EXERGY ANALYSIS OF ENERGY CONVERSION PROCESSES

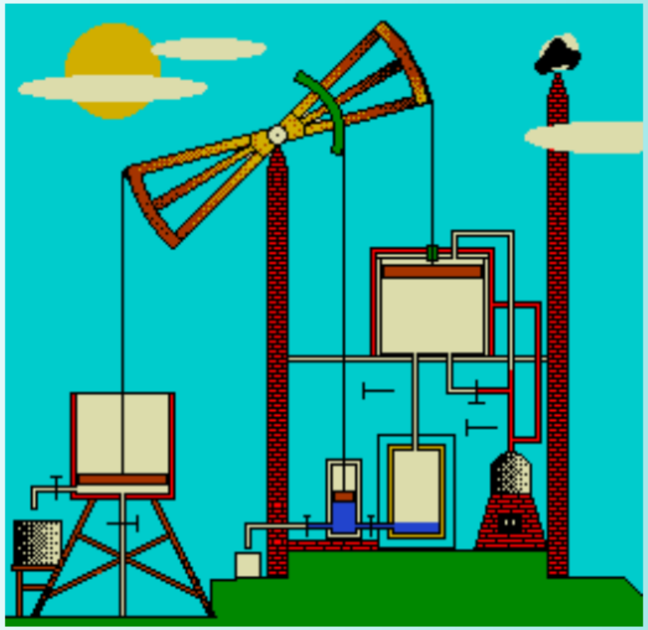
Efficiency

2nd Law

Quality

Quantity

1st Law

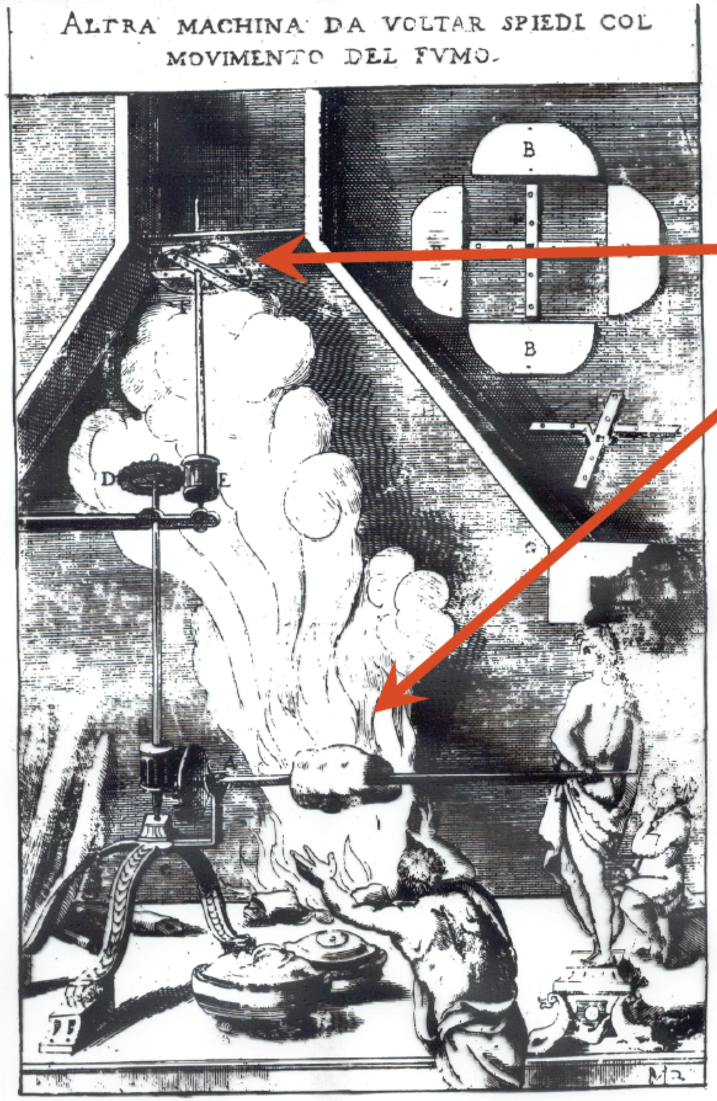


Newcomen Engine (1712)

**Le plus important est la détermination de la temperature à laquelle il est le plus convenable d'employer la vapeur.
En effet, plus ele est chaude, plus sa force élastique est considérable, et par conséquent plus elle produit d'effort sur la surface du piston qu'elle presse, le vide étant toujour de l'autre coté.
Mais aussi il faut consommer plus de charbon pour produire une vapeur plus chaude; en sorte que le profit ou le désavantage de la température est un element à determiner.**

(J. B. Biot, Traité de Physique Expérimentale et Mathématique, tome 4, p. 739, Paris, Deterville, 1816.)

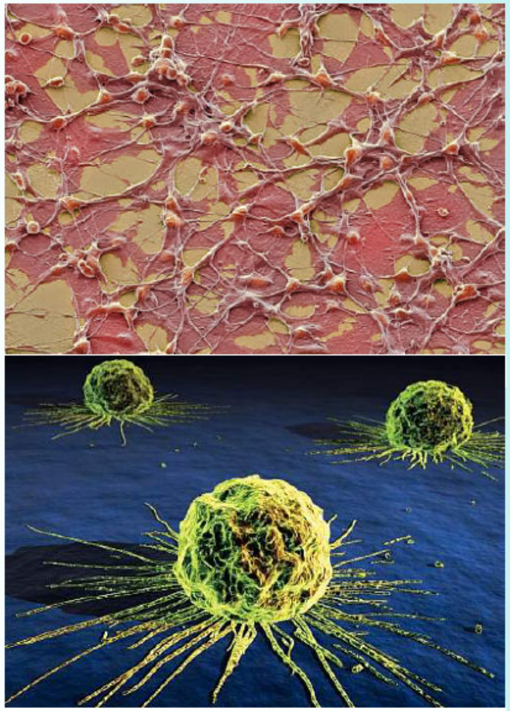
Efficiency and Sustainability



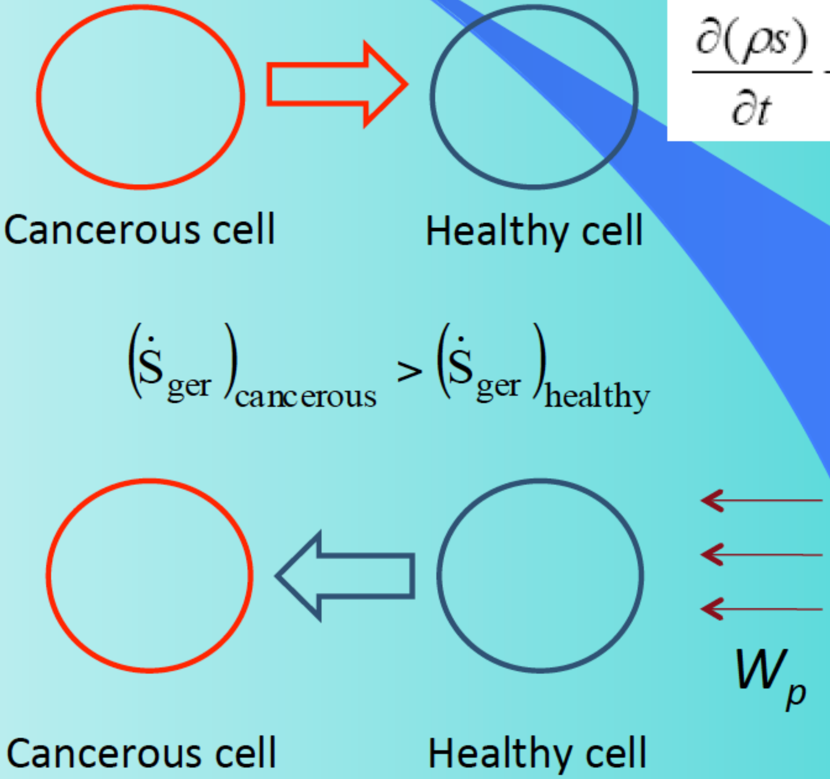
Sustainable use of energy resources?

Novo Teatro di Machine et Edificii, Vittorio Zonca, In Padova, appresso Pietro Bertelli, 1607

Entropy Generation and Pathology



Cancerous cell



Entropy transfer

Cancerous cell

Healthy cell

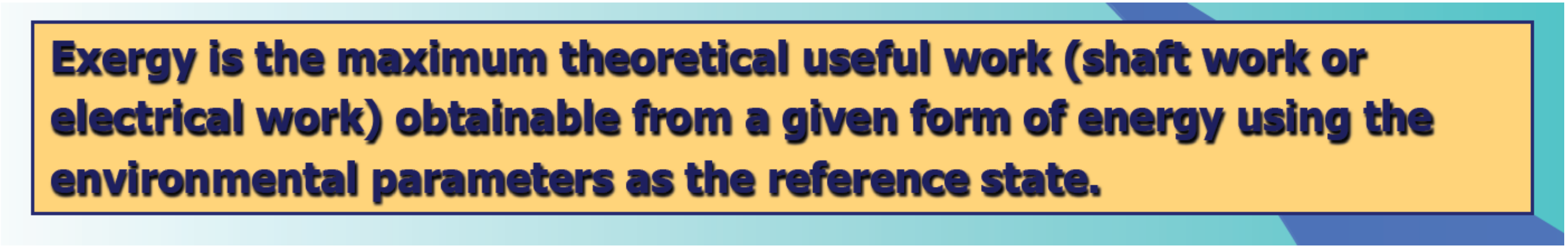
$$\frac{\partial(\rho s)}{\partial t} + \nabla \cdot \mathbf{j}_s = \sigma_s$$

$$\left(\dot{S}_{\text{ger}}\right)_{\text{cancerous}} > \left(\dot{S}_{\text{ger}}\right)_{\text{healthy}}$$

W_p

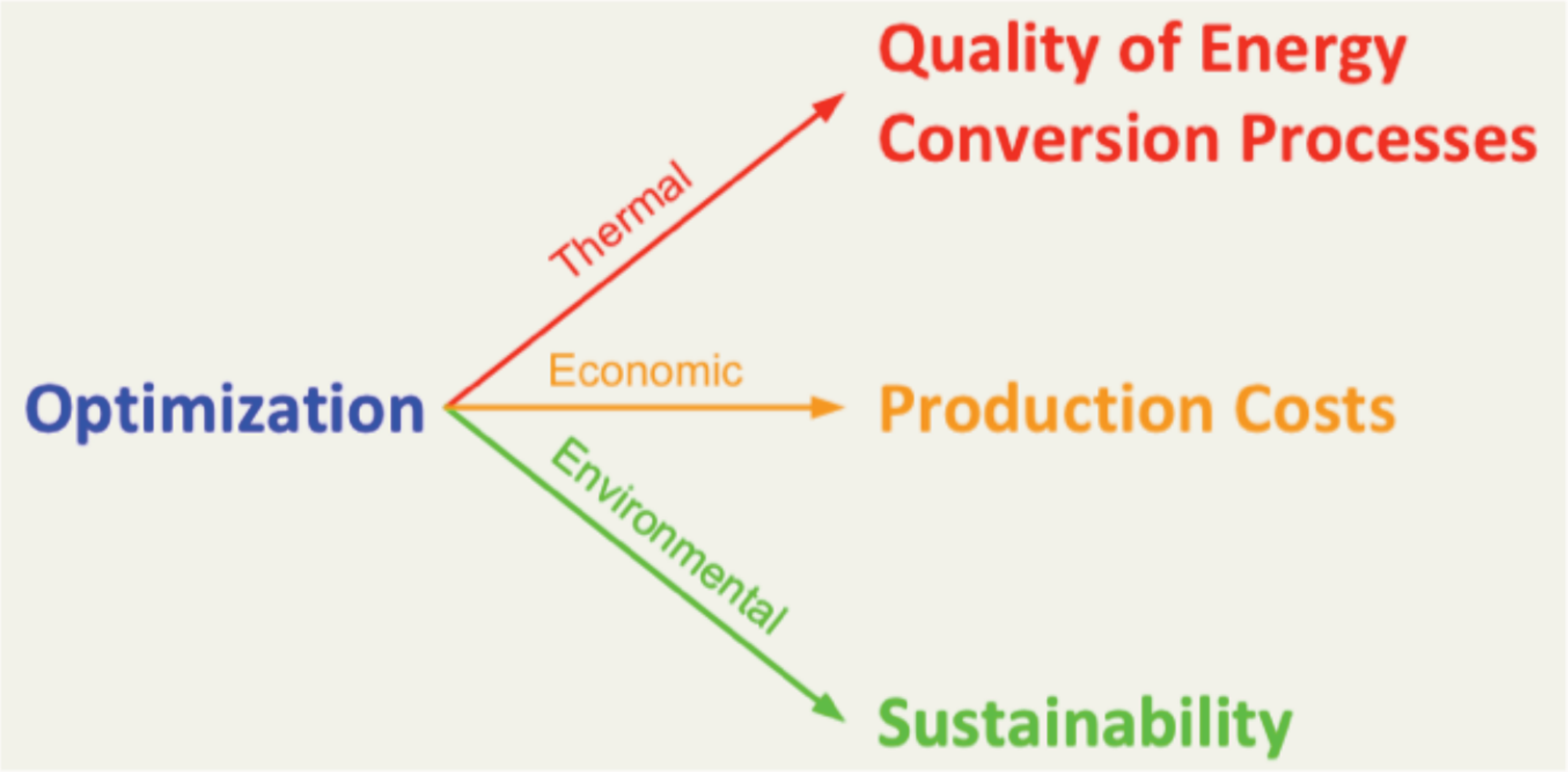
Cancerous cell

Healthy cell



Exergy is the maximum theoretical useful work (shaft work or electrical work) obtainable from a given form of energy using the environmental parameters as the reference state.

Exergy and Optimization



Optimization

Thermal

Quality of Energy Conversion Processes

Economic

Production Costs

Environmental

Sustainability

Performance Assessment

- Synthesis plant
- Mass, energy, entropy, and exergy balances
- Thermodynamic and transport properties
- Equipment and processes models (effectiveness, efficiency, performance charts, conversion rates)
- Process simulator (EES, GSP, Gate Cycle, Aspen Hysys, tailor-made)
- Costs
- Performance parameters: energy, exergy, economy, environment

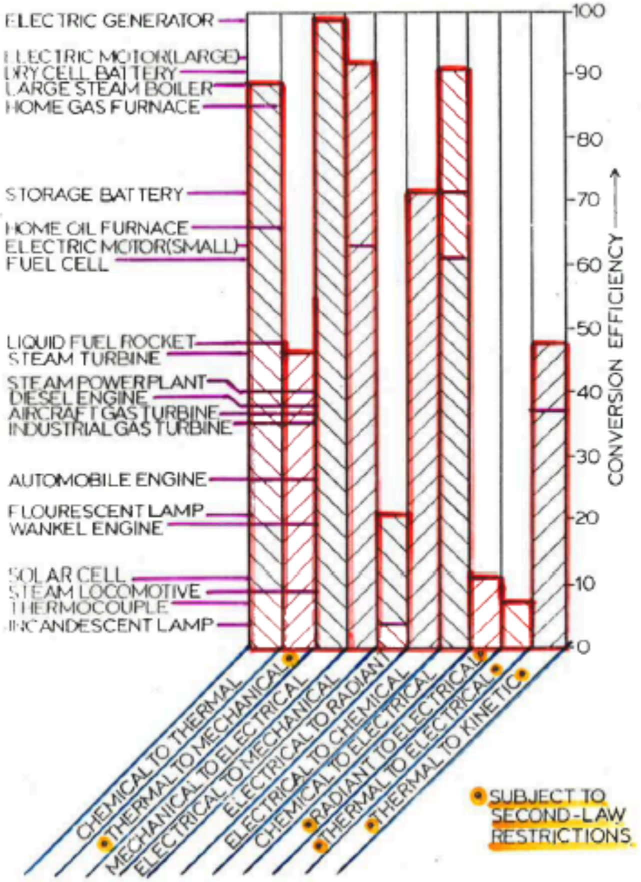
LETE Applications

* Petroleum Primary Processing Plants (Offshore and Onshore)
* Petroleum Refinery Plant
* Sugar, Alcohol and Electricity Production Plant
* Aircraft Systems Design
* Hydrogen Production Unit
* Acetaldehyde Production Plant
* Combined Cycle Power Plant
* Trigeneration plants
* BIGCC Cogeneration System
* Biodiesel Production
* Human Body and its Systems

Table 2.1 Exergy and related concepts [16]

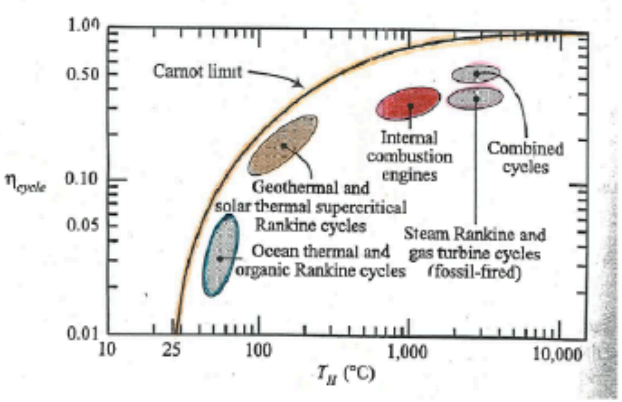
Year	Author	Designation
1824	Carnot	Puissance motrice du feu
1872	Thomson (Kelvin)	Motivity
1873	Gibbs	Available energy of the body and medium
1881	Gouy	Energie utilisable
1898	Stodola	Freie technische energie
1925	Debaufre	Available energy
1935	Bonsjakovic	Technische arbeitsfähigkeit
1944	Thring	Virtue of energy
1953	Schmidt	Technische maximale arbeit
1955	Gibert	Energie non dégradée
1956	Grigull	Ekthalpie
1956	Rant	EXERGIE

2a Lei e Trabalho Máximo

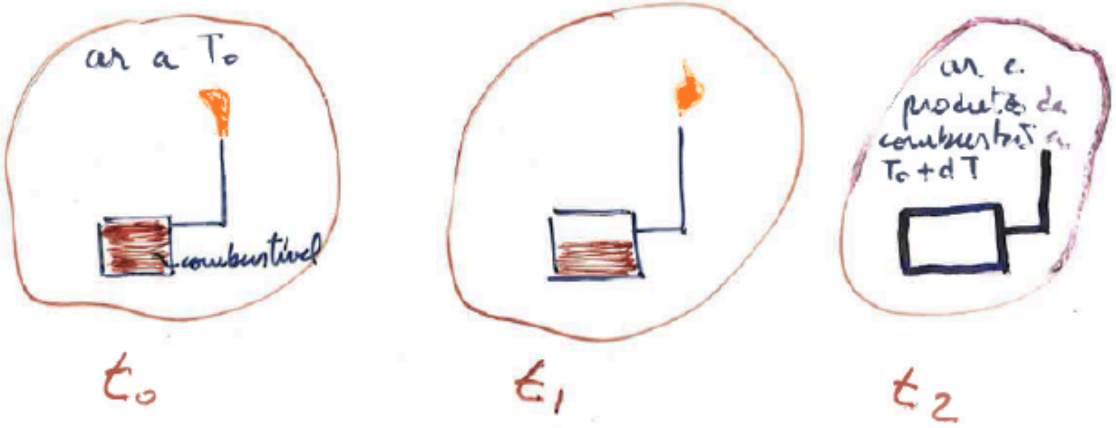


Comparative efficiencies of energy converters

RENDIMENTOS TÍPICOS DE MÁQUINAS TÉRMICAS



Utilidade da Energia



→ energia = cte.

