ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Turbulência fantástica

aantunha

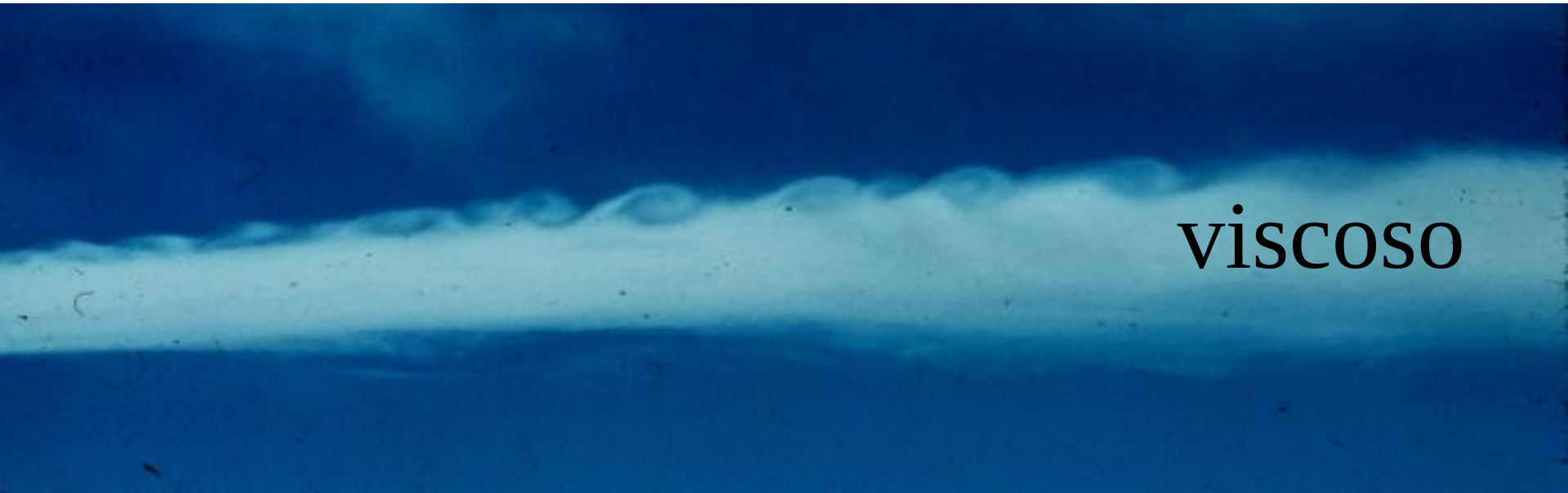
Demócrito 460 a 385 a.C.

Os mundos vem a ser como se segue: muitos corpos de todos os tipos e figura movem-se “por abscisão a partir do infinito” em um grande vazio; eles se juntam lá e produzem um único turbilhão (ou vórtice, dinê), no qual, colidindo uns com os outros e se revolvendo de todos os jeitos, eles começam a se separar, juntando semelhante com semelhante. Mas quando a sua multitude os impede de continuar rodando em equilíbrio, aqueles que são finos vão para fora , em direção ao vazio em volta, como se fossem sugados, enquanto o resto “permanece junto” e, embaraçando-se , une os seus movimentos e produz uma primeira estrutura esférica

atomismo

Essa estrutura permanece separada como uma “membrana”, que contém em si todos os tipos de corpos; e, enquanto eles rodopiam em volta, em razão da resistência do meio, a membrana ao redor torna-se fina, enquanto os átomos contíguos continuam flutuando juntos em razão do contato com o turbilhão. Assim, a terra veio a ser, com os átomos que ficaram no meio permanecendo lá juntos. Novamente a membrana abrangente é aumentada por causa da atração dos corpos externos. Conforme ela se move dentro do turbilhão, agarra tudo aquilo que toca. Alguns desses corpos que se embaraçam formam uma estrutura que primeiro é úmida e turva mas, conforme elas giram com o turbilhão do todo, eles se secam e depois se inflamam, para formar a substância dos corpos celestes.

vácuo

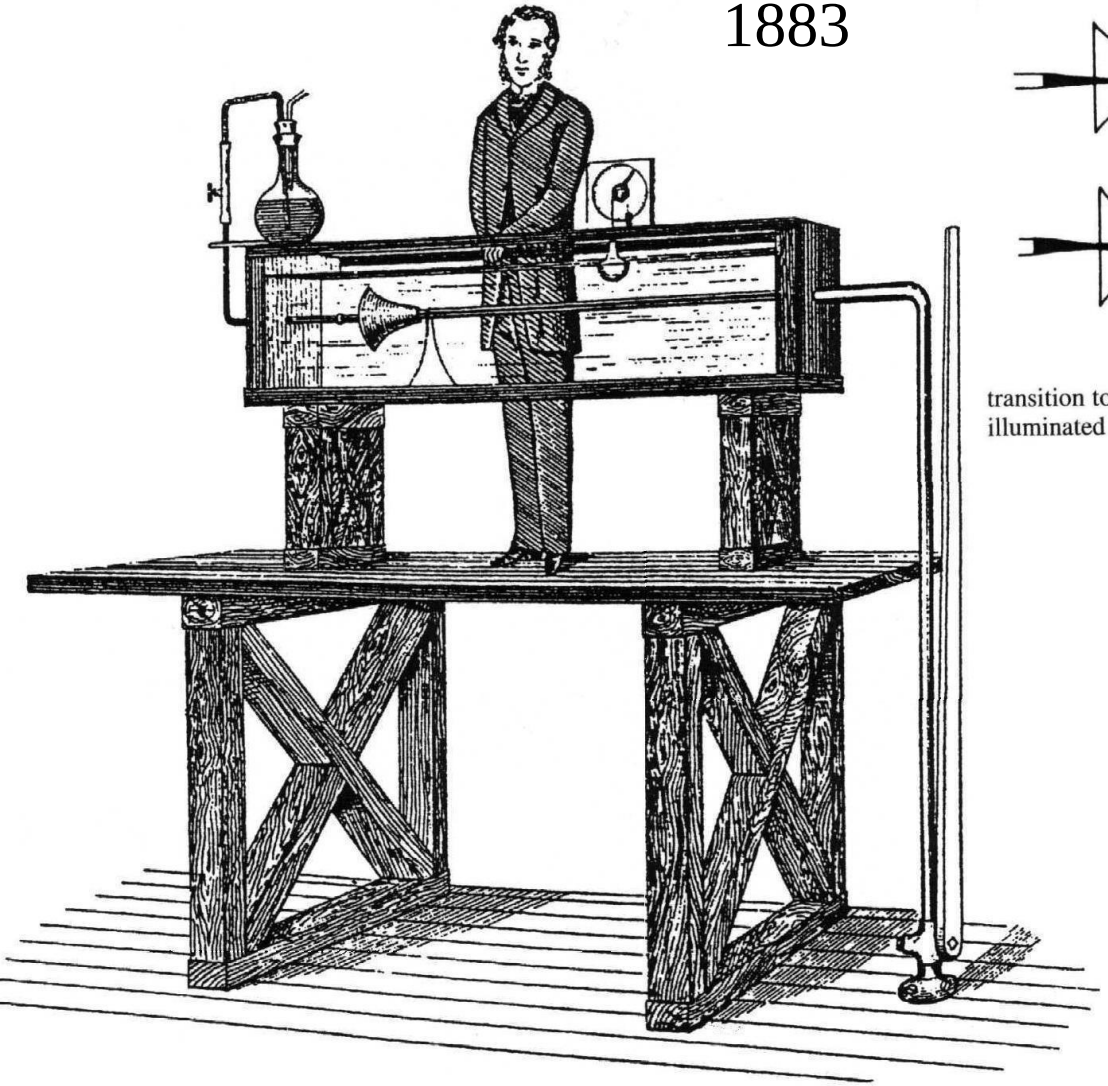

viscoso

da Vinci

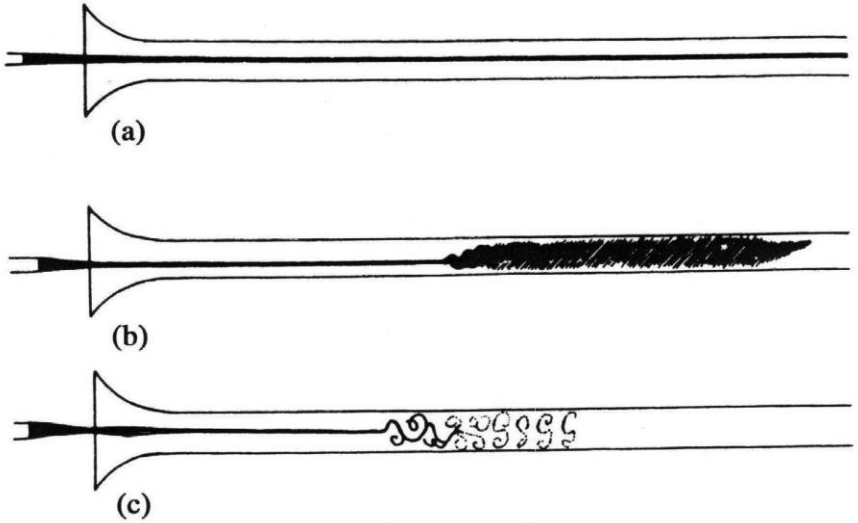
1452-1519

Reynolds

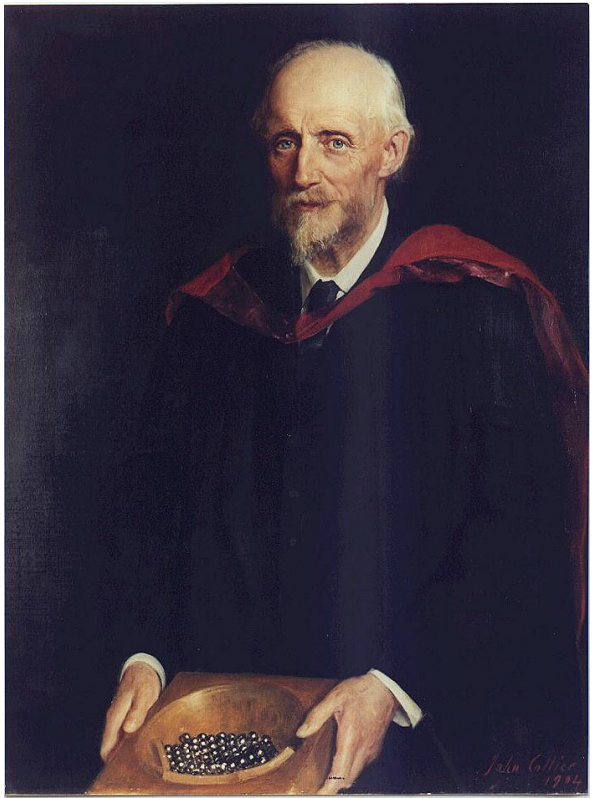
1883



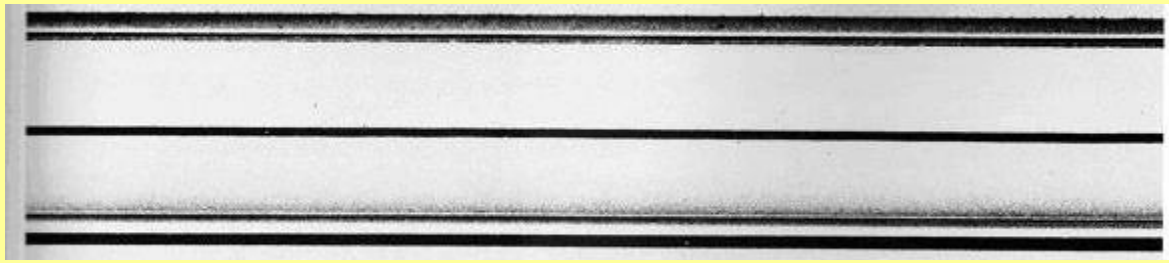
The configuration of Reynolds's experiment on flow along a pipe.



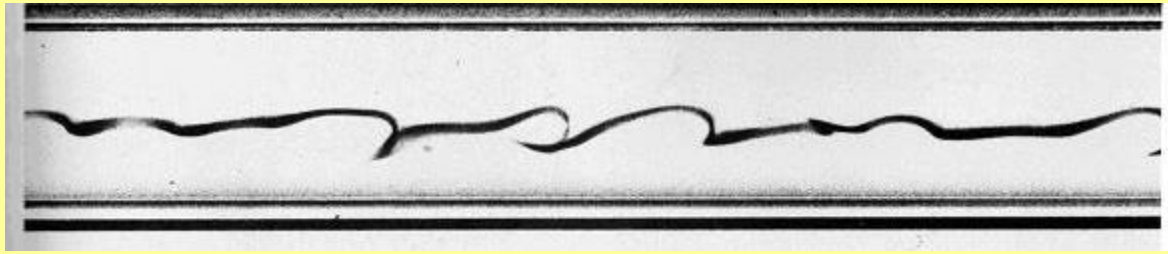
Sketches of (a) laminar flow in a pipe, indicated by a dye streak; (b) transition to turbulent flow in a pipe; and (c) transition to turbulent flow as seen when illuminated by a spark. (From Reynolds, 1883, Figs. 3, 4 and 5.)



Reynolds



rotacional

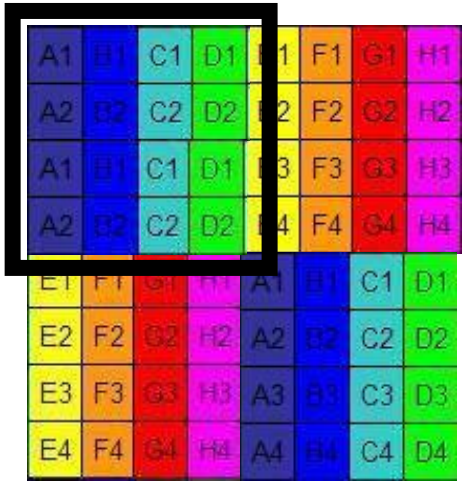


3D

irreversível

padeiro

A1 B1 C1 D1 E1 F1 G1 H1
A2 B2 C2 D2 E2 F2 G2 H2
A3 B3 C3 D3 E3 F3 G3 H3
A4 B4 C4 D4 E4 F4 G4 H4
A1 B1 C1 D1 E1 F1 G1 H1
A2 B2 C2 D2 E2 F2 G2 H2
A3 B3 C3 D3 E3 F3 G3 H3
A4 B4 C4 D4 E4 F4 G4 H4


A1 B1 C1 D1 E1 F1 G1 H1
A2 B2 C2 D2 E2 F2 G2 H2
A1 B1 C1 D1 E3 F3 G3 H3
A2 B2 C2 D2 E4 F4 G4 H4
E1 F1 G1 H1 A1 B1 C1 D1
E2 F2 G2 H2 A2 B2 C2 D2
E3 F3 G3 H3 A3 B3 C3 D3
E4 F4 G4 H4 A4 B4 C4 D4

padeiro

A1	B1	C1	D1	E1	F1	G1	H1
A2	B2	C2	D2	E2	F2	G2	H2
A3	B3	C3	D3	E3	F3	G3	H3
A4	B4	C4	D4	E4	F4	G4	H4
A5	B5	C5	D5	E5	F5	G5	H5
A6	B6	C6	D6	E6	F6	G6	H6
A7	B7	C7	D7	E7	F7	G7	H7
A8	B8	C8	D8	E8	F8	G8	H8

1

A1	B1	C1	D1	E1	F1	G1	H1
A2	B2	C2	D2	E2	F2	G2	H2
A3	B3	C3	D3	E3	F3	G3	H3
A4	B4	C4	D4	E4	F4	G4	H4
E5	F5	G5	H5	A5	B5	C5	D5
E6	F6	G6	H6	A6	B6	C6	D6
E7	F7	G7	H7	A7	B7	C7	D7
E8	F8	G8	H8	A8	B8	C8	D8

2

A1	B1	C1	D1	E1	F1	G1	H1
A2	B2	C2	D2	E2	F2	G2	H2
C3	D3	A3	B3	G3	H3	E3	F3
C4	D4	A4	B4	G4	H4	E4	F4
E5	F5	G5	H5	A5	B5	C5	D5
E6	F6	G6	H6	A6	B6	C6	D6
G7	H7	E7	F7	C7	D7	A7	B7
G8	H8	E8	F8	C8	D8	A8	B8

3

A1	B1	C1	D1	E1	F1	G1	H1
B2	A2	D2	C2	F2	E2	H2	G2
C3	D3	A3	B3	G3	H3	E3	F3
D4	C4	B4	A4	H4	G4	F4	E4
E5	F5	G5	H5	A5	B5	C5	D5
F6	E6	H6	G6	B6	A6	D6	C6
G7	H7	E7	F7	C7	D7	A7	B7
H8	G8	F8	E8	D8	C8	B8	A8

4

sucessivamente

A1	B1	C1	D1	E1	F1	G1	H1
B2	A2	D2	C2	F2	E2	H2	G2
C3	D3	A3	B3	G3	H3	E3	F3
D4	C4	B4	A4	H4	G4	F4	E4
E5	F5	G5	H5	A5	B5	C5	D5
F6	E6	H6	G6	B6	A6	D6	C6
G7	H7	E7	F7	C7	D7	A7	B7
H8	G8	F8	E8	D8	C8	B8	A8

1

A1	B1	C1	D1	E1	F1	G1	H1
B2	A2	D2	C2	F2	E2	H2	G2
C3	D3	A3	B3	G3	H3	E3	F3
D4	C4	B4	A4	H4	G4	F4	E4
A5	B5	C5	D5	E5	F5	G5	H5
B6	A6	D6	C6	F6	E6	H6	G6
C7	D7	A7	B7	G7	H7	E7	F7
D8	C8	B8	A8	H8	G8	F8	E8

2

A1	B1	C1	D1	E1	F1	G1	H1
B2	A2	D2	C2	F2	E2	H2	G2
A3	B3	C3	D3	E3	F3	G3	H3
B4	A4	D4	C4	F4	E4	H4	G4
A5	B5	C5	D5	E5	F5	G5	H5
B6	A6	D6	C6	F6	E6	H6	G6
A7	B7	C7	D7	E7	F7	G7	H7
B8	A8	D8	C8	F8	E8	H8	G8

3

A1	B1	C1	D1	E1	F1	G1	H1
A2	B2	C2	D2	E2	F2	G2	H2
A3	B3	C3	D3	E3	F3	G3	H3
A4	B4	C4	D4	E4	F4	G4	H4
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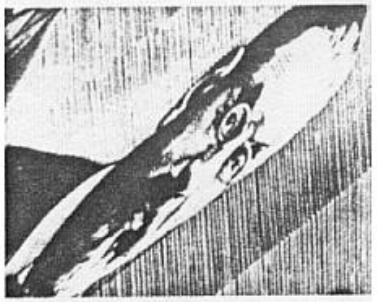
A6	B6	C6	D6	E6	F6	G6	H6
A7	B7	C7	D7	E7	F7	G7	H7
A8	B8	C8	D8	E8	F8	G8	H8

4

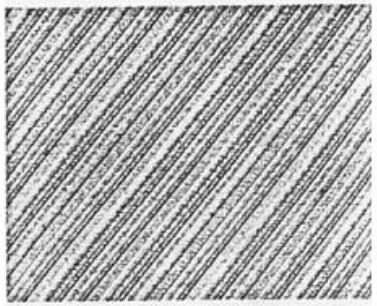
não destrói informação => revertível => não serve

Poincaré

N = 0



N = 1



N = 10

N = 48

N = 241

Figure 4.3. The stretching and folding of change is illustrated by this computer mosaic of the father of modern chaos, Henri Poincaré. Poincaré's image was digitalized by physicist James Crutchfield so it could be stretched mathematically as if it were painted on a sheet of rubber. Crutchfield uses the image to show how positive feedback or iteration can transform things. Iterating the computer formula, Poincaré's image is stretched on the sheet diagonally, and the leftover part is reinserted on the other side. The number above each panel tells how many iterations of this process have taken place. As the iterations go on Poincaré's face is scrambled randomly until it's completely homogenized.

However, as the folding operation continues, it may happen that some of the points come close enough to their initial positions for the image to reappear. In other words, a brief intermittency of order occurs before the iterative folding shears the points apart again. Crutchfield's equation makes a momentary return to a state close to the initial conditions (known to scientists as the "Poincaré recurrence") much more likely than it would be in typical chaotic transformations. In "typical" chaos, the chances of Poincaré's face showing up again as the system iterates are astronomically small, particularly if there is any background interference. A small spike in the electrical signal to the computer, for example, would become folded in by the iterations and would destroy the original information.

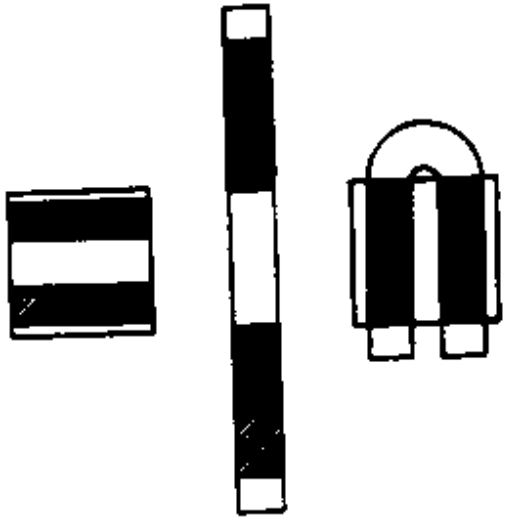


Fig. 36. Representation of the Smale horseshoe map.

