Vamos agora discutir a Distribuição Gaussiana para o Movimento Browniano (MB) com $p=q$ e $p\neq q$

$$P(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-\frac{(n-\langle n\rangle)^2}{2\sigma_n^2}} \quad \text{ou} \quad P(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}}\, e^{-\frac{(x-\langle x\rangle)^2}{2\sigma_x^2}}$$

n = nº de passo p/ direita

$$\begin{cases} \langle n\rangle = Np \\ \langle n^2\rangle = Np(q+Np) \\ \sigma_n^2 = \langle n^2\rangle - \langle n\rangle^2 \end{cases}$$

$p = 0{,}6$ e $q = 1-p = 0{,}4$

$N = 6250$ passos

$\langle n\rangle = Np = 6250 \times 0{,}6 = 3750$

$\langle n^2\rangle = 6250\times 0{,}6\,(0{,}4 + 6250\times 0{,}6)$

$\langle n^2\rangle = 14\,064\,000$

$\sigma_n^2 = \langle n^2\rangle - \langle n\rangle^2 = 14064000 - 3750^2$

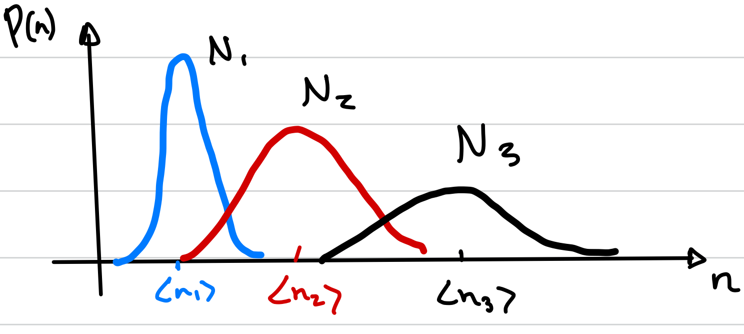
$\sigma_n^2 = 1500$

$$P(n) = 0{,}010\, e^{-(n-3750)^2/3000}$$

varia com N

se $N = 15000$

$\langle n\rangle \propto N \qquad N_1 < N_2 < N_3$



x = deslocamento $= (n-m)\ell$

m = nº passos p/ esquerda $= N-n$

Então $x = [n-(N-n)]\ell = (n-N+n)\ell$

$x = 2\ell n - N\ell \Rightarrow dx = 2\ell\, dn$

$$\underbrace{P(x)\,dx}_{\text{Gaussina também}} = \underbrace{P(n)\,dn}_{\text{gaussiana}}$$

$t = N\tau \quad \therefore \quad N = \dfrac{t}{\tau}$

$\langle x\rangle = \langle 2\ell n - N\ell\rangle$

$\langle x\rangle = 2\ell\langle n\rangle - N\ell$

$\langle x\rangle = 2\ell Np - N\ell$

$\langle x\rangle = \ell N(2p-1)$

como $p+q=1$

$\langle x\rangle = \ell N(2p-p-q)$

$\langle x\rangle = \ell N(p-q)$

$$\boxed{\langle x\rangle = \ell t(p-q)/\tau}$$

$\langle x^2\rangle = \langle(2\ell n - N\ell)^2\rangle$

$\langle x^2\rangle = \langle 4\ell^2n^2 - 4\ell^2Nn + N^2\ell^2\rangle$

$\langle x^2\rangle = 4\ell^2\langle n^2\rangle - 4\ell^2N\langle n\rangle + N^2\ell^2$

para $p=q \Rightarrow \langle x\rangle = 0$

$p\neq q \Rightarrow \langle x\rangle = \ell t(p-q)/\tau$, ou seja, cresce com t.