→ utilidade da energia diminui

$T_1 = 450°C$

$p_1 = 30\,bar$

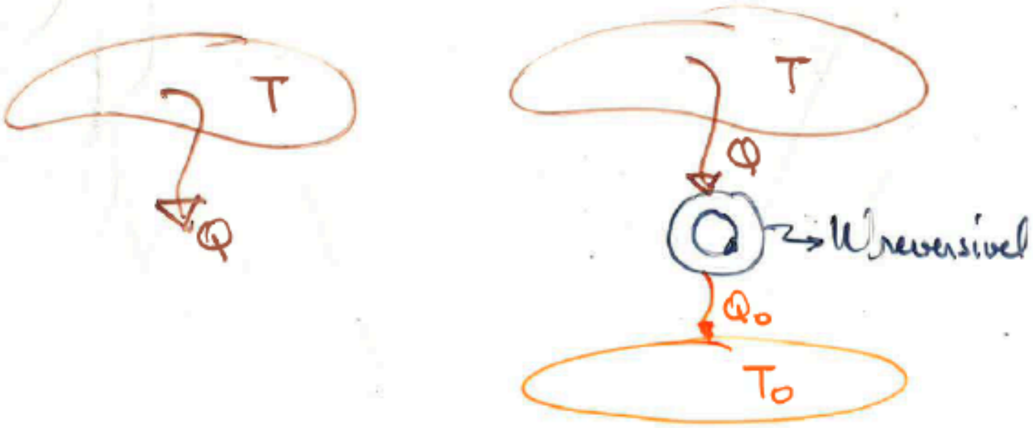
$h_1 = 3343{,}3\ \frac{kJ}{kg}$

$p_2 = 5\,bar$

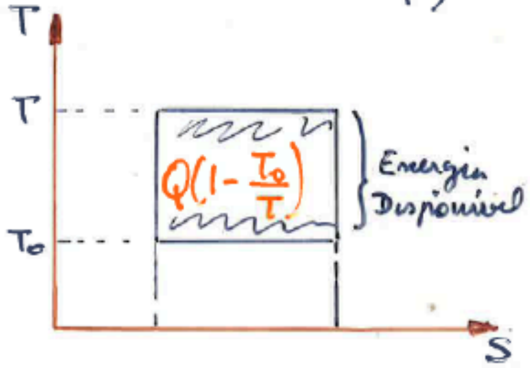
$h_{2,isc} = 2866{,}3\ \frac{kJ}{kg}$

$W_{irr} = 477{,}0\ \frac{kJ}{kg}$

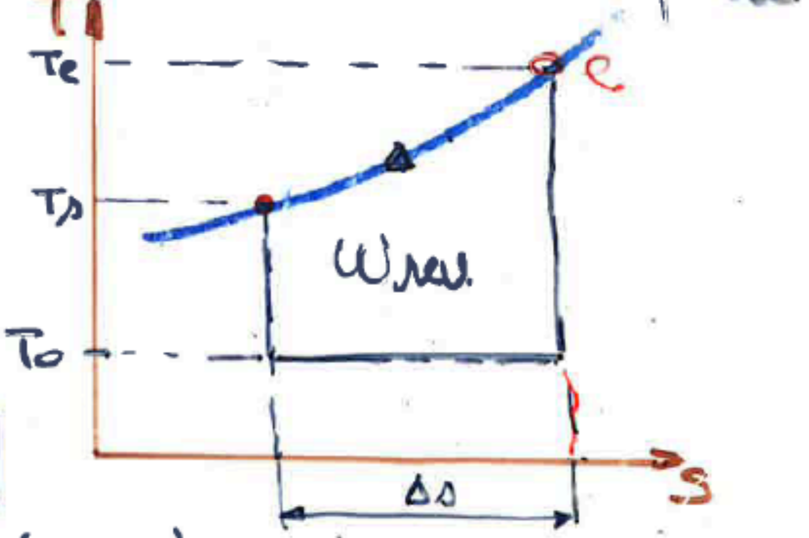
Trabalho Reversível



$$W_{\text{reversível}} = Q\left(1-\frac{T_0}{T}\right)$$



Trabalho Reversível



$$\dot{Q} = (h_e - h_s)\dot{m}$$

$$\dot{W}_{rev} = \dot{Q} - T_o \dot{m} \Delta s$$

$$\dot{W}_{rev} = \dot{m}\left[h_e - h_s - T_o(s_e - s_s)\right]$$

$$w_{rev} = \frac{\dot{W}_{rev}}{\dot{m}} = h_e - h_s - T_o(s_e - s_s)$$

2.2 Exergy and Exergy Balance

2.2.1 *Reversible Work*

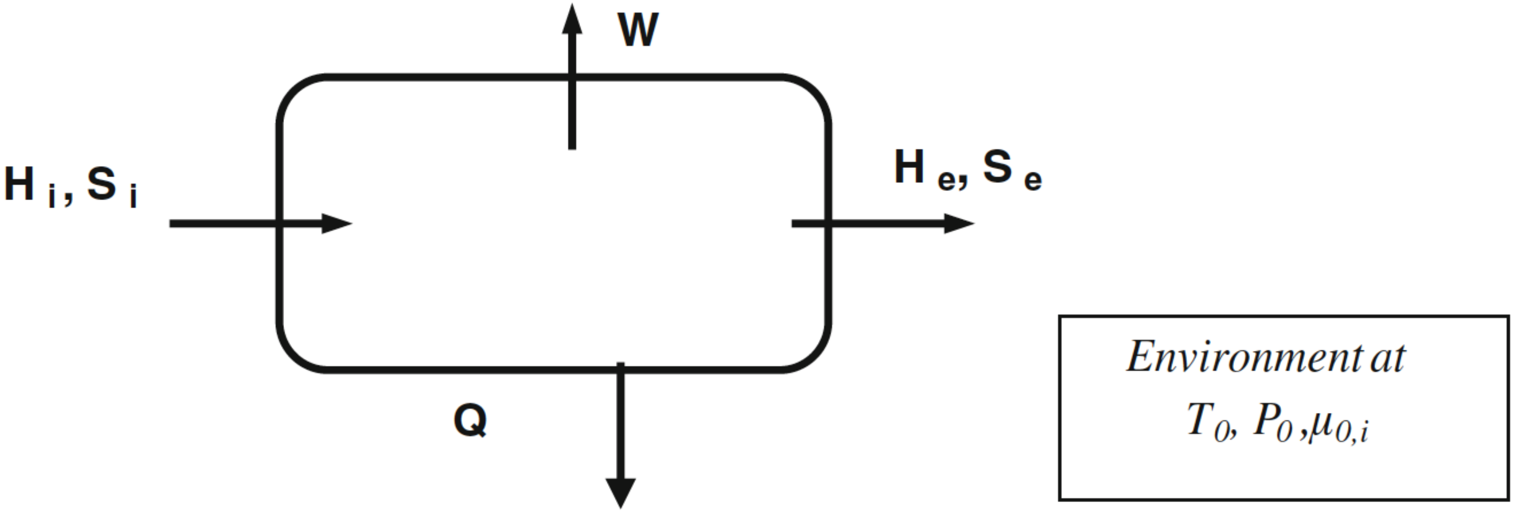


Fig. 2.1 Control volume with one inlet and one outlet

For the chosen control volume, one could ask the following question: ‘What is the maximum work that can be performed giving the same thermodynamic states at the inlet and outlet sections of the control volume?’ This maximum work is determined under the following conditions:

- all the processes within the control volume are reversible;
- the maximum work that can be generated using the heat rejected by the control volume is the one obtained from a Carnot engine, operating between temperatures T_i and T_e and the environment temperature where the control volume is placed (T_0).

$$q = h_\mathrm{e} - h_\mathrm{i} + w \tag{2.1}$$

$$s_\mathrm{e} - s_\mathrm{i} = \int \frac{\delta q}{T} + \mathrm{s}_\mathrm{ger} \tag{2.2}$$

$$q_0 = h_\mathrm{e} - h_\mathrm{i} + w_\mathrm{max} \tag{2.3}$$

$$s_\mathrm{e} - s_\mathrm{i} = q_0/T_0 \tag{2.4}$$

$$w_\mathrm{max} = h_\mathrm{i} - h_\mathrm{e} - T_0(s_\mathrm{i} - s_\mathrm{e}) \tag{2.5}$$

This development could be done for the original condition where there are irreversible processes inside the control volume as well as the original values of w and q. In this case, the maximum work could be obtained in a similar way: including a Carnot engine that operates by using q and rejecting q_0' to the environment.

For the reversible engine, it can be written:

$$\oint \frac{\delta Q}{T} = 0 = \int_{T_\mathrm{i}}^{T_\mathrm{e}} \left(\frac{\delta q}{T}\right) + \int \left(\frac{\delta q_0'}{T_0}\right) \tag{2.6}$$

$$s_\mathrm{e} - s_\mathrm{i} = q_0'/T_0 + s_\mathrm{ger} \tag{2.7}$$

$$q_0' = h_{\mathrm{e}} - h_{\mathrm{i}} + w + w_{\mathrm{mr}} \tag{2.8}$$

$$w_{\mathrm{mr}} = q - q_0' \tag{2.9}$$

$$T_0(s_{\mathrm{e}} - s_{\mathrm{i}}) - T_0\, s_{\mathrm{ger}} = h_{\mathrm{e}} - h_{\mathrm{i}} + w + q\eta_{mr} \tag{2.10}$$

$$w_{\mathrm{mr}} = q\eta_{\mathrm{mr}} \tag{2.11}$$

$$h_{\mathrm{i}} - h_{\mathrm{e}} - T_0(s_{\mathrm{i}} - s_{\mathrm{e}}) = w + q\eta_{\mathrm{mr}} + T_0\, s_{\mathrm{ger}}$$

$$\eta_{\mathrm{mr}} = 1 - \frac{T_0}{q} \int_{T_{\mathrm{i}}}^{T_{\mathrm{e}}} \left(\frac{\delta q}{T} \right) \tag{2.12}$$

It is evident that, as shown by Eq. 2.5, the term $[h_{\mathrm{i}} - h_{\mathrm{e}} - T_0(s_{\mathrm{i}} - s_{\mathrm{e}})]$ also characterizes a potential for doing work that in the occurrence of irreversible processes is reduced by the product $T_0 s_{\mathrm{ger}}$, or the so-called irreversibility. Eventually, this potential is completed destroyed, or:

$$h_{\mathrm{i}} - h_{\mathrm{e}} - T_0(s_{\mathrm{i}} - s_{\mathrm{e}}) = T_0\ s_{\mathrm{ger}} \tag{2.13}$$

2.2.2 *Exergy and Exergy Balance*

Equation 2.10 can simply be obtained by multiplying the entropy balance by $-T_0$ and adding this equation to the energy balance. The result, a linear combination of the entropy and energy balances, gives:

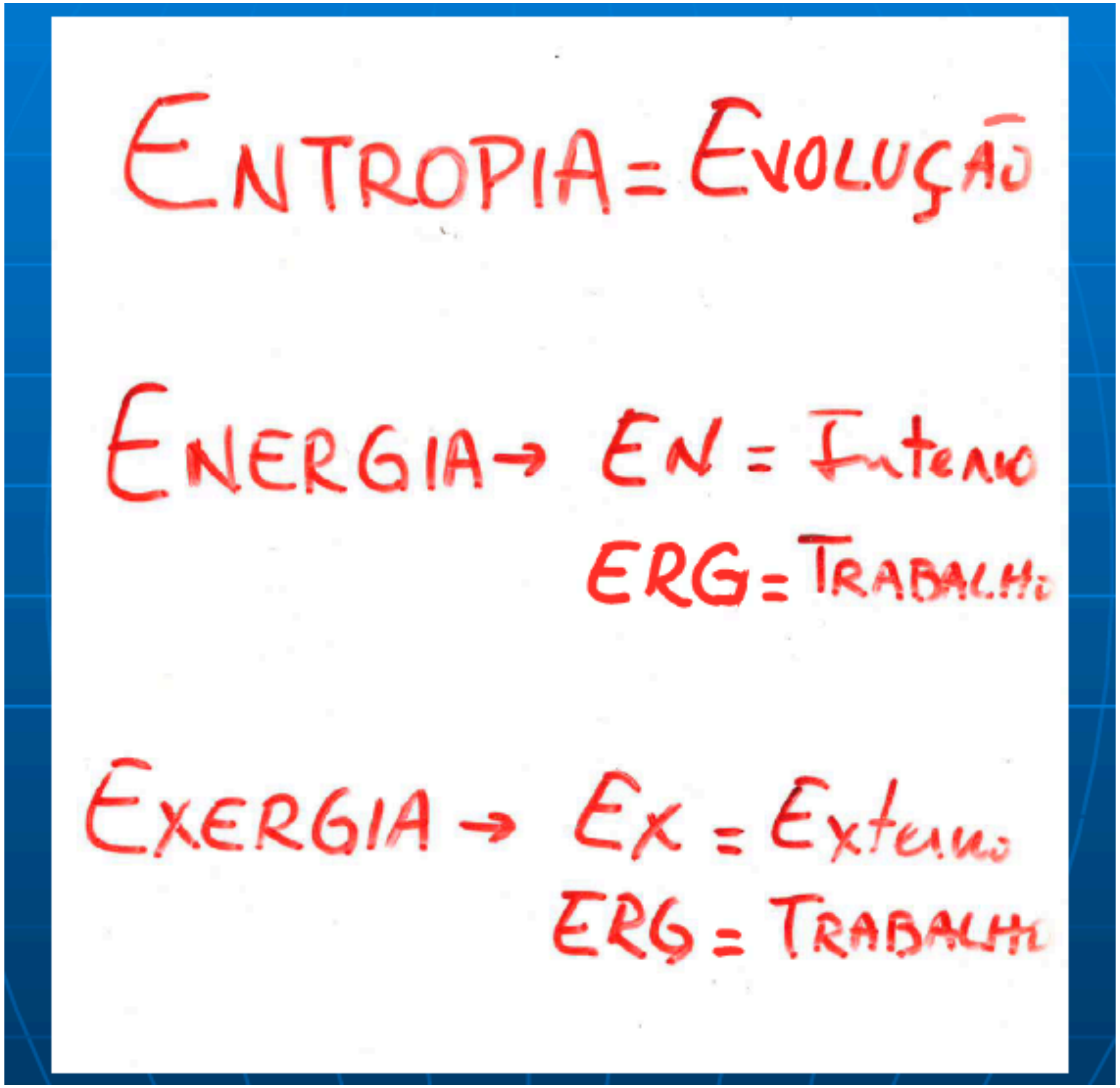
$$[h_\mathrm{i} - h_\mathrm{e} - T_0(s_\mathrm{i} - s_\mathrm{e})] = w + q\left[1 - \frac{T_0}{q}\int_{T_\mathrm{i}}^{T_\mathrm{e}}\left(\frac{\delta q}{T}\right)\right] + T_0\, s_\mathrm{ger} \tag{2.14}$$

In the case where all processes inside the control volume are reversible, the control volume interacts only with the environment, and the mass flow rate at the outlet section is in thermodynamic equilibrium with the environment ($h = h_0$ and $s = s_0$), Eq. 2.14 becomes the expression of the maximum reversible work associated to the inlet flow rate state of the control volume:

$$[h_\mathrm{i} - h_\mathrm{e} - \mathrm{T}_0(s_\mathrm{i} - s_\mathrm{e})] = w + q\left[1 - \frac{T_0}{q}\int_{T_\mathrm{i}}^{T_\mathrm{e}}\left(\frac{\delta q}{T}\right)\right] = w_\mathrm{max} \tag{2.15}$$

This maximum reversible work is called specific exergy of the flow rate at the inlet section of the control volume:

$$b = [h_\mathrm{i} - h_0 - T_0(s_\mathrm{i} - s_0)] \tag{2.16}$$



ENTROPIA = EVOLUÇÃO

ENERGIA → EN = Interno
ERG = TRABALHO

EXERGIA → EX = Externo
ERG = TRABALHO

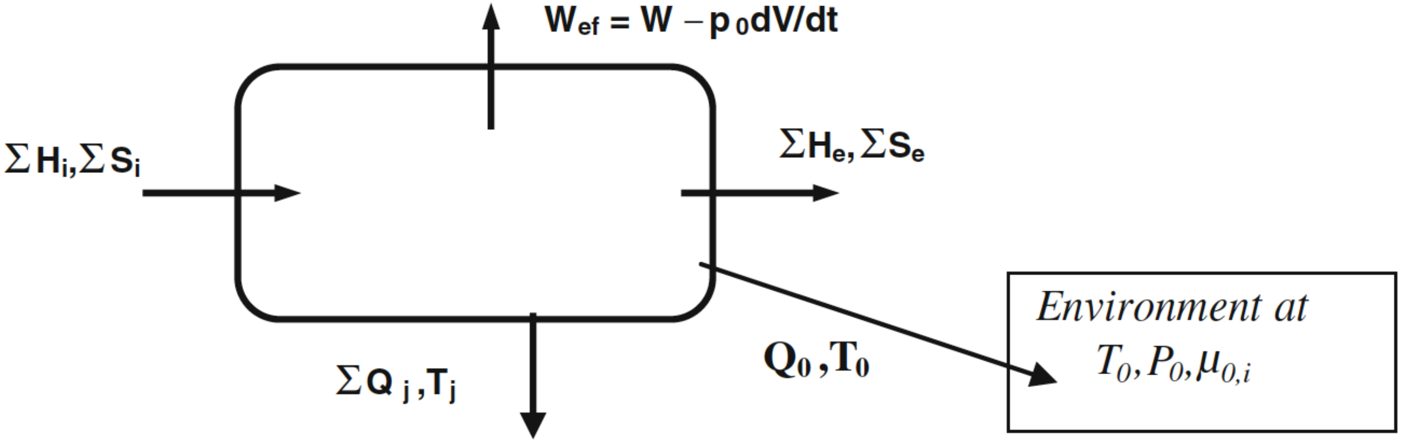
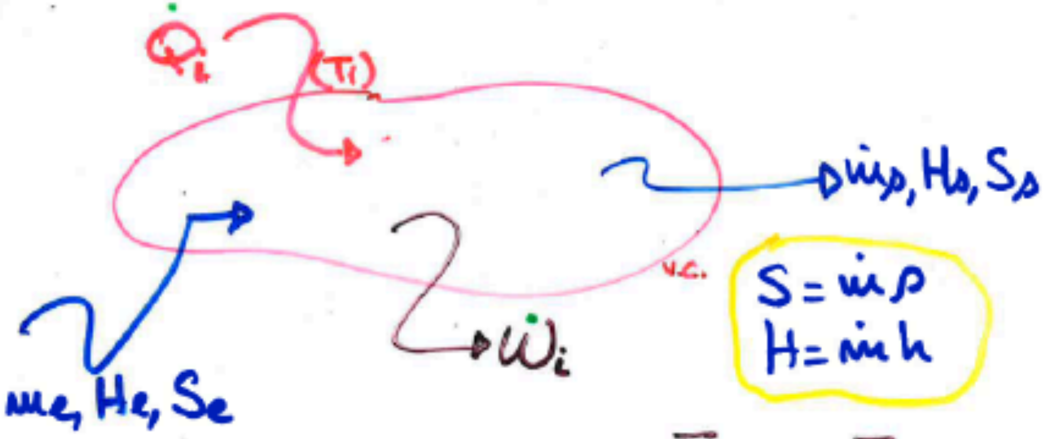


