EX. 130

a) $f\delta_0' = c_0\delta_0 + c_1\delta_0'$

DEMO: $f\delta_0'(\varphi) = \delta_0'(f\varphi) = -\delta_0\left(\frac{d}{dx}(f\varphi)\right) = -\delta_0(f'\varphi + f\varphi')$

$= -f'(0)\varphi(0) - f(0)\varphi'(0) = -f'(0)\delta_0(\varphi) + f(0)\delta_0'(\varphi)$

$$f\delta_0' = \underbrace{-f'(0)}_{c_0}\delta_0 + \underbrace{f(0)}_{c_1}\delta_0'$$

CUIDADO! SINAL

$$\frac{d}{dx}\mu(\varphi) = -\mu\left(\frac{d\varphi}{dx}\right)$$

b) $f\delta_0^{(k)} = \sum_{j=0}^{k} c_{kj}\frac{d^j}{dx^j}\delta_0$

DEMO: $f\delta_0^{(k)}(\varphi) = \delta^{(k)}(f\varphi) = (-1)^k\delta_0\left(\frac{d^k}{dx^k}(f\varphi)\right) = (-1)^k\delta_0\left(\sum_{j=0}^{k}\binom{k}{j}f^{(k-j)}\varphi^{(j)}\right)$

$= (-1)^k\sum_{j=0}^{k}\binom{k}{j}f^{(k-j)}(0)\underbrace{\varphi^{(j)}(0)}_{\delta_0\left(\frac{d^j}{dx^j}\varphi\right) = (-1)^j\delta_0^{(j)}(\varphi)} = (-1)^k\sum_{j=0}^{k}\binom{k}{j}f^{(k-j)}(-1)^j\delta_0^{(j)}(\varphi)$

$$= \sum_{j=0}^{k}\underbrace{\binom{k}{j}(-1)^{k+j}f^{(k-j)}(0)}_{c_{kj}}\delta_0^{(j)}(\varphi)$$

$c_{kj} = (-1)^{k+j}\binom{k}{j}f^{(k-j)}(0).$

Ex. 29

$$\sum_{|\alpha| \le m} a_\alpha(x) \partial^\alpha u(x+x_0) = \sum_{|\alpha| \le m} a_\alpha(x+x_0) \partial^\alpha u(x+x_0) \underset{\forall x_0}{} \Rightarrow a_\alpha = \text{ct.}$$

DEMO: $\partial^\alpha (x^\beta) = \frac{\beta!}{(\beta-\alpha)!} x^{\beta-\alpha}$, $\alpha \le \beta$

$u(x) = x^\beta$, $|\beta| \le m$

$$\underbrace{\sum_{|\alpha| \le m}} a_\alpha(x) \frac{\beta!}{(\beta-\alpha)!} (x+x_0)^{\beta-\alpha} = \sum_{|\alpha| \le m} a_\alpha(x+x_0) \frac{\beta!}{(\beta-\alpha)!} (x+x_0)^{\beta-\alpha}.$$

$$\sum_{\alpha \le \beta} a_\alpha(x) \frac{\beta!}{(\beta-\alpha)!} (x+x_0)^{\beta-\alpha} = \sum_{\alpha \le \beta} a_\alpha(x+x_0) \frac{\beta!}{(\beta-\alpha)!} (x+x_0)^{\beta-\alpha}$$

ESCOLHEMOS $x = -x_0$

$\beta \ne \alpha$ $(x+x_0)^{\beta-\alpha} = 0$. $\beta = \alpha$ $(x+x_0)^{\overset{=0}{\beta-\alpha}} = 1$.

$$\Rightarrow a_\alpha(-x_0) \frac{\beta!}{\cancel{(\beta-\alpha)!}} = a_\alpha(0) \frac{\beta!}{\cancel{(\beta-\alpha)!}} \Rightarrow \overline{a_\alpha(-x_0) = a_\alpha(0)}$$

$\forall x_0 \in \mathbb{R}^n$.

$\Rightarrow a_\alpha \in$ CONST.

EX. 66 a) $Lu > 0 \Rightarrow u$ NÃO TEM MÁXIMO LOCAL

DEMO: SUPONHA QUE O MÁXIMO LOCAL x_0 PERTENÇA A U, ENTÃO

x_0 É PONTO CRÍTICO I) $\nabla u(x_0) = 0$.

