit of the good and pays him the amount of his bid. Derive a symmetric Bayesian Nash equilibrium of this mechanism. [Hint: Look for one in which an agent's bid is a linear function of his type.]

(c) What is the social choice function that is implemented by this mechanism? Verify that it is Bayesian incentive compatible. Is it individually rational [which here requires that $U_i(\theta_i) \geqslant \theta_i$ for all θ_i and $i = 1, 2$]? Intuitively, why is there a difference from the conclusion of the Myerson Satterthwaite theorem? [See Cramton, Gibbons, and Klemperer (1987) for a formal analysis of these "partnership division" problems.]

Exercise 10 (**MWG 23.E.7**). . Consider a bilateral trade setting in which the buyer's and seller's valuations are drawn independently from the uniform distribution on [0, 1].

(a) Show that if $f(.)$ is a Bayesian incentive compatible and interim individually rational social choice function that is ex post efficient, the sum of the buyer's and seller's expected utilities under $f(.)$ cannot be less than 5/6.

(b) Show that, in fact there is no social choice function (whether Bayesian incentive compatible and interim individually rational or not) in which the sum of the buyer's and seller's expected utilities exceeds 2/3.

Exercise 11. Enuncie e demonstre a Proposição 23.E.1 (Myerson-Satterwaite Theorem).