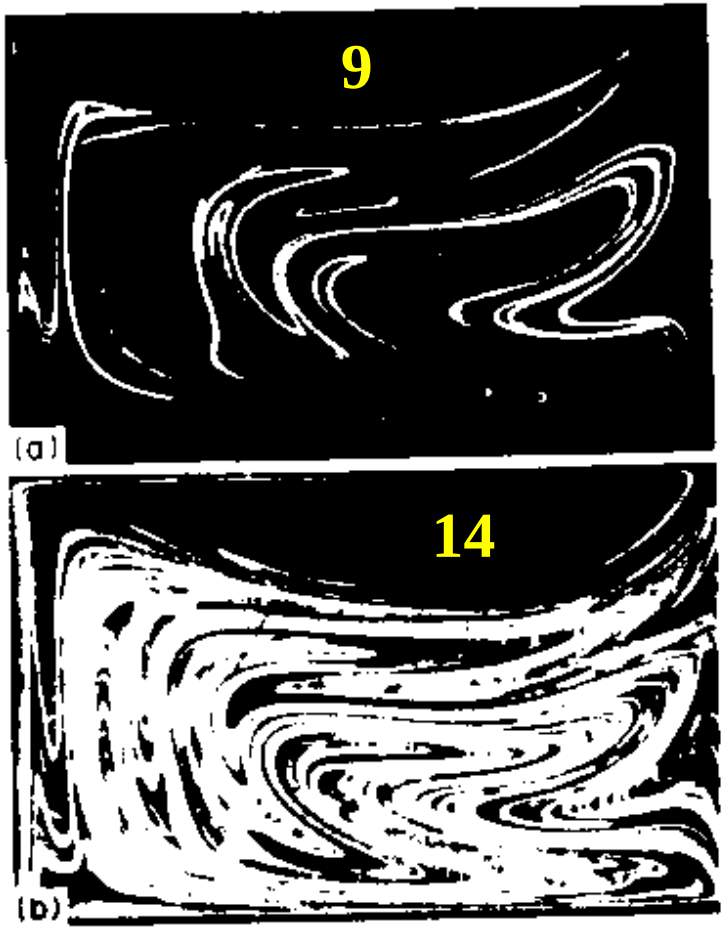
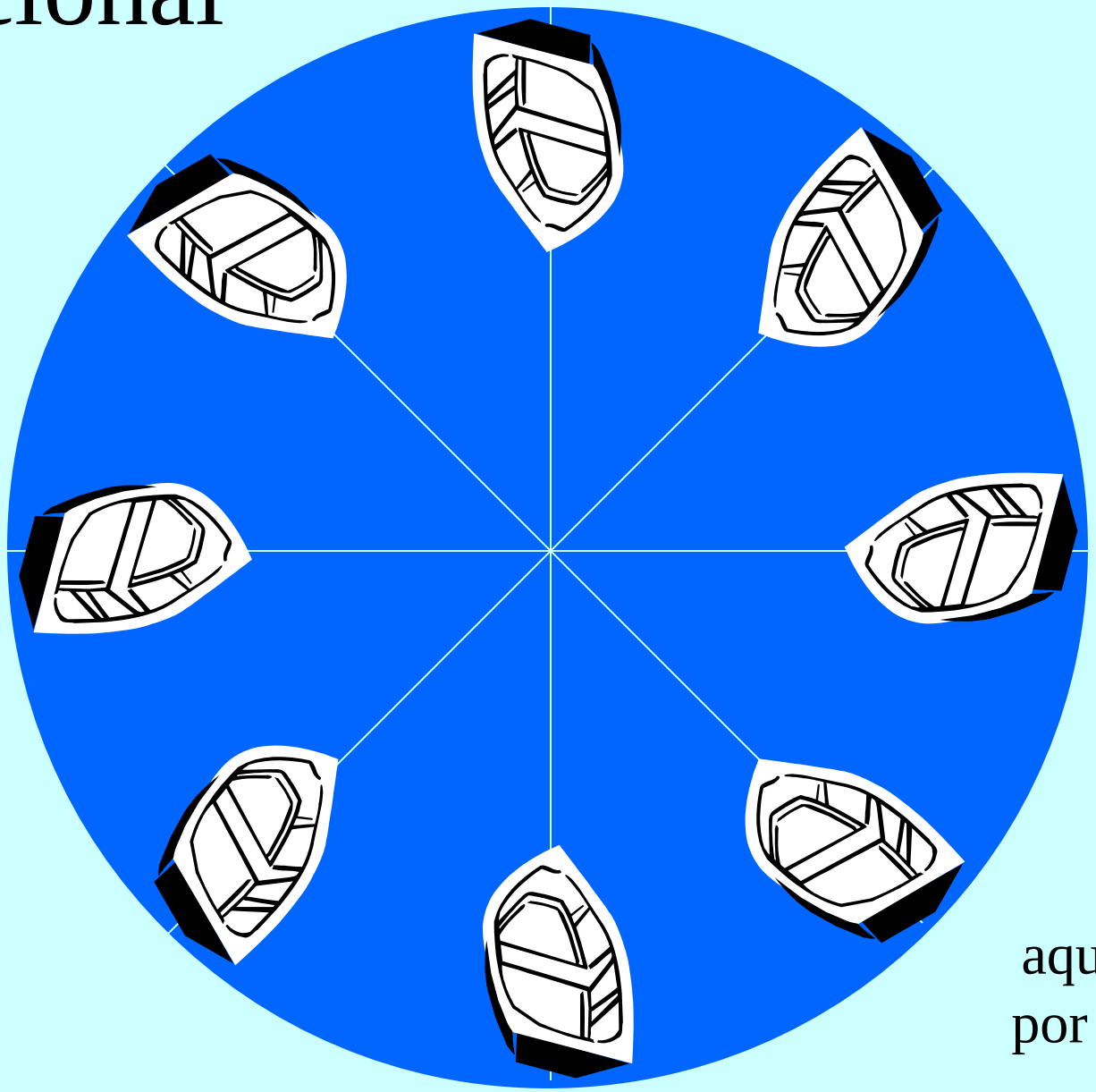


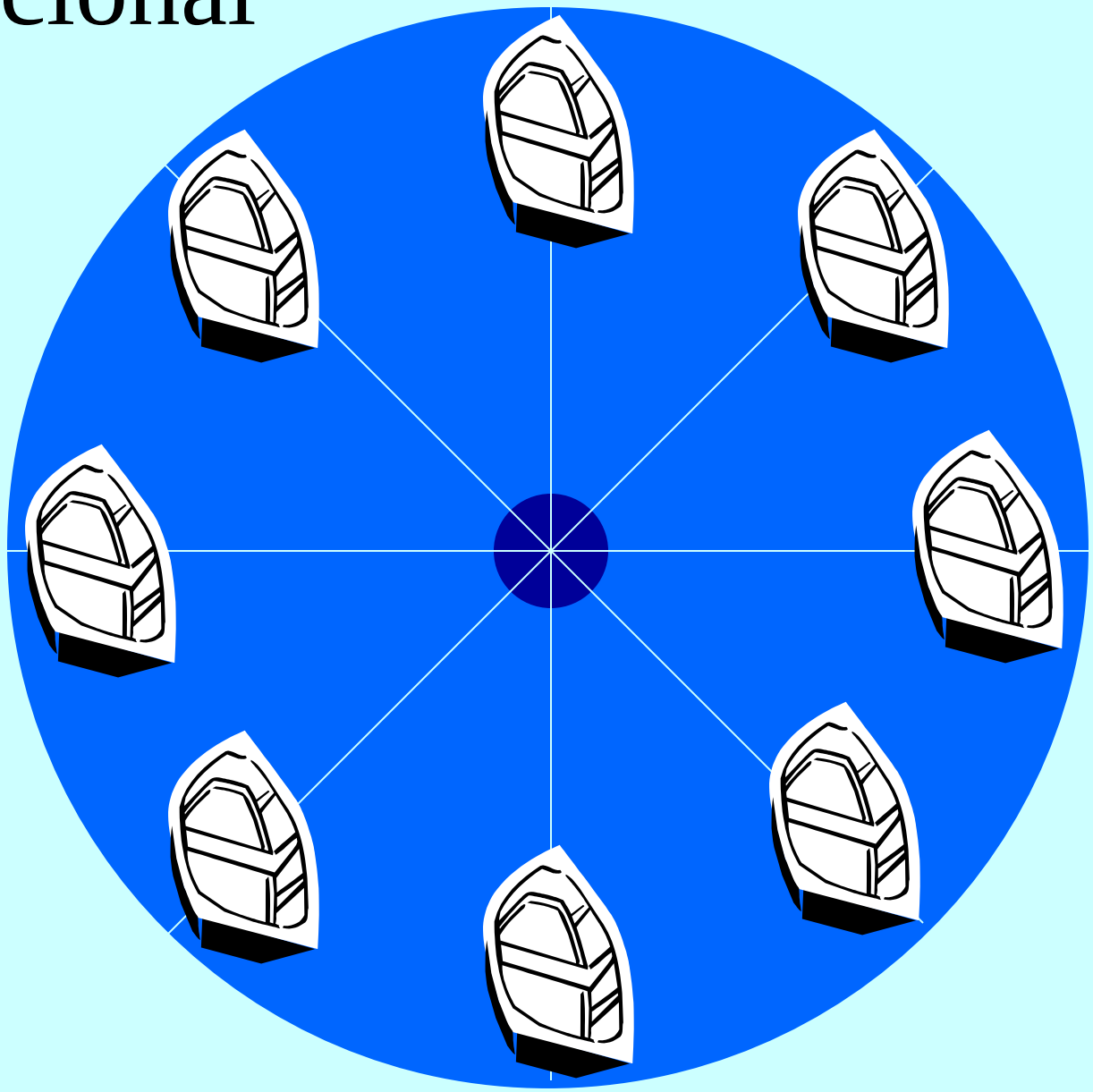
Fig. 37. Mixing of a blob of a tracer in low Reynolds number cavity flow (an improved version of the apparatus described in Chien *et al.* (1986). The fluid is glycerine and the tracer is a fluorescent dye: illumination is provided by ultraviolet light. The system mixes by periodic operation of the upper and lower boundaries moving in sinusoidal fashion (corotating) with a period of 30 s. (a) represents the system after nine cycles whereas (b) represents the system after 14 cycles. Note the almost exponential mixing growth and the "holes" of unmixed material (regular region). Even though "holes" deform by repeated application of the cycles they do not disappear even for long times. From Leong (1987).

rotacional

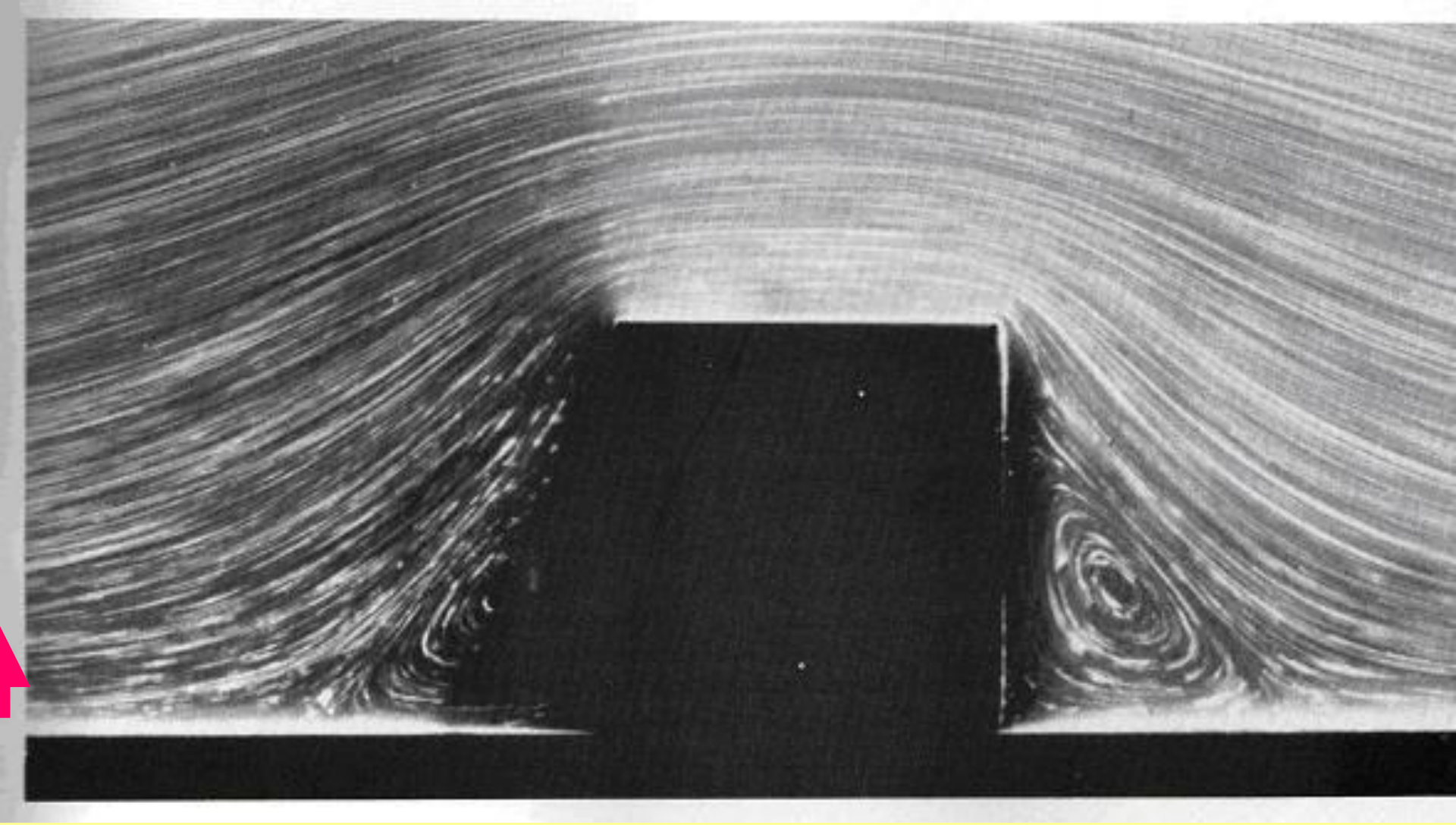


aquário girando
por muito tempo

irrotacional



ralo

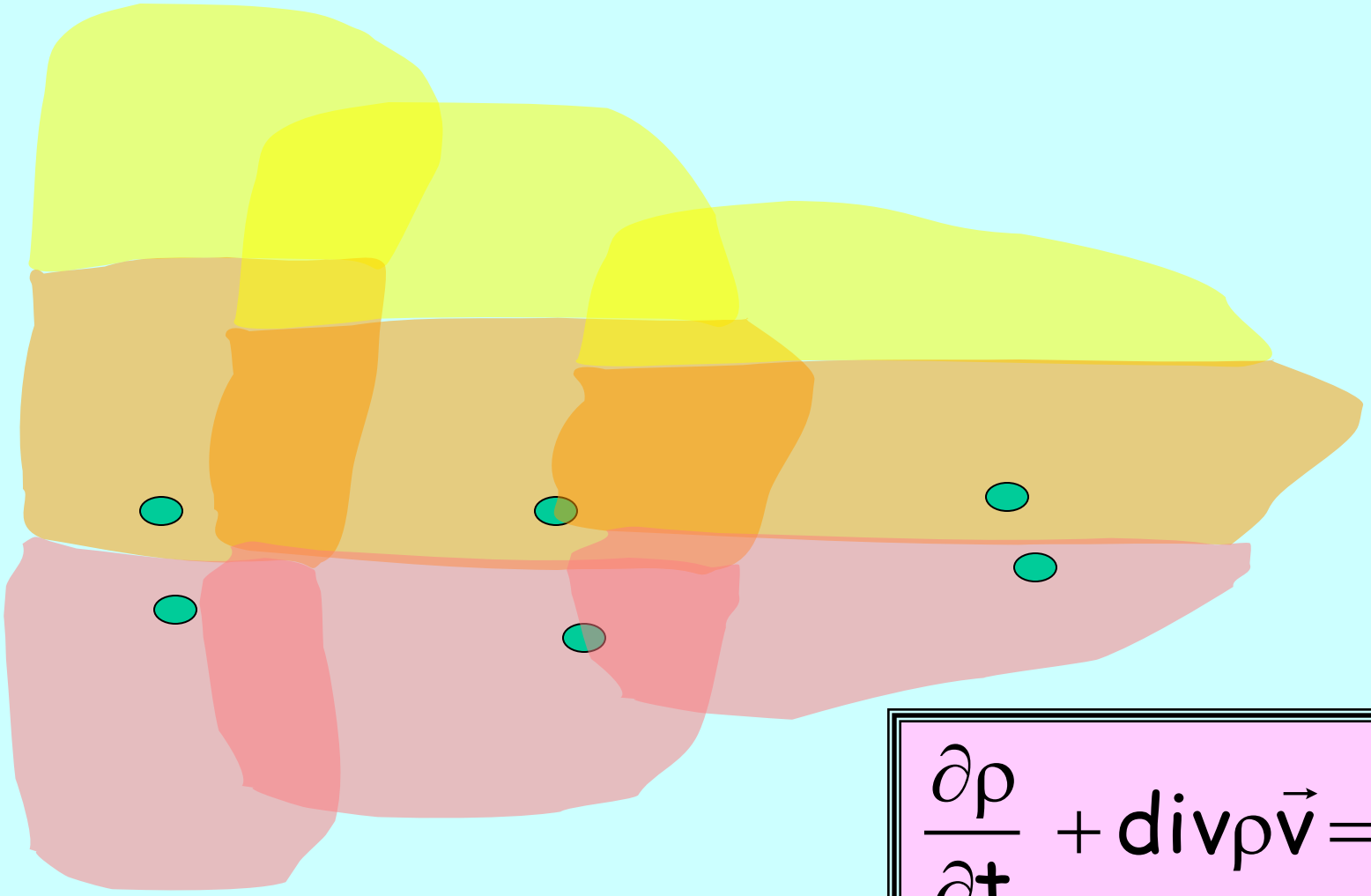


creeping flow

altamente viscoso Re = 0,02

rotacional
laminar

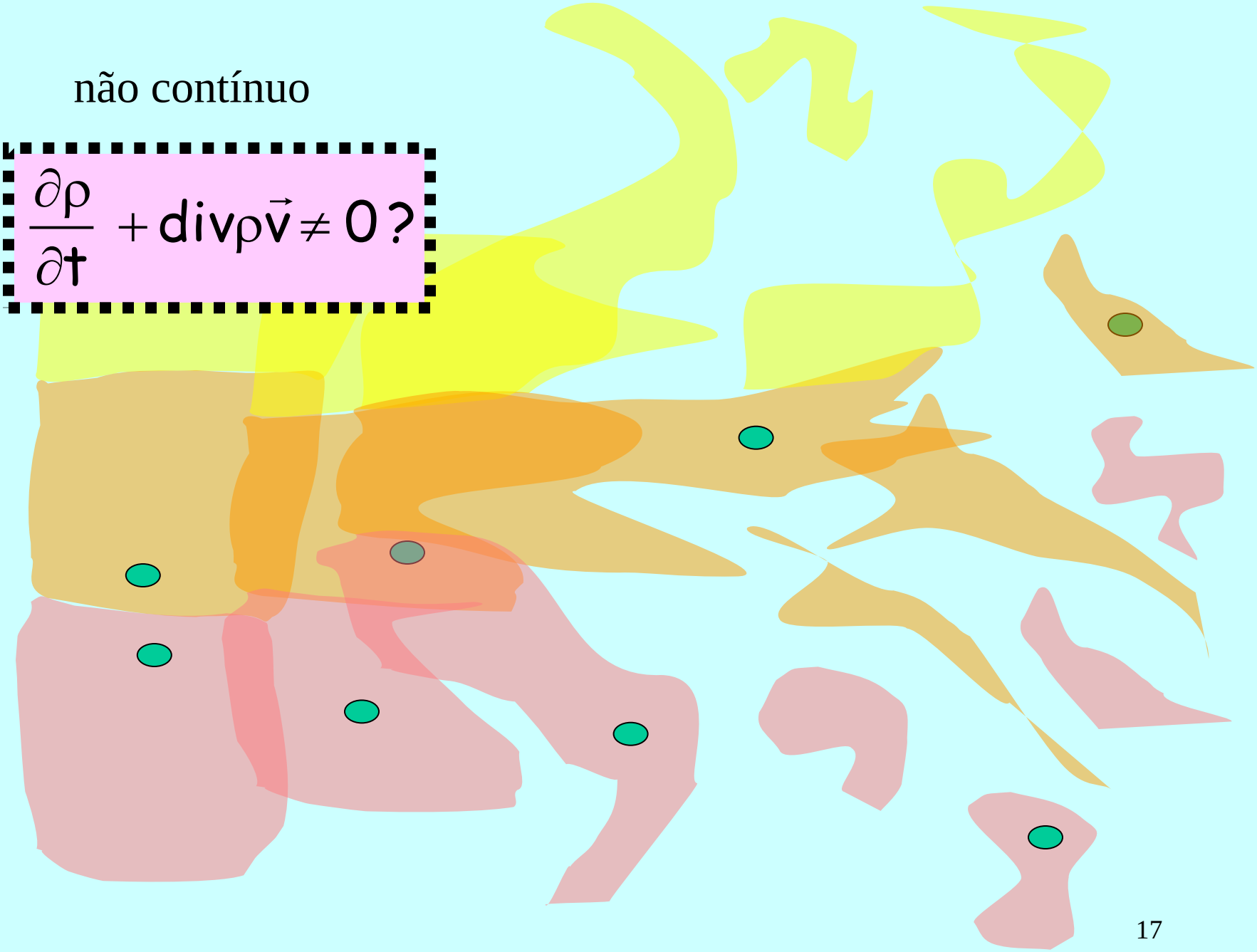
sempre conexo e vizinhos $\rightarrow$ contínuo