$$\langle x^2\rangle = 4\ell^2\langle n^2\rangle - 4\ell^2 N\langle n\rangle + N^2\ell^2$$
$$\langle x^2\rangle = 4\ell^2(Np(q+Np)) - 4\ell^2 N(Np) + N^2\ell^2$$
$$\langle x^2\rangle = 4\ell^2(Npq + N^2p^2) - 4\ell^2N^2p + N^2\ell^2$$
$$\langle x^2\rangle = 4\ell^2Npq + 4\ell^2N^2p^2 - 4\ell^2N^2p + N^2\ell^2$$
$$\langle x^2\rangle = 4\ell^2Npq + \ell^2N^2(4p^2 - 4p + 1)$$
$$\langle x^2\rangle = 4\ell^2Npq + \ell^2N^2(2p-1)^2$$
$$\langle x^2\rangle = 4\ell^2Npq + \ell^2N^2(2p-(p+q))^2$$
$$\langle x^2\rangle = 4\ell^2Npq + \ell^2N^2(p-q)^2$$

$$\boxed{\langle x^2\rangle = \frac{4\ell^2 t}{\tau}pq + \frac{\ell^2t^2}{\tau^2}(p-q)^2}$$

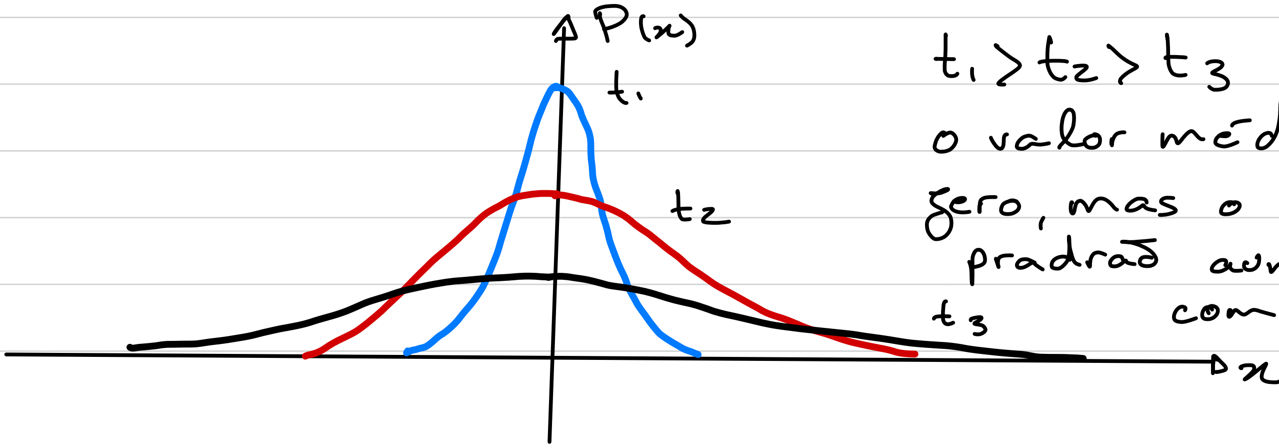
para $p = q = \frac{1}{2}$ $\quad \langle x^2\rangle = \dfrac{\ell^2 t}{\tau}$ $\quad$ e $\quad \langle x\rangle = 0$

então $\sigma_n^2 = \langle x^2\rangle - \cancel{\langle x\rangle^2}^{\to 0} = \dfrac{\ell^2 t}{\tau} = 2Dt$

onde $D = \dfrac{\ell^2}{2\tau}$ = coef. de difusão para $p=q=\frac{1}{2}$

$$\boxed{P(x) = \frac{1}{\sqrt{4\pi D t}}\, e^{-x^2/4Dt}}$$

Distribuição Gaussiana para $p = q = \frac{1}{2}$



$t_1 > t_2 > t_3$
o valor médio é zero, mas o desvio pradrão aumenta com t.

Para $p \neq q$, vamos achar a expressão geral:

$\langle x \rangle = 2l\langle n \rangle - Nl$

$\langle x^2 \rangle = 4l^2\langle n^2 \rangle - 4l^2N\langle n \rangle + N^2l^2$

$\langle n \rangle = Np$

$\langle n^2 \rangle = Np(q + Np)$

$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$

$\sigma_x^2 = 4l^2\langle n^2 \rangle - 4l^2N\langle n \rangle + N^2l^2 - (2l\langle n \rangle - Nl)^2$

$\sigma_x^2 = 4l^2\langle n^2 \rangle - \cancel{4l^2N\langle n \rangle} + \cancel{N^2l^2} - 4l^2\langle n \rangle^2 + \cancel{4l^2N\langle n \rangle} - \cancel{N^2l^2}$