HESSIANO II) $H(x_0) \leq 0 \Leftrightarrow$ AUTOVALORES DE $\left(\frac{\partial^2 u}{\partial x_i \partial x_j}(x_0)\right)$ SÃO ≤ 0.

$$H(x_0) = \frac{\partial^2 u}{\partial x_i \partial x_j}$$

I) LOGO $\displaystyle\sum_{j=1}^{n} b_j(x_0) \frac{\partial u}{\partial x_j}(x_0) = 0$.

II) SABEMOS QUE $\exists\ P$ ORTOGONAL T.Q. $P^T H(x_0) P = D = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}, \lambda_j \leq 0$

$P \in \mathbb{R}^{n \times n}$, POIS $H(x_0)$ É SIMÉTRICO.

$H(x_0) = P^T D P$.

ASSIM, $H(x_0) = P D P^T \Leftrightarrow \displaystyle\frac{\partial^2 u}{\partial x_i \partial x_j}(x_0) = \sum_{l=1}^{n}\sum_{k=1}^{n} P_{il} \lambda_l \delta_{lk} P_{jk}$

$$= \sum_{l=1}^{n} P_{il} \lambda_l P_{jl}$$

CONCLUÍMOS QUE $\displaystyle\sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(x_0) \frac{\partial^2 u}{\partial x_j \partial x_k}(x_0)$

$$= \sum_{l=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(x_0) P_{jl} \lambda_l P_{kl}$$

$$= -\sum_{l=1}^{n}\left(\sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(x_0) \underbrace{\sqrt{|\lambda_l|} P_{jl}}_{:= \xi_{j,l}} \underbrace{\sqrt{|\lambda_l|} P_{kl}}_{:= \xi_{k,l}}\right) \leq 0$$

$\hookrightarrow \sum a_{jk} \xi_j \xi_k \geq 0$.

$\lambda_l = -|\lambda_l|$, POIS $\lambda_l \leq 0$.

ASSIM, CONCLUÍMOS QUE

$$Lv(x_0) = \underbrace{\sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(x_0)\frac{\partial^2 v}{\partial x_j \partial x_k}(x_0)}_{\leq 0} + \underbrace{\sum_{j=1}^{n} b_j(x_0)\frac{\partial v}{\partial x_j}(x_0)}_{=0} \leq 0.$$

OBTEMOS UMA CONTRADIÇÃO, POIS $Lv(x_0) > 0$. LOGO O MÁXIMO $\notin U$

b) $w(x) = e^{-M|x-x_0|^2}$, $x_0 \notin U$.

$$Lw(x) = \sum_{j=1}^{n}\sum_{k=1}^{n} a_{jk}(x)\frac{\partial^2 w}{\partial x_j \partial x_k}(x) + \sum_{j=1}^{n} b_j(x)\frac{\partial w}{\partial x_j}(x)$$

$$= \sum_{j=1}^{n}\sum_{k=1}^{n} M^2 a_{jk}(x)(x_j - x_{0j})(x_k - x_{0k}) - M\sum_{j=1}^{n} b_j(x)(x_j - x_{0j})$$

$$\geq M^2 a_0 |x-x_0|^2 - M \max|b_j(x)|\,|x-x_0|$$

$$= M|x-x_0|\left(Ma_0|x-x_0| - \max|b_j(x)|\right)$$

$$\geq \underbrace{M|x-x_0|}_{>0}\underbrace{\left(Ma_0 d(x_0,U) - \max\|b_j\|_{L^\infty(U)}\right)}_{>0\text{, SE } M \text{ É GRANDE}} > 0 \text{, SE } M \text{ É GRANDE}$$

c) $L(u+\varepsilon w) = \underset{\overset{\shortparallel}{0}}{Lu} + \varepsilon \underset{>0}{Lw} > 0.$

$\max_{\bar U}(u+\varepsilon w) = \max_{\partial U}(u+\varepsilon w)$, PELO ITEM a)