$$\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\vec{v} = 0$$

não contínuo

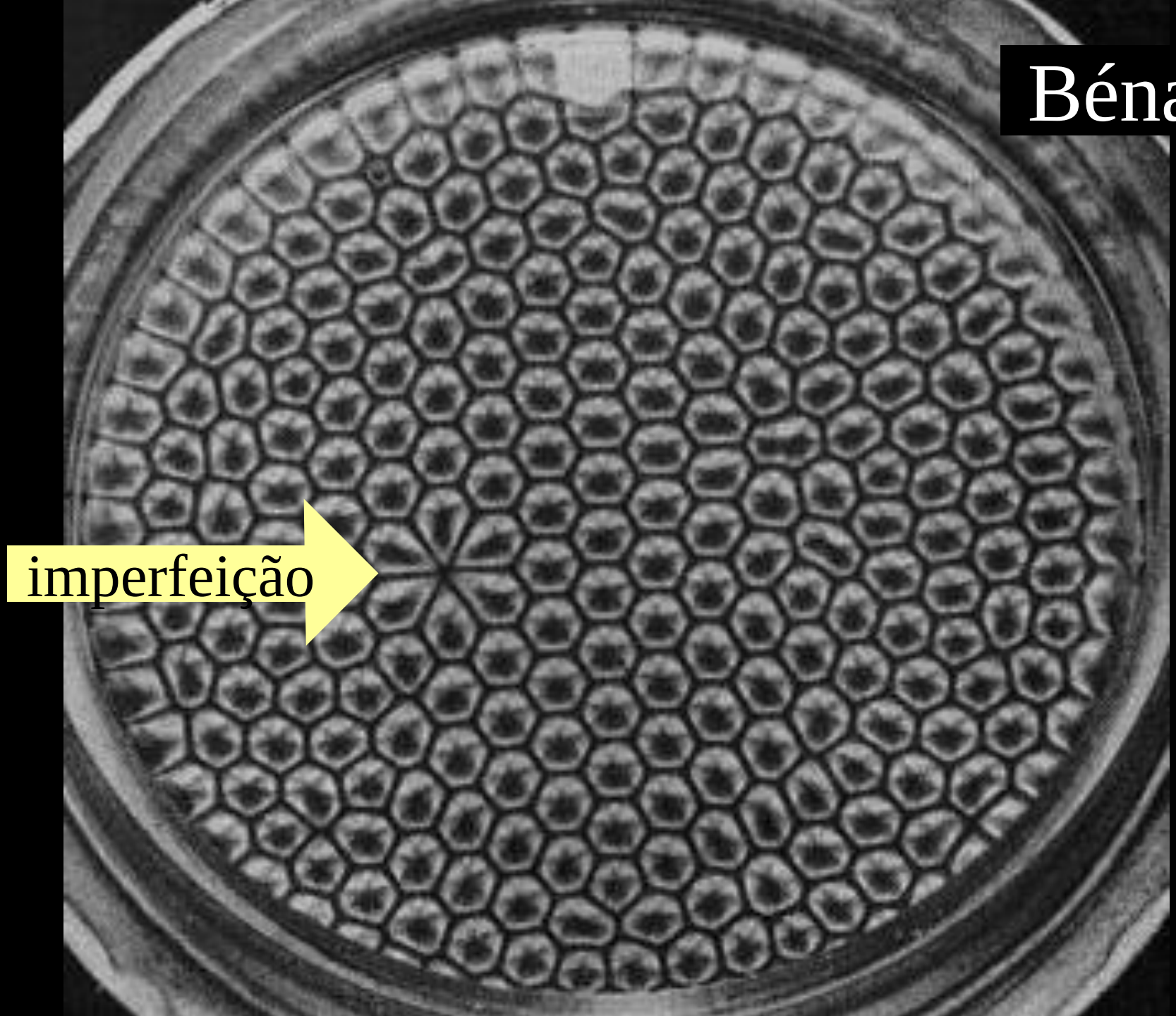
$$\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\vec{v} \neq 0\,?$$



descontínuo

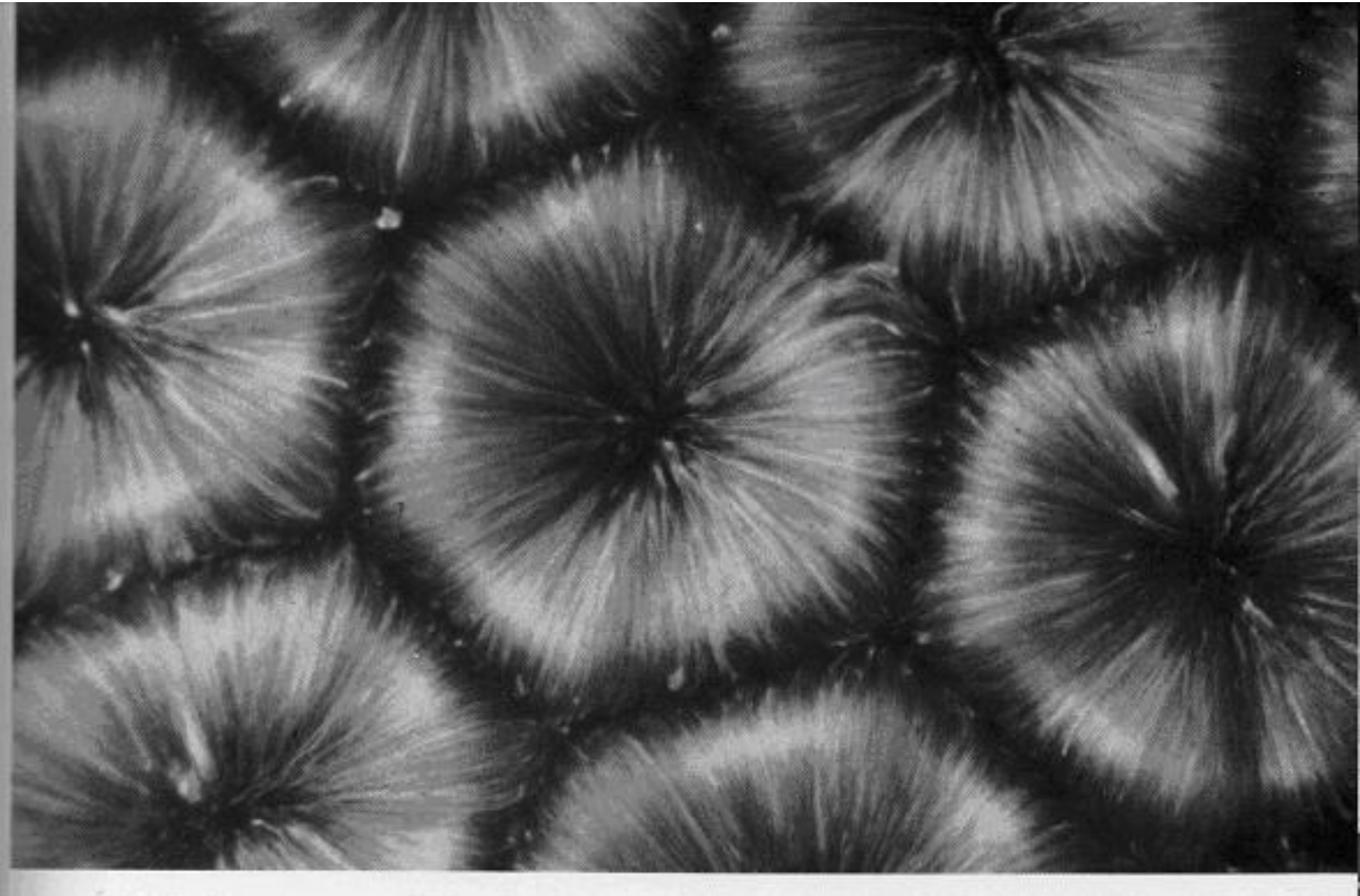
1 ou 2D tubos de corrente laminares

3D: tubos de corrente turbulentos


Bénard
imperfeição

oleo de silicone 1mm

zoom 25 x



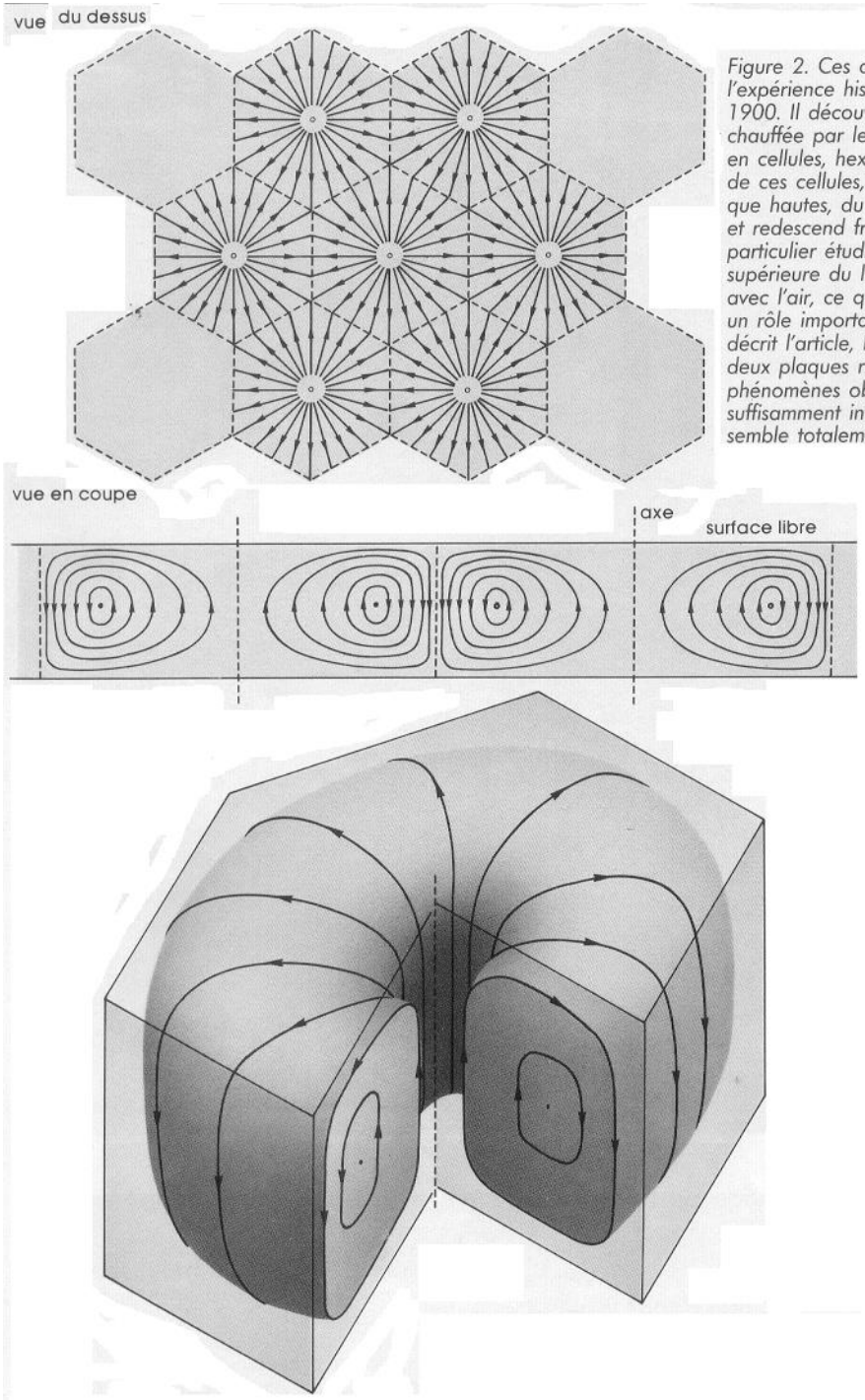
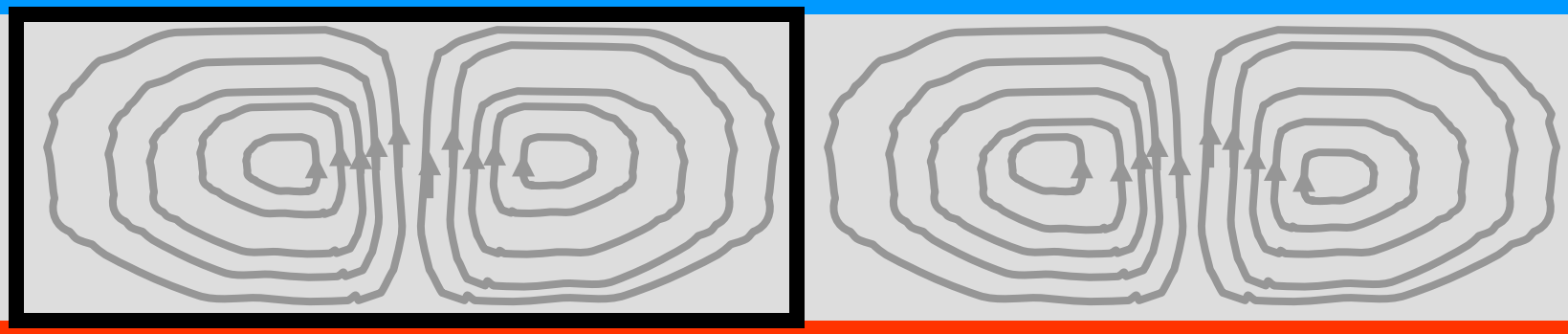


Figure 2. Ces dessins représentent l'expérience historique que réalisa Bénard en 1900. Il découvrit qu'une couche fluide, chauffée par le bas, s'organise spontanément en cellules, hexagonales ici. Dans chacune de ces cellules, environ deux fois plus larges que hautes, du fluide chaud monte au centre et redescend froid sur les bords. Dans le cas particulier étudié par Bénard, la surface supérieure du liquide était en contact libre avec l'air, ce qui faisait jouer à la capillarité un rôle important. Dans les expériences que décrit l'article, le fluide est enfermé entre deux plaques rigides, ce qui simplifie les phénomènes observés, mais le chauffage est suffisamment intense pour que l'écoulement semble totalement désordonné.