Fig. 2.2 Control volume with several inlets and outlets, heat transfer, and temperatures at T_i

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{E} + P_0 V - T_0 S) = \frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t} = \sum_\mathrm{j}\left(1 - \frac{T_0}{T_\mathrm{j}}\right)Q_\mathrm{j} - W_\mathrm{ef} + \sum_\mathrm{i} m_\mathrm{i}(h_\mathrm{i} - T_0 s_\mathrm{i}) - \sum_\mathrm{e} m_\mathrm{e}(h_\mathrm{e} - T_0 s_\mathrm{e}) - T_0 S_\mathrm{ger} \tag{2.17}$$

BALANÇO DE EXERGIA

(Regime Permanente, $\Delta E.C = \Delta EP \approx 0$)



$$\boxed{\begin{aligned} S &= \dot{m}s \\ H &= \dot{m}h \end{aligned}}$$

1ª Lei: $\sum \dot{Q}_i - \sum \dot{W}_i = \sum H_s - \sum H_e$ +

2ª Lei: $\sum \frac{\dot{Q}_i}{T_i} + \dot{S}_{ger} = \sum S_s - \sum S_e$ $(\times -T_0)$

$$\sum \dot{Q}_i\left(1 - \frac{T_0}{T_i}\right) - \sum \dot{W}_i - T_0 \dot{S}_{ger} = \sum H_s - \sum H_e - T_0\left(\sum S_s - \sum S_e\right)$$

Balanço de Exergia

$$\underbrace{\sum \dot{Q}_i\left(1 - \frac{T_0}{T_i}\right)}_{\text{exergia associada à transferência}} - \underbrace{\sum \dot{W}_i}_{\text{exergia pura}} - \underbrace{T_0 \dot{S}_{ger}}_{\text{exergia destruída}} = \underbrace{\sum B_s - \sum B_e}_{\text{variação dos fluxos de exergia}}$$

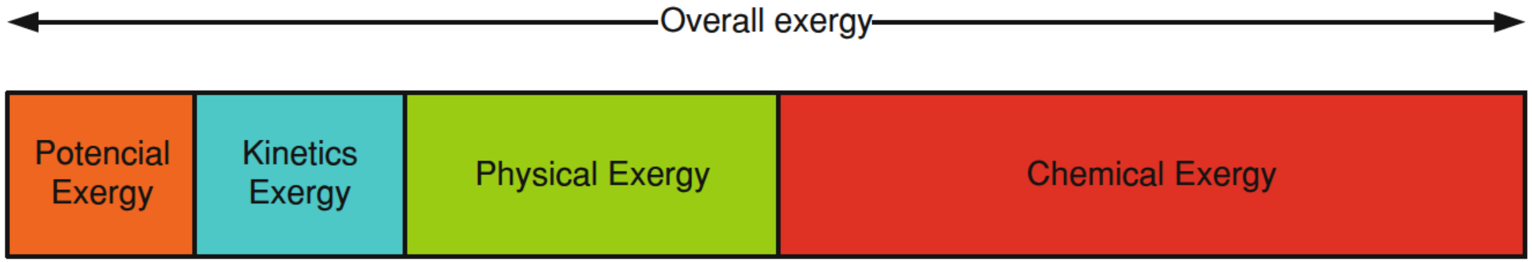


Fig. 2.3 Exergy components

$$b = b_{\text{kin}} + b_{\text{pot}} + b_{\text{ph}} + b_{\text{ch}} \tag{2.18}$$

$$b_{\text{kin}} = \frac{\upsilon^2}{2} \tag{2.19}$$

$$b_{\text{pot}} = gz \tag{2.20}$$

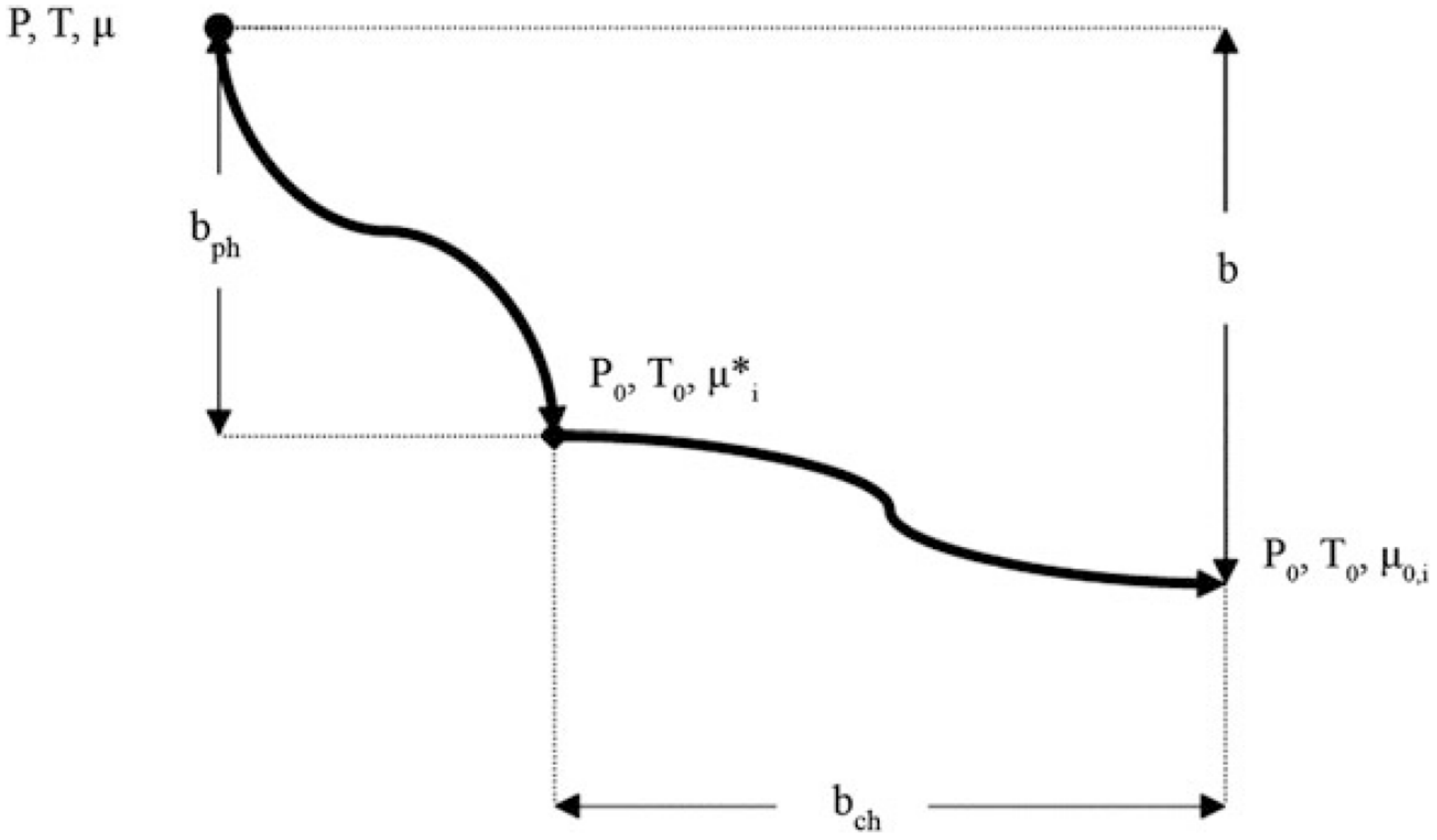


Fig. 2.4 Physical, chemical, and total exergy

Physical Exergy

$$b^{s}_{ph} = u - u^{*} + P_{0}\,(v - v^{*}) - T_{0}\,(s - s^{*}) \tag{2.21}$$

$$b_{ph} = h - h^{*} - T_{0}\,(s - s^{*}) \tag{2.22}$$

Chemical Exergy

$$B^{s}_{\mathrm{ch}} = U^{*} + P_{0}\,V^{*} - T_{0}S^{*} - \sum_{i=1}^{n} \mu_{0,i}\,N_{i} = \sum_{i=1}^{n} (\mu_{i}^{*} - \mu_{0,i})N_{i} \tag{2.25}$$

$$\overline{b}_{\mathrm{ch}} = \overline{h}^{*} - T_{0}\,\overline{s}^{*} - \sum_{i=1}^{n} \mu_{0,i}\,x_{i} \tag{2.26}$$

2.2.3 Chemical Exergy Calculation

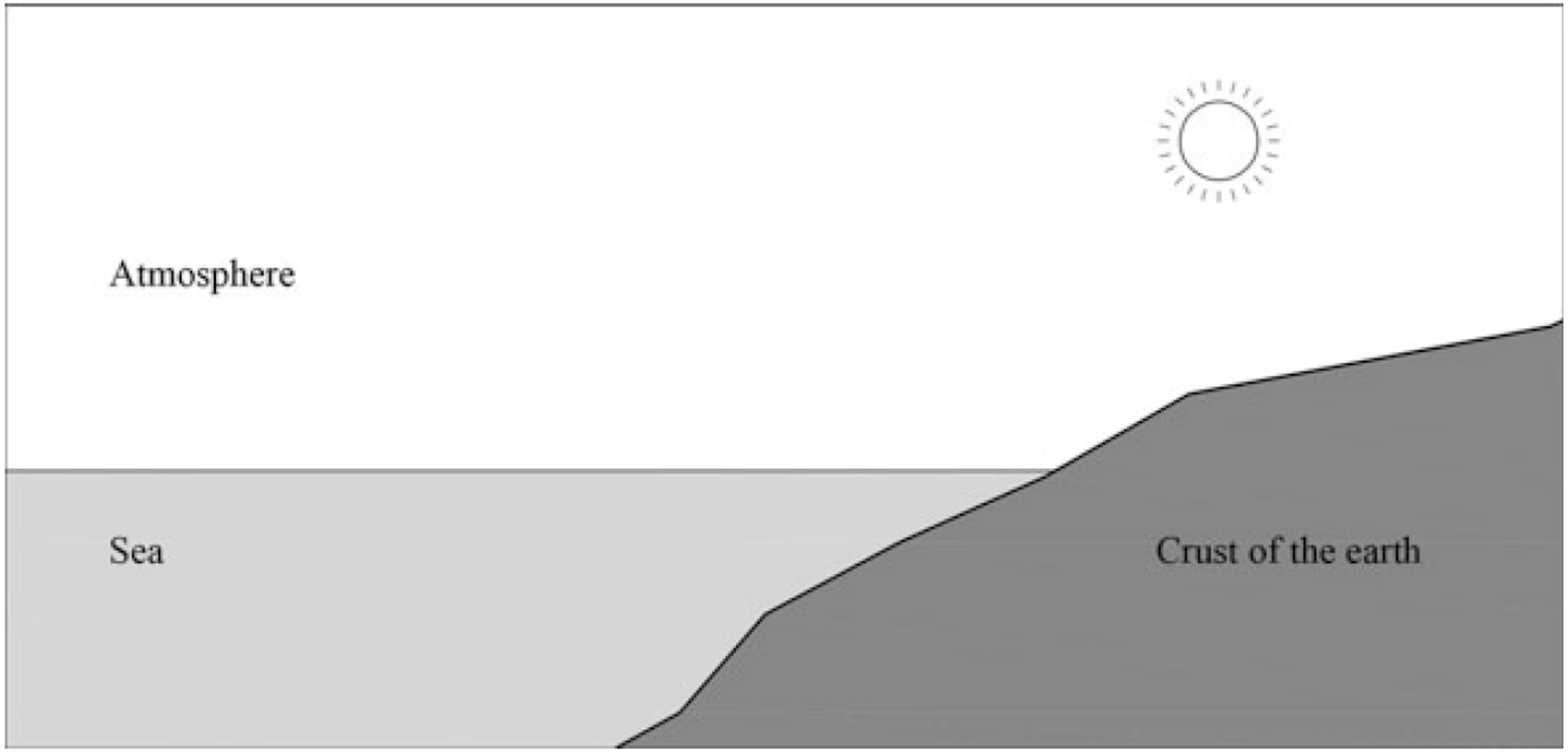


Fig. 2.5 Components of the environment proposed by Szargut et al. [19]

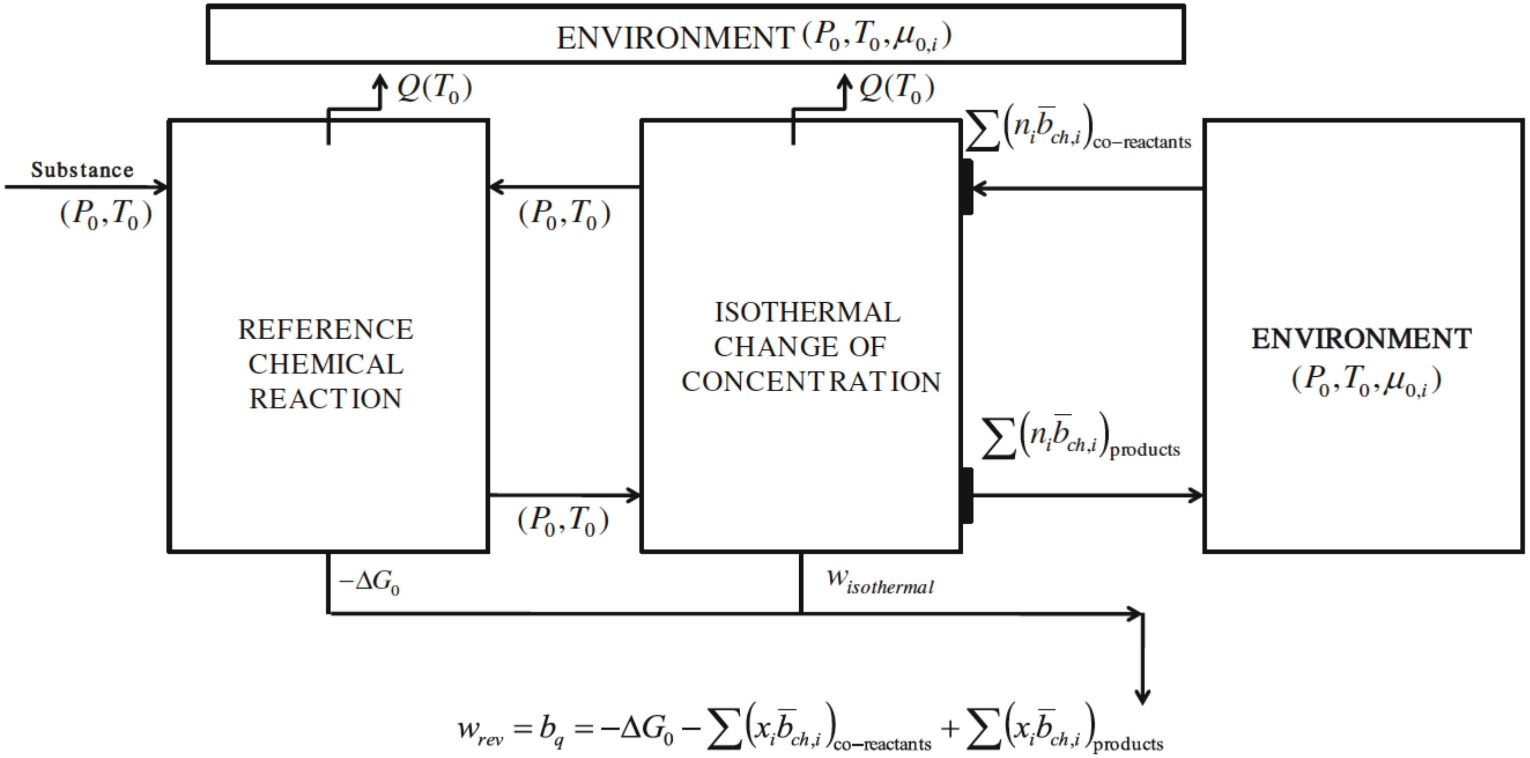


Fig. 2.6 Chemical exergy determination of substances that does not exist in the reference environment

$$\bar{b}_{\mathrm{ch}} = -\Delta G_0 - \left\lfloor \sum_i x_i \; \bar{b}_{\mathrm{ch,i}} \right\rfloor \text{coreactants} + \left\lfloor \sum_j x_j \; \bar{b}_{\mathrm{ch},j} \right\rfloor \text{products} \tag{2.30}$$

with,

$$\Delta G_0 = \sum_j v_j \; g_j - \sum_k v_k \; g_k \tag{2.31}$$

$$b_{\mathrm{ch}} = \phi \mathrm{LHV} \tag{2.33}$$

EXAMPLE 1

A mercury condenser and steam boiler of a binary power plant is shown below. The heat transfer rate is 5 MW and there are not heat losses to the environment. Using data given in the figure calculate the thermal exergy rate transferred from mercury, thermal exergy rate transferred to water and the destroyed exergy rate (T_0=290 K).

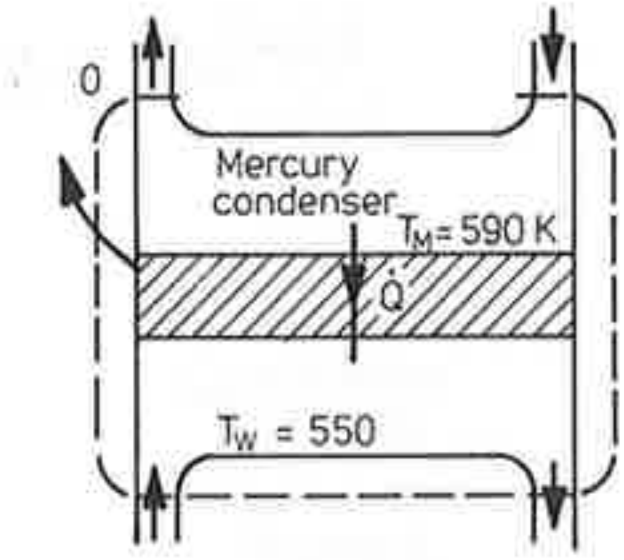
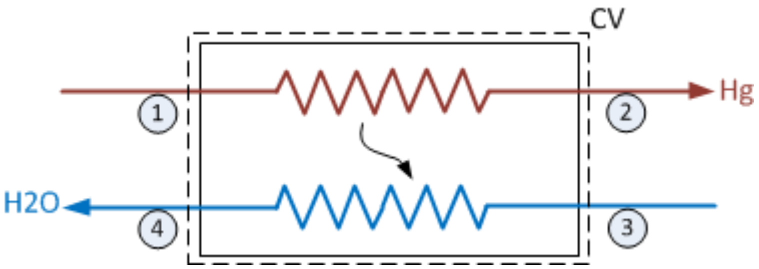


Fig. Scheme of a mercury condenser and steam boiler of a binary power plant.