$\sigma_x^2 = 4l^2(\langle n^2 \rangle - \langle n \rangle^2)$

$\boxed{\sigma_x^2 = 4l^2\sigma_n^2}$

$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2$

$\sigma_n^2 = Npq + \cancel{N^2p^2} - \cancel{N^2p^2}$

$\boxed{\sigma_x^2 = 4l^2Npq} \Leftarrow \sigma_n^2 = Npq$

Sumário: Lembrando que $x = 2ln - Nl$ e $t = N\tau$

$P(n) = \dfrac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(n-\langle n \rangle)^2}{2\sigma_n^2}}$

$\langle n \rangle = Np$

$\langle n^2 \rangle = Np(q + Np)$

$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2 = Npq$

$P(n) = \dfrac{1}{\sqrt{2\pi Npq}} e^{-(n-Np)^2/2Npq}$

$P(x) = \dfrac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\langle x \rangle)^2}{2\sigma_x^2}}$

$\langle x \rangle = 2l\langle n \rangle - Nl = lt(p-q)/\tau$

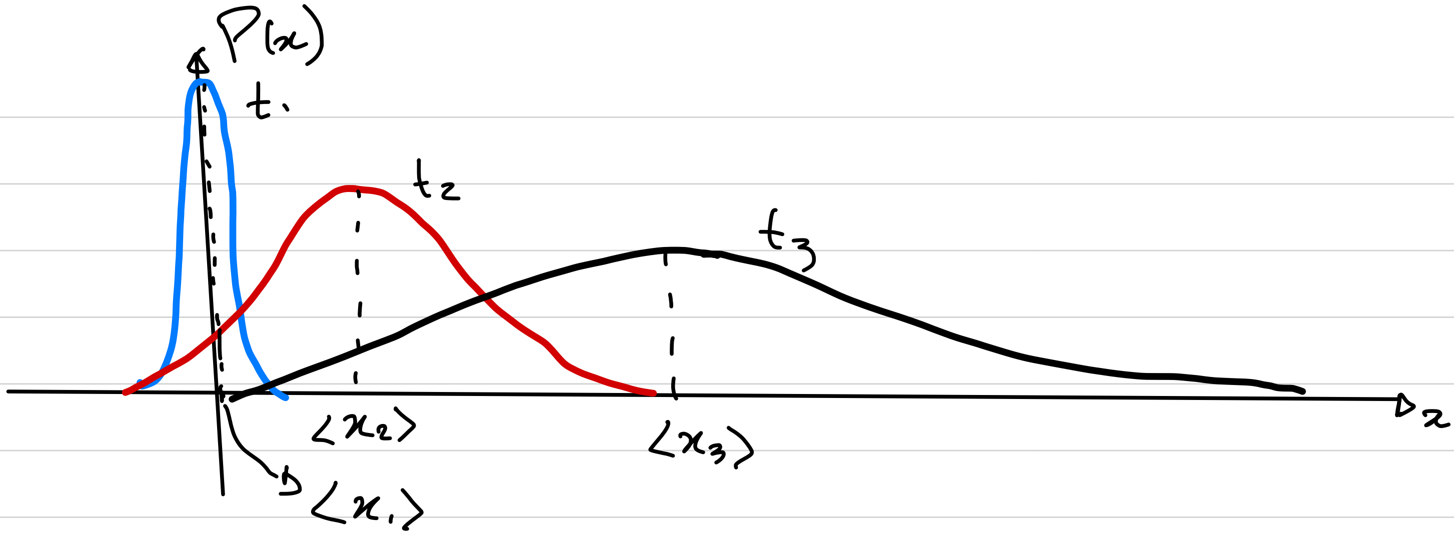
$\langle x^2 \rangle = 4l^2\langle n^2 \rangle - 4l^2N\langle n \rangle + N^2l^2$

$= \dfrac{4l^2t}{\tau}pq + \dfrac{l^2t^2}{\tau^2}(p-q)$

$\sigma_x^2 = 4l^2\sigma_n^2$

$= \dfrac{4l^2t}{\tau}pq = 2Dt$

$P(x) = \dfrac{1}{\sqrt{4\pi Dt}} e^{-\frac{(x - lt(p-q)/\tau)^2}{4Dt}}$



O valor médio de x ($\langle x \rangle$) cresce com t e
e a variância de x (σ_x^2) também.