frio

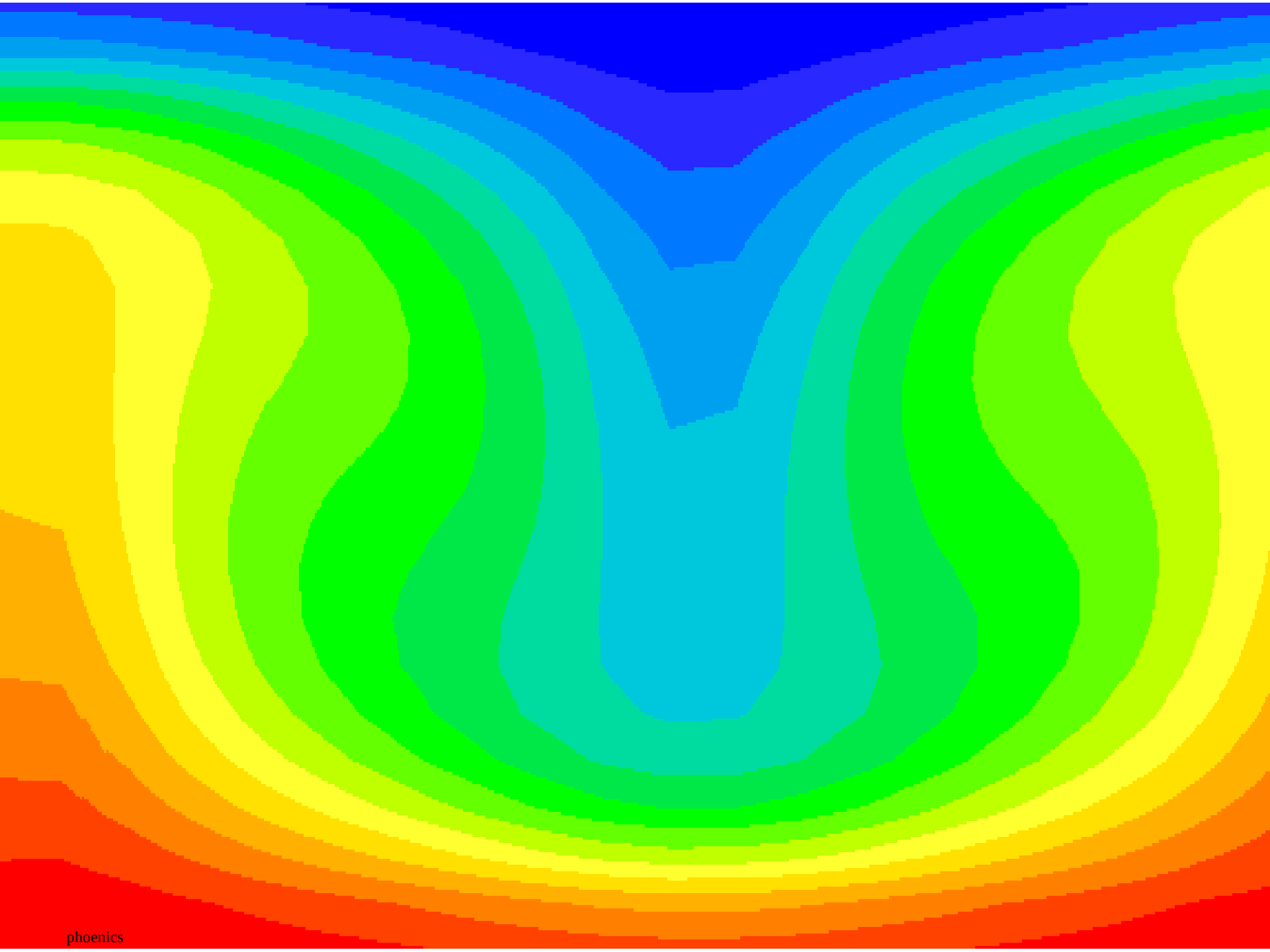
óleo

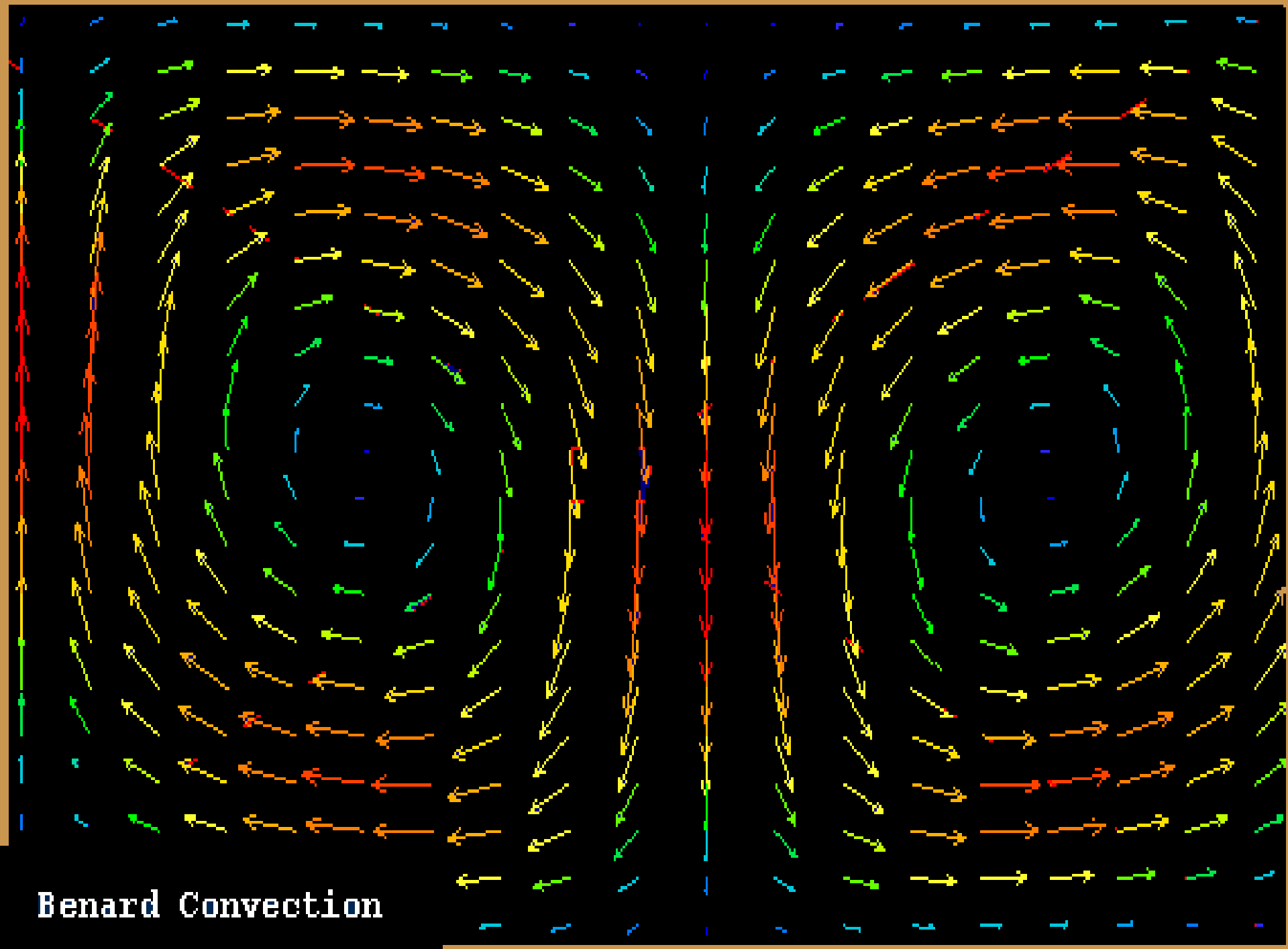


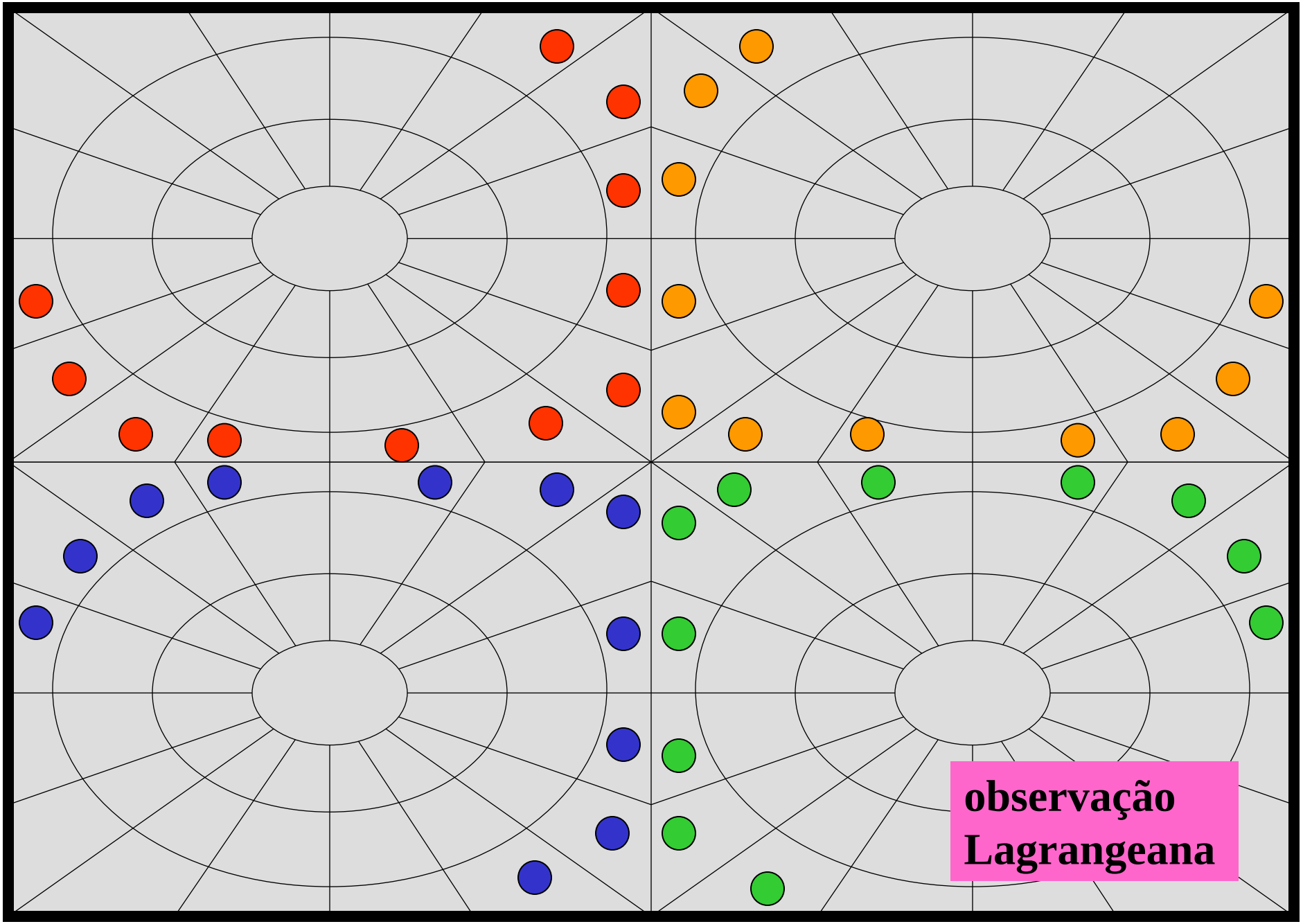
quente

Células de Bénard

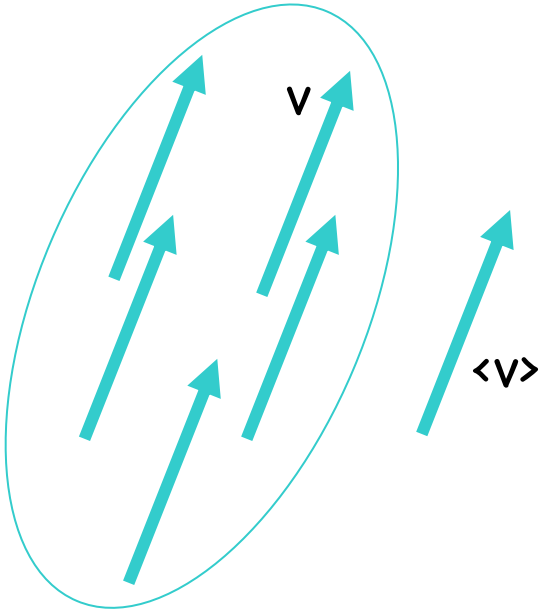
esquema


phoenics


Benard Convection



Lagrange

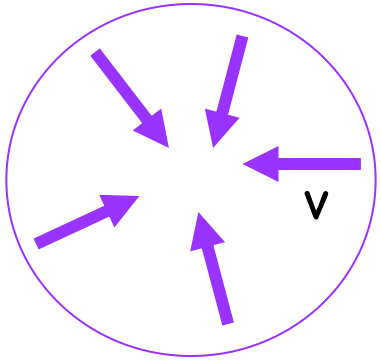


$Ec^e = 5mv^2/2$

$Ec^i = 0$

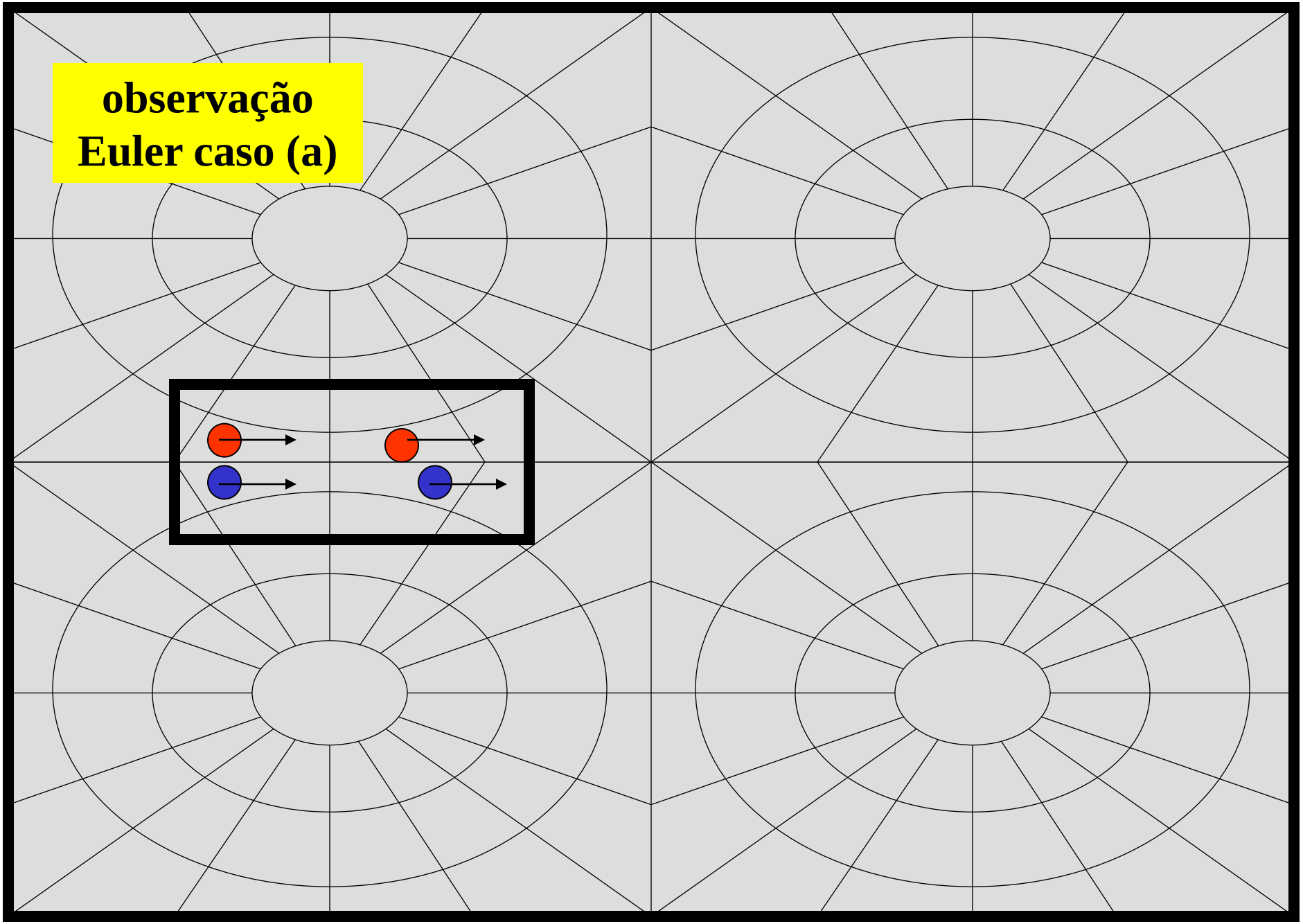
comparação

<v> = 0



$Ec^e = 0$

$Ec^i = 5mv^2/2$


observação
Euler caso (a)

velocidade média

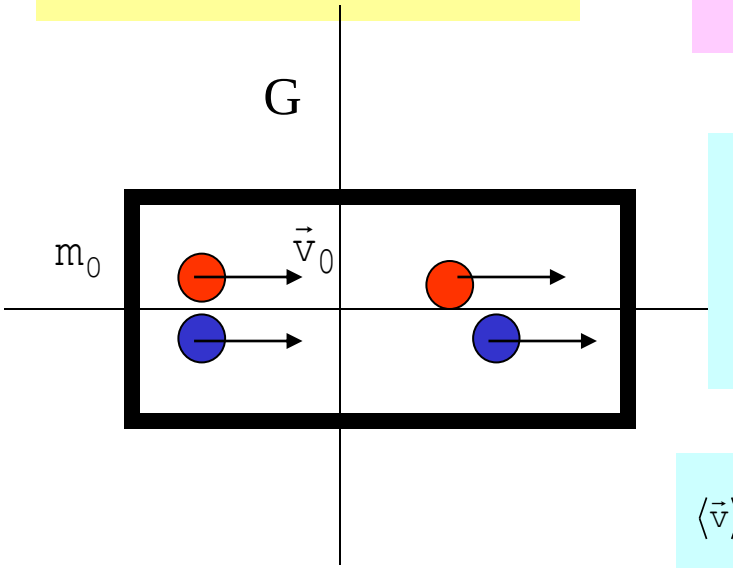
$$\langle \vec{v} \rangle = \frac{\sum_k m_k \vec{v}_k}{\sum_k m_k}$$

variância

$$\langle v^2 \rangle = \frac{\sum_k m_k (\vec{v}_k . \vec{v}_k)}{\sum_k m_k}$$

Intensidade de Turbulência

$$I = \frac{\sqrt{\langle v'^2 \rangle}}{\sqrt{\langle v^2 \rangle}}$$



caso (a)

velocidade relativa

$$\vec{v}_k = \langle \vec{v} \rangle + \vec{v}'_k$$

$$\langle \vec{v}'_k \rangle = 0$$

$$\langle v^2 \rangle = \langle \vec{v}.\vec{v} \rangle = \langle \vec{v} \rangle^2 + \langle \vec{v}'^2 \rangle$$

$$\langle \vec{v} \rangle_a = \frac{1+1+1+1}{4\, m_o} m_o v_o \vec{I} = v_o \vec{I}$$

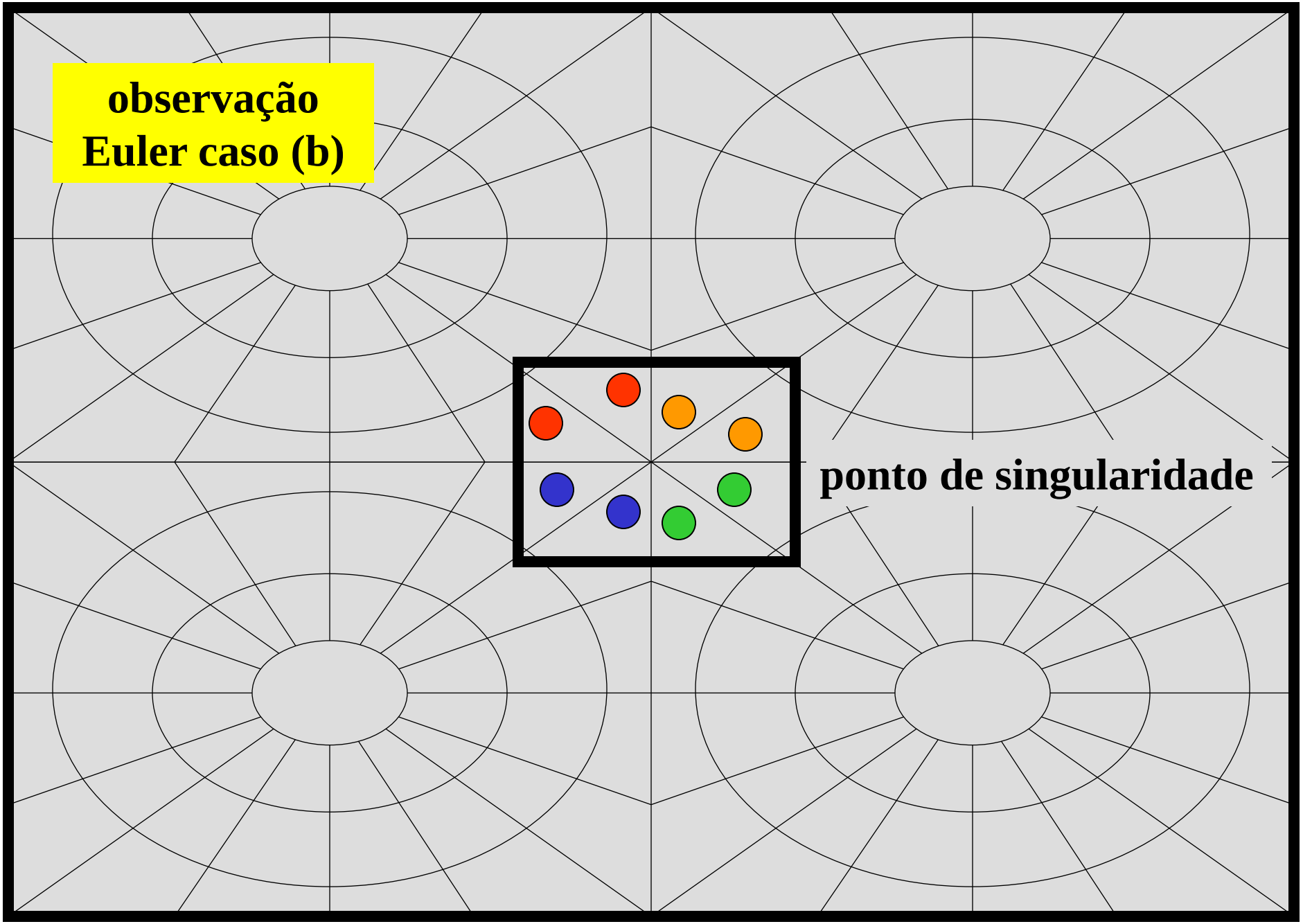
$$\langle \vec{v}' \rangle_a = 0 + 0 + 0 + 0 = 0$$

$$\langle v^2 \rangle_a = \frac{1+1+1+1}{4\, m_o} m_o v_o^2 = v_o^2$$

K

$$\langle \vec{v}'^2 \rangle_a = 0 + 0 + 0 + 0 = 0$$

$$I_a = \frac{\sqrt{0}}{\sqrt{v_o^2}} = 0$$


observação
Euler caso (b)
ponto de singularidade

velocidade média

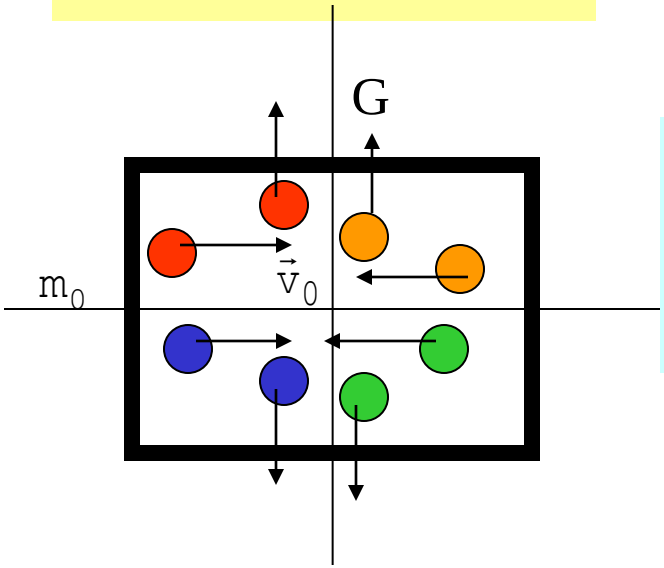
$$\langle \vec{v} \rangle = \frac{\sum_k m_k \vec{v}_k}{\sum_k m_k}$$

variância

$$\langle v^2 \rangle = \frac{\sum_k m_k (\vec{v}_k . \vec{v}_k)}{\sum_k m_k}$$

Intensidade de Turbulência

$$I = \frac{\sqrt{\langle v'^2 \rangle}}{\sqrt{\langle v^2 \rangle}}$$



velocidade relativa

$$\vec{v}_k = \langle \vec{v} \rangle + \vec{v}'_k$$

$$\langle \vec{v}'_k \rangle = 0$$

$$\langle v^2 \rangle = \langle \vec{v}.\vec{v} \rangle = \langle \vec{v} \rangle^2 + \langle \vec{v}'^2 \rangle$$

caso (b)

singularidade

$$\langle \vec{v} \rangle_b = \frac{(2-2)\vec{I} + (2-2)\vec{J}}{8\, m_o} m_o v_o = 0$$

$$\langle \vec{v}' \rangle_b = (2-2) + (2-2) = 0$$

$$\langle v^2 \rangle_b = \frac{8\,(1)}{8\, m_o} m_o v_o^2 = v_o^2$$

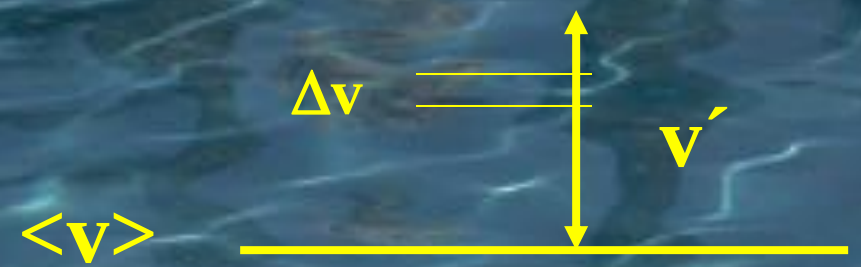
K

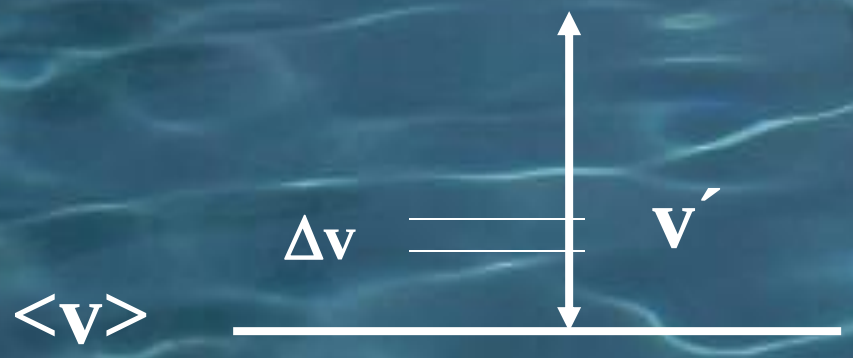
$$\langle \vec{v}'^2 \rangle_b = 8\, v_o^2 / 8 = v_o^2$$