Solution:



No pressure drop for both flows
For the chosen C.V.:

- Mass balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}_{Hg}$ and $\dot{m}_3 = \dot{m}_4 = \dot{m}_{H2O}$
- Energy balance: $\dot{m}_{Hg}(h_1 - h_2) = \dot{m}_{H2O}(h_4 - h_3) = \dot{Q} = 5\ MW$
- Exergy balance: $\dot{m}_1 b_1 + \dot{m}_3 b_3 = \dot{m}_2 b_2 + \dot{m}_4 b_4 + \dot{B}_{dest}$

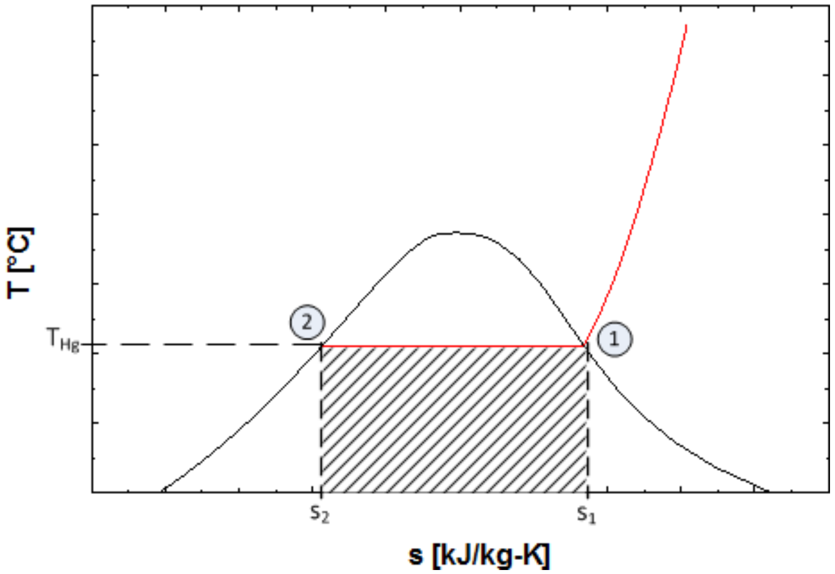
  or $\dot{m}_{Hg}(b_1 - b_2) = \dot{m}_{H2O}(b_4 - b_3) + \dot{B}_{dest}$

  with

$$b_1 - b_2 = h_1 - h_2 - T_0(s_1 - s_2) = (h_1 - h_2)\left(1 - T_0\frac{s_1 - s_2}{h_1 - h_2}\right)$$

$$b_4 - b_3 = h_4 - h_3 - T_0(s_4 - s_3) = (h_4 - h_3)\left(1 - T_0\frac{s_4 - s_3}{h_4 - h_3}\right)$$

$$\dot{m}_{Hg}(b_1 - b_2) = \underbrace{\dot{m}_{Hg}(h_1 - h_2)}_{\dot{Q}}\left(1 - \frac{T_0}{\left(\frac{h_1 - h_2}{s_1 - s_2}\right)}\right)$$



As

$$Tds = dh - v\underbrace{dp}_{0}$$ (pure substance)

$$T = \frac{dh}{ds}$$

Then:

$$\dot{m}_{Hg}(b_1 - b_2) = \dot{Q}\left(1 - \frac{T_0}{T_{Hg}}\right) = \dot{Q}\theta_{Hg}$$

Analogously: $\dot{m}_{H2O}(b_4 - b_3) = \dot{Q}\theta_{H2O}$

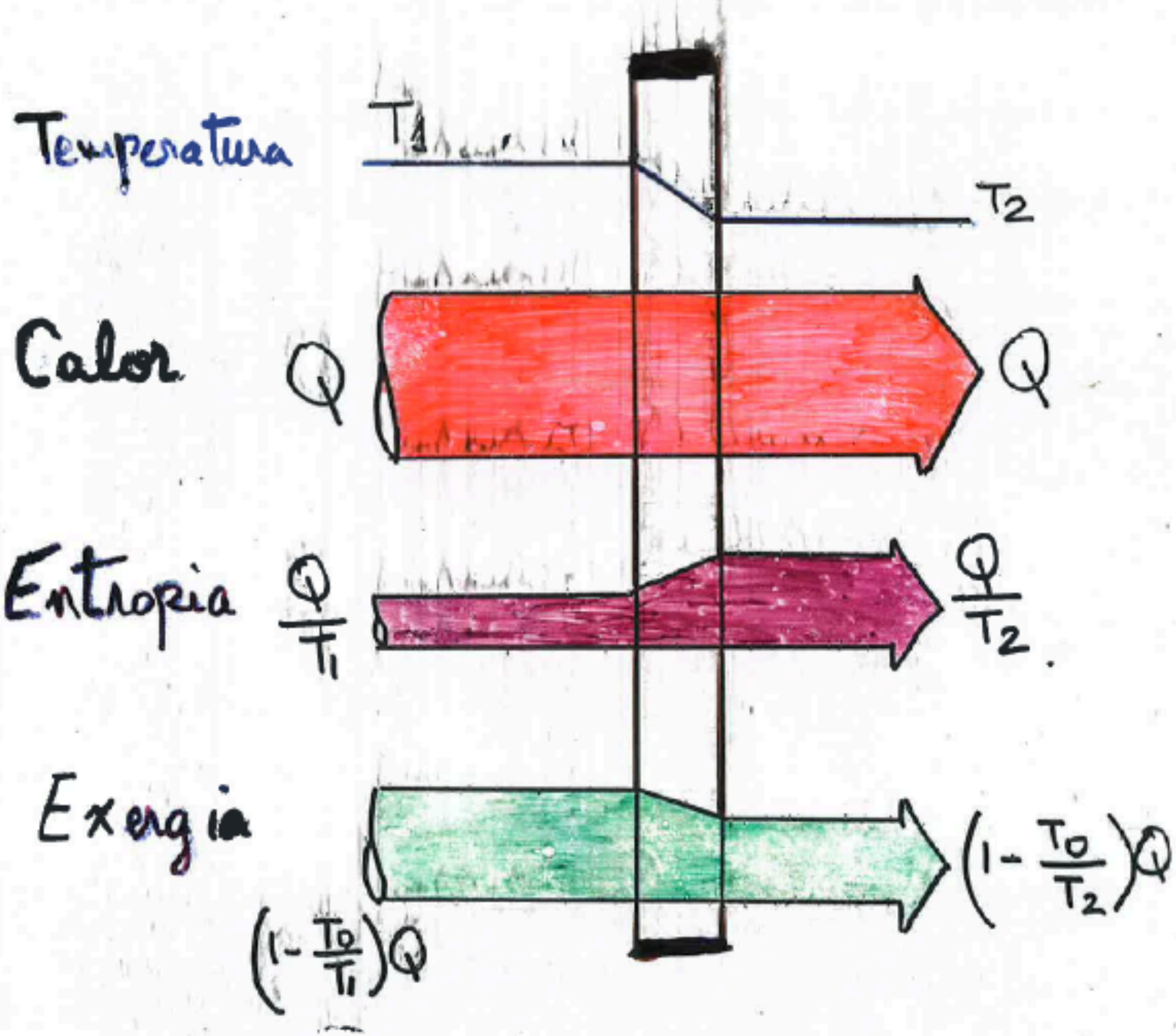
Then: $\dot{B}_{dest} = \dot{Q}(\theta_{Hg} - \theta_{H2O})$

$$\dot{m}_{Hg}(b_1 - b_2) = 2{,}54\ MW$$

$$\dot{m}_{H2O}(b_4 - b_3) = 2{,}36\ MW$$

$$\dot{B}_{dest} = 0{,}18\ MW$$

TRANSFERÊNCIA DE CALOR



EXAMPLE 2

Example 2.1 Determine the chemical exergy of methane. The reference reaction for the determination of the chemical exergy of methane is the combustion reaction of methane with oxigen:

$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$

First procedure

$$\bar{b}_{\mathrm{ch\,CH_4}} = -\Delta G_0 - 2b_{\mathrm{ch\,O_2}} + 2b_{\mathrm{ch\,H_2O}} + b_{\mathrm{ch\,CO_2}}$$

$-\Delta G_0 =$ Enthalpy (CH_4; $T = 25$ °C) + 2 Enthalpy (O_2; $T = 25$ °C) − Enthalpy (CO_2; $T = 25$ °C) − 2 Enthalpy (H_2O; $T = 25$ °C) − 298.15 [Entropy (CH_4; $T = 25$ °C; $P = 101.325$ kPa) + 2 Entropy (O_2; $T = 25$ °C; $P = 101.325$ kPa) − Entropy (CO_2; $T = 25$ °C; $P = 101.325$ kPa) − 2 Entropy (H_2O; $T = 25$ °C; $P = 101.325$ kPa)]

$h_{CH_4} = -74{,}595$ kJ/kmol
$s_{CH_4} = 186.3$ kJ/kmol K

Table 2.2 Properties of some ideal gases of the reference atmosphere

	h (T_0 = 25 °C) (kJ/kmol)	s (T_0 = 25 °C, P_0 = 101.325 kPa) (kJ/kmol K)	s (T = 25 °C, $P = P_{00}$) (kJ/kmol K)	b_{ch} (kJ/kmol)
CO_2	−393,486	213.7	280.3[a]	19,867
H_2O	−241,811	188.7	220.6[b]	9,494
N_2	0	191.5	193.9[c]	720
O_2	0	205.0	218.4[d]	3,974

[a] $P_{00\,CO_2} = 0.03$ kPa
[b] $P_{00\,H_2O} = 2.2$ kPa
[c] $P_{00\,N_2} = 75.78$ kPa
[d] $P_{00\,O_2} = 20.39$ kPa

$$\bar{b}_{ch\,CH_4} = 831,862\,\mathrm{kJ/kmol\ CH_4}$$

Second procedure

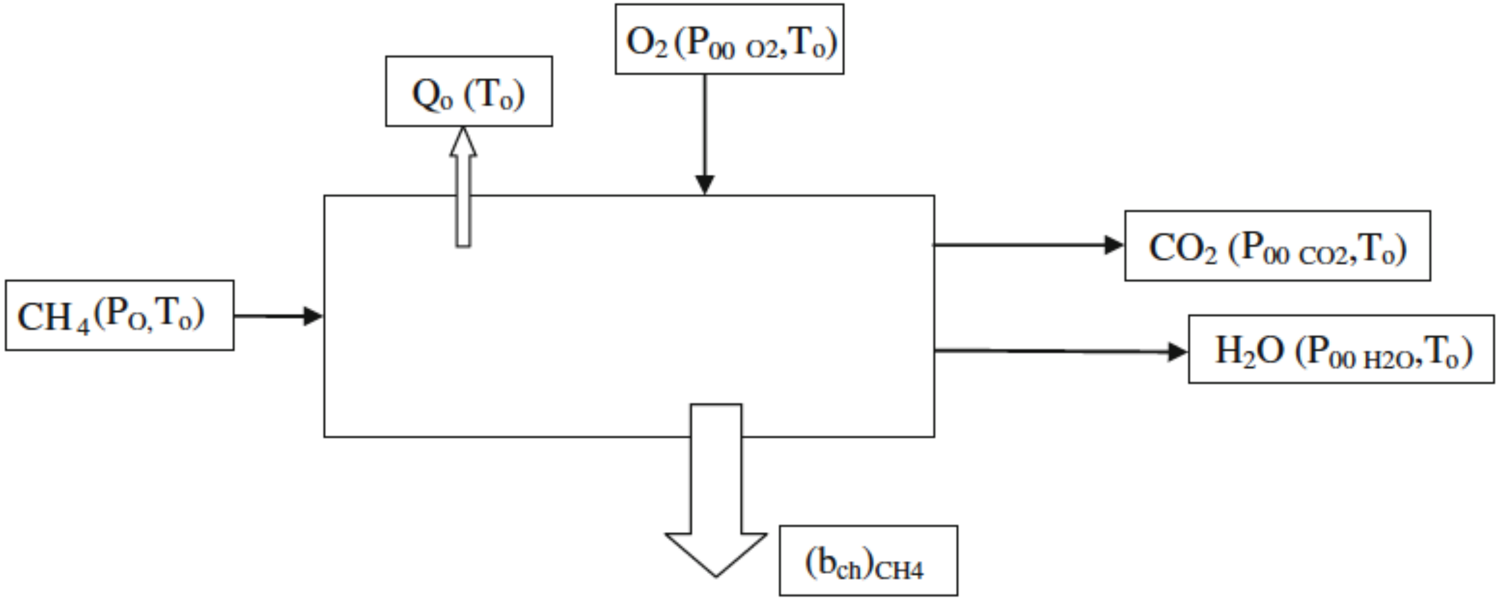


Fig. 2.7 Thermodynamic scheme for calculation of the chemical exergy of methane (maximum work capacity of methane at P_0 and T_0)

$\bar{b}_{\text{ch}\,CH_4}$ = Enthalpy (CH_4; $T = 25$ °C) + 2 Enthalpy (O_2; $T = 25$ °C) – Enthalpy (CO_2; $T = 25$ °C) – 2 Enthalpy (H_2O; $T = 25$ °C) – 298.15 [Entropy (CH_4; $T = 25$ °C; $P = 101.325$ kPa) + 2 Entropy (O_2; $T = 25$ °C; $P = 20.43$ kPa)

– Entropy (CO_2; $T = 25$ °C; $P = 0.03347$ kPa) – 2 Entropy (H_2O; $T = 25$ °C; $P = 2.2$ kPa)]

$$\bar{b}_{\text{Ch}\,CH_4} = 831{,}874\,\text{kJ/kmol}\,CH_4$$

EXAMPLE 3

Example 2.2 Methane is burnt with air. The mixture air methane enters the reactor at $T_i = T_0 = 25$ °C and $P_i = 101.325$ kPa. The combustion products leave the reactor, considered adiabatic, at $P_e = 101.325$ kPa and at the adiabatic flame temperature (T_e). Determine the exergy destruction per kmol of methane, considering that reactants and products can be modeled as ideal gases and with percent excess air ranging from 0 up to 180 % ($1 \le \alpha \le 2.8$).

$$\mathrm{CH_4} + 2\alpha(\mathrm{O_2} + 3.762\mathrm{N_2}) \rightarrow 2\mathrm{H_2O} + \mathrm{CO_2} + 7.524\alpha\mathrm{N_2} + 2(\alpha - 1)\mathrm{O_2}$$

$$H_{\text{reactants}} = H_{\text{products}}$$

$$B_{\text{dest}} = B_{\text{reactants}} - B_{\text{products}} = 298.15\left(S_{\text{products}} - S_{\text{reactants}}\right) = T_0 S_{\text{ger}}$$

$$B_{\text{reactants}} = H_{\text{reactants}} - H_{\text{reactants0}} - 298.15\left(S_{\text{reactants}} - S_{\text{reactants0}}\right)$$

$$B_{\text{product}} = H_{\text{product}} - H_{\text{products0}} - 298.15\left(S_{\text{product}} - S_{\text{products0}}\right)$$

The relative exergy destroyed during the combustion reaction can be calculated as:

$$\Delta_{\text{comb}} = B_{\text{dest}} / B_{\text{reactants}}$$

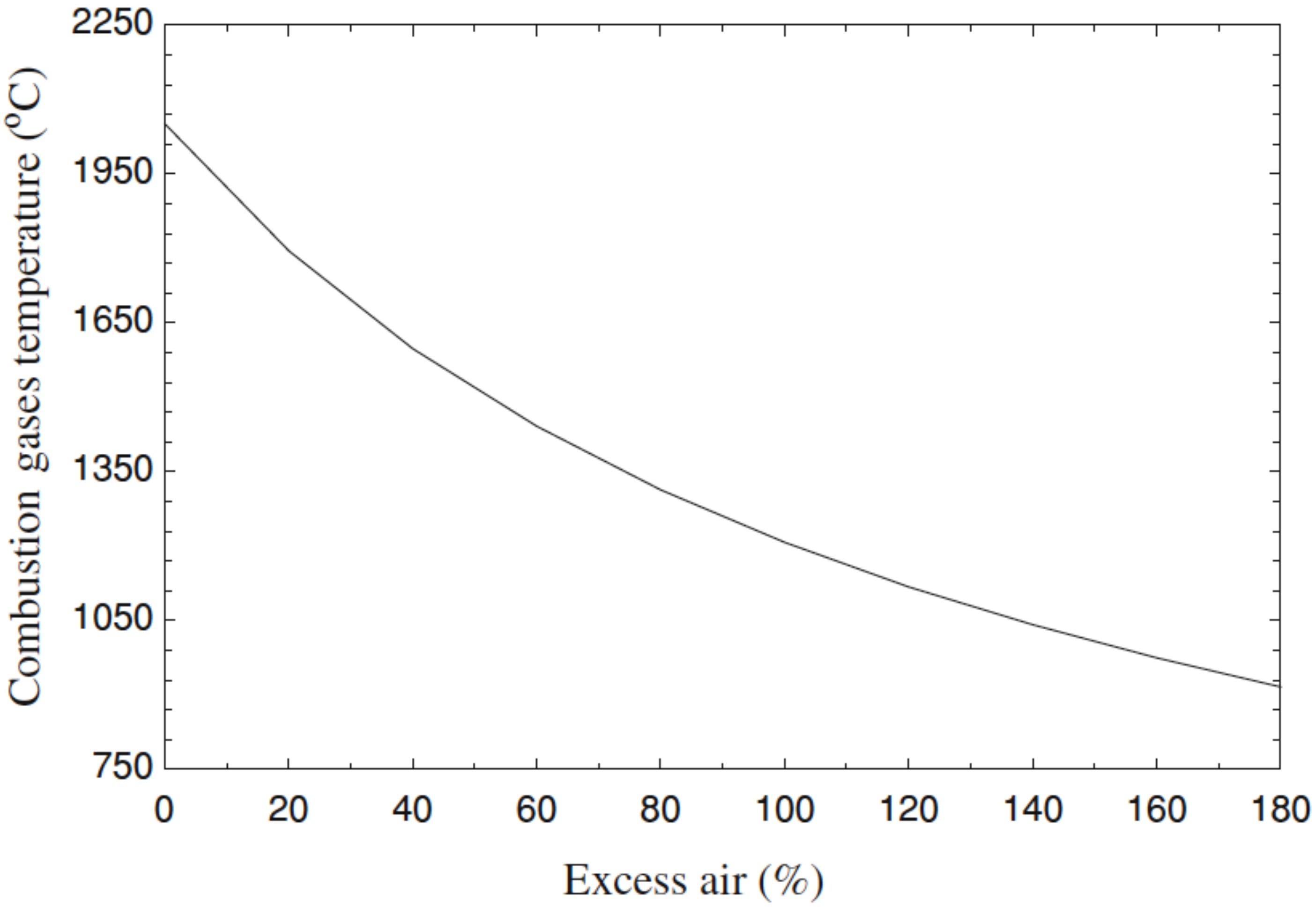


Fig. 2.8 Temperature of combustion gases as a function of percentage of excess air

