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Novembro de 2020

Microeconomia II
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Seção 23.C - Dominant Strategy Implementation

Exercise 1. Consider the quasilinear setting in which

$$u_i(x, \theta_i) = v_i(k, \theta_i) + \bar{m}_i + t_i$$

and

$$\mathbb{X} = \left\{ (k, t_1, \cdots, t_I) \in K \times \mathbb{R}^I \middle| \sum_i t_i \leqslant 0 \right\}.$$

Show that no dictatorial social choice function exists.

Exercise 2. State and prove the Revelation Principle for Dominant Strategies. Give the intuition of this result and comment on its importance.

Exercise 3. Usando a notação da seção 23.C, suponha que

- $\mathbb{X}$ é finito e contém pelo menos 3 elementos
- $\mathscr{R}_i = \mathscr{P}$ para todo agente i
- $f(\Theta) = \mathbb{X}$.

Demonstre que a FES $f(\cdot)$ é ditatorial se, e somente se, ela é truthfully implementável em estratégias dominantes.

Exercise 4 (MWG 23.C.1). Verify that if preference reversal property [condition (23.C.6)] is satisfied for all i and all θ_i', θ_i'', and θ_{-i} then $f(.)$ is truthfully implementable in dominant strategies.

Exercise 5 (MWG 23.C.2). Show that for any I, when X contains two elements (say, $X = \{x_1, x_2\}$, then any majority voting social choice function [i.e., a social choice function that always chooses alternative x_i if more agents prefer x_i over x_j than prefer x_j over x_i (it may select either x_1 or x_2 if the number of agents preferring x_1 over x_2 equals the number preferring x_2 over x_1)] is truthfully implementable in dominant strategies.

Exercise 6 (MWG 23.C.3). Show that when $\mathscr{R}_i = \mathscr{P}$ for all i, any ex post efficient social choice function $f(.)$ has $f(\Theta) = X$

Exercise 7 (MWG 23.C.4). Show that if $f : \Theta \to X$ is truthfully implementable in dominant strategies when the set of possible types is Θ_i for $i = 1, ...I$, then when each agent $i's$ set of possible types is $\hat{\Theta}_i \subset \Theta_i$ (for $i = 1, ..., I$) the social choice function $\hat{f} : \hat{\Theta} \to X$ satisfying $\hat{f}(\theta) = f(\theta)$ for all $\theta in \hat{\Theta}$ is truthfully implementable in dominant strategies.

Exercise 8 (MWG 23.C.8). Suppose that $I = 2$, $X = \{a, b, c, d, e\}$, $\Theta_1 = \{\theta_1', \theta_1''\}$, and $\Theta_2 = \{\theta_2', \theta_2''\}$ and taht the agents' possible preferences are ($a - b$ means that alternatives a and b are indifferent):

$\succsim_1(\theta_1')$	$\succsim_1(\theta_1'')$	$\succsim_2(\theta_2')$	$\succsim_2(\theta_2'')$
a–b	a	a–b	a
c	b	c	b
d	d	d	d
e	c	e	c
	e		e

Consider the social choice function

$$f(\theta) = \begin{cases} b & \text{if } \theta = (\theta_1', \theta_2') \\ a & \text{otherwise} \end{cases}$$

(a) Is $f(.)$ ex post efficient?

(b) Does it satisfy the property identified in Proposition 23.C.2?

(c) Examine the direct revelation mechanism that truthfully implement $f(.)$. Is truth telling each agent's unique (weakly) dominant strategy? Show that if an agent chooses this untruthful (weakly) dominant strategy, then $f(.)$ is not implemented.

Exercise 9 (MWG 23.C.10). (B. Holmstrom) Consider the quasilinear environment studied in section 23.C. Let $k^*(.)$ denote any project decision rule that satisfies (23.C.7). Also define the function $V^*(\theta) = \sum_i v_i(k^*\theta), \theta_i)$.

(a) Prove that there exists an ex post efficient social choice function [i.e., one that satisfies condition (23.C.7) and the budget balance condition (23.C.12)] that is truthfully implementable in dominant strategies if and only if the function $V^*(.)$ can be written as $V^*(\theta) = \sum_i V(\theta_{-i})$ for some functions $V_1(.), ..., V_I(.)$ having the property that $V_i(.)$ depends only on θ_{-i} for all i.

(b) Use the result in part **(a)** to show that when $I = 3$ $K = \mathbb{R}$, $\Theta_i = \mathbb{R}_+$ for all i, and $v_i(k, \theta_i) = \theta_i k - (\frac{1}{2})k^2$ for all i an ex post efficient social choice function exists that is truthfully implementable in dominant strategies. (This result extends to any $I > 2$.)

(c) Now suppose that the $v_i(k, \theta_i)$ functions are such that $V^*(.)$ is an I-times continuously differentiable function. Argue that a necessary condition for an ex post efficient social choice function to exist is that, at all θ,

$$\frac{\delta^I V^*(\theta)}{\delta\theta_1 ... \delta\theta_I} = 0$$

(In fact, this is a sufficient condition as well.)

(d) Use the result in **(c)** to verify that, under the assumptions made in the small type discussion all the end of Section 23C, when $I = 2$ no ex post efficient social choice is truthfully implementable in dominant strategies.

Exercise 10 (MWG 23.C.11). Consider a quasilinear environment, but now suppose that each agent i has a Bernoulli utility function of the form $u_i(v_i(k, \theta_i) + \bar{m}_i + t_i$ with $u_i'(.) > 0$. That is, preferences over certain outcomes take a quasilinear form, but risk preferences are unrestricted. Verify that Proposition 23.C.4 is unaffected by this change.

Exercise 11. Usando a notação da seção 23.C, suponha que

- $k = (y_1, y_2, ..., y_I)$
- $\mathbb{K} = \{(y_1, y_2, ..., y_I) : y_i \in \{0, 1\}, \forall i$ e $\sum_{i=1}^{I} y_i = 1\}$
- $v_i(k, \theta_i) = \theta_i y_i$.
- uma FES é $f : \Theta \mapsto \mathbb{X}$, ou seja, $\forall \theta \in \Theta$

$$f(\theta) = [k(\theta, t_1(\theta), ..., t_I(\theta)]$$

tal que $k(\theta) \in \mathbb{K}$ e $\sum_{i=1}^{I} t_i(\theta) \leqslant 0$

(a) Mostre que se a FES $f(.)$ é ex post eficiente, então $\forall \theta \in \Theta, k(\theta)$ satisfaz

$$\sum_{i=1}^{I} v_i[k(\theta), \theta_i] \geqslant \sum_{i=1}^{I} v_i[k, \theta_i], \qquad \forall k \in \mathbb{K} \tag{23.C.7}$$

(b) Mostre que se $f(.)$ é ex post eficiente, então $\sum_{i=1}^{I} t_i(\theta) = 0$

(c) Mostre que se $f(.)$ satisfaz (23.C.7) e $\sum_{i=1}^{I} t_i(\theta) = 0$, então $f(.)$ é ex post eficiente.