$$I_a = \frac{\sqrt{v_o^2}}{\sqrt{v_o^2}} = 1$$

$$0 \le I \le 1$$

$$v = \langle v \rangle + v'$$

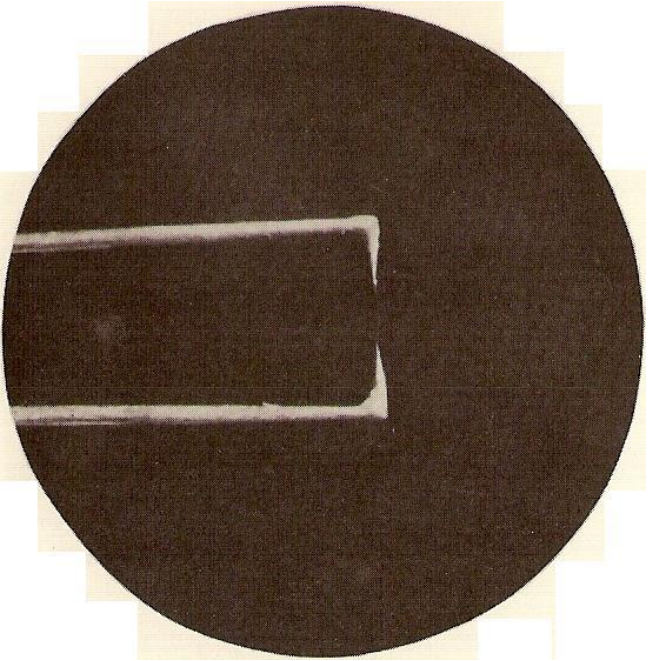




$$f_v = f_v(t, r)$$

$$f_v = f_v[\, v\, ,\, v+\Delta v\,]$$

f_v = Probality Density Function = PDF



anemômetro de fio quente

$$\vec{v} = \vec{v}' + \overline{\vec{v}}$$

osciloscópio

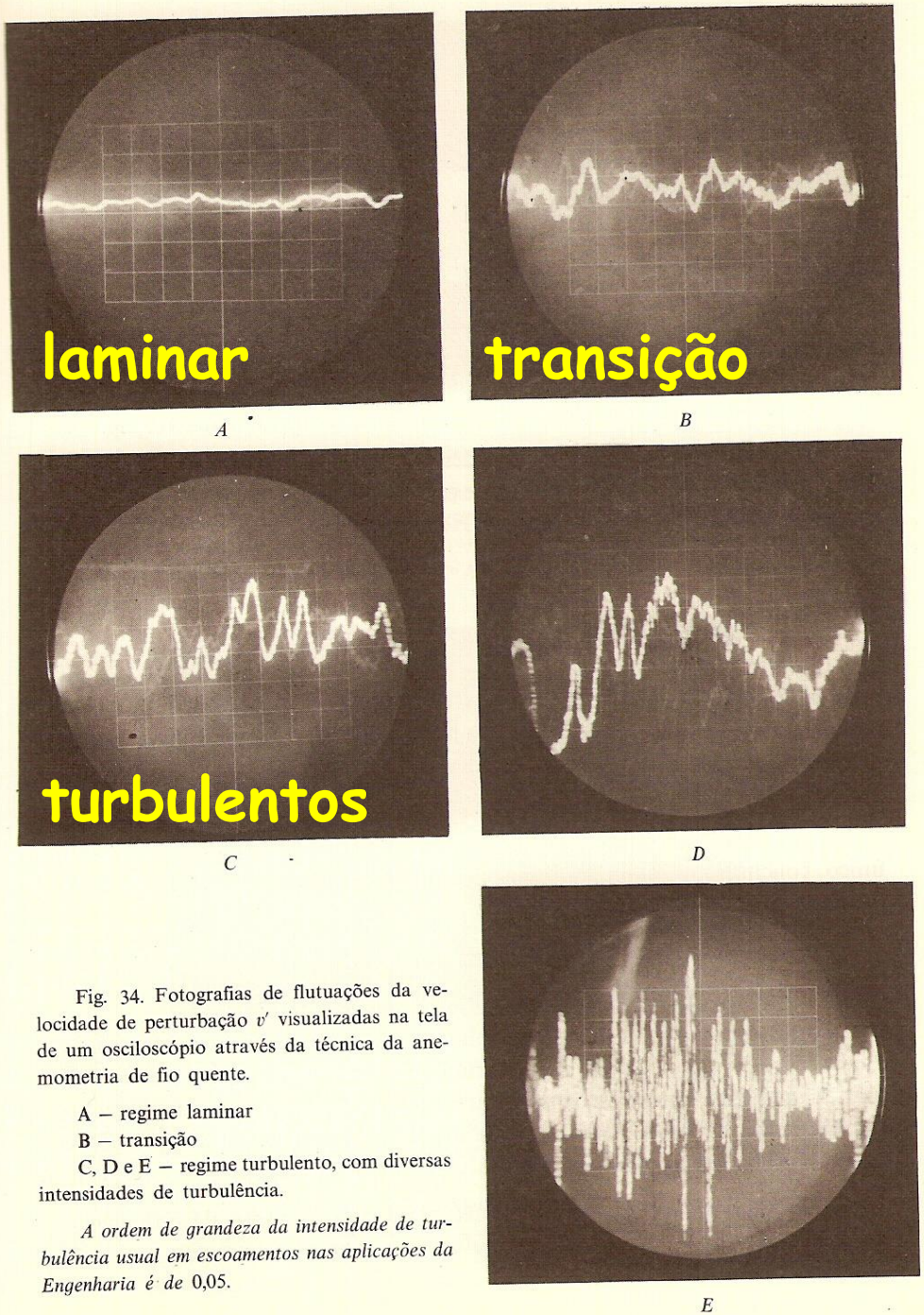
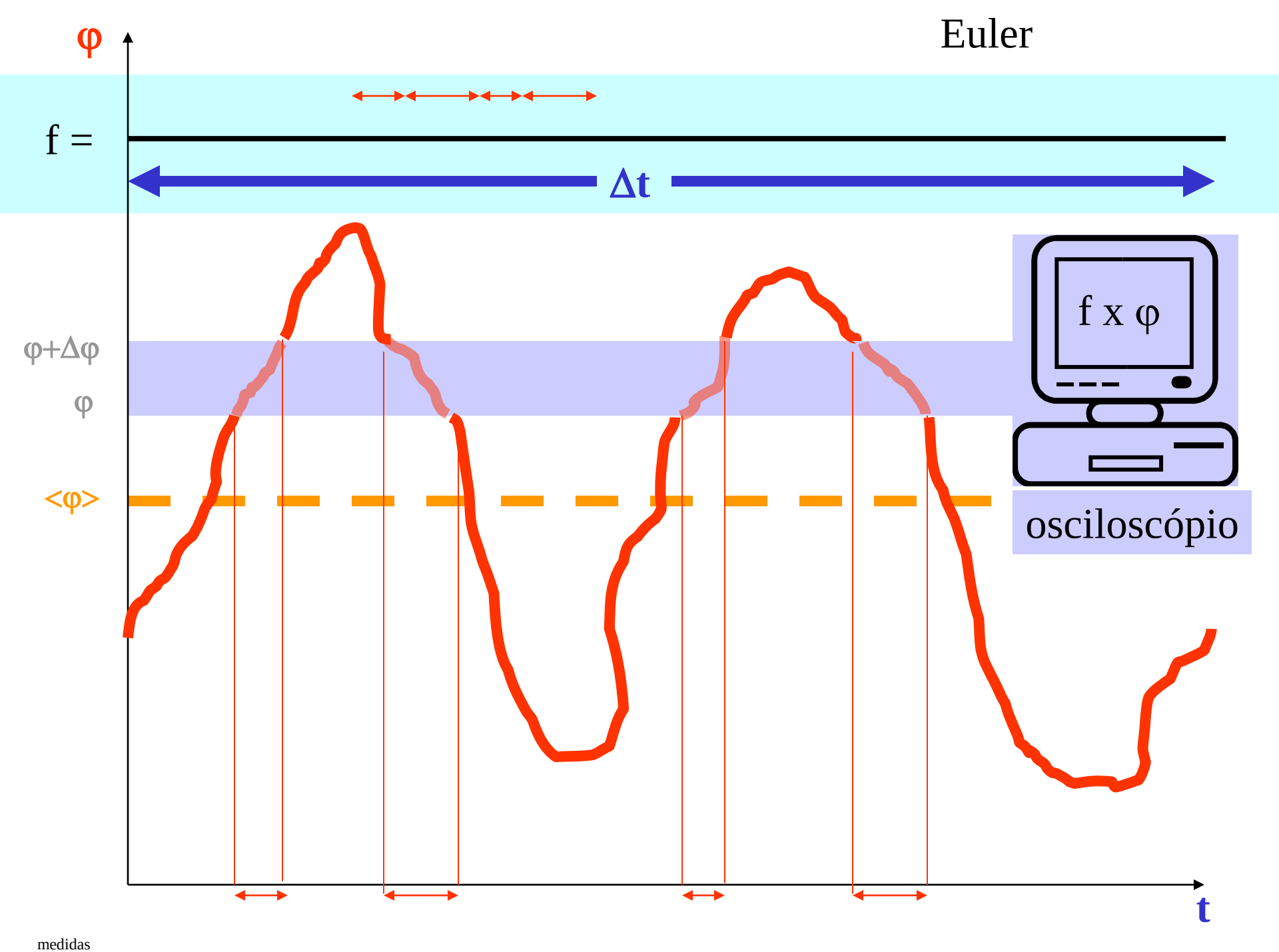


Fig. 34. Fotografias de flutuações da velocidade de perturbação v' visualizadas na tela de um osciloscópio através da técnica da anemometria de fio quente.

A – regime laminar
B – transição
C, D e E – regime turbulento, com diversas intensidades de turbulência.

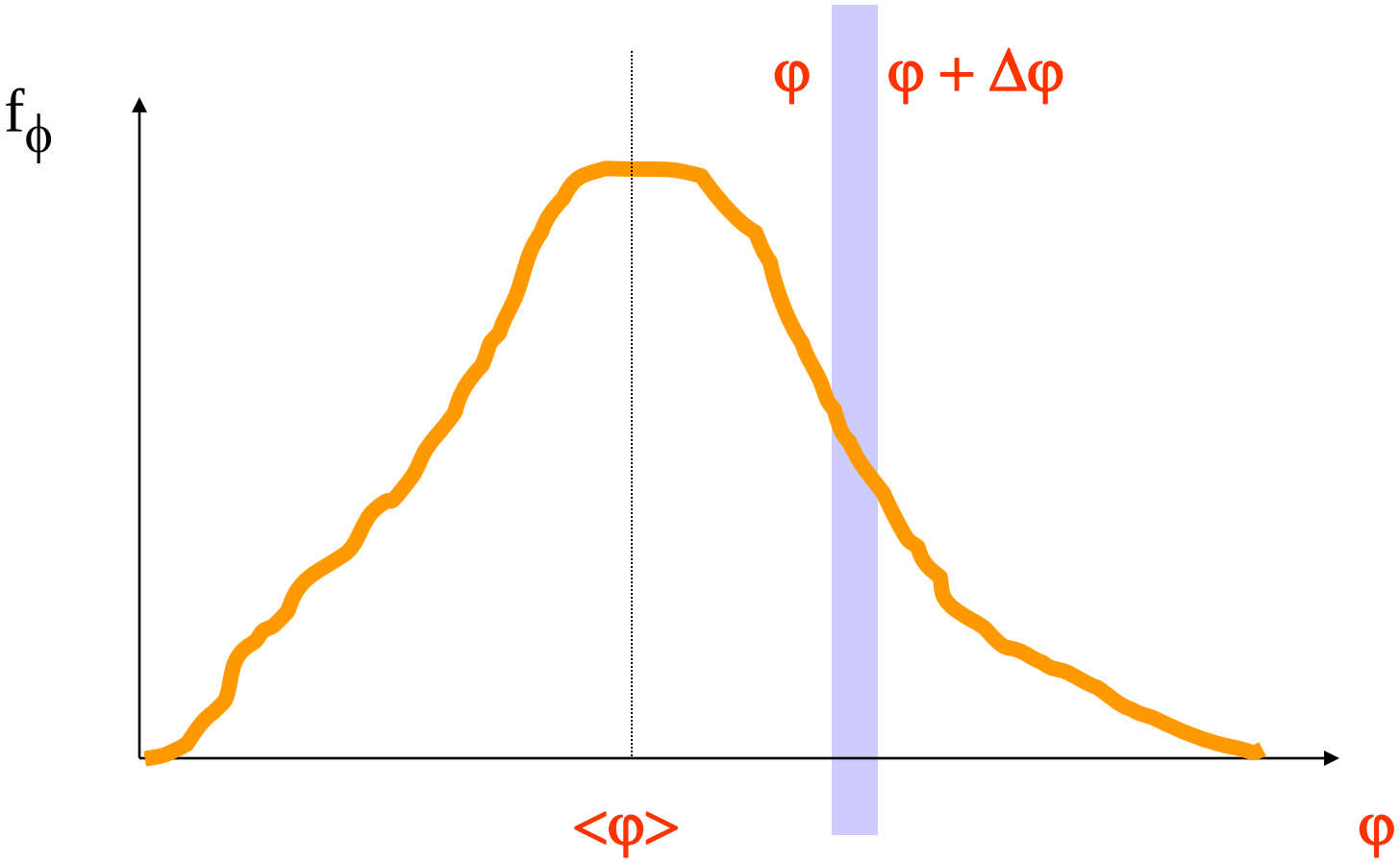
A ordem de grandeza da intensidade de turbulência usual em escoamentos nas aplicações da Engenharia é de 0,05.


φ
Euler
f =
Δt
φ+Δφ
φ
<φ>
f x φ
osciloscópio
t

f_ϕ = Probality Density Function - PDF

PDF

$$f_\phi = f_\phi\,[\,\varphi\,,\,\varphi+\Delta\varphi\,] = f_\phi(t,r)$$



Reynolds

$$\rho \frac{D\varphi}{Dt} = \frac{D\rho\varphi}{Dt} - \rho\varphi \operatorname{div}\vec{v} = \rho\left(\frac{\partial\varphi}{\partial t} + \vec{v}.\operatorname{grad}\varphi\right) = \frac{\partial\rho\varphi}{\partial t} + \operatorname{div}\rho\vec{v}\varphi = -\operatorname{div}\vec{j}_\phi + \dot{\sigma}_{V_\phi}$$

$$\varphi = 1 \qquad \frac{D\rho}{Dt} + \rho \operatorname{div}\vec{v} = \frac{\partial\rho}{\partial t} + \operatorname{div}\rho\vec{v} = 0$$

hipótese $\qquad \vec{j}_\phi = -\rho\ \lambda_\alpha \operatorname{grad}\varphi$

$$\rho \frac{D\varphi}{Dt} = \frac{\partial\rho\varphi}{\partial t} + \operatorname{div}\rho\vec{v}\varphi = \operatorname{div}\left(\rho\lambda_\phi \operatorname{grad}\varphi\right) + \dot{\sigma}_{V_\phi}$$

$$\rho = \rho' + \overline{\rho} \qquad \vec{v} = \vec{v}' + \overline{\vec{v}} \qquad \varphi = \varphi' + \overline{\varphi}$$

médias

$$\langle \xi \rangle = \frac{1}{\Delta\tau} \int_{t}^{t+\Delta\tau} \xi \, dt$$

flutuações

$$\langle \xi' \rangle = 0$$

soma

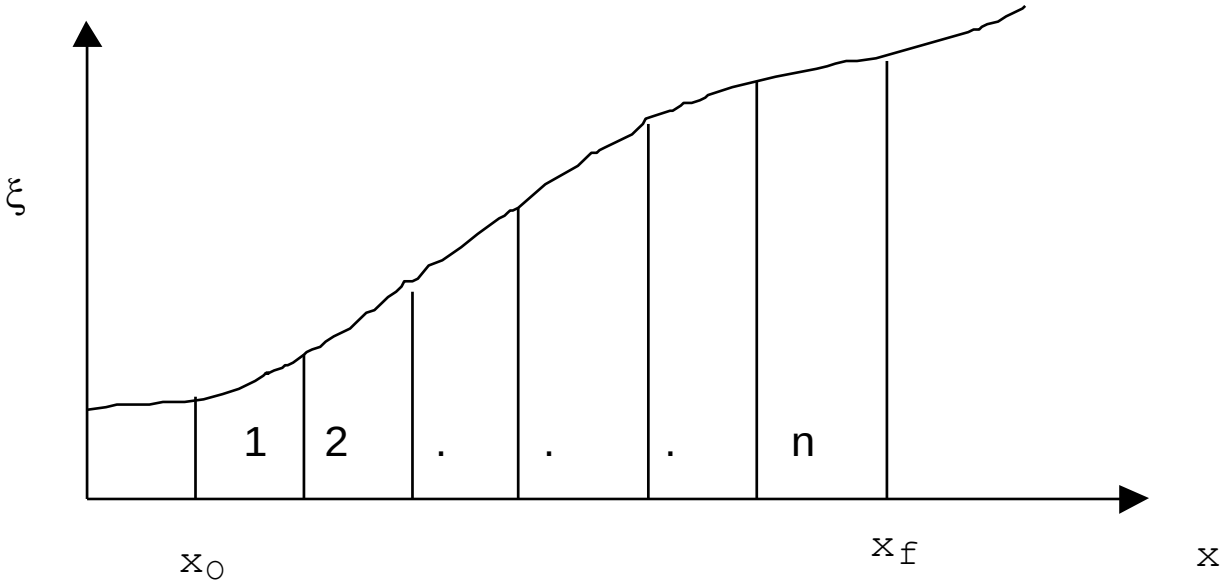
$$\langle \xi \rangle = \langle \chi + \zeta \rangle = \frac{1}{\Delta\tau} \int_{t}^{t+\Delta\tau} (\chi + \zeta) \, dt = \frac{1}{\Delta\tau} \int_{t}^{t+\Delta\tau} \chi \, dt + \frac{1}{\Delta\tau} \int_{t}^{t+\Delta\tau} \zeta \, dt = \langle \chi \rangle + \langle \zeta \rangle$$

$$\langle \xi \rangle = \langle \bar{\xi} \rangle = \bar{\bar{\xi}} = \bar{\xi} \quad ; \quad \overline{\xi'} = 0 \quad ; \quad \overline{\bar{\xi} + \bar{\zeta}} = \bar{\xi} + \bar{\zeta}$$

produto

$$\overline{\bar{\xi}\,\bar{\zeta}} = \bar{\xi}\,\bar{\zeta} \quad ; \quad \overline{\bar{\xi}\,\zeta'} = 0$$

A integração de propriedades turbulentas



$$\int_{x_o}^{x_f} \xi(x)\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} \xi\left(x_f - \frac{i\left(x_f - x_o\right)}{n} \right) \frac{\left(x_f - x_o\right)}{n}$$

$$\boxed{\overline{\int \xi\, dx} = \int \overline{\xi}\, dx}$$

A derivação de propriedades turbulentas

Euler

$$\overline{\frac{\partial \xi}{\partial t}} = \frac{\partial \bar{\xi}}{\partial t} \quad ; \quad \overline{\frac{\partial \xi'}{\partial t}} = 0$$

espaço

$$\overline{\vec{\mathrm{grad}}\,\xi} = \vec{\mathrm{grad}}\,\bar{\xi} \quad ; \quad \overline{\mathrm{grad}\,\vec{\xi}'} = 0$$

$$\overline{\mathrm{div}\,\vec{v}} = \mathrm{div}\,\bar{\vec{v}} \quad ; \quad \overline{\mathrm{div}\,\vec{v}'} = 0$$

Lagrange

$$\frac{D\bar{\xi}}{Dt} = \left\langle \frac{D\bar{\xi}}{Dt} \right\rangle + \vec{v}'\,.\,\vec{\mathrm{grad}}\,\bar{\xi}$$

Euler -> Lagrange

$$\left\langle \frac{D\bar{\xi}}{Dt} \right\rangle = \frac{\partial \bar{\xi}}{\partial t} + \bar{\vec{v}}\,.\,\vec{\mathrm{grad}}\,\bar{\xi}$$

A equação da continuidade Euleriana turbulenta

$$\frac{\partial\left(\bar{\rho}+\rho'\right)}{\partial t}+\operatorname{div}\left(\bar{\rho}+\rho'\right)\left(\bar{\vec{v}}+\vec{v}'\right)=0$$

$$\frac{\partial\,\bar{\rho}}{\partial t}+\frac{\partial\,\rho'}{\partial t}+\operatorname{div}\left(\bar{\rho}\;\bar{\vec{v}}+\bar{\rho}\;\vec{v}'+\rho'\;\bar{\vec{v}}+\rho'\;\vec{v}'\right)=0$$

integrando entre t e $t+\Delta t$

$$\left\langle\frac{\partial\,\bar{\rho}}{\partial t}\right\rangle+\left\langle\frac{\partial\,\rho'}{\partial t}\right\rangle+\left\langle\operatorname{div}\left(\bar{\rho}\;\bar{\vec{v}}+\bar{\rho}\;\vec{v}'+\rho'\;\bar{\vec{v}}+\rho'\;\vec{v}'\right)\right\rangle=0$$

$$\frac{\partial\,\bar{\rho}}{\partial t}+\frac{\partial\,\overline{\rho'}}{\partial t}+\operatorname{div}\bar{\rho}\,\bar{\vec{v}}+\operatorname{div}\overline{\bar{\rho}\,\vec{v}'}+\operatorname{div}\overline{\rho'\,\bar{\vec{v}}}+\operatorname{div}\overline{\rho'\,\vec{v}'}=0$$

$$\frac{\partial\,\bar{\rho}}{\partial t}+\operatorname{div}\bar{\rho}\,\bar{\vec{v}}+\operatorname{div}\overline{\rho'\,\vec{v}'}=0$$

FLUIDO incompressível $\boxed{\rho' \cong 0 \quad ?}$

FLUIDO incompressível

$$\rho' = 0 \quad ; \quad \frac{\partial \bar{\rho}}{\partial t} = 0 \quad ; \quad \mathrm{grad}\,\bar{\rho} = 0$$

$$\mathrm{div}\,\bar{\vec{v}} = 0$$

$$\frac{\partial \bar{\rho}}{\partial t} + \mathrm{div}\,\bar{\rho}\,\bar{\vec{v}} = 0$$

alternativamente: Favre

$$\tilde{\vec{v}} \equiv \frac{\overline{\rho\,\vec{v}}}{\bar{\rho}}$$

$$\frac{\partial \rho \varphi}{\partial t} + \mathrm{div}\, \rho \vec{v} \varphi = - \mathrm{div}\, \vec{j}_\phi + \dot{\sigma}_{V_\phi}$$

$$\frac{\partial \bar{\rho} \left(\varphi' + \bar{\varphi}\right)}{\partial t} + \mathrm{div}\, \bar{\rho} \left(\varphi' + \bar{\varphi}\right) \left(\vec{v}' + \bar{\vec{v}}\right) = \mathrm{div}\, \bar{\rho} \left(\lambda_\phi \mathrm{gr\vec{a}d}\left(\varphi' + \bar{\varphi}\right)\right) + \dot{\sigma}'_{V_\phi} + \overline{\dot{\sigma}_{V_\phi}}$$

$$\frac{\partial \bar{\rho} \varphi'}{\partial t} + \frac{\partial \bar{\rho} \bar{\varphi}}{\partial t} + \mathrm{div}\, \bar{\rho} \left(\varphi' \vec{v}' + \bar{\varphi} \vec{v}' + \varphi' \bar{\vec{v}} + \bar{\varphi} \bar{\vec{v}}\right) = \mathrm{div}\, \bar{\rho} \lambda_\phi \mathrm{gr\vec{a}d}\left(\varphi' + \bar{\varphi}\right) + \overline{\dot{\sigma}_{V_\phi}}$$

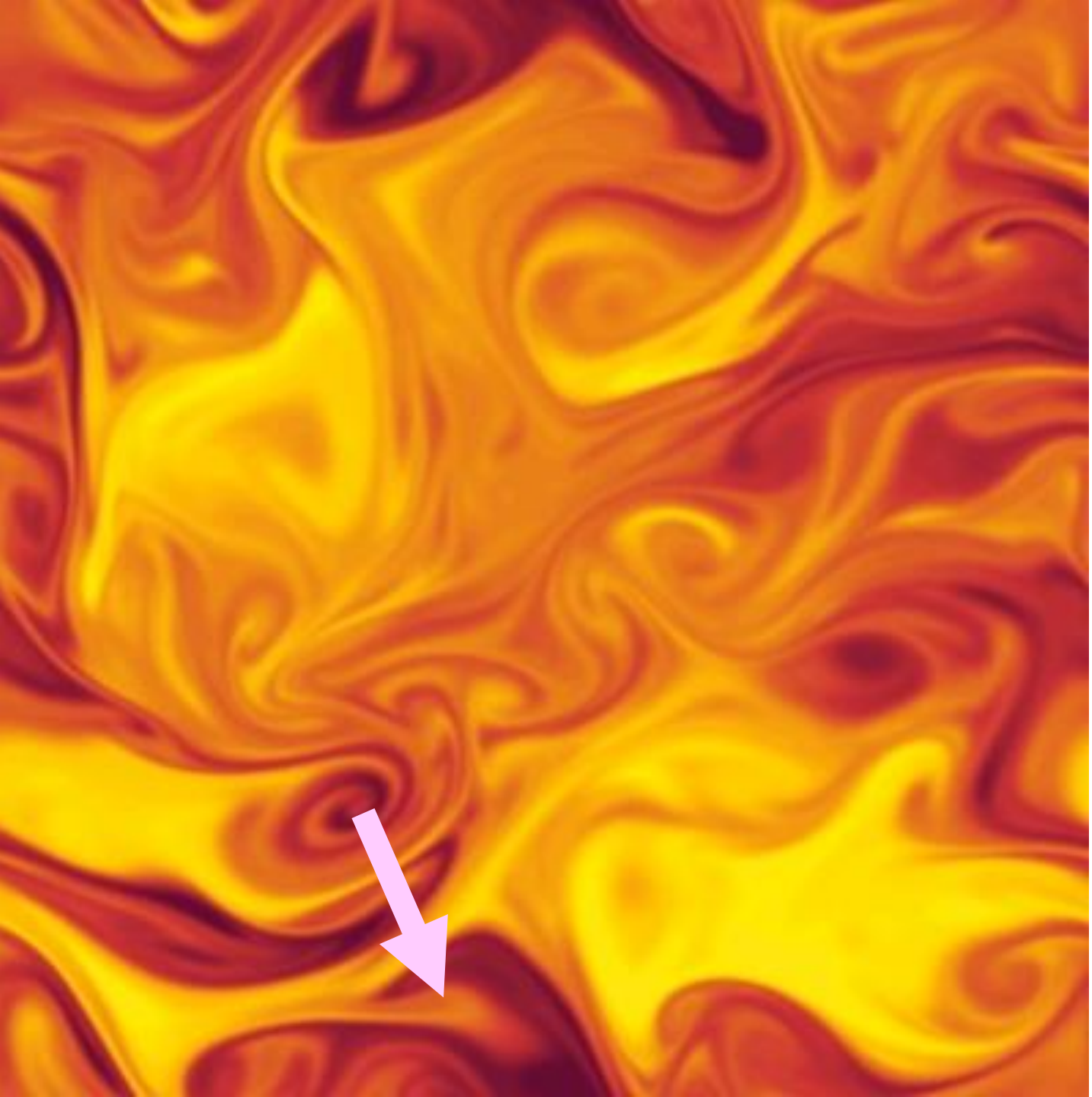
integrando entre t e $t + \Delta t$

$$\frac{\partial \bar{\rho} \langle \varphi' \rangle}{\partial t} + \frac{\partial \bar{\rho} \langle \bar{\varphi} \rangle}{\partial t} + \mathrm{div}\, \bar{\rho} \left[\langle \varphi' \vec{v}' \rangle + \langle \bar{\varphi} \vec{v}' \rangle + \langle \varphi' \bar{\vec{v}} \rangle + \langle \bar{\varphi} \bar{\vec{v}} \rangle \right] =$$
$$= \mathrm{div}\, \bar{\rho} \lambda_\phi\, \mathrm{gr\vec{a}d} \left(\langle \varphi' \rangle + \langle \bar{\varphi} \rangle \right) + \left\langle \overline{\dot{\sigma}_{V_\phi}} \right\rangle$$

$$\frac{\partial \bar{\varphi}}{\partial t} + \mathrm{div} \left[\overline{\vec{v}' \varphi'} + \bar{\vec{v}} \bar{\varphi} \right] = \mathrm{div} \left[\lambda_\phi\, \mathrm{gr\vec{a}d}\, \bar{\varphi} \right] + \overline{\dot{\sigma}_{M_\phi}}$$

$$\overline{\vec{v}' \varphi'}$$

e agora ?

