

# CAPÍTULO 5

## FLAMBAGEM DE COLUNAS (VIGAS)



$$\frac{d^2 v(x)}{dx^2} = \frac{M(x)}{EI}$$

$$v(x) \rightarrow \epsilon_x \rightarrow \sigma_x$$

PLACAS

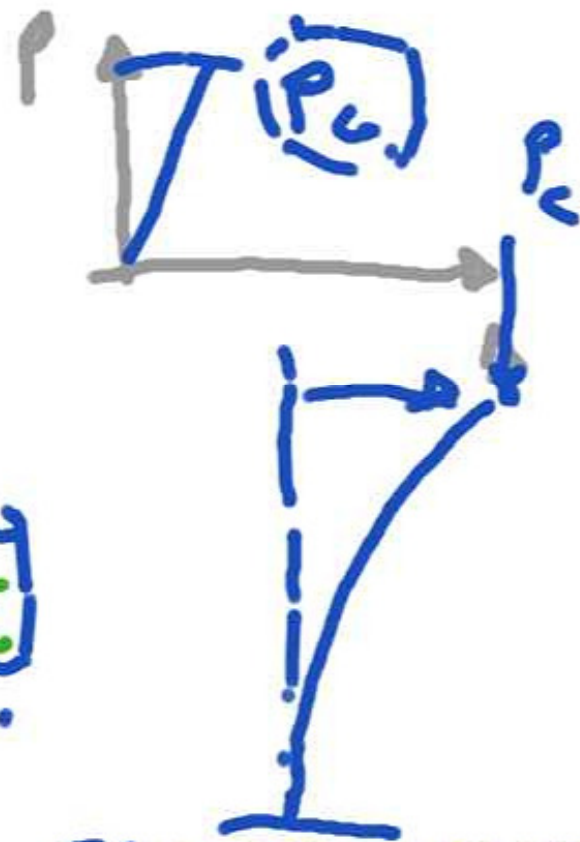
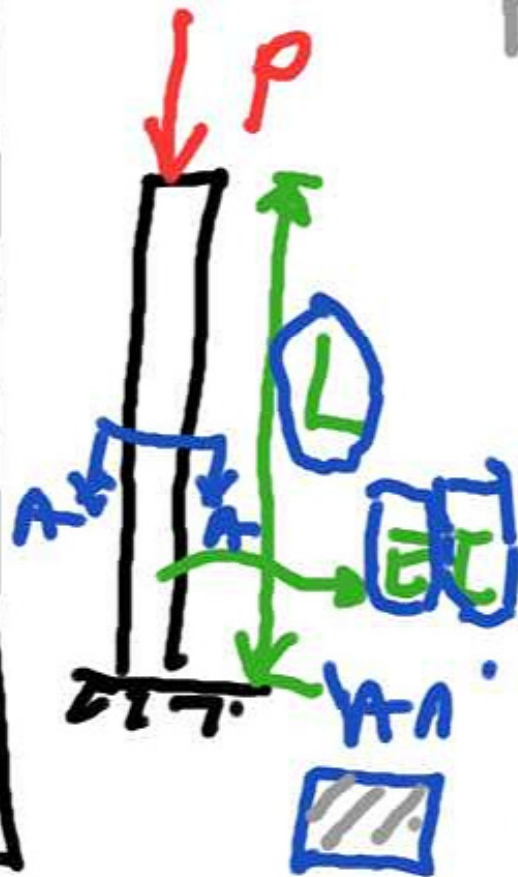
$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$$