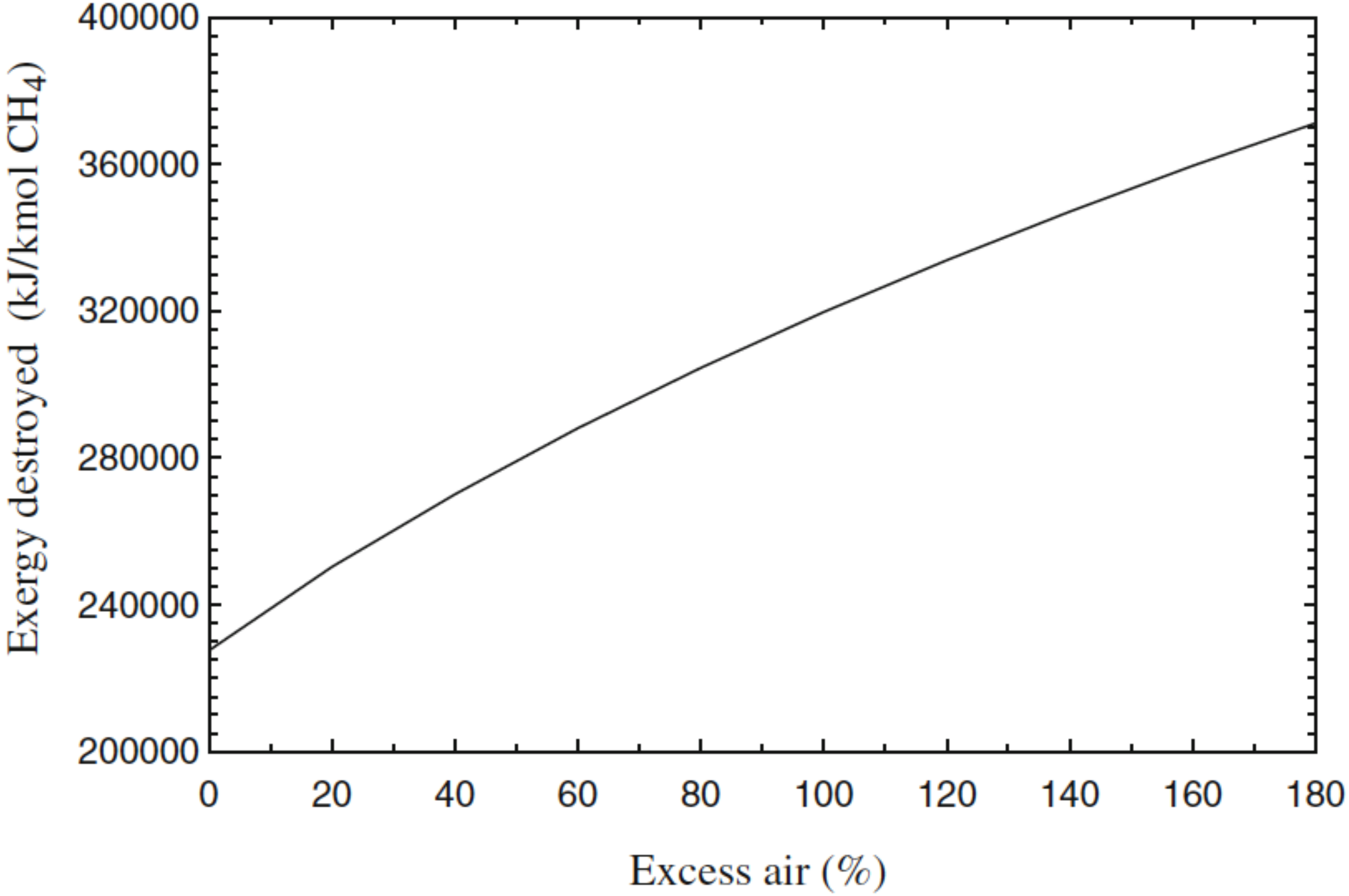


Fig. 2.9 Exergy destroyed in the reactor as a function of percentage of excess air

2.3 Exergy: Graphical Representations and Exergy Diagrams

Diagram Carnot Factor-Enthalpy

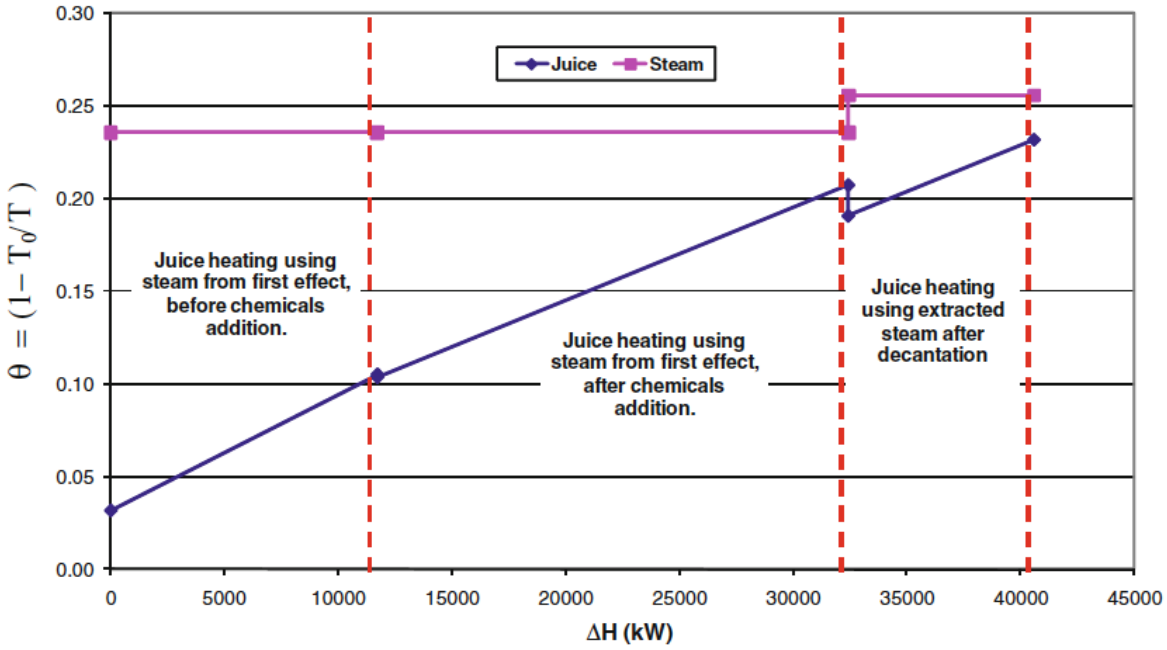


Fig. 2.11 Diagram Carnot factor (θ)–ΔH for a heating juice system of a sugar and alcohol mill [14]

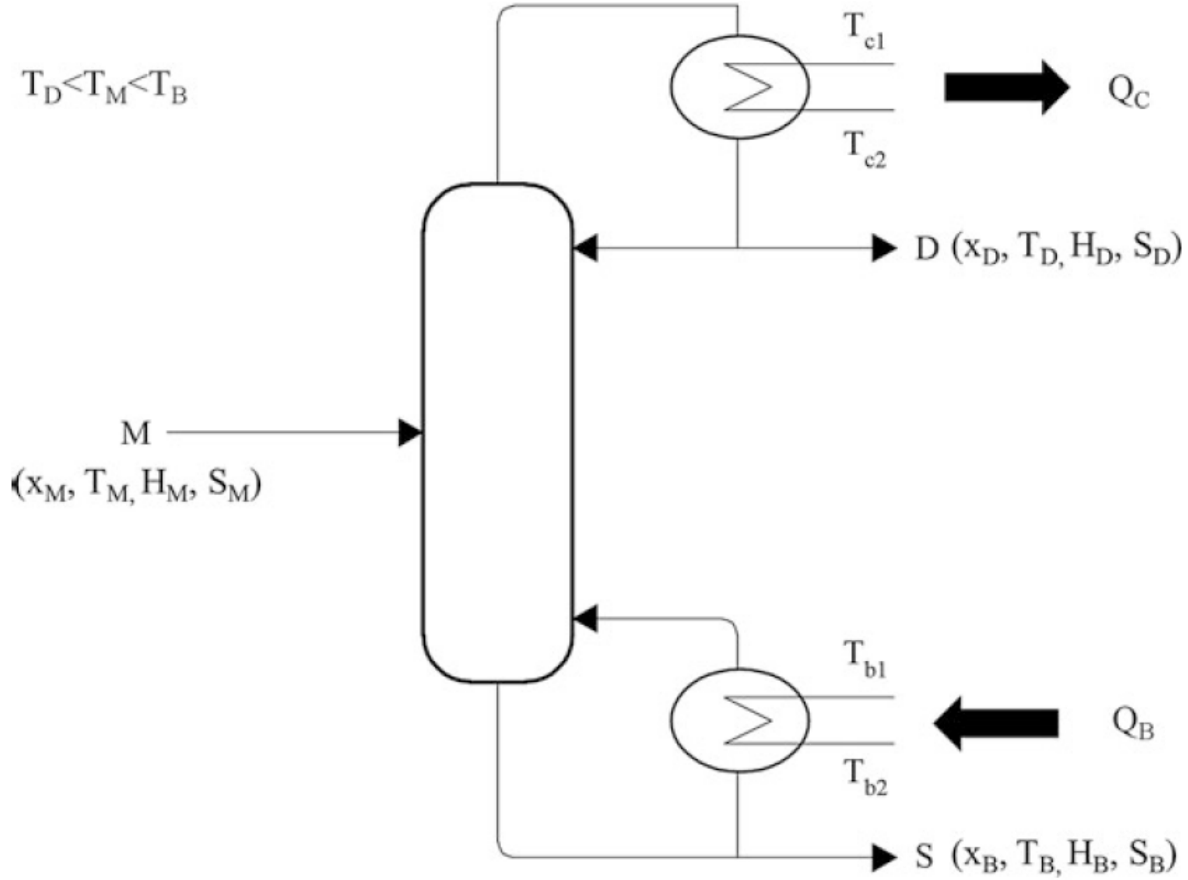


Fig. 2.12 Typical scheme of a binary distillation set [9]

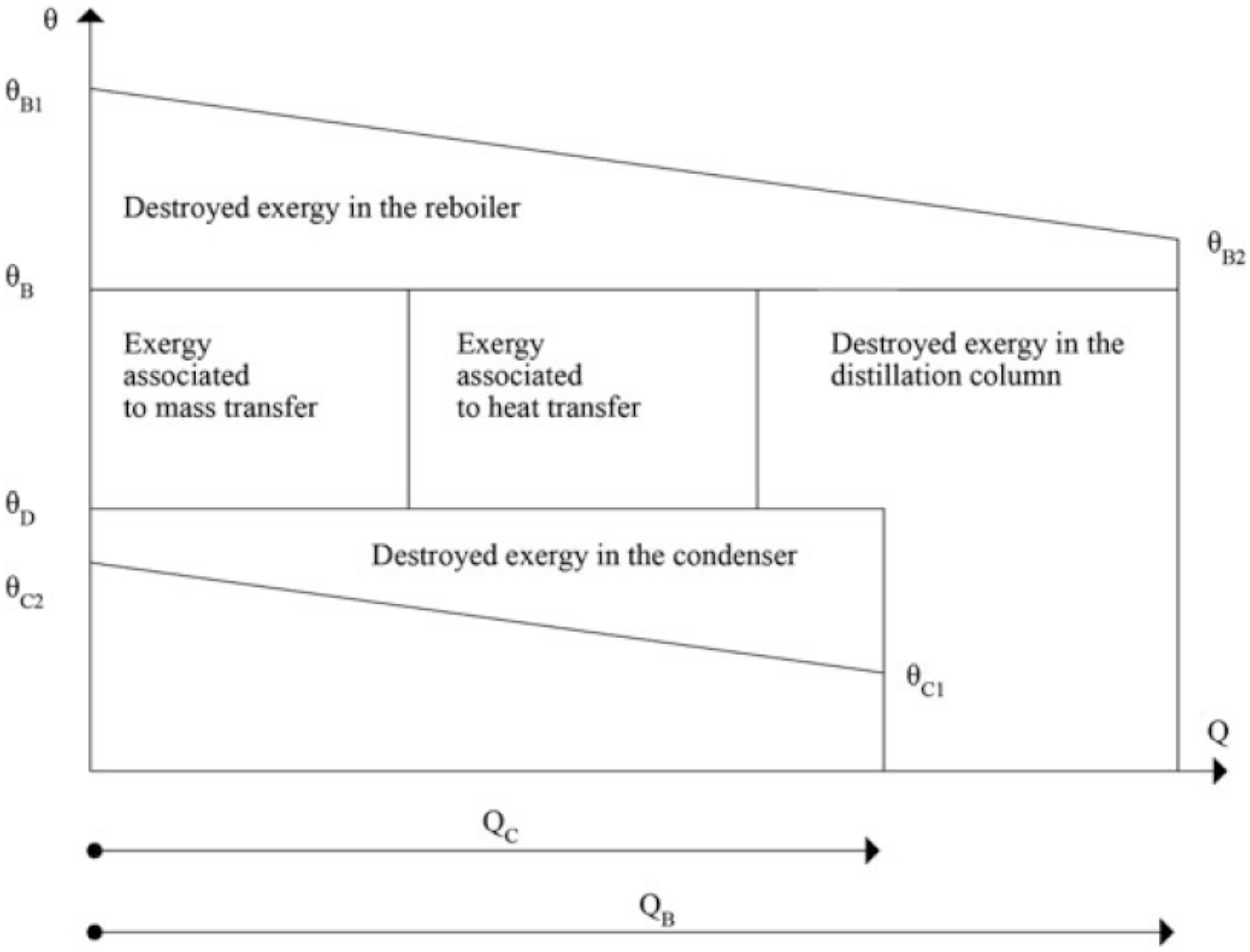


Fig. 2.13 Diagram Carnot factor—enthalpy of a binary mixture distillation process [9]

Diagram Exergy–Enthalpy

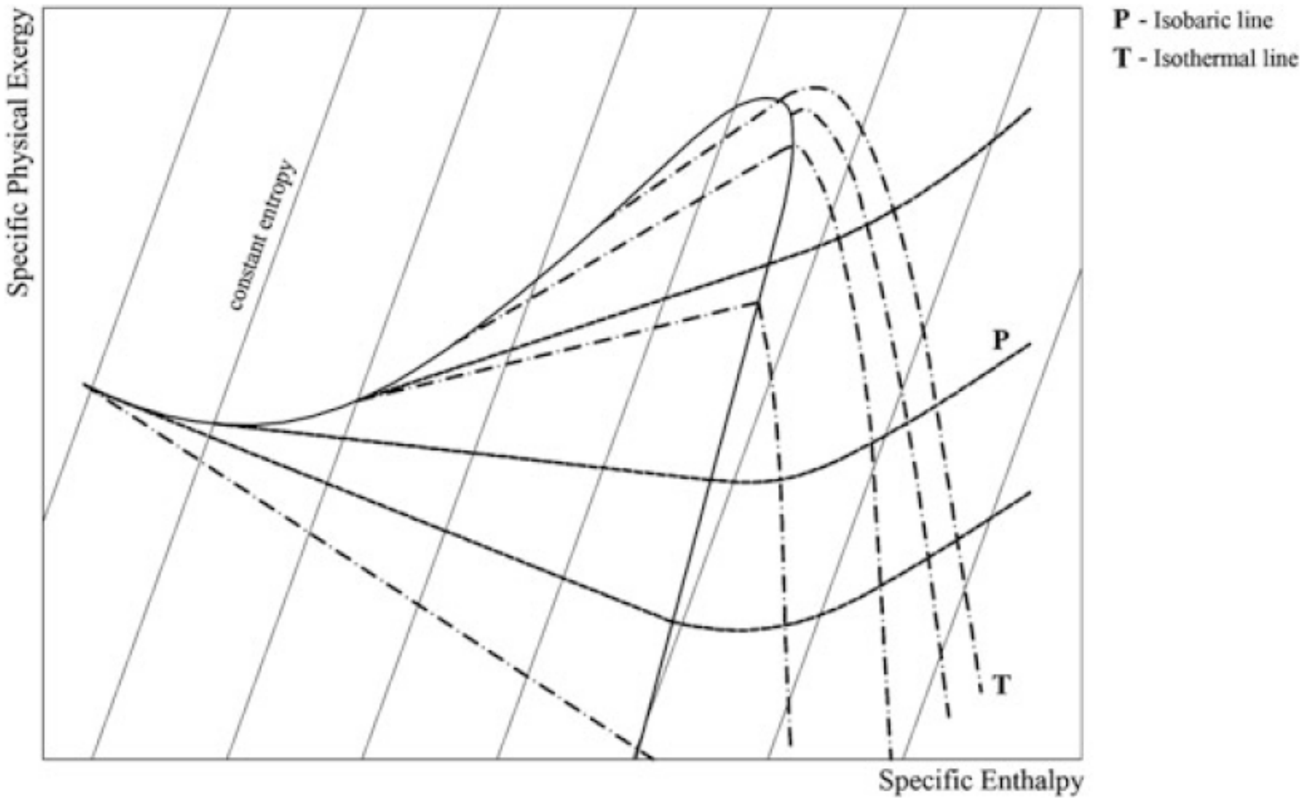
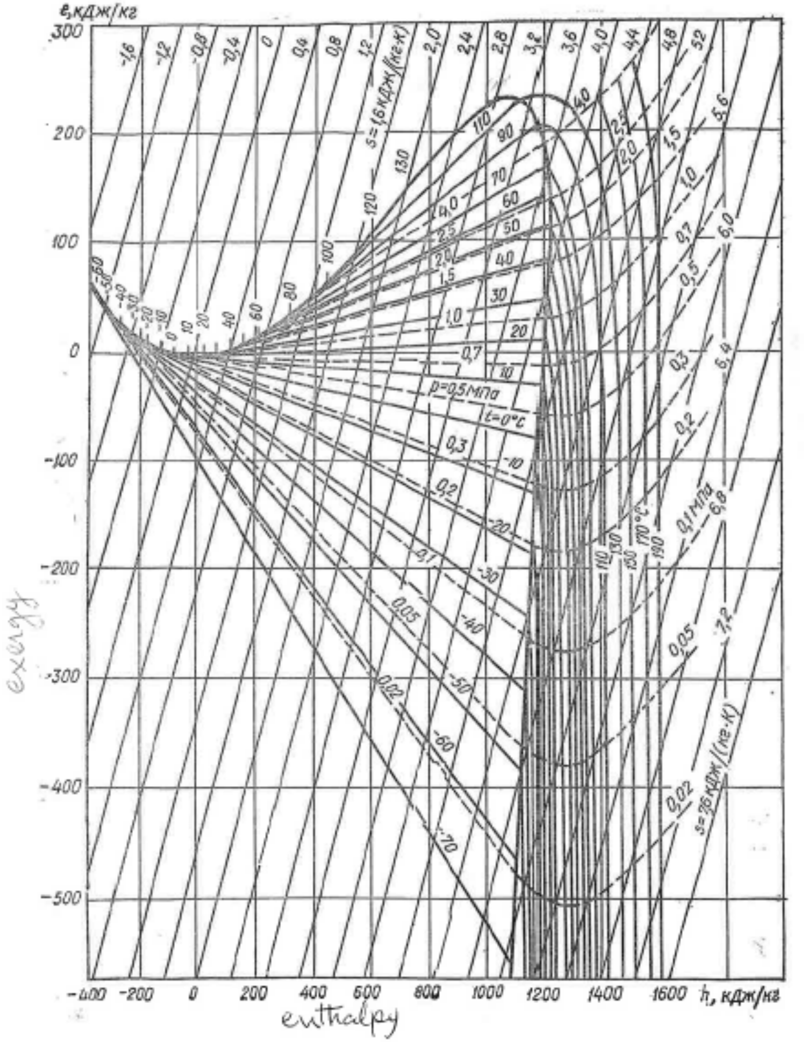