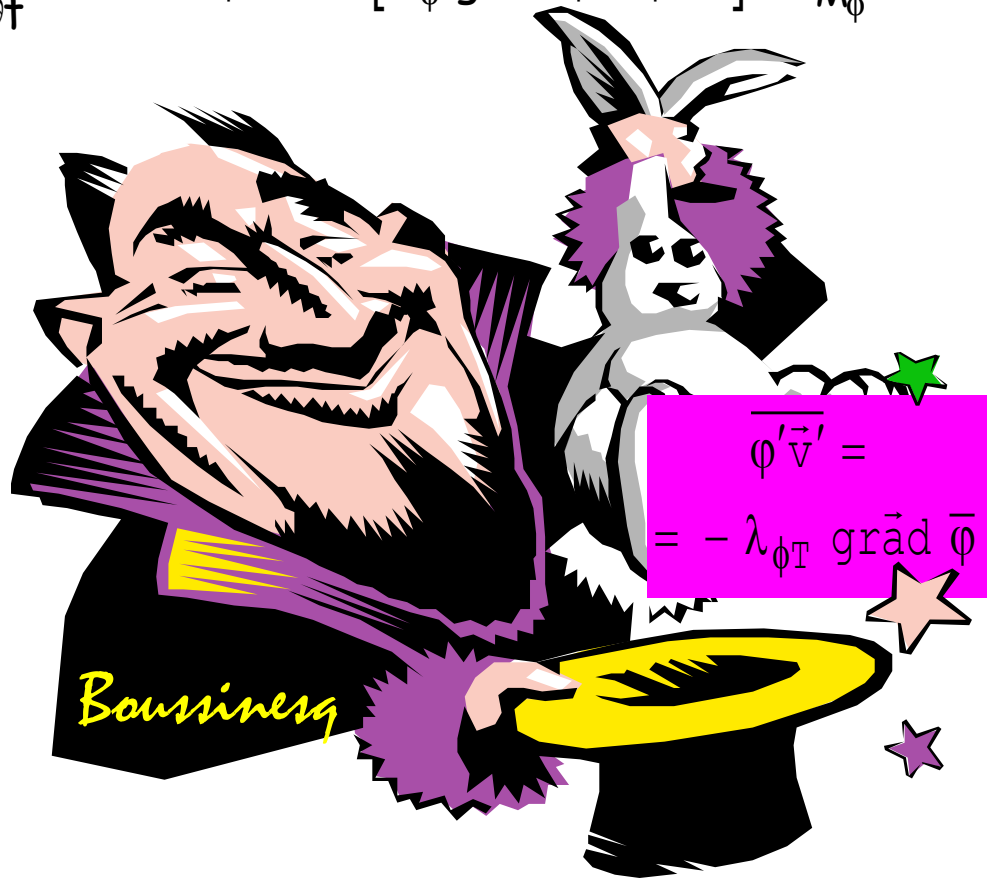


$$\overline{\vec{v}'\varphi'}$$

$$\mathrm{gr}\vec{\mathrm{a}}\mathrm{d}\,\overline{\varphi}$$

$$\frac{\partial \overline{\varphi}}{\partial t} + \mathrm{div}\left[\overline{\vec{v}'\varphi'} + \overline{\vec{v}}\,\overline{\varphi}\right] = \mathrm{div}\left[\lambda_\phi\ \mathrm{gr\vec{a}d}\,\overline{\varphi}\right] + \overline{\dot{\sigma}_{M_\phi}}$$

$$\frac{\partial \overline{\varphi}}{\partial t} + \mathrm{div}\,\overline{\vec{v}}\,\overline{\varphi} = \mathrm{div}\left[\lambda_\phi\ \mathrm{gr\vec{a}d}\,\overline{\varphi} - \overline{\varphi'\vec{v}'}\right] + \overline{\dot{\sigma}_{M_\phi}}$$



$$\overline{\varphi'\vec{v}'} =$$

$$= -\lambda_{\phi T}\ \mathrm{gr\vec{a}d}\ \overline{\varphi}$$

fechou!

$$\frac{\partial \overline{\varphi}}{\partial t} + \mathrm{div}\,\overline{\vec{v}}\,\overline{\varphi} = \mathrm{div}\left[\lambda_\phi + \lambda_{\phi T}\right]\mathrm{gr\vec{a}d}\,\overline{\varphi} + \overline{\dot{\sigma}_{M_\phi}}$$

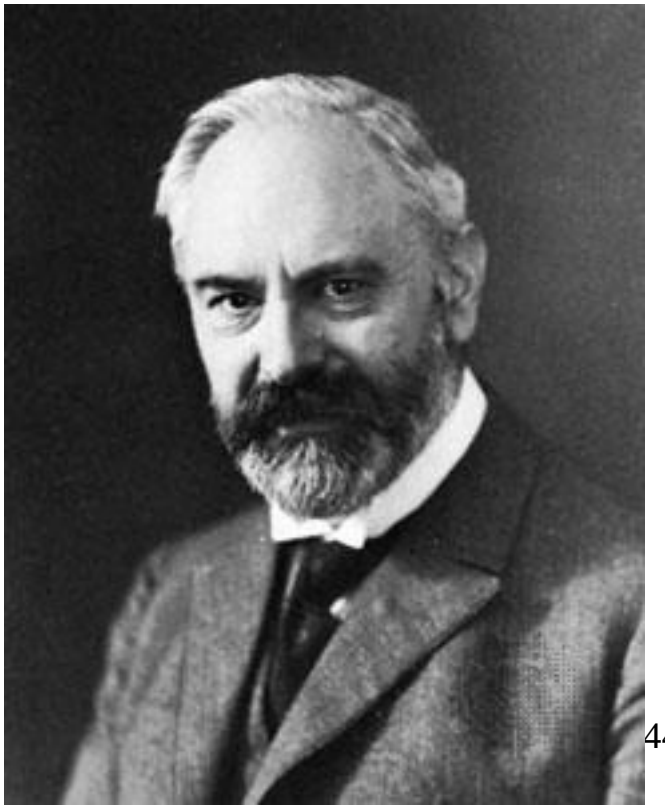
sendo a difusividade turbulenta >> laminar:

$$\left[\lambda_\phi + \lambda_{\phi T}\right] \approx \lambda_{\phi T}$$

$$\frac{\partial \overline{\varphi}}{\partial t} + \mathrm{div}\, \overline{\vec{v}}\, \overline{\varphi} = \mathrm{div}\, \lambda_{\phi T}\, \mathrm{grad}\, \overline{\varphi} + \overline{\dot{\sigma}_{M_\phi}}$$

voltou à poderosa

agora super-poderosa



$$\lambda_{\vec{v}T} = \ell_m^2 \left| \mathrm{grad}\, \overline{\vec{v}} \right|$$

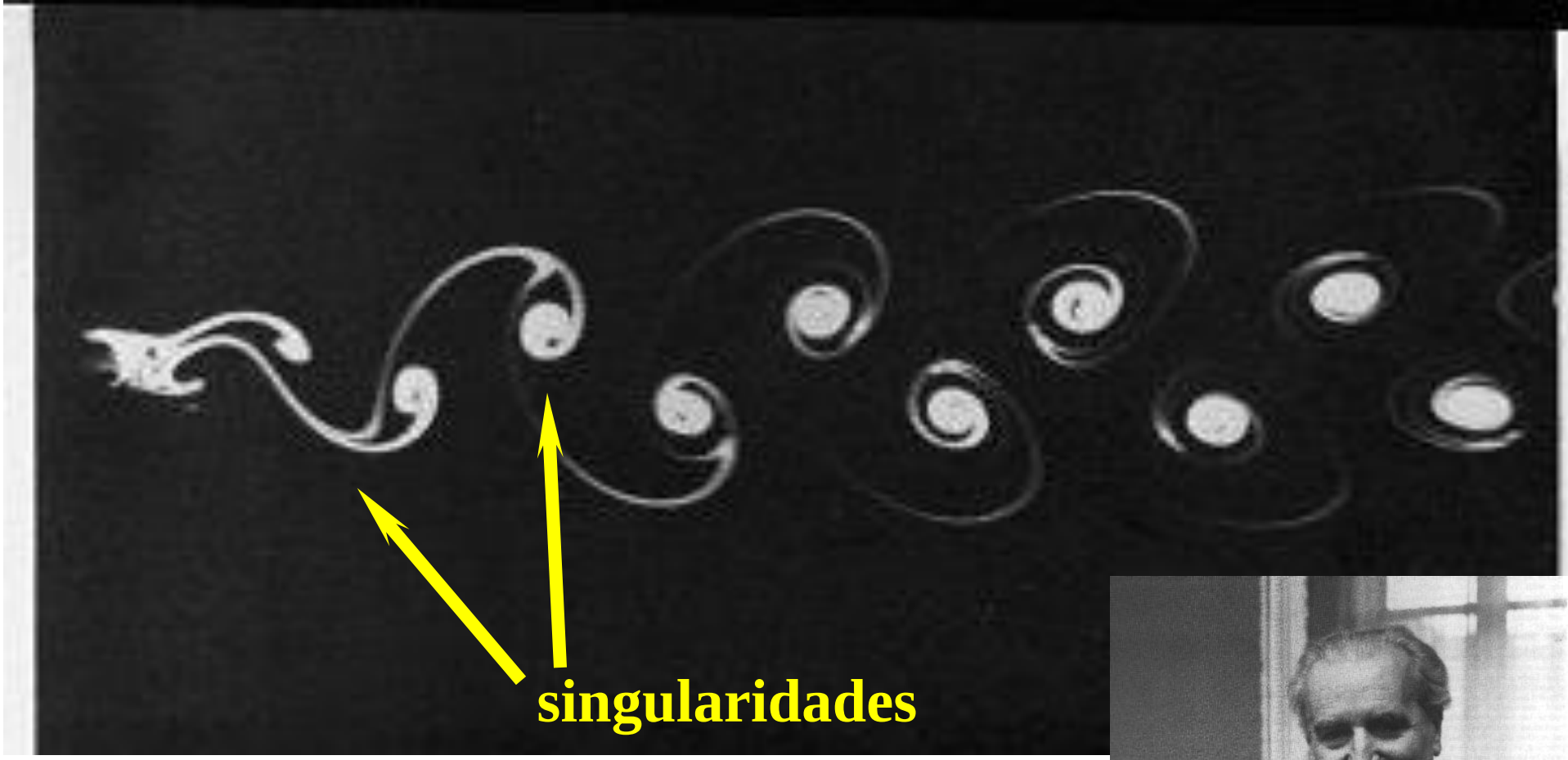
Prandtl mixing lenght

hipótese

sendo $\varphi = v$:

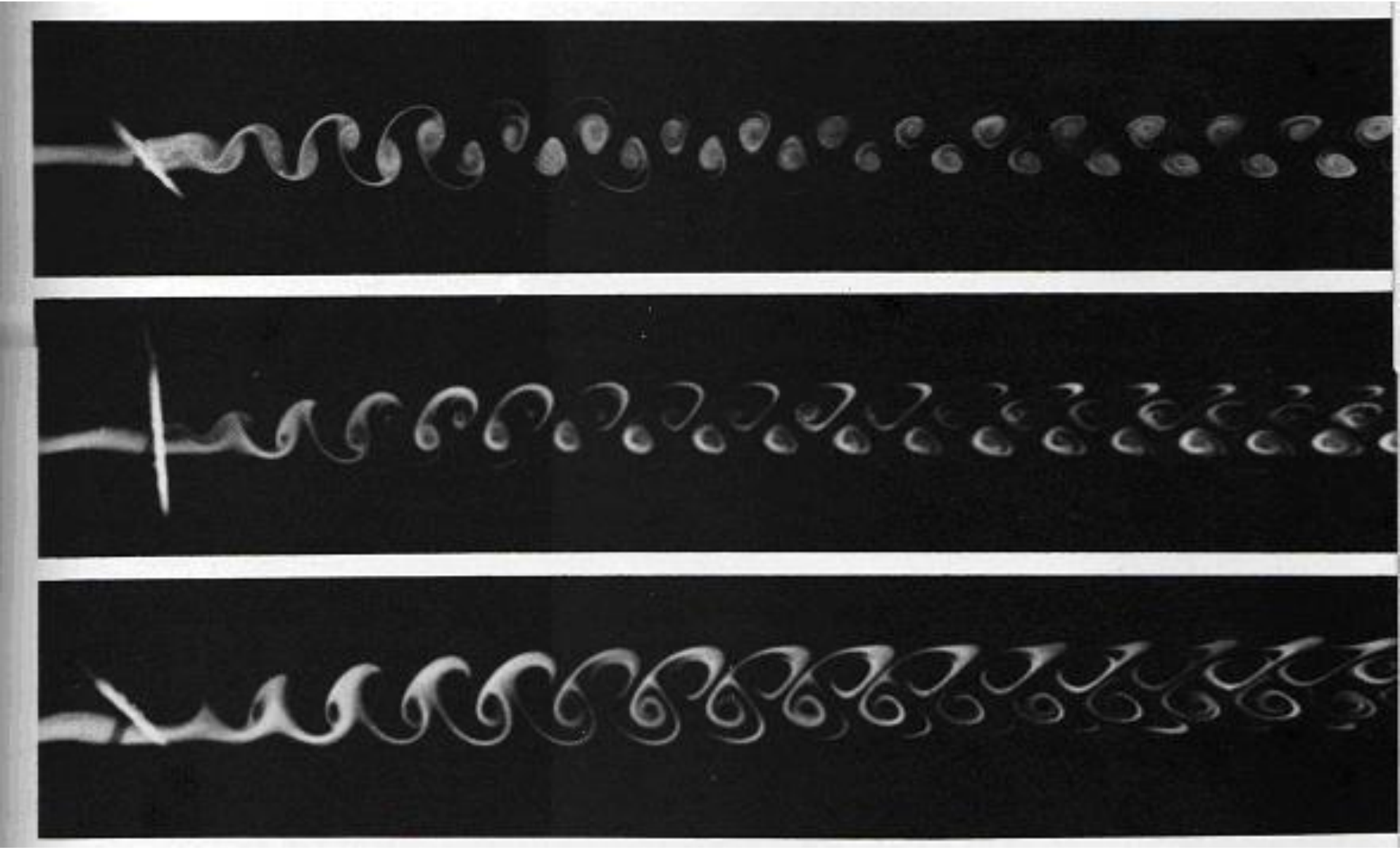
$$\frac{\partial \vec{\bar{v}}}{\partial t} + \vec{div}\,\vec{\bar{v}}\;\vec{\bar{v}} = \vec{div}\;\nu_T\;\vec{grad}\,\vec{\bar{v}} - \frac{\vec{grad}\,\bar{p}}{\bar{\rho}} + \vec{\bar{g}}$$

Prandtl mixing lenght	$\lambda_{\vec{v}T} = \nu_T = \frac{\mu_T}{\rho} = \ell_m^2 \left\| \vec{grad}\;\vec{\bar{v}} \right\|$
similaridade Prandtl	$\nu_T = c_\mu \ell \sqrt{K}$
k	$\boxed{K} \rightarrow k = \overline{e_{c_T}} = \frac{\overline{\vec{v}'.\vec{v}'}}{2}$ $\frac{Dk}{Dt} = div\left(\frac{\nu_T}{\sigma_k}\,\vec{grad}\,k\right) + \nu_T \left\| \vec{grad}\;\vec{v} \right\|^2 - c_D \frac{k^{3/2}}{\ell}$
k ε	$\nu_T = C_\mu\, K^2/\varepsilon$



cilindro
Re = 105

Von Karman
vortex



vortex Re = 100

Modelos de Turbulência

Reynolds Stress $\vec{\vec{R}} = \overline{\vec{v}'.\vec{v}'}$

sete equações diferenciais	$\frac{D\vec{\vec{R}}}{Dt} = -\vec{\mathrm{div}}\left[\vec{\vec{j}}_{\vec{\vec{R}}} + \vec{\vec{\pi}}_{\vec{\vec{R}}} + \vec{\vec{\Omega}}_{\vec{\vec{R}}}\right] + \overline{\vec{\vec{\dot{\sigma}}}_{M_{\vec{\vec{R}}}}} - \overline{\vec{\vec{\dot{\varepsilon}}}_{M_{\vec{\vec{R}}}}}$
algébrico	$\vec{\vec{R}} = \frac{2}{3} K \vec{\vec{\delta}} + \left[\frac{C_D}{C_1 - 1 + p/\varepsilon}\right]\left(\overline{\vec{\vec{\dot{\sigma}}}_{M_{\vec{\vec{R}}}}} - \frac{2}{3} p \vec{\vec{\delta}}\right)\frac{K}{\varepsilon}$
2 eq. K ε	$\nu_T = C_\mu K^2/\varepsilon$
0 eq. Prandtl mixing lenght	$\nu_T = \ell_m^2 \left\lvert \vec{\mathrm{grad}}\, \overline{\vec{v}} \right\rvert$
large eddy simulation LES	por hora apenas fornecem parâmetros

tabela

Potência dissipada por unidade de volume

P = (dW) / (dV . dt) = df . dr /dt /dV = df . dv /dV

$$\dot{P} = \left(d\vec{f}\,.\,d\vec{v}\right)/\,dV = \left(-\vec{\vec{\zeta}}\,.\,d\vec{S}\right).\,\left(\vec{\vec{grad}}\,\vec{v}\,.\,d\vec{r}\right)/\,\left(d\vec{r}\,.\,d\vec{S}\right) =$$

$$= -\vec{\vec{\zeta}} : \vec{\vec{grad}}\,\vec{v} = -\underbrace{\left(-\mu\,\vec{\vec{grad}}\,\vec{v}\right)}_{\text{Newtoniano}} : \vec{\vec{grad}}\,\vec{v} = \mu\,\Phi_v$$

Rayleight

$$\Phi_v = \vec{\vec{grad}}\,\vec{v} : \vec{\vec{grad}}\,\vec{v}$$

Rayleight

retangular

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2$$

cilíndrico

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \phi}\right]^2 + \left[\frac{1}{r}\frac{\partial v_z}{\partial \phi} + \frac{\partial v_\phi}{\partial z}\right]^2 +$$

$$+\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2 - \frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}\right]^2$$

esférico

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)^2\right] +$$

$$+\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2 + \left[\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2 +$$

$$-\frac{2}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}\left(v_\theta\ \mathrm{sen}\,\theta\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2$$

Rayleight

$$P = \mu \Phi_v = \rho \nu \left(\vec{\vec{\mathrm{grad}}}\, \vec{v} : \vec{\vec{\mathrm{grad}}}\, \vec{v} \right)$$

turbulento

$$\vec{\vec{\zeta}}_T = -\rho \ \nu_T \ \ \vec{\vec{\mathrm{grad}}}\, \bar{\vec{v}}$$

comprimento de mistura de Prandtl

$$\nu_T = \ell_m^2 \left| \vec{\vec{\mathrm{grad}}}\, \bar{\vec{v}} \right|$$

dissipação de energia cinética de turbulência

$$\boxed{P_k = -\bar{\rho}\, \ell_m^2 \left(\vec{\vec{\mathrm{grad}}}\, \bar{\vec{v}} : \vec{\vec{\mathrm{grad}}}\, \bar{\vec{v}} \right)}$$

$$\frac{\partial \bar{k}}{\partial t} + \mathrm{div}\,\bar{\vec{v}}\,\bar{k} = \mathrm{div}\,\frac{\nu_T}{\sigma_k}\,\vec{\mathrm{grad}}\,\bar{k} + P_k - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} + \mathrm{div}\,\bar{\vec{v}}\,\varepsilon = \mathrm{div}\,\frac{\nu_T}{\sigma_\varepsilon}\,\vec{\mathrm{grad}}\,\varepsilon + C_{\varepsilon 1}\,\frac{\varepsilon}{\bar{k}}\,P_k - C_{\varepsilon 2}\,\frac{\varepsilon^2}{\bar{k}}$$