$$w(x, y)$$

$$\epsilon_x, \epsilon_y, \gamma_{xy}$$

VIGAS-COLUNAS  
- COMPRESSÃO -



FLAMBAGEM

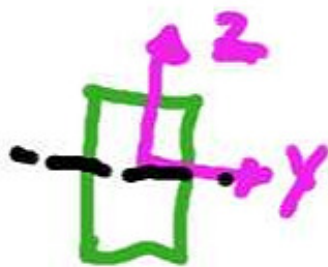
# FLAMBAGEM

$$EI \frac{d^2 v(x)}{dx^2} = -P v(x)$$



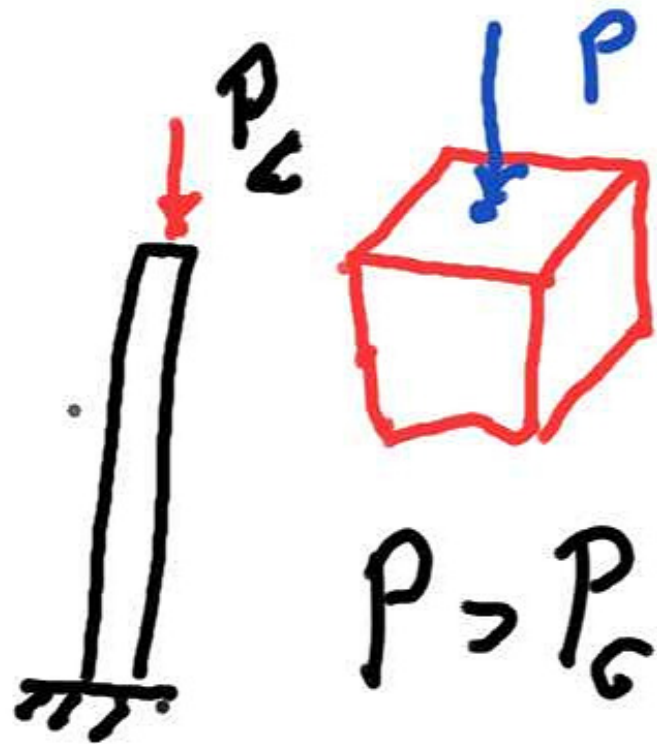
$$M = P v(x)$$

CONDIÇÃO CONTORNO



$$I_y \rightarrow I_{\min}$$
$$I_z \rightarrow I_{\min}$$

$$P_c = \frac{\pi^2 EI_{\min}}{L^2}$$



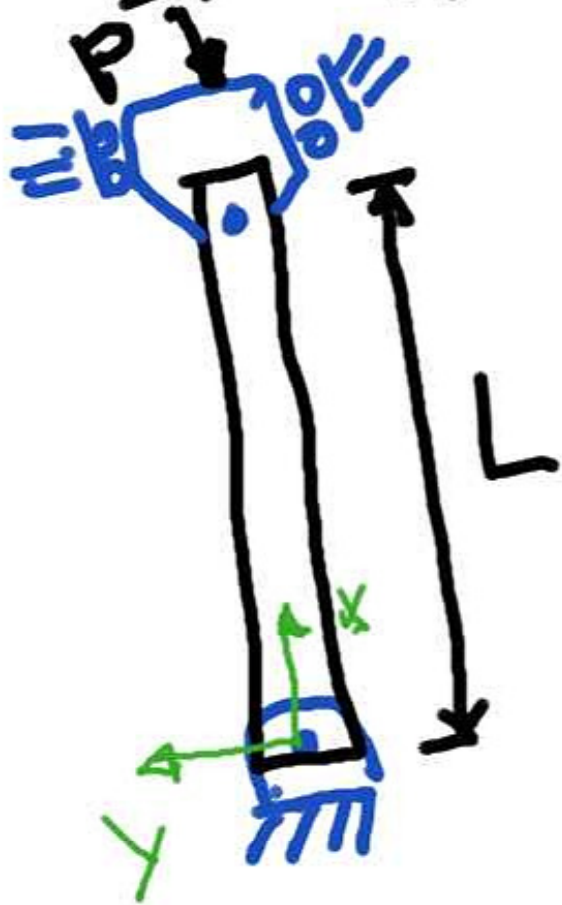
ESTABILIDADE DO EQUILÍBRIO



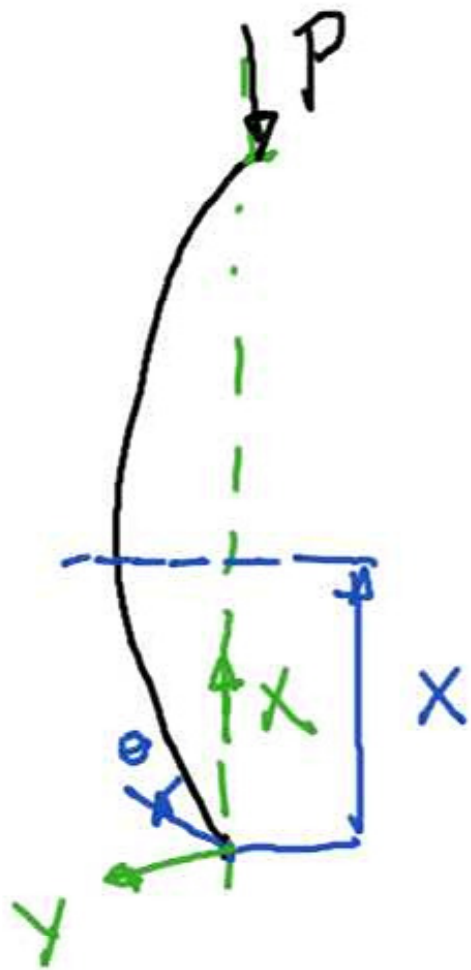
INSTÁVEL

ESTÁVEL

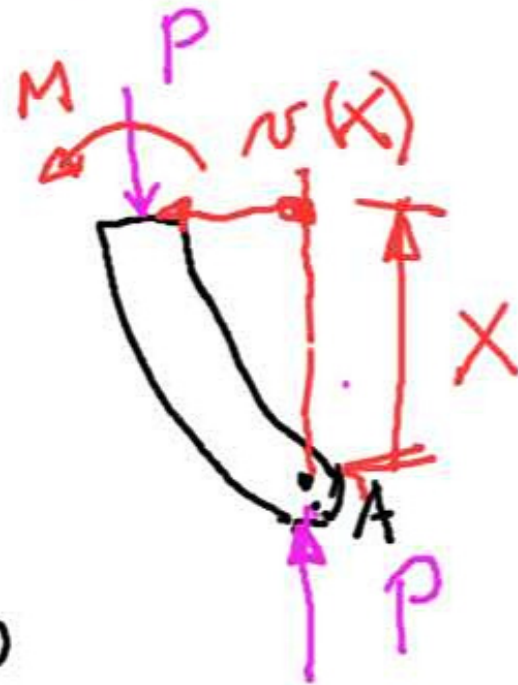
# INSTABILIDADE DE COLUNAS



- COLUNA RETA
- MAT. LINEAR-ELÁSTICO
- COLUNA SIMÉTRICA NO PLANO  $XY$
- DEFLEXÃO NO PLANO  $XY$



D.C.L



$$\sum M_A = 0$$

$$M(x) + P v(x) = 0$$

$$M(x) = -P v(x)$$

• A DEFLEXÃO  $v(x)$  É OBTIDA PELA TEORIA DE VIGAS

$$\frac{d^2 v(x)}{dx^2} = \frac{M(x)}{EI}$$

$$M(x) = -Pv(x)$$



$$EI \frac{d^2 v(x)}{dx^2} + P_{CR} v(x) = 0 \quad P = P_{CR}$$

$$v(x) = ?$$

$$P_{CR} = ?$$

+ C. CONTORNO

$$v(x) = 0$$

$$v(L) = 0$$

$$\Rightarrow y'' + a y' + a_0 y = 0$$

$$y = e^{\lambda x} \quad y' = \lambda e^{\lambda x} \quad y'' = \lambda^2 e^{\lambda x}$$

↓

$$e^{\lambda x} \neq 0 \quad | \quad \underbrace{\lambda^2 + a\lambda + a_0}_{=0} = 0$$

$$\lambda^2 + a\lambda + a_0 = 0$$

EQ.