Fig. 2.14 Diagram specific physical exergy—specific entalphy



NH_3 Exergy-enthalpy diagram

Grassmann Diagram

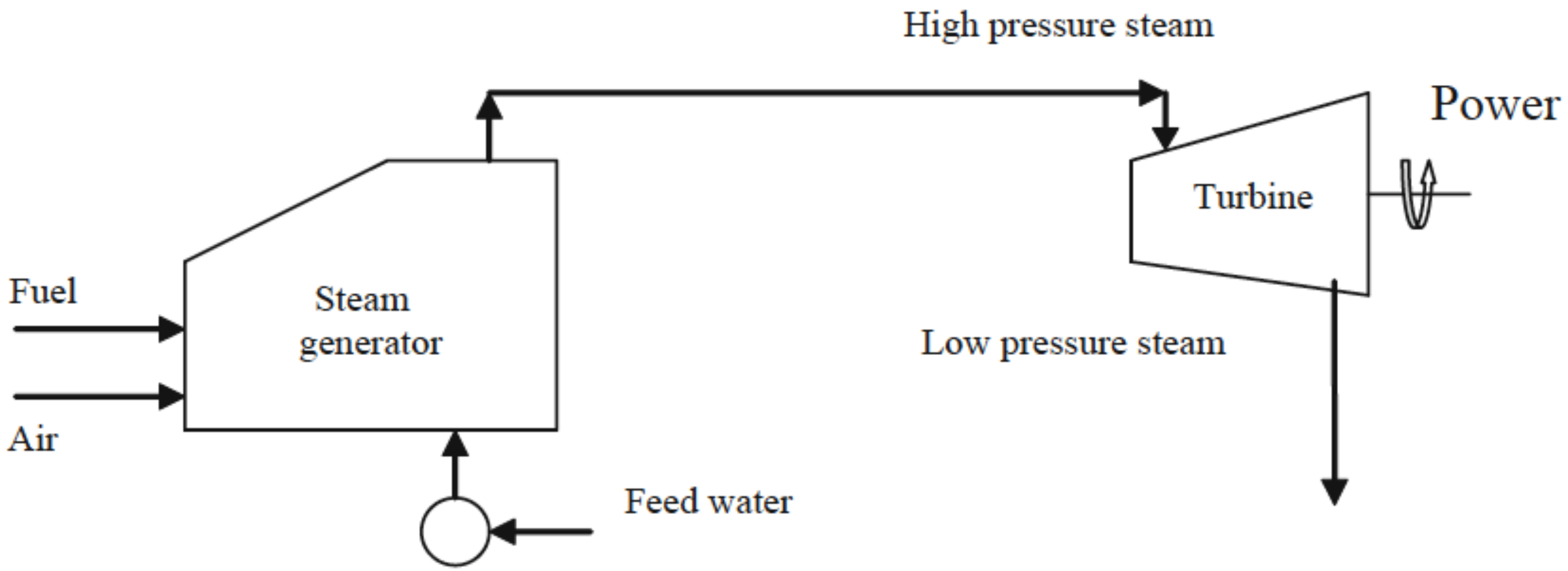


Fig. 2.19 Simplified cogeneration plant

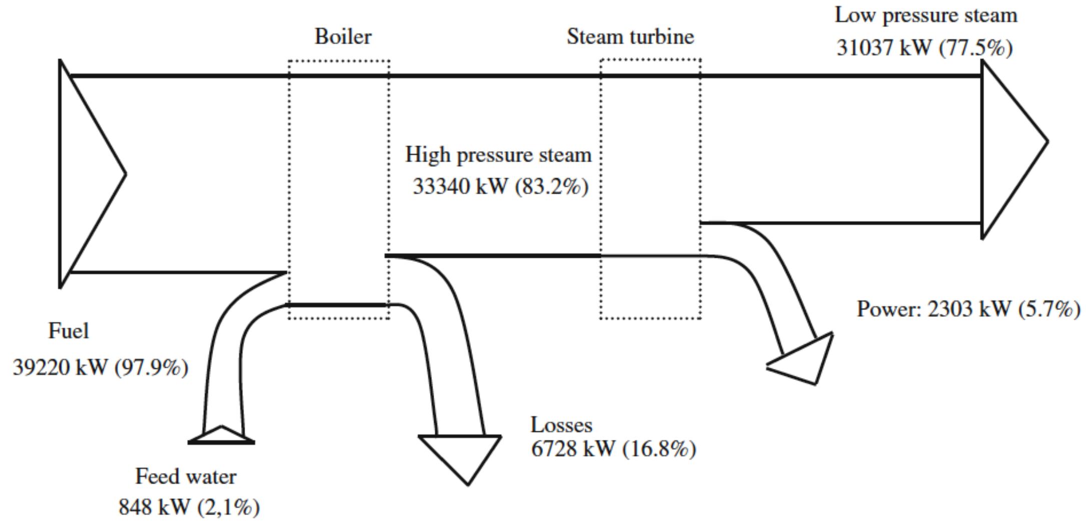


Fig. 2.20 Sankey diagram of a cogeneration plant

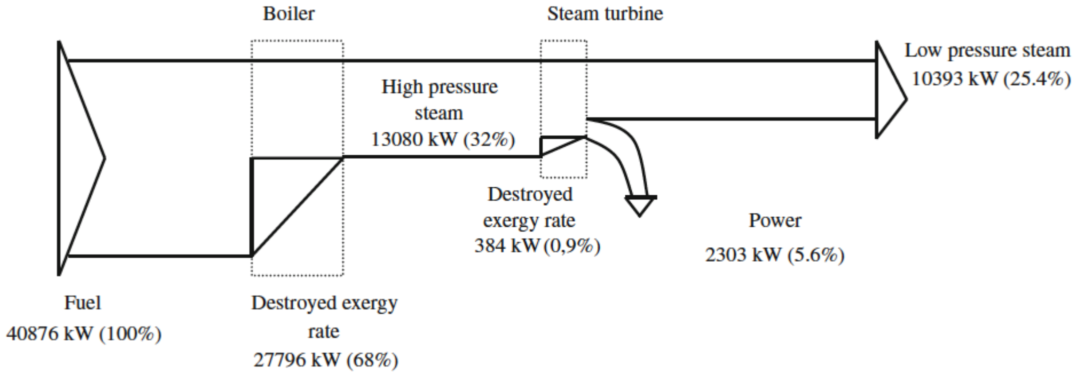


Fig. 2.21 Grassmann diagram of the cogeneration plant

2.4 Exergy Efficiency

2.4.1 *Balance of the Energy Value*

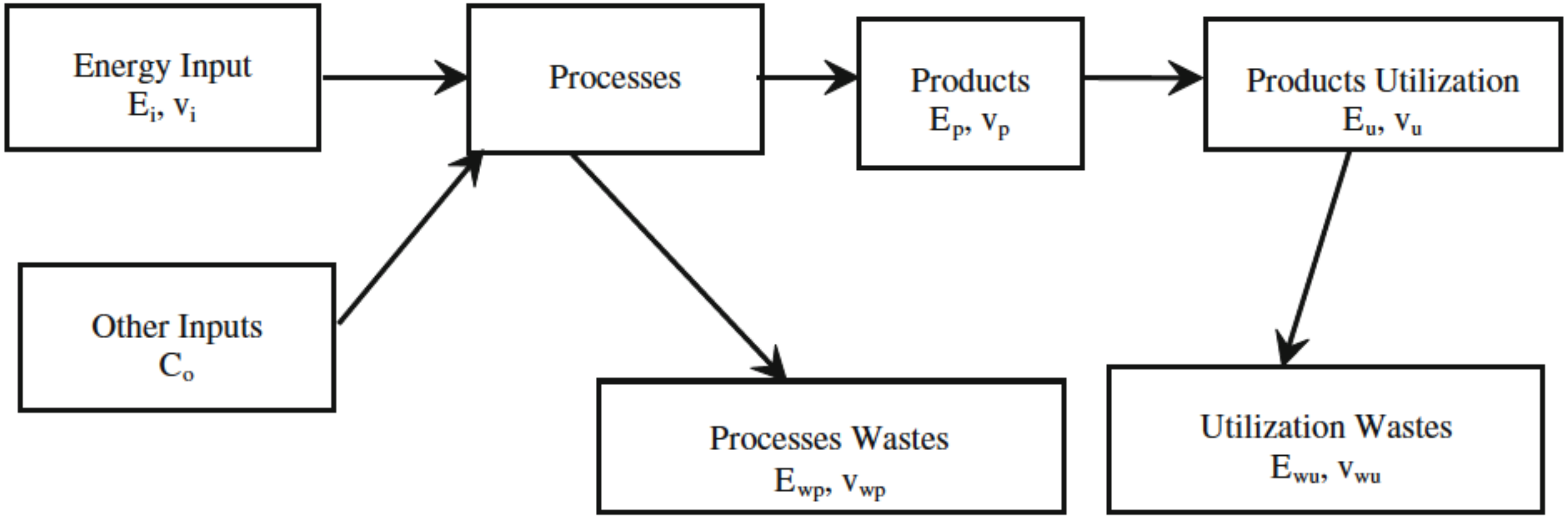


Fig. 2.22 Energy and value balance

$$E_{\mathrm{i}} = E_{\mathrm{p}} + E_{\mathrm{w}} \tag{2.49}$$

The balance of value of this process can be written as:

$$E_i v_i + C_{\mathrm{o}} = E_p v_p + E_w v_{\mathrm{w}} \tag{2.50}$$

$$C_{\mathrm{o}} = E_{\mathrm{p}}(v_{\mathrm{p}} - v_{\mathrm{i}}) + E_{\mathrm{w}}(v_{\mathrm{w}} - v_{\mathrm{i}}) \tag{2.51}$$

$$C_{\mathrm{o}} = \frac{C_{\mathrm{o}}}{E_{\mathrm{u}}} = \underbrace{(v_{\mathrm{p}} - v_{\mathrm{i}})}_{\text{value gain of the products}} + \overbrace{\frac{E_{\mathrm{W}}}{E_{\mathrm{u}}}(v_{\mathrm{w}} - v_{\mathrm{i}})}^{\text{value loss of the wastes}} \tag{2.52}$$

General Definition of Efficiency

$$\eta = \sum(\text{value of products}) / \sum(\text{value of inputs}) \qquad (2.53)$$

For an energy system whose objective is to perform a given energy conversion, its efficiency can be given as:

$$\eta = \frac{\sum(E_u v_u)}{\sum(E_i v_i)} \qquad (2.54)$$

The first scale postulates that every energy form has the same value.

$$v_a = v_u = v_w = v_i = v_f \qquad (2.55)$$

A second value scale consists in associating to a given energy quantity its corresponding value of exergy, for instance:

- For work:

$$v_W = 1 \tag{2.56}$$

- For heat:

$$v_Q = \left(1 - \frac{T_0}{T}\right) \tag{2.57}$$

- For an enthalpy change of a pure substance:

$$v_H = 1 - T_0\left(\frac{s_1 - s_2}{h_1 - h_2}\right) \tag{2.58}$$

Exergy Efficiency

The first expression (Eq. 2.59), called degree of perfection by Szargut et al. [19] is useful to evaluate chemical processes:

$$\eta_{\mathrm{P}} = \frac{\text{Exergy of useful products}}{\text{Feeding exergy}} \tag{2.59}$$

The second expression is similar to Eq. 2.59 and is indicated in the analysis of thermal processes:

$$\eta_{\mathrm{b1}} = \frac{\text{Useful exergy effect}}{\text{Driving exergy}} \tag{2.60}$$

For thermal engines and refrigerating system that interact with the environment, Eq. 2.60 can be rewritten as:

$$\eta_{\mathrm{b2}} = \frac{\eta_{\mathrm{e}}}{\eta_{\mathrm{carnot}}} = \frac{\eta_{\mathrm{e}}}{\eta_{\mathrm{e\ max}}} \tag{2.61}$$

The third expression is recommended to be used to quantify the performance of processes and equipment where the only effect is the exergy destruction, such as the flow of steam trough an expansion valve, or the heat rejection that takes place in a condenser of a thermal power plant, or in the condenser of a refrigerating system. These processes are usually called pure dissipative processes.

$$\eta_{\mathrm{b2}} = \frac{\sum (\text{exit exergy})}{\sum (\text{inlet exergy})} \tag{2.62}$$

EXAMPLE 4

Consider a process, which requires a thermal load of 50 kW at 70°C. Determine the energy, and exergy efficiency (T_0 = 25°C) of the following technical heating options:

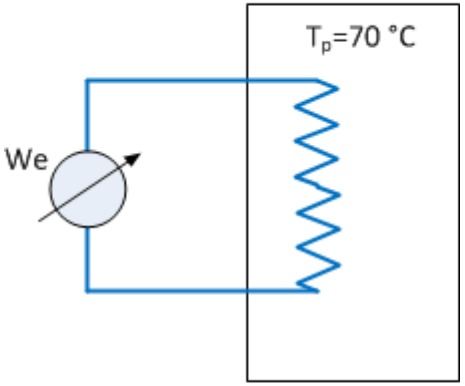
a) An electric heater.
b) A hot water waste flow from another process available at 100°C (suppose the heat exchanger outlet temperature of the hot water is 80°C).
c) Steam generated in a boiler operating with methane and with 80% thermal efficiency;
d) A compression heat pump with COP=3.0.
e) A compression heat pump with COP=3.0 driven by a gas turbine with 0.20 overall thermal efficiency (including turbine, generator and transmission). Suppose the fuel is methane.

Solution

$\dot{Q}_p = 50\ kW\ @70°C\ ;\ T_0 = 298\ K$

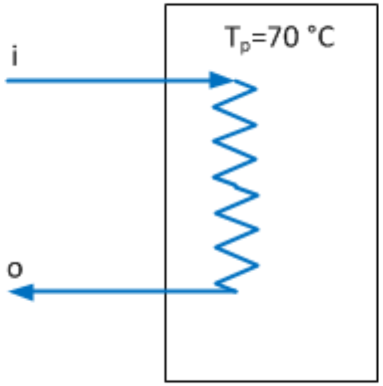
a) Electric heater

$$\dot{W}_e = \dot{Q}_p$$
$$\eta_e = \frac{\dot{Q}_p}{\dot{W}_e} = 1{,}00$$
$$\eta_b = \frac{\dot{Q}_p\left(1 - \frac{T_0}{T_p}\right)}{\dot{W}_e} = \theta_p = 1 - \frac{T_0}{T_p} = 0{,}131$$



b) Waste heat

$$T_i = 100\ ℃ \quad ; \quad T_o = 80\ ℃$$
$$\dot{Q}_p = \dot{m}c(T_i - T_o) = \dot{m}(h_i - h_o)$$
$$\left(\Delta\dot{B}\right)_{\substack{waste\\heat}} = \dot{m}(b_i - b_o) = \dot{m}\big(h_i - h_o - T_0(s_i - s_o)\big)$$



$$\left(\Delta\dot{B}\right)_{\substack{waste\\heat}} = \dot{m}\left(c(T_i - T_o) - T_0(s_i - s_o)\right)$$

$ds = \frac{cdT}{T}$ and $s_i - s_o = cln\frac{T_i}{T_o}$

$$\therefore \left(\Delta\dot{B}\right)_{\substack{waste\\heat}} = \dot{m}c(T_i - T_o)\left(1 - \frac{T_0}{\left(\frac{T_i - T_o}{ln\,T_i/T_o}\right)}\right)$$