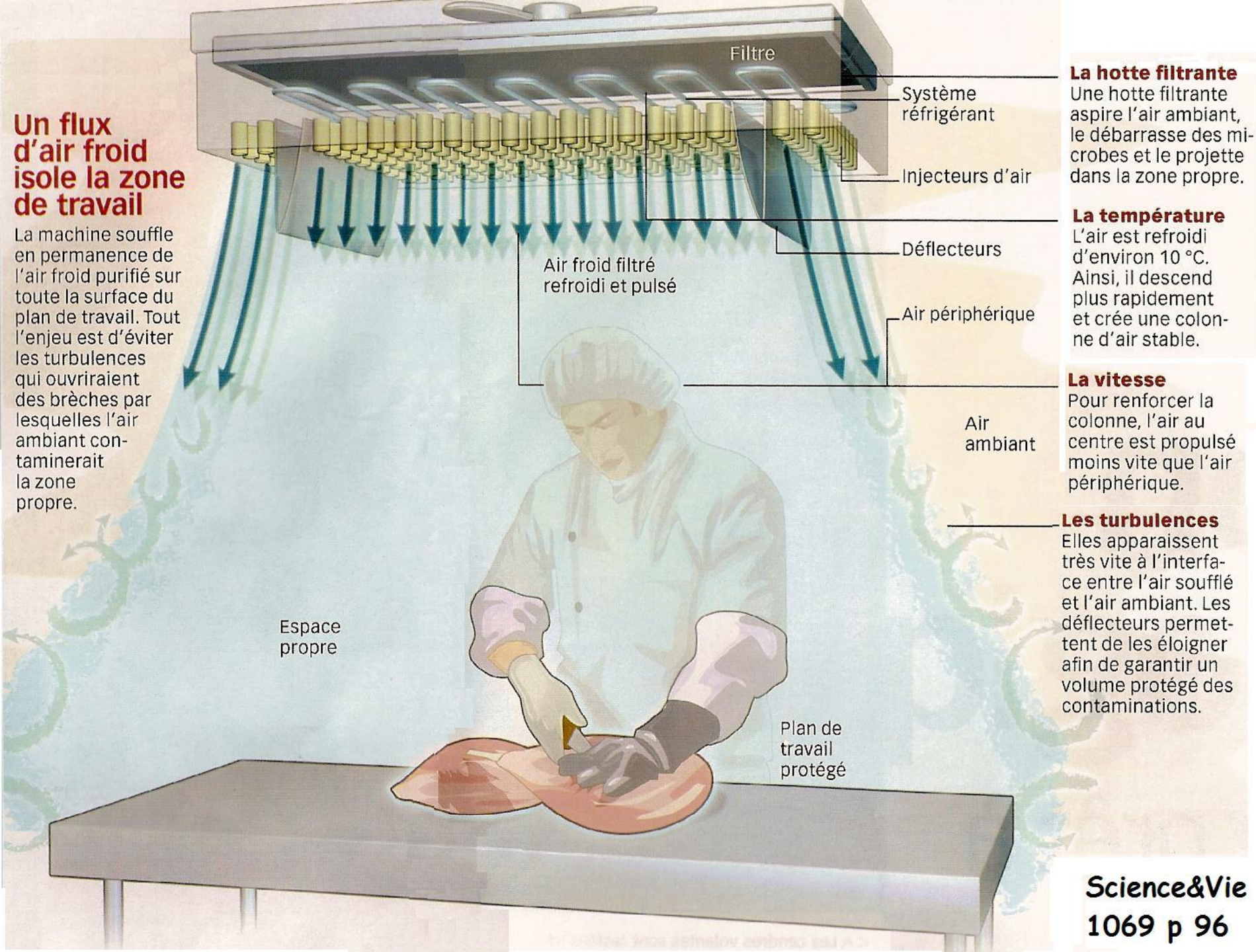
$$P_k = -\bar{\rho}\,\nu_T\left(\vec{\mathrm{grad}}\,\bar{\vec{v}} : \vec{\mathrm{grad}}\,\bar{\vec{v}}\right)$$

$$\nu_T = C_\mu\,\frac{\bar{k}^2}{\varepsilon}$$

$\sigma_k = 1{,}0$; $\sigma_\varepsilon = 1{,}217$;

$C_{\varepsilon 1} = 1{,}44$; $C_{\varepsilon 2} = 1{,}92$; $C_\mu = 0.09$

constantes experimentais



Un flux d'air froid isole la zone de travail

La machine souffle en permanence de l'air froid purifié sur toute la surface du plan de travail. Tout l'enjeu est d'éviter les turbulences qui ouvriraient des brèches par lesquelles l'air ambiant contaminerait la zone propre.

La hotte filtrante
Une hotte filtrante aspire l'air ambiant, le débarrasse des microbes et le projette dans la zone propre.

La température
L'air est refroidi d'environ 10 °C. Ainsi, il descend plus rapidement et crée une colonne d'air stable.

La vitesse
Pour renforcer la colonne, l'air au centre est propulsé moins vite que l'air périphérique.

Les turbulences
Elles apparaissent très vite à l'interface entre l'air soufflé et l'air ambiant. Les déflecteurs permettent de les éloigner afin de garantir un volume protégé des contaminations.

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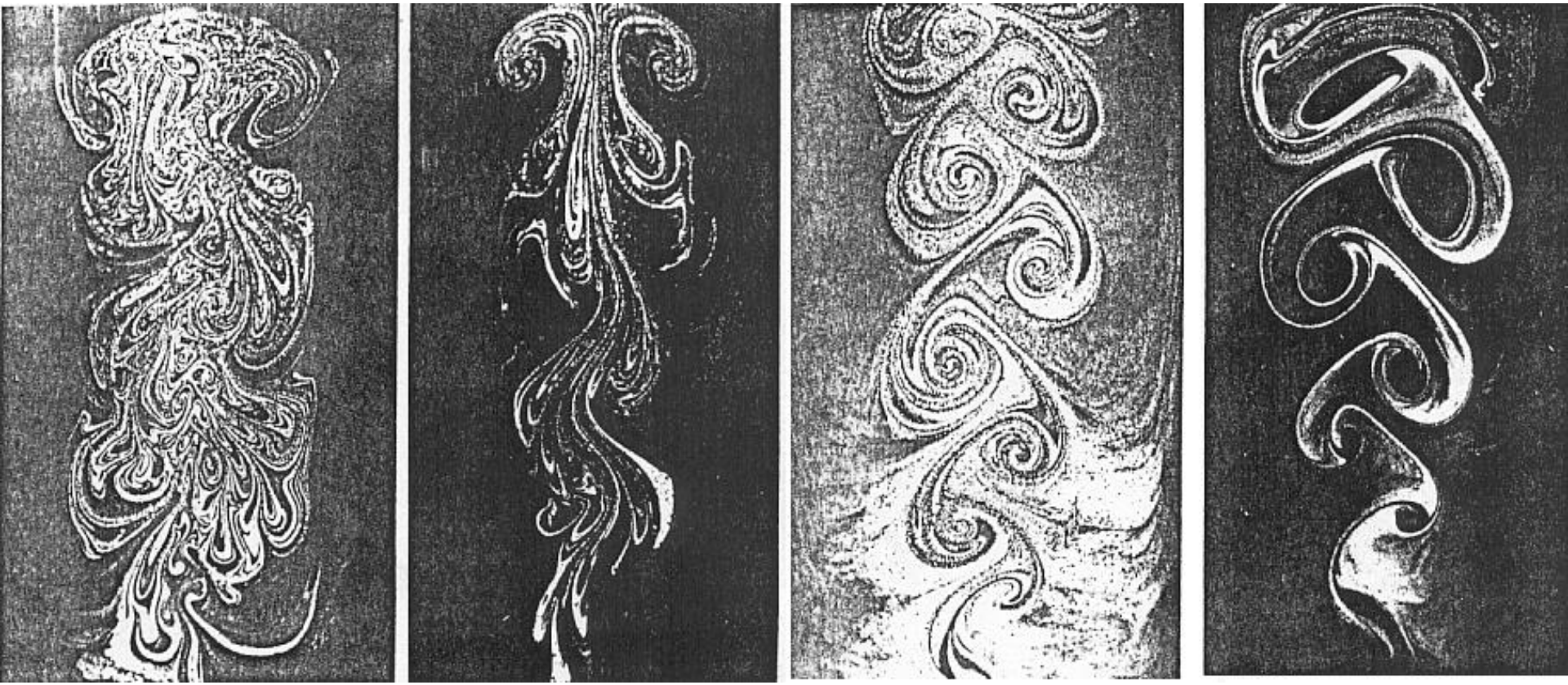
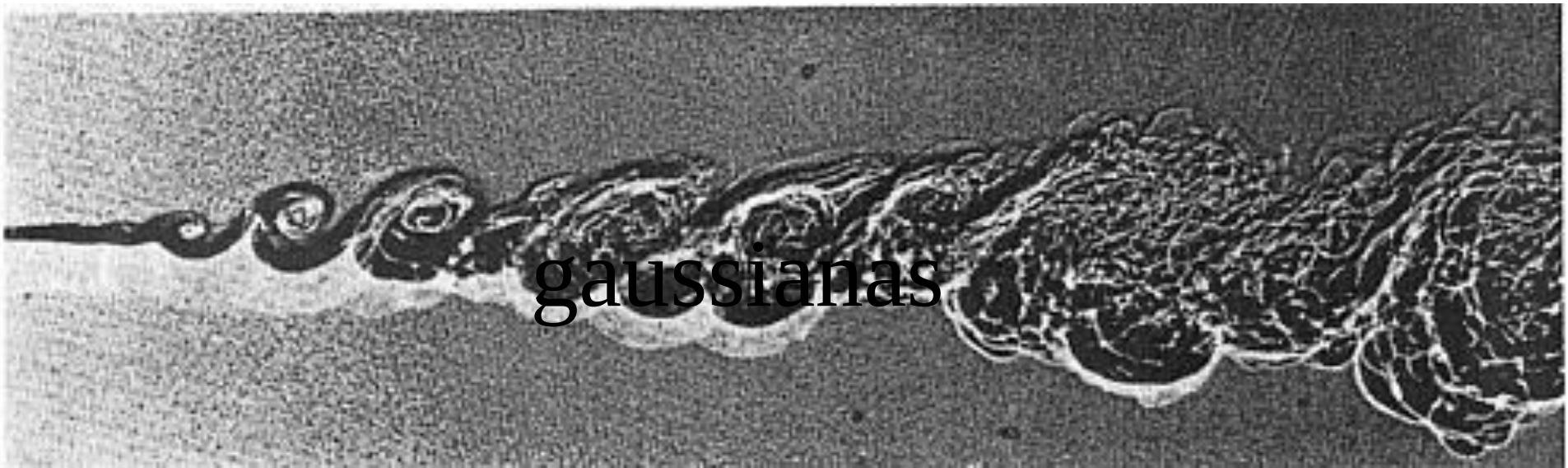
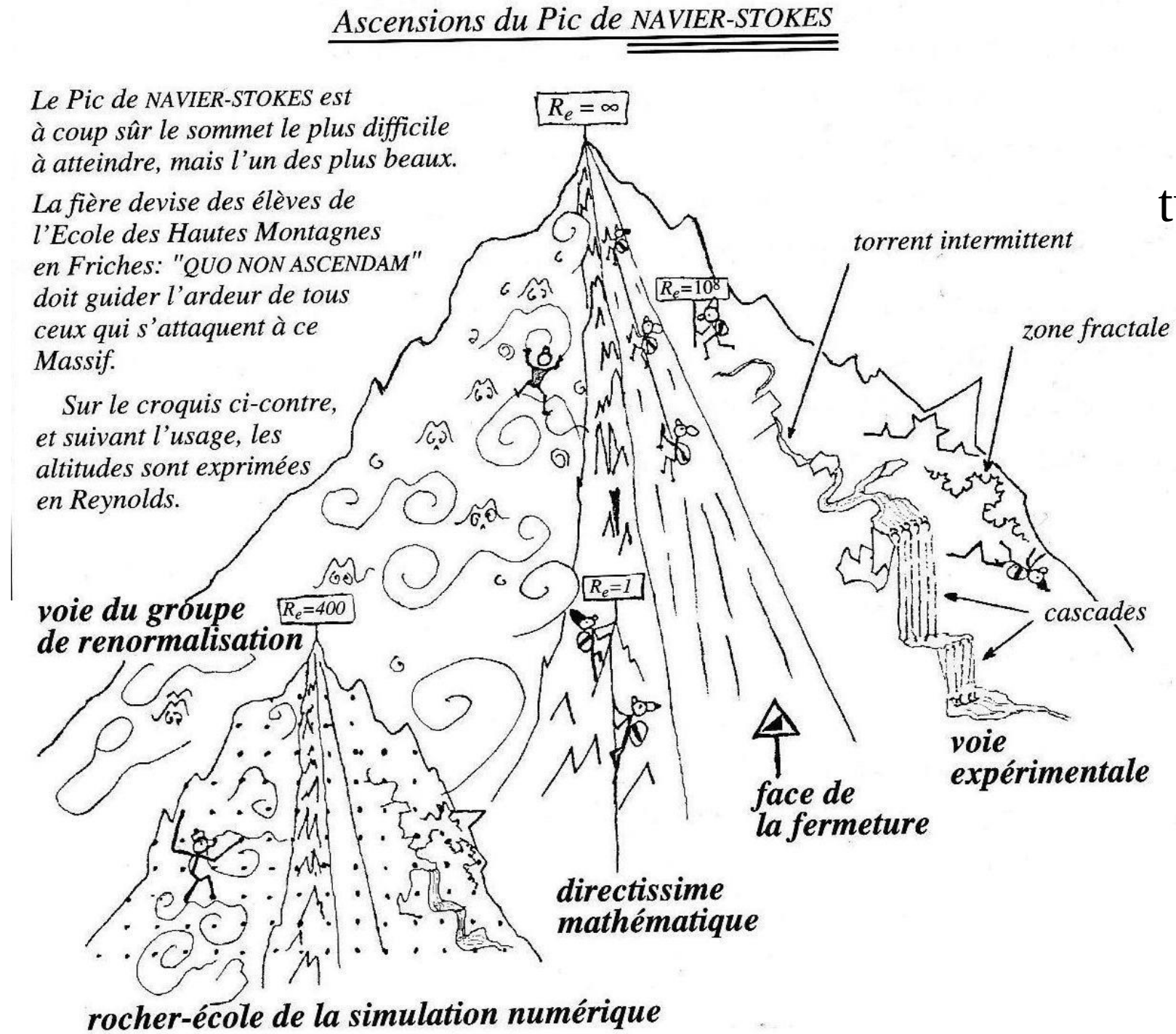
1069 p 96



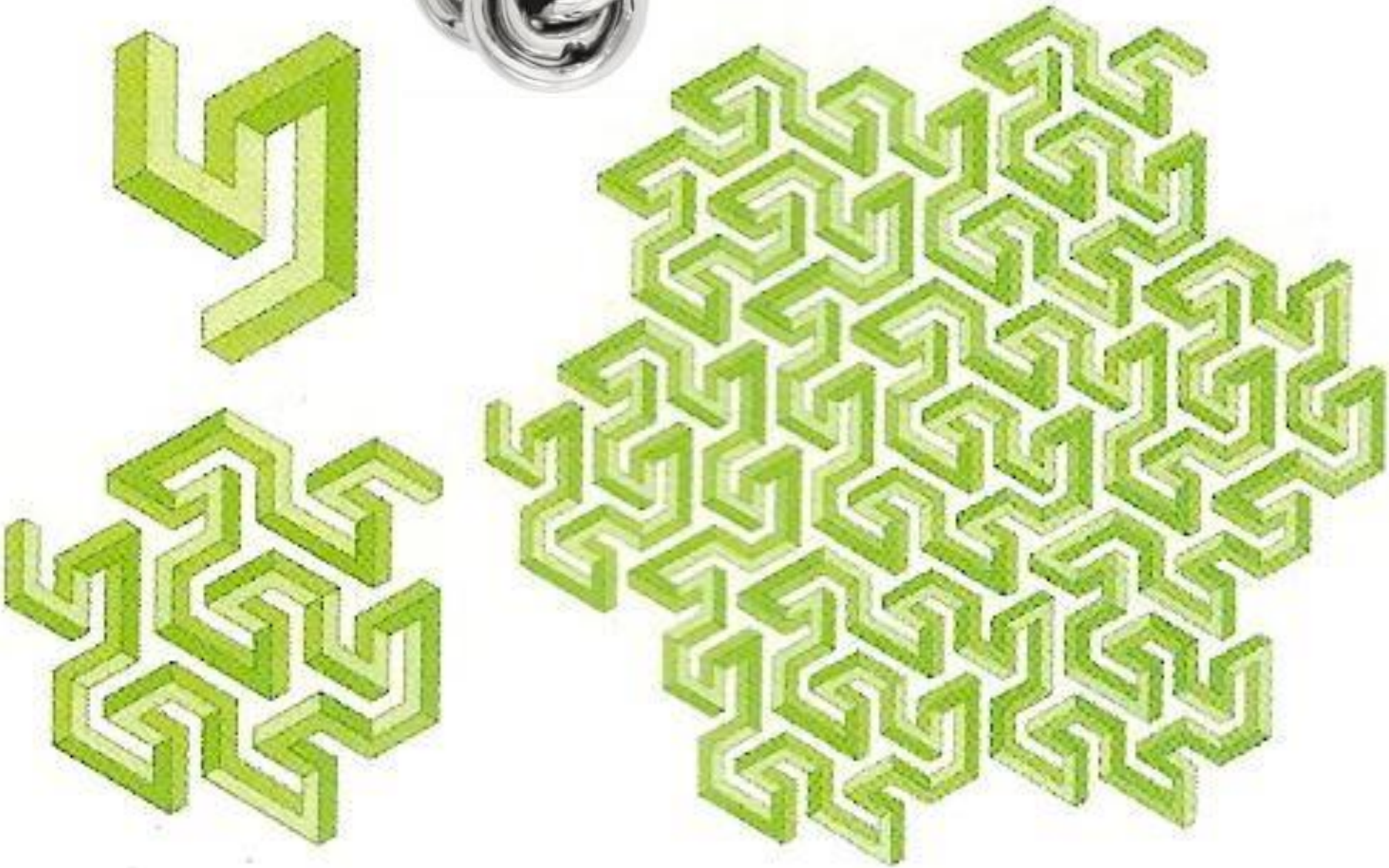


gaussianas

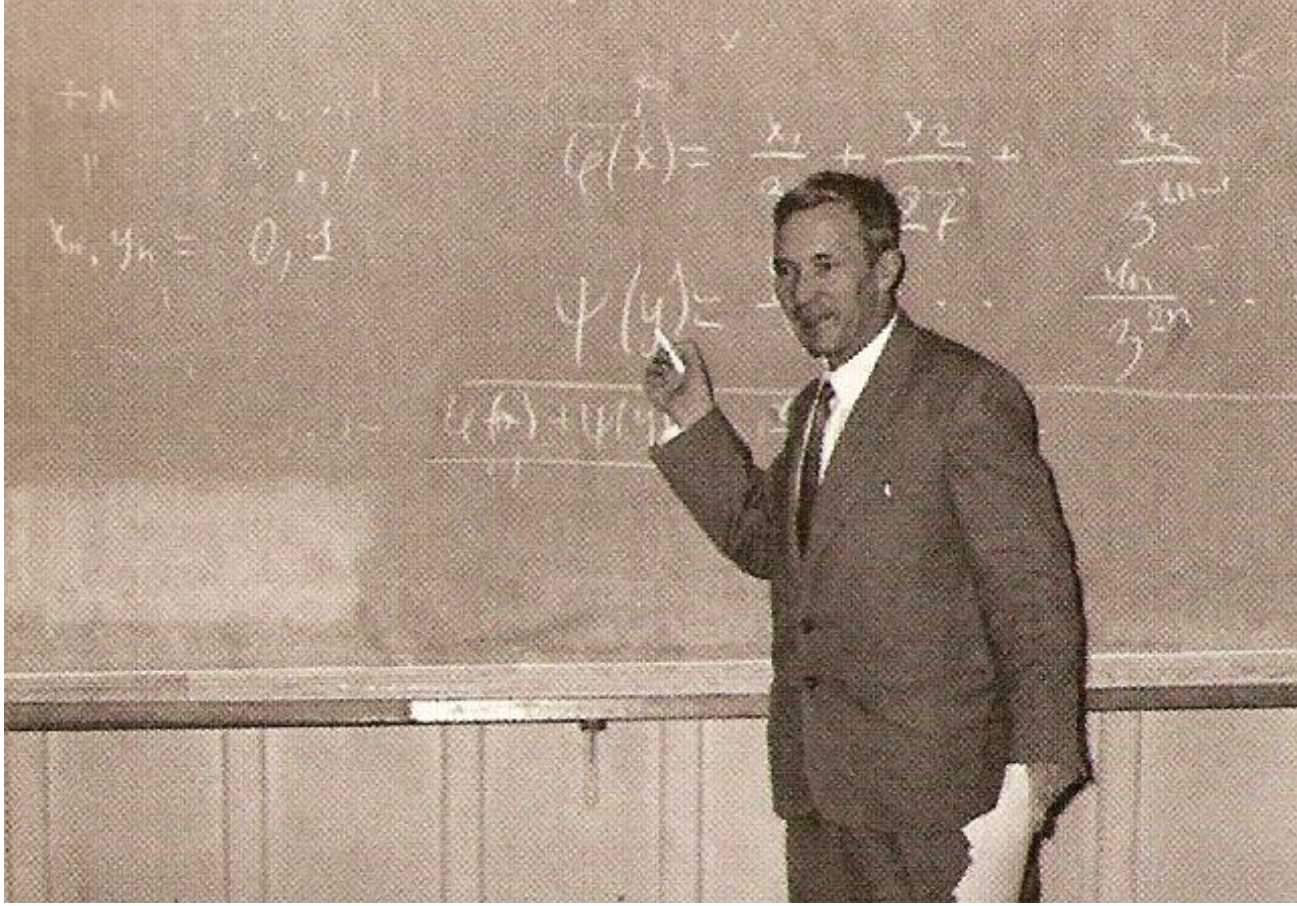
guia do alpinista turbulento 200?