CHARACTER

$$v'' + \frac{P}{EI} v = 0$$

$$\lambda^2 = \frac{P}{EI}$$

$$v'' + \lambda^2 v = 0$$

$$v(x) = c_1 \sin \lambda x + c_2 \cos \lambda x$$

$$\lambda \quad c_1 \quad c_2 \quad \} ?$$



$$v(L) = 0$$

$$v(x) = c_1 \sin \lambda x + c_2 \cos \lambda x$$

$$v(0) = 0$$

$$\hookrightarrow v(0) = c_1 \sin \lambda 0 + c_2 \cos \lambda 0$$

$$c_2 = 0$$

$$v(L) = 0 \rightarrow v(L) = c_1 \sin \lambda L = 0$$

$$c_1 = 0$$

$$\sin \lambda L = 0$$

$$\sin \lambda L = 0$$



$$\lambda L = n\pi \rightarrow$$

$$\lambda_n = \frac{n\pi}{L}$$

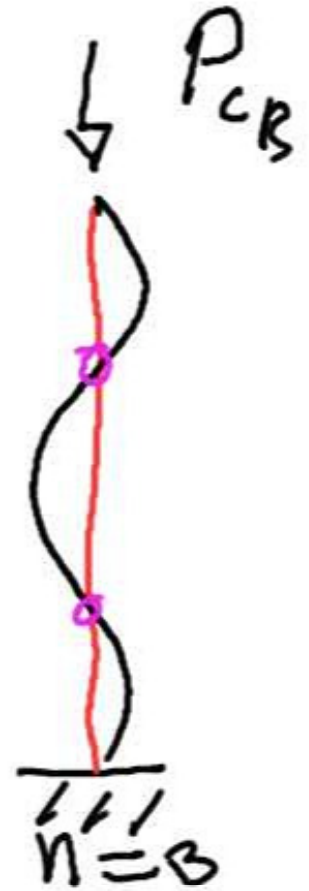
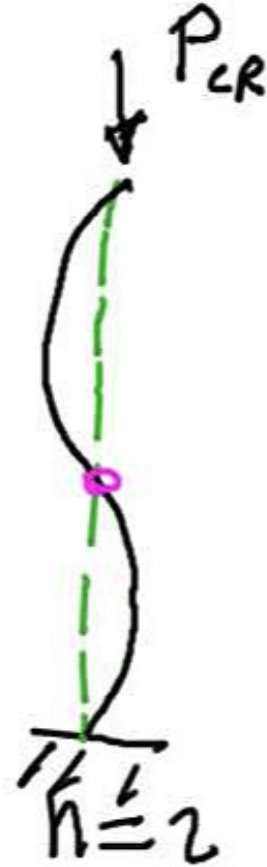
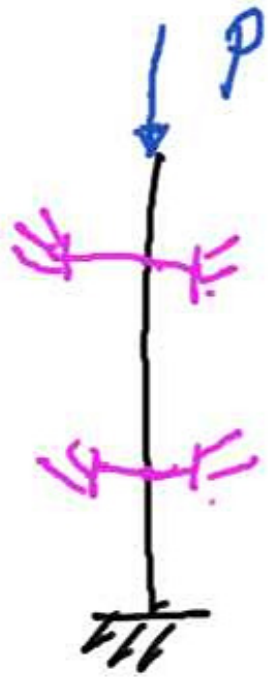
$$\lambda^2 = \frac{P_c}{EI}$$

$$n = 1, 2, 3, \dots$$

CARGA  
CRÍTICA  
EULER

$$P_{CR} = \frac{n^2 \pi^2 EI}{L^2}$$

# MODOS DE FLAMBAGEM



$n=1$

$\sigma^2$