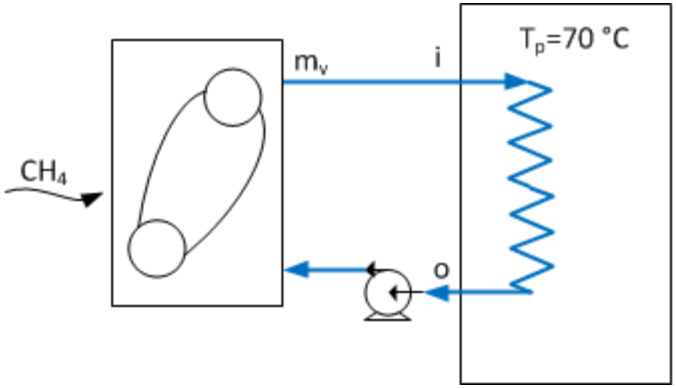
$$\bar{T}_{ln} = \frac{T_i - T_o}{ln\,T_i/T_o} = 362{,}9\ K$$

$$\left(\Delta\dot{B}\right)_{\substack{waste\\heat}} = \dot{Q}_p\left(1 - \frac{T_0}{\bar{T}_{ln}}\right) = \dot{Q}_p\bar{\theta}_{ln}$$

$$\eta_e = \frac{\dot{Q}_p}{\dot{m}c(T_i - T_o)} = 1{,}00$$

$$\eta_b = \frac{\dot{Q}_p}{\left(\Delta\dot{B}\right)_{\substack{waste\\heat}}} = \frac{\dot{Q}_p}{\bar{\theta}_{ln}} = 0{,}732$$

c) Steam



Neglect pump power

$$\eta_e = 0{,}80$$

$$\dot{Q}_p = \dot{m}_v(h_i - h_o)$$

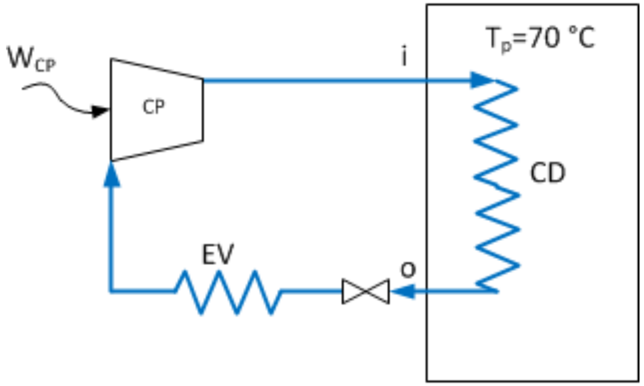
$$\eta_e = \frac{\dot{m}_v(h_i - h_o)}{\dot{m}_{CH4}(heating\ value)_{CH4}}$$

$$\eta_b = \frac{\dot{Q}_p\theta_p}{\dot{m}_{CH4}(b_{ch})_{CH4}} = \frac{\dot{m}_v(h_i - h_o)\theta_p}{\dot{m}_{CH4}(heating\ value)_{CH4}\cdot\varphi}$$

$$\varphi = \frac{b_{ch}}{heating\ value} = 1{,}0372\ \text{(Szargut)}$$

$$\eta_b = 0{,}101$$

d) Heat pump

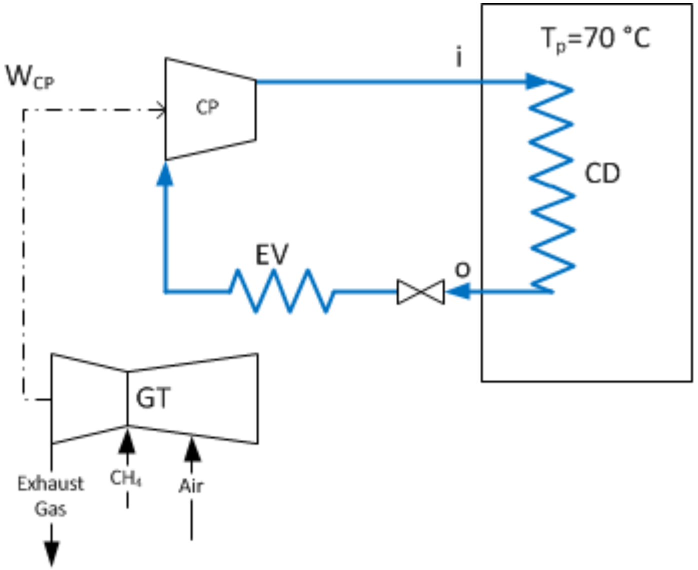


$$\eta_e = COP = \frac{\dot{Q}_{CD}}{\dot{W}_{CP}} = \frac{\dot{Q}_p}{\dot{W}_{CP}} = 3{,}00$$

$$\eta_b = \frac{\dot{Q}_p \theta_p}{\dot{W}_{CP}} = COP \cdot \theta_p = 0{,}393$$

$$\eta_e = 0{,}80$$

e) Heat pump with gas turbine



$$\eta_{GT} = 0{,}20$$

$$COP = 3{,}00$$

$$\eta_e = \frac{\dot{Q}_p}{\dot{m}_{CH4}(heating\ value)_{CH4}}$$

$$\eta_e = \frac{\dot{Q}_p}{\dot{W}_{CP}/\eta_{GT}} = \eta_{GT} \cdot COP = 0{,}60$$

$$\eta_b = \frac{\dot{Q}_p \theta_p}{\frac{\dot{W}_{CP}}{\eta_{GT}} \cdot \varphi} = \frac{1}{\varphi} \eta_{GT} \cdot COP \cdot \theta_p$$

$$\eta_b = 0{,}076$$

OVERALL COMPARATION

Alternative	***a***	***b***	***c***	***d***	***e***
η_e	*1,00*	*1,00*	*0,80*	*3,00*	*0,60*
η_b	*0,131*	*0,732*	*0,101*	*0,393*	*0,076*

Environmental Exergy Efficiency ($\eta_{b,env}$)

$$\eta_{\mathrm{b,env}} = \frac{B_{\mathrm{product}}}{B_{\mathrm{nat,res}} + B_{\mathrm{processing}} + B_{\mathrm{utilities}} + B_{\mathrm{deactivation}} + B_{\mathrm{disposal}}} \tag{2.63}$$

where:

B_{product}	exergy rate of the useful effect of a process
$B_{\mathrm{nat,\ res}}$	exergy rate of the natural resources consumed by the processes
$B_{\mathrm{processing}}$	exergy rate or flow rate required for extraction and preparation of the natural resources
$B_{\mathrm{utilities}}$	exergy rate or flow rate required by the utilities of the process
B_{disposal}	exergy rate or flow rate related to waste disposal of the process
$B_{\mathrm{deactivation}}$	destroyed exergy rate of additional natural resources during waste deactivation

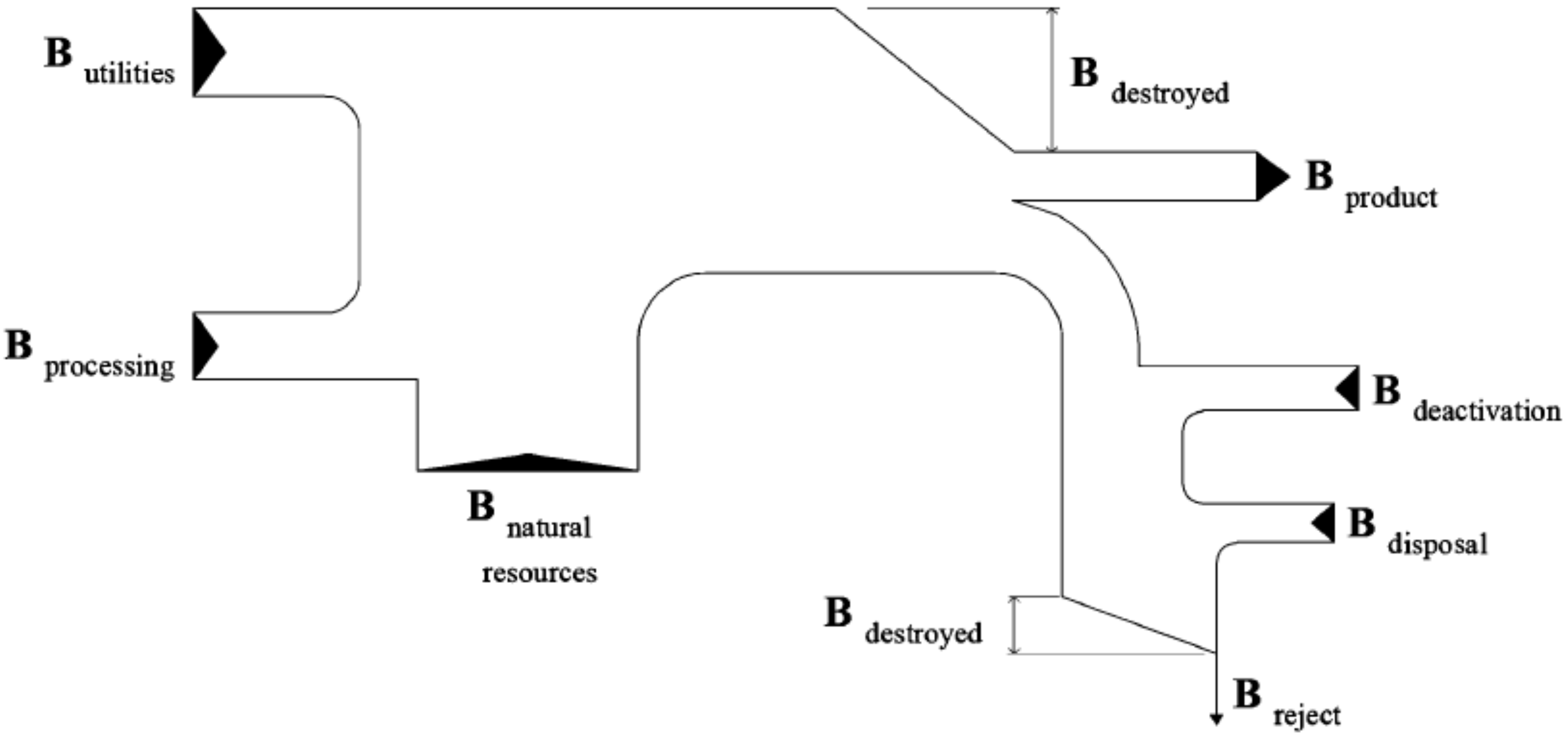


Fig. 2.25 Exergy balance of energy conversion processes

Exergy and Renewability Analysis

Currently, human development is based on the use of fossil fuels at a greater rate than that at which the deposit of fossil fuels have been generated. Also, since total recycling is not possible, it is imperative to seek for technologies that make better use of exergy available from all sources, including the so-called "renewable sources". Thus, two aspects should be taken into account whenever discussing renewability of any product:

- Origin of its energy source;
- Efficiency of the energy conversion processes.

The use of the concept of reversible processes, the one that having occurred can be reversed to the initial system and surroundings states, in the analysis of the renewability of energy conversion processes can contribute to develop such analysis in a rational basis by using thermodynamic parameters.

Renewability Exergy Index

$$\lambda = \frac{\sum B_{\text{product}}}{B_{\text{fossil}} + B_{\text{destroyed}} + B_{\text{deactivation}} + B_{\text{disposal}} + \sum B_{\text{emissions}}} \tag{2.98}$$

where:

- B_{product} represents the net exergy associated to the products and by-products.
- B_{fossil} is the non-renewable exergy consumed on production processes chain.
- $B_{\text{destroyed}}$ is the exergy destroyed inside the system, punishing the process for its inefficiencies.
- $B_{\text{deactivation}}$ is the deactivation exergy for treating wastes, when they are carried to equilibrium conditions with the environment. It accounts for exergy required for passing the streams leaving the system, considered as wastes, to no harmful environmental conditions.
- B_{disposal} is exergy rate or flow rate related to waste disposal of the process.
- $B_{\text{emissions}}$ is the exergy of wastes that are not treated or deactivated.

Depending on the value of the renewability exergy index, it indicates that:

- Processes with $0 \leq \lambda < 1$ are environmentally unfavorable.
- For internal and externally reversible processes with non-renewable inputs, $\lambda = 1$.

- If $\lambda > 1$, the process is environmentally favorable, and additionally, increasing λ implies that the process is more environmentally friendly.
- When $\lambda \rightarrow \infty$, it means that the process is reversible with renewable inputs and no wastes are generated.

When λ is higher than 1, the exergy of the products could be used to restore the environment to its conditions before the process and yet have a net output of exergy. In this sense, the natural cycles would only be responsible for the recycling of the renewable sources, considered as those able to be recycled by natural cycles without causing immediate harm to the environment in human lifetime basis.

Example: thermal power plant

$$\eta_\mathrm{b} = \frac{W_\mathrm{net}}{B_\mathrm{fuel}} \tag{2.99}$$

$$\lambda_f = \frac{W_\mathrm{net}}{B_\mathrm{fuel} + B_\mathrm{dest} + B_\mathrm{fluegases}} = \frac{\eta_\mathrm{b}}{(2 - \eta_\mathrm{b})} \tag{2.100}$$

$$\lambda_r = \frac{W_\mathrm{net}}{B_\mathrm{dest} + B_\mathrm{fluegases}} = \frac{\eta_\mathrm{b}}{(1 - \eta_\mathrm{b})} \tag{2.101}$$

Note that, one can verify that, as stated before:

- $\lambda = 1$ when $\eta_\mathrm{b} = 1$, for a reversible power plant using fossil fuel, and
- $\lambda \to \infty$ when $\eta_\mathrm{b} = 1$, for a reversible power plant using renewable fuel

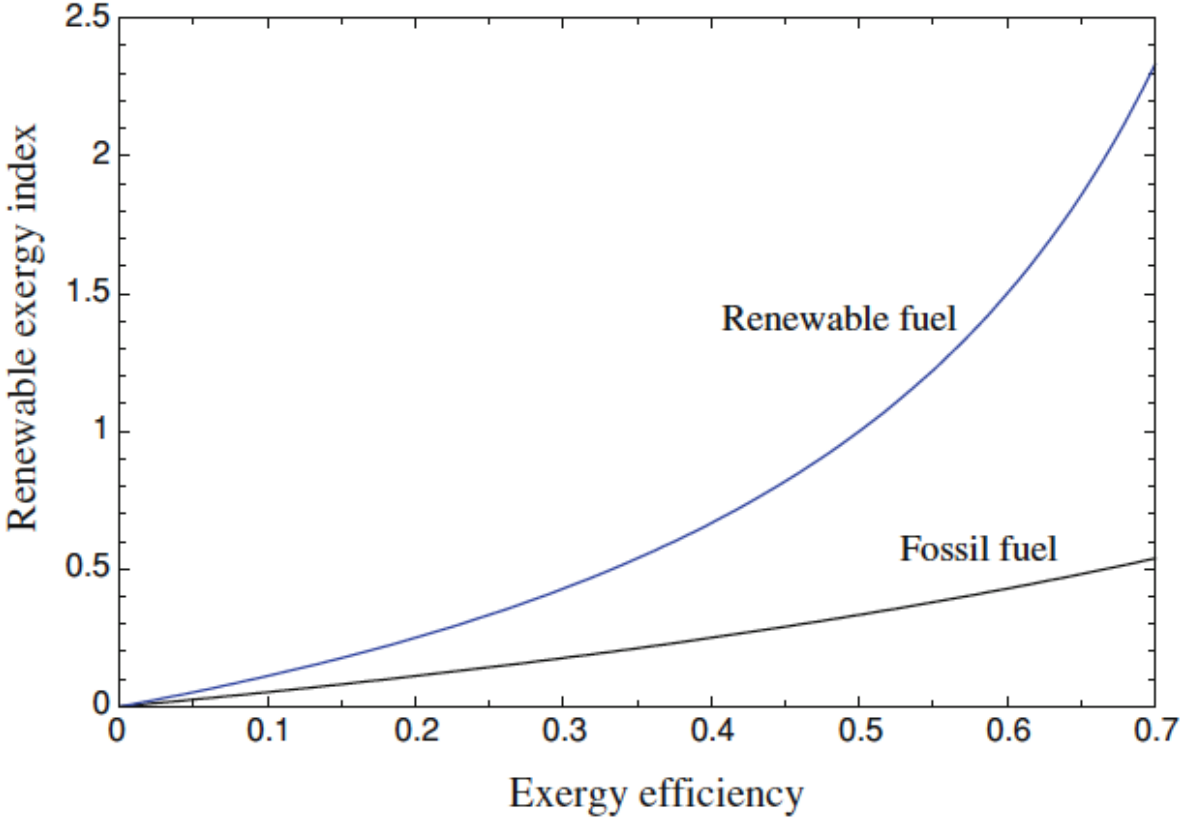


Fig. 2.31 Dependence of λ with η_b for thermal power plants using fossil and renewable fuels

Considering that a conventional power plant using coal as fuel has thermal efficiency of about 35 % for a Rankine subcritical plant and up to 50 % for a supercritical one, and that a combined cycle plant using natural gas can have thermal efficiency up to 60 %, the λ values for these types of power plants range from 0.18 to 0.43.

In case the supercritical power plant could operate with sugarcane bagasse as fuel (steam generate @ 600 °C and 294 bar, as it will be discussed in Chap. 6), the new value of λ would be 0.38, for an exergy efficiency of 0.28 [14].

Finally, it is important to note that this index evaluates the renewability of a given energy conversion process and not a product. In this sense it is questionable the concept of renewable product usually employed to characterize energy sources and fuels.

Exergy Costing

Cost Balance

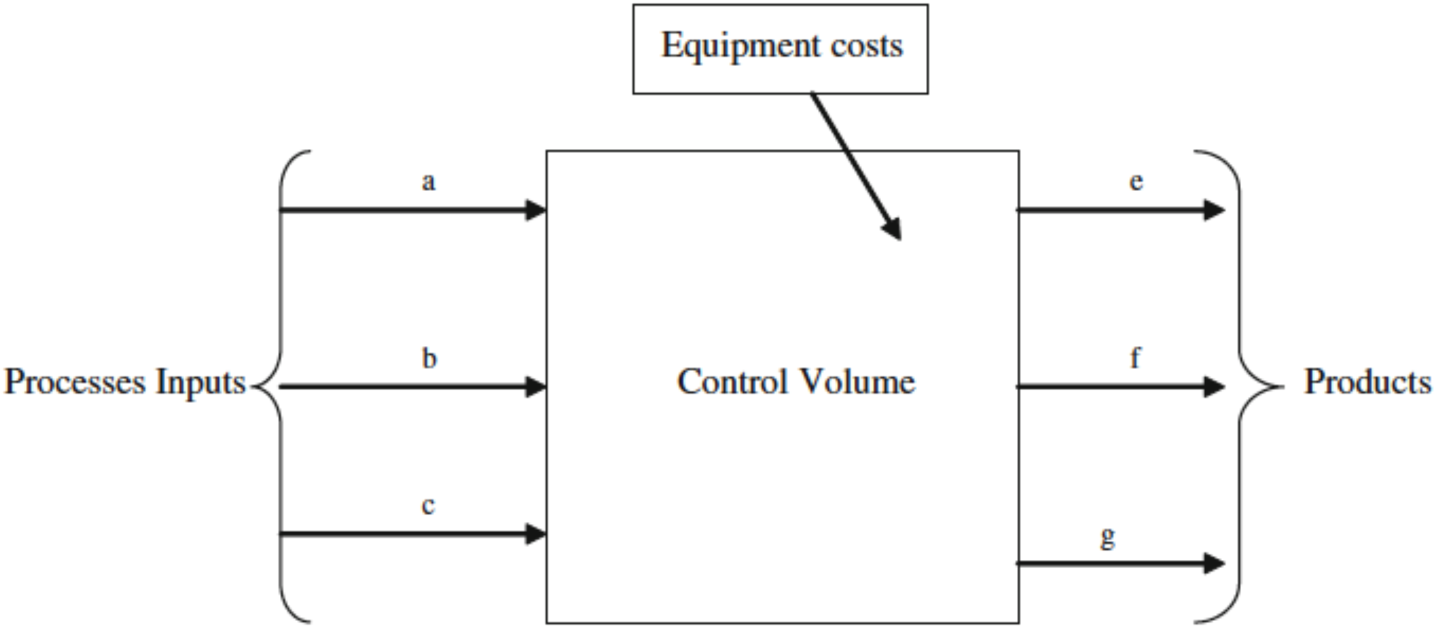


Fig. 2.28 Control volume for cost balance application

$$\Sigma(\text{Rate of expenses}) = \Sigma(\text{Cost rate of the products}) \tag{2.64}$$

$$C_{\mathrm{a}} + C_{\mathrm{b}} + C_{\mathrm{c}} + C_{\mathrm{equipment}} = C_{\mathrm{e}} + C_{\mathrm{f}} + C_{\mathrm{g}} \tag{2.65}$$

$$c_i = {}^{C_i}\!/_{N_i} \tag{2.66}$$

$$c_{\mathrm{a}}N_{\mathrm{a}} + c_{\mathrm{b}}N_{\mathrm{b}} + c_{\mathrm{c}}N_{\mathrm{c}} + C_{\mathrm{equipment}} = c_{\mathrm{e}}N_{\mathrm{e}} + c_{\mathrm{f}}N_{\mathrm{f}} + c_{\mathrm{g}}N_{\mathrm{g}} \tag{2.67}$$

$$c_i = \frac{\text{Overall production cost}}{N_i\,\text{product units}} \tag{2.68}$$