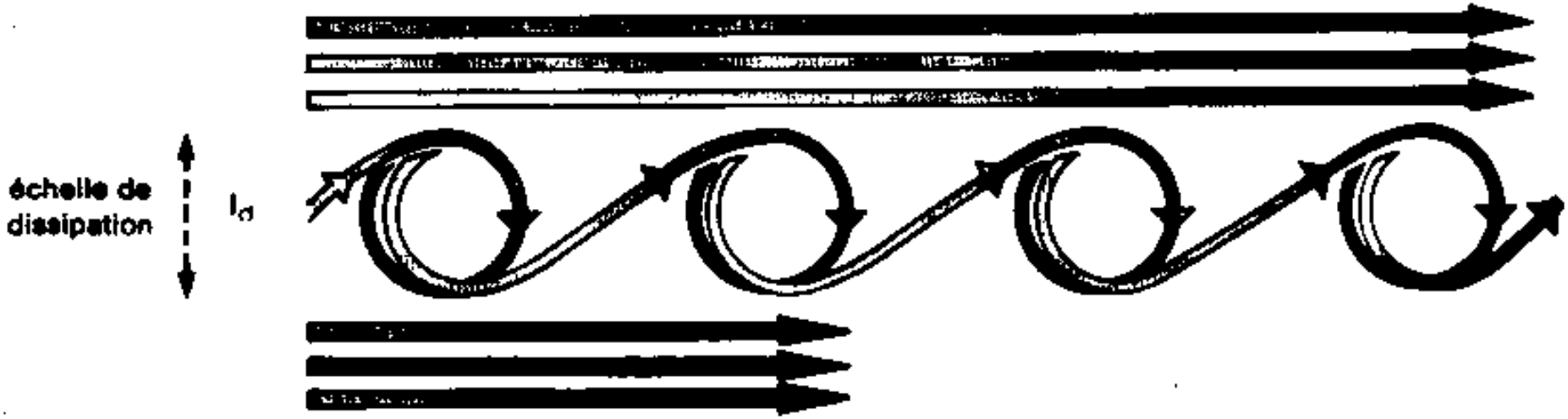


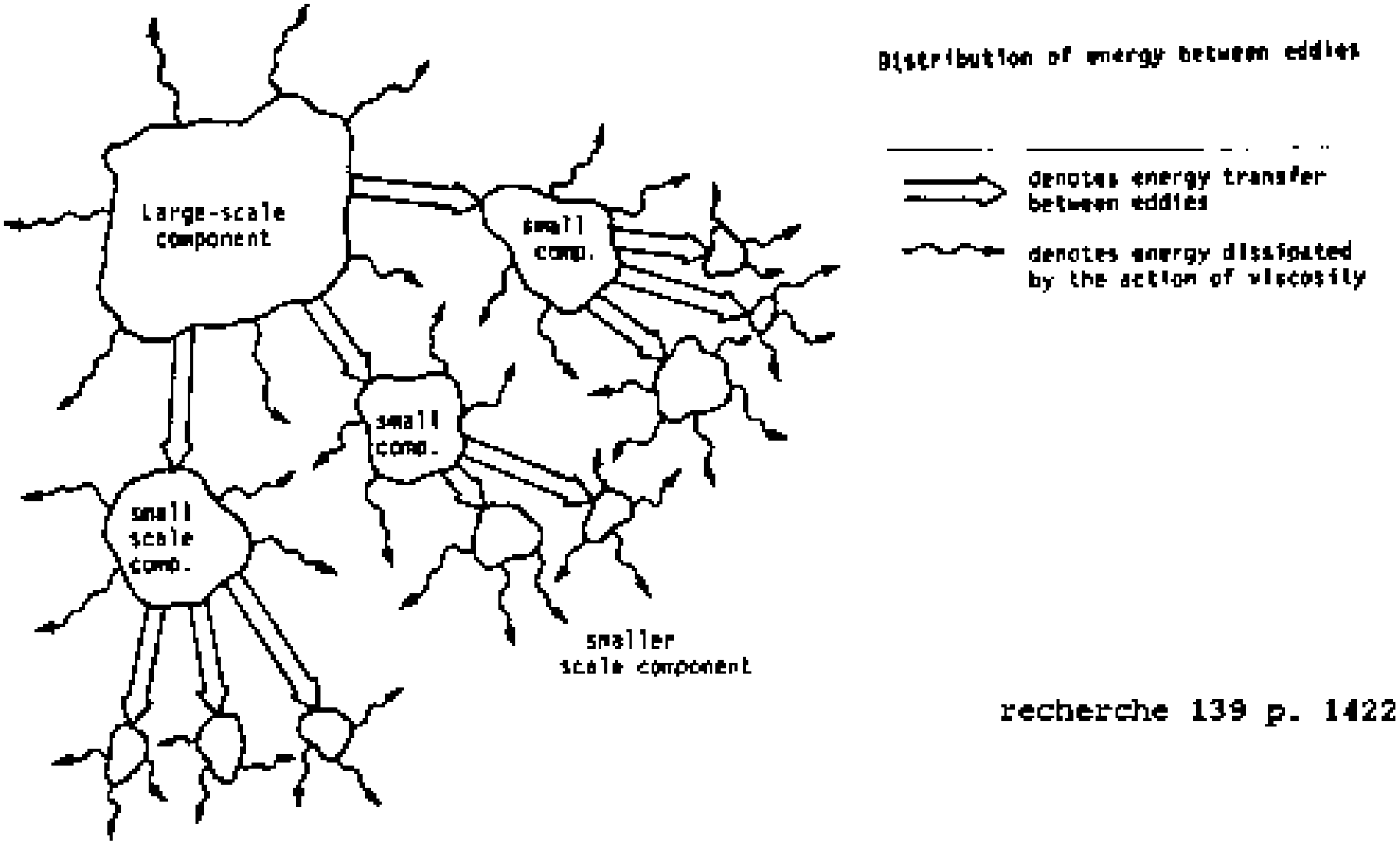
fractais



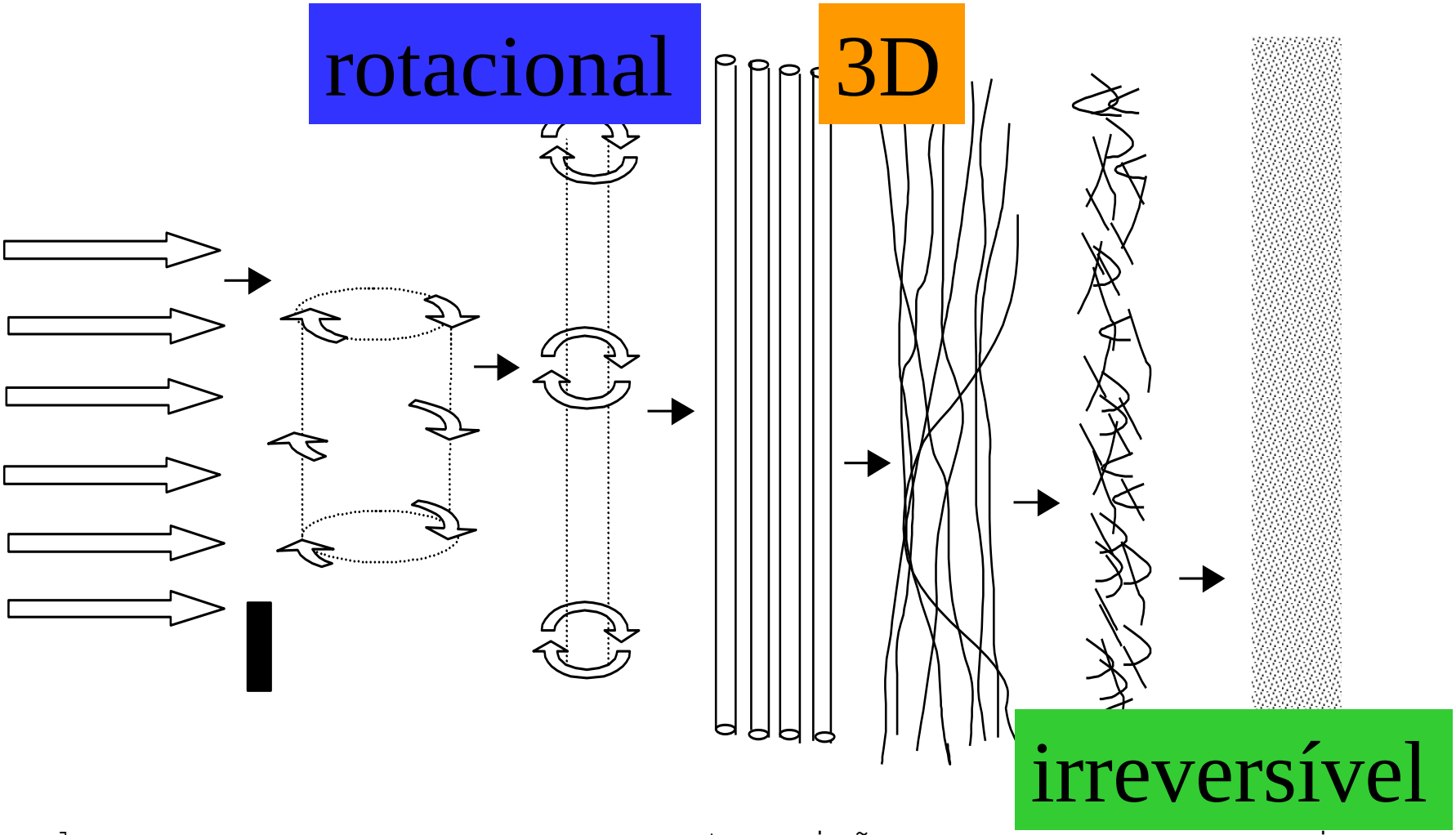
Kolmogorov






Distribution of energy between eddies
denotes energy transfer between eddies
denotes energy dissipated by the action of viscosity
Large-scale component
small comp.
small comp.
small scale comp.
smaller scale component
recherche 139 p. 1422

Kolmogorov

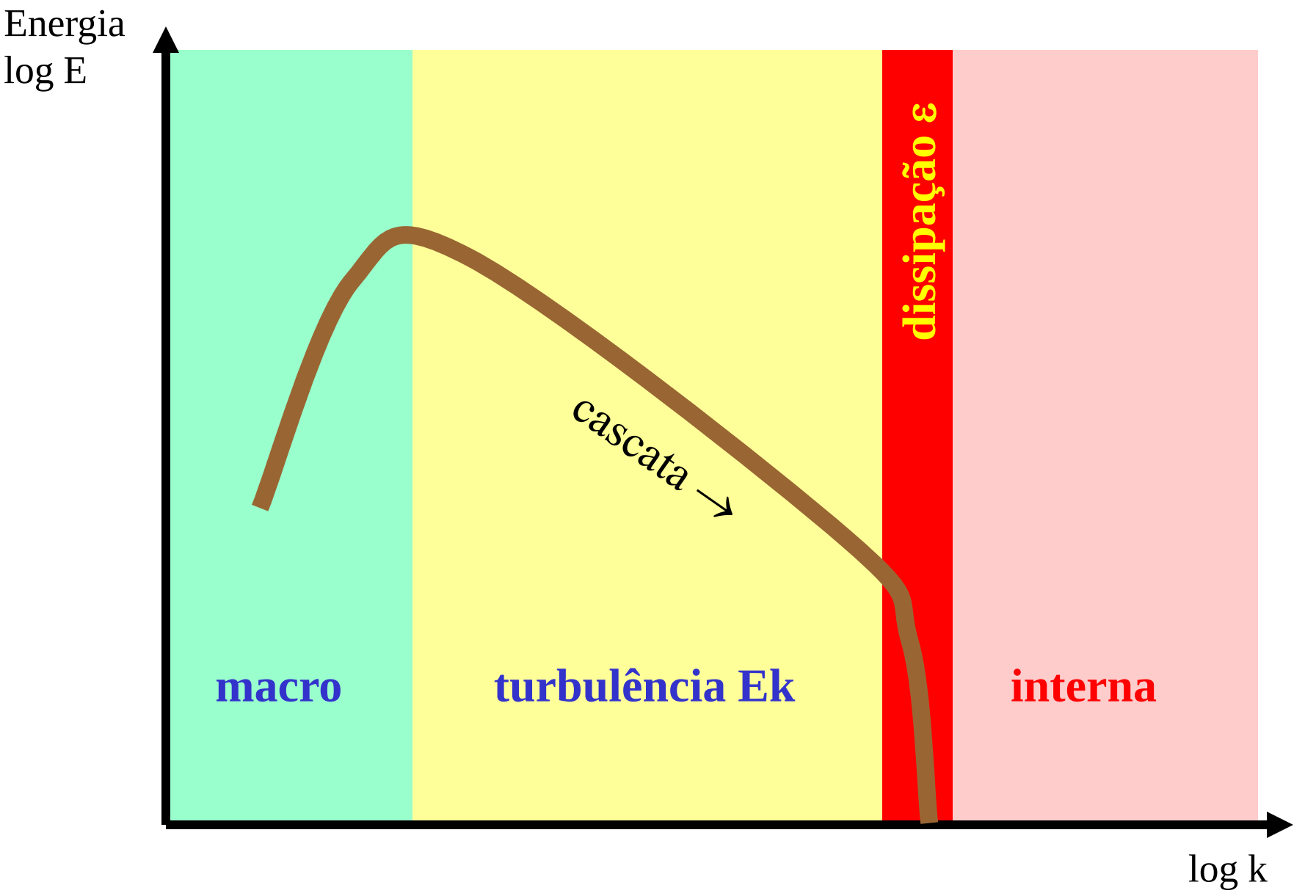


escala: macro transição micro
energia cinética macro turbulenta micro
proporcional: **v** ←---------- k ----------→ T

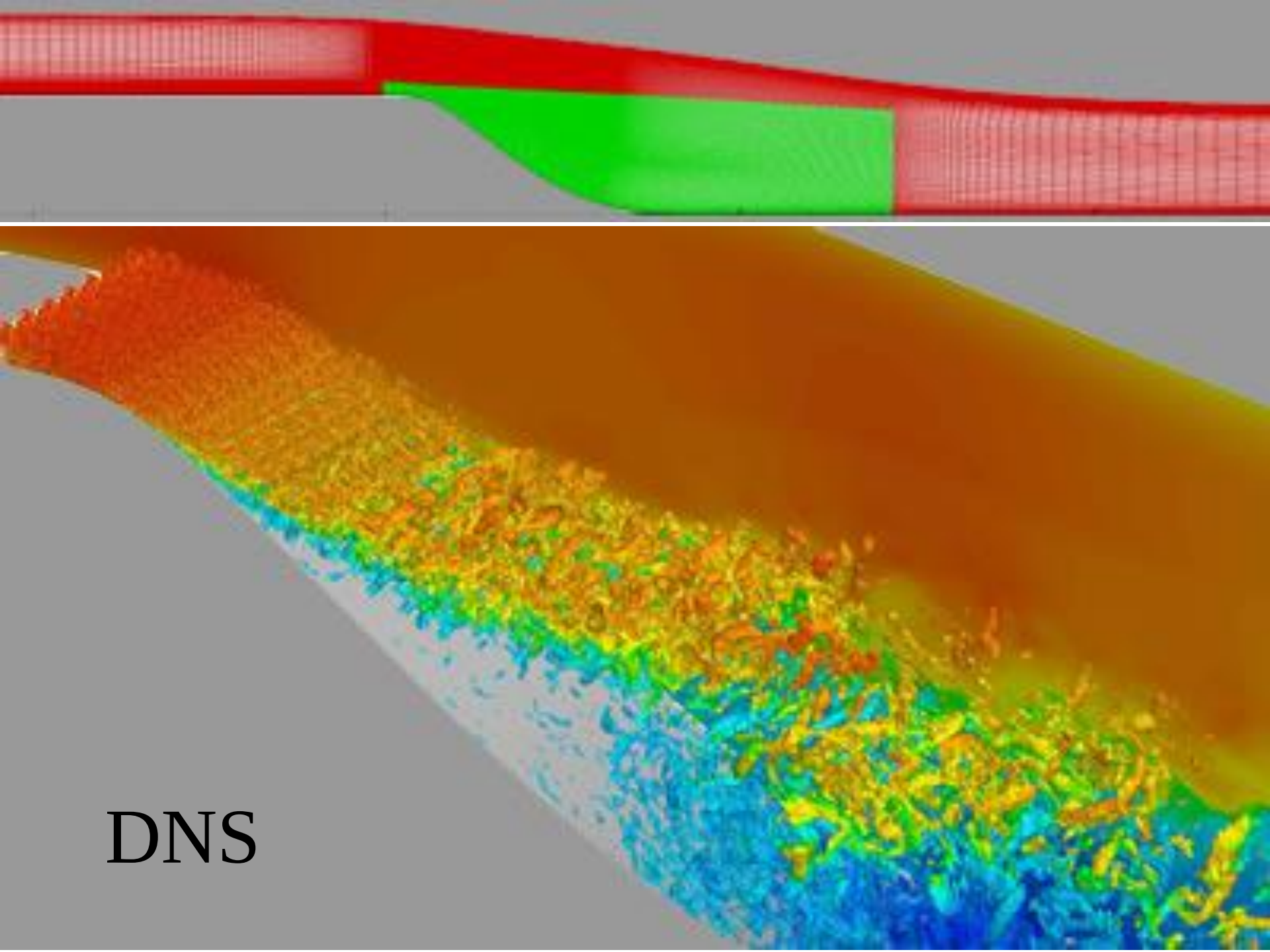
escalas de turbulências

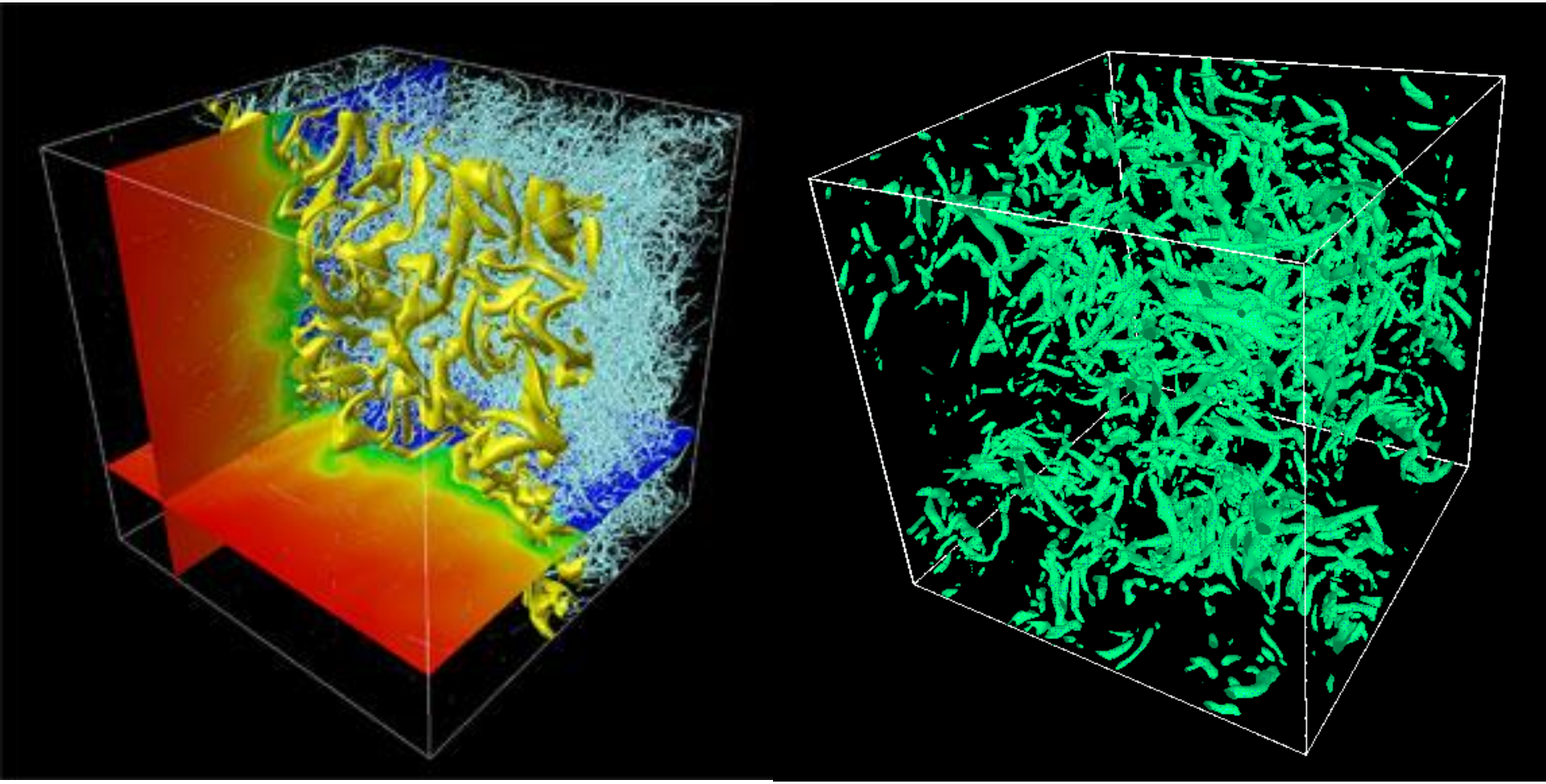
Escalas de Kolmogorov – menores escalas de turbulência

- Comprimento $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ $\frac{\eta}{l_0} = \mathrm{Re}^{-3/4}$
- Velocidade $u_\eta = (\nu\varepsilon)^{1/4}$ $\frac{u_\eta}{u_0} = \mathrm{Re}^{-1/4}$
- Tempo $\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$ $\frac{\tau_\eta}{\tau_0} = \mathrm{Re}^{-1/2}$
- Reynolds $\mathrm{Re}_\eta = \frac{u_\eta \eta}{\nu} = 1$


Energia
log E
dissipação ε
cascata →
macro
turbulência Ek
interna
log k
número de onda

cascata


DNS



Navier Stokes