Exergy-Based Cost Partition Criteria

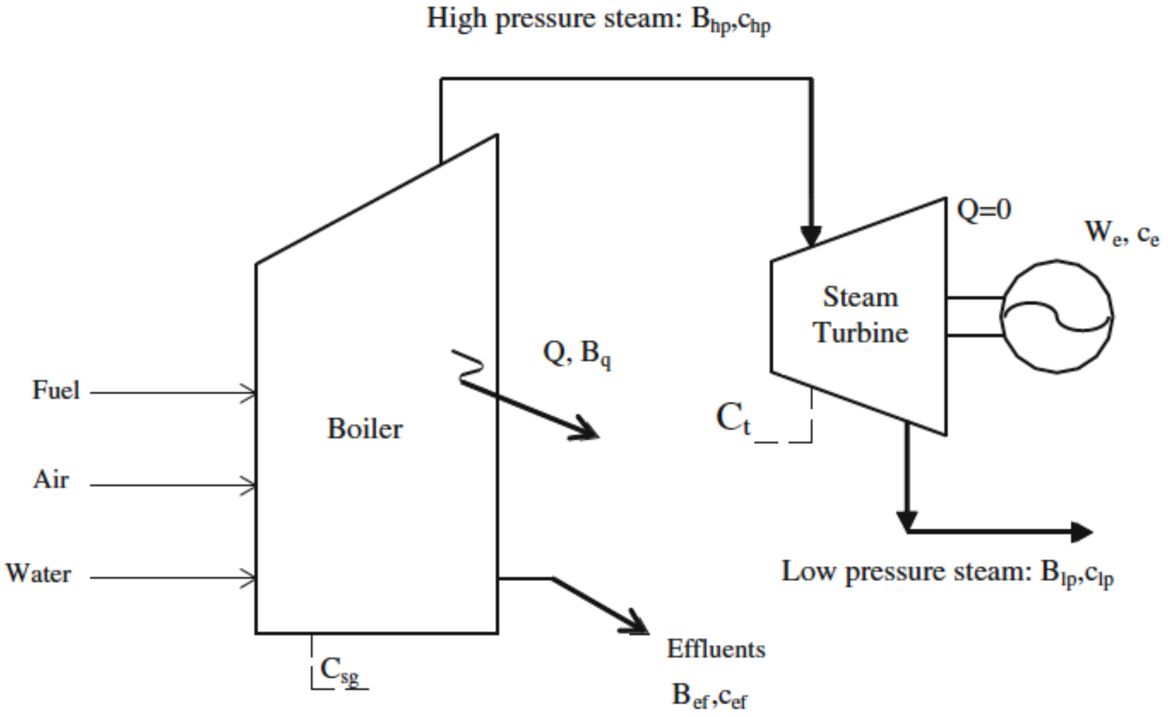


Fig. 2.29 Cogeneration system

The cost balances for the steam generator and the steam turbine are given, respectively, by Eqs. 2.69 and 2.70.

$$c_{\mathrm{hp}}B_{\mathrm{hp}} + c_{\mathrm{ef}}B_{\mathrm{ef}} + c_{\mathrm{q}}B_{\mathrm{q}} = c_{\mathrm{fuel}}B_{\mathrm{fuel}} + c_{\mathrm{air}}B_{\mathrm{air}} + c_{\mathrm{water}}B_{\mathrm{water}} + C_{\mathrm{sg}} \tag{2.69}$$

$$c_{\mathrm{e}}W_{\mathrm{e}} + c_{\mathrm{lp}}B_{\mathrm{lp}} = c_{\mathrm{hp}}B_{\mathrm{hp}} + C_t \tag{2.70}$$

$$c_{\mathrm{ef}} = c_{\mathrm{q}} = 0 \tag{2.71}$$

Equality method

$$c_{\mathrm{e}} = c_{\mathrm{lp}} \tag{2.72}$$

$$c_{\mathrm{e}} = c_{\mathrm{lp}} = c_{\mathrm{hp}}\left(\frac{B_{\mathrm{hp}}}{B_{\mathrm{lp}} + W_{\mathrm{e}}}\right) + \frac{C_t}{(B_{\mathrm{lp}} + W_{\mathrm{e}})} \tag{2.73}$$

$$\eta_{\mathrm{b\,turbine}} = \frac{W_{\mathrm{e}} + B_{\mathrm{lp}}}{B_{\mathrm{hp}}} \tag{2.74}$$

Extraction method

$$c_{\mathrm{hp}} = c_{\mathrm{lp}} \tag{2.75}$$

$$(c_{\mathrm{hp}})_{\mathrm{m}} = c_{\mathrm{hp}} b_{\mathrm{hp}} \tag{2.76}$$

$$(c_{\mathrm{lp}})_{\mathrm{m}} = c_{\mathrm{lp}} b_{\mathrm{lp}} \tag{2.77}$$

$$(c_{\mathrm{hp}})_{\mathrm{m}} > (c_{\mathrm{lp}})_{\mathrm{m}}$$

$$c_{\mathrm{e}} = c_{\mathrm{hp}} \left(\frac{B_{\mathrm{hp}} - B_{\mathrm{lp}}}{W_{\mathrm{e}}} \right) + \frac{C_{\mathrm{t}}}{W_{\mathrm{e}}} \tag{2.79}$$

$$\eta_{\mathrm{b\,turbine}} = \frac{W_{\mathrm{e}}}{B_{\mathrm{hp}} - B_{\mathrm{lp}}} \tag{2.80}$$

Application of the Thermoeconomic Analysis

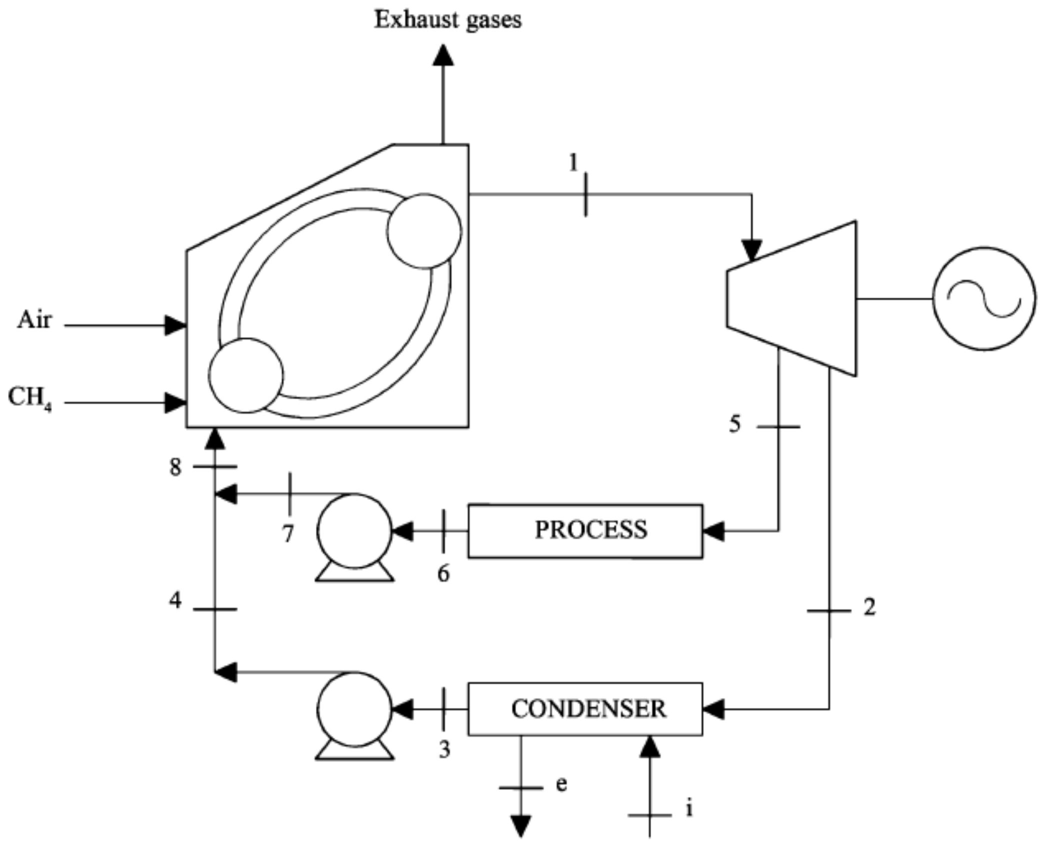


Fig. 2.30 Studied cogeneration plant [15]

The plant operational data are the following:

- Steam generation (section 1): 490 °C at 100 bar;
- Percent excess air in the boiler: 100 %
- Methane flow rate: 1 kg/s;
- Methane cost: US$ 10.00/MWh (US$ 144.00/t)
- Methane lower heating value: 50,146 kJ/kg;
- Stack gases temperature: 240 °C;

- Air temperature: 25 °C;
- Water temperature at the condenser inlet: 25 °C;
- Water temperature at the condenser outlet: 45 °C;
- Extraction mass ratio: ${}^{m_5}/_{m_1} = 0.5$;
- Steam state at turbine section 5: 240 °C@2 bar;
- Condensation pressure: 0.07 bar;
- Quality at turbine section 2: 0.97;
- Pumps Isentropic efficiency: 80 %.
- Environment conditions: 25 °C and 1 bar

Table 2.6 Cogeneration plant data [15]

Section	Mass flow rate (kg/s)	Pressure (bar)	Temperature (K)	Specific enthalpy (kJ/kg)	Specific entropy (kJ/kg K)	Specific exergy (kJ/kg)
1	13.94	100.00	763.20	3348.00	6.56	1396.00
2	6.97	0.07	312.20	2499.00	8.04	106.10
3	6.97	0.07	312.20	163.40	0.56	1.24
4	6.97	100.00	313.00	175.90	0.57	11.41
5	6.97	2.00	513.20	2950.00	7.67	668.50
6	6.97	2.00	393.40	504.70	1.53	53.06
7	6.97	100.00	394.80	517.70	1.54	64.06
8	13.94	100.00	354.10	346.80	1.08	29.36

Exergy balance:

$$\sum_{\text{inlet}} m_{\mathrm{i}}\, b_{\mathrm{i}} + Q_{\mathrm{VC}} \left(1 - \frac{T_0}{T}\right) = W_{\mathrm{VC}} + \sum_{\text{inlet}} m_e\, b_e + I_{\mathrm{VC}} \tag{2.83}$$

Energy efficiency:

$$\eta_{\mathrm{e}} = \frac{W_{\mathrm{net}} + Q_{\mathrm{process}}}{m_{\mathrm{CH_4}}\, \mathrm{LHV}} \tag{2.84}$$

Exergy efficiency:

$$\eta_{\mathrm{b}} = \frac{W_{\mathrm{net}} + B_{\mathrm{process}}}{m_{\mathrm{CH_4}}\, b_{\mathrm{CH_4}}} \tag{2.85}$$

The components capital cost can by estimated by using, for every component, the so-called exponential form as described by Bohem [2] and presented by the cost curve given in Eq. 2.86, where C_r is the cost of reference size component, C_{eq} is the cost of a size of interest, α is the exponent on cost size relationship, S_r is the component reference size, and S is the component size of interest (see Table 2.8).

$$C_{eq} = C_r \left(\frac{S}{S_r} \right)^{\alpha} \tag{2.86}$$

The following hypotheses are considered in the determination of the components cost rates:

- Capital recovery period (N): 20 years;
- Interest rate (i): 15 % per year;
- Annual operating hours: 8000;
- Annual operational and maintenance factor ($f_{O\&M}$): 1.06

Table 2.7 Performance parameters of the cogeneration plant [15]

Performance Parameter
Turbine power: 8,684 kW
Pumping power: 87 and 90 kW
Net power plant: 8,507 kW
Process heat rate: 17,041 kW
Process exergy rate: 4,288 kW
Plant energy efficiency: 51 %
Plant exergy efficiency: 25 %

Table 2.8 Cost components parameters [15]

Component	α	Cr (U$1000)	Sr	S	
Steam generator	0.59	320	10_5	m	(lb/h)
Steam turbine	0.68	25	10_3	W	(kW)
Condenser	0.55	3	10	W	(kW)
Pumps	0.58	7.5	100	W	(kW)

Table 2.9 Components costs [15]

Equipment	C_{eq} (US$)	C_{eqt} (US$/s)
Steam generator	339603	0.001997
Steam turbine	108713	0.000639
Condenser	124001	0.000729
Pump 1	6941	0.000040
Pump 2	7070	0.000041

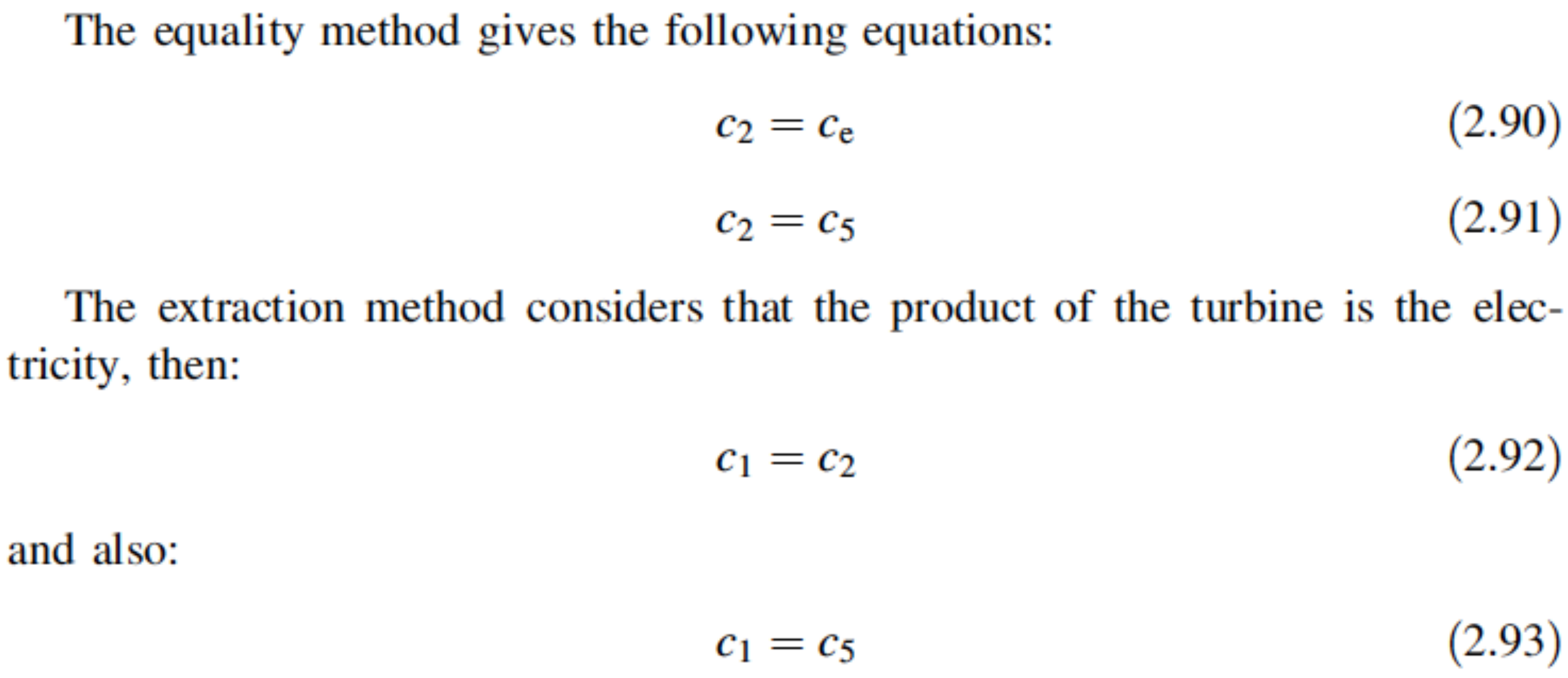
With these parameters the cost rates are obtained by Eq. 2.87 and 2.88 and the components values are presented in Table 2.9:

$$C = \frac{C_{\mathrm{eq}} f_a f_{O\&M}}{3600\,8000} \tag{2.87}$$

$$f_{\mathrm{a}} = \frac{i}{1 - (1 + i)^{-N}} \tag{2.88}$$

With this information the cost balances can now be written for every component:

$$\sum_{\mathrm{inlet}} c_{\mathrm{i}} B_{\mathrm{i}} + C = \sum_{\mathrm{outlet}} c_{\mathrm{e}} B_{\mathrm{e}} \tag{2.89}$$



The equality method gives the following equations:

$$c_2 = c_\mathrm{e} \tag{2.90}$$

$$c_2 = c_5 \tag{2.91}$$

The extraction method considers that the product of the turbine is the electricity, then:

$$c_1 = c_2 \tag{2.92}$$

and also:

$$c_1 = c_5 \tag{2.93}$$

For the transferred exergy to the process:

$$c_5 = c_6 \tag{2.94}$$

The cost of the expended exergy in the process, c_p, is determined by the process cost balance:

$$c_5 B_5 = c_p B_p + c_6 B_6 \tag{2.95}$$

The annual overall cost rate of the cogeneration plant is done by Eq. 2.96.

$$C_{\text{overall}} = \sum_{\text{component}} C + c_{\text{CH}_4} B_{\text{CH}_4} = \text{US\$4},\ 258,013/\text{year} \tag{2.96}$$

This cost rate must be distributed between both products of the plant: electricity and heat transferred to the process, according to the chosen cost partition criteria:

$$C_{\text{total}} = C_{\text{electricity}} + C_{\text{heat}} \tag{2.97}$$

Table 2.10 Results of the thermoeconomic analysis [15]

Cost	Extraction method	Equality method
1 (US$/t)	11.38	11.63
2 (US$/t)	0.86	1.23
3 (US$/t)	0.97	1.33
4 (US$/t)	1.14	1.48
5 (US$/t)	5.45	7.72
6 (US$/t)	0.43	0.61
7 (US$/t)	0.61	0.77
8 (US$/t)	0.88	1.13
Electricity (US$/MWh)	47.77	41.60
Process heat (US$/MWh)	29.35	41.60

By the extraction method:

$$C_{\text{total}} = 47.77 \cdot 8.5 \cdot 8,000 + 29.35 \cdot 4.3 \cdot 8,000 = 4,258,000/\text{year}$$

By the equality method:

$$C_{\text{total}} = 41.60 \cdot 8.5 \cdot 8,000 + 41.60 \cdot 4.3 \cdot 8,000 = 4,259,840/\text{year}$$

The small difference of overall values is due to some approximations done during the calculations.