TOMANDO $\varepsilon \to 0$, OBTEMOS $\max_{\bar U} u = \max_{\partial U} u$

253 (PROVA 2019)

a) $\frac{d}{dt}\int_V \sigma(x)\, u(x,t)\, dx = -\int_{\partial V} F\cdot n\, ds + \int_V f.$

$$\int_V \sigma \frac{\partial u}{\partial t} = -\int_V \nabla\cdot F + \int_V f$$

$$\int_V \left(\sigma \frac{\partial u}{\partial t} + \nabla\cdot F - f\right) dx = 0, \quad \forall V$$

$$\Rightarrow \sigma \frac{\partial u}{\partial t} + \nabla F - f = 0. \quad \forall x.$$

$F = -k\nabla u$

$$\sigma \frac{\partial u}{\partial t} - \nabla\cdot(k\nabla u) - f = 0$$

$$\sigma \frac{\partial u}{\partial t} = \nabla\cdot(k\nabla u) + f.$$

USAMOS 1) $v : U \to \mathbb{R}$ CONTÍNUA

$\int_V v\, dx = 0.\ \forall\ V \subset \bar{V} \subset U \Rightarrow v \equiv 0$ EM U

V DE CLASSE C^1.

2) TEO. DIVERGÊNCIA

$$\sigma \frac{\partial u}{\partial t} = \nabla \cdot (k \nabla u) + f$$
$$= \nabla k \cdot \nabla u + k \Delta u + f.$$
$$a = \frac{k}{\sigma}, \quad b = \frac{1}{\sigma} \nabla k, \quad g = \frac{f}{\sigma}.$$

b) $\frac{\partial u}{\partial t} = a \Delta u + b \cdot \nabla u + g$, $\underline{a > 0}$

SE $g < 0$ $\quad \max_{U_T} u = \max_{\Gamma_T} u$ $\quad$ MAX  = MAX 

$U_T = U \times [0,T]$ $\quad \Gamma_T = U \times \{0\} \cup \partial U \times]0,T]$

SUPONHA QUE $\max_{U_T} u = u(x_0, t_0)$, $(x_0, t) \in U_T \setminus \Gamma_T$.

1) $x \mapsto u(x, t_0)$ ASSUME MÁXIMO EM $\underline{x_0 \in U}$.

LOGO $\nabla u(x_0, t_0) = 0$

$$\underbrace{\left(\frac{\partial^2 u}{\partial x_i \partial x_j}(x_0, t_0)\right)}_{\text{AUTOVALORES } \leq 0} \leq 0 \Rightarrow \operatorname{Tr}\underbrace{\left(\frac{\partial^2 u}{\partial x_i \partial x_j}\right)}_{\text{SOMA DOS AUTOVAL...}} \leq 0 \Rightarrow \Delta u(x_0, t_0) \leq 0$$

2) $t \mapsto u(x_0, t)$ ASSUME MÁXIMO EM $t_0 \in]0, T]$.

SE $t_0 < T$, ENTÃO $\frac{\partial u}{\partial t}(x_0, t_0) = 0$

SE $t_0 = T$, ENTÃO $u(x_0, t_0) - u(x_0, t_0 - h) \geq 0$, $\forall h > 0$.

$u(x_0, t_0 - h) - u(x_0, t_0) \leq 0$ $\quad \frac{u(x_0, t_0 - h) - u(x_0, t_0)}{-h} \geq 0$ $\quad \frac{\partial u}{\partial t}(x_0, t_0) \geq 0$

$$\underbrace{\underbrace{\frac{\partial u}{\partial t}}_{\geq 0} - \underbrace{\Delta u}_{\geq 0} - \underbrace{b \cdot \nabla u}_{\geq 0}}_{\geq 0} = \underbrace{\underbrace{g}_{<0}}_{<0}$$

ABSURDO.

$$\max_{U_n} u = \max_{\Gamma_T} u$$

b) $w(x) = e^{\lambda x_1}$

$$\underbrace{a(x)\, \Delta w(x) + b(x) \cdot \nabla w(x)}\ \overset{?}{>} 0$$

$$a(x)\, \lambda^2 e^{x_1 \lambda} + b(x) \cdot (\lambda e^{\lambda x_1}, 0, \dots, 0)$$

$b = (b_1, \dots, b_n)$

$$\lambda^2 a(x) e^{\lambda x_1} + \lambda\, b_1(x)\, e^{\lambda x_1}$$

$\lambda > 0$

$$\underbrace{\lambda^2}_{>0} \left(\underbrace{a(x) + \frac{b_1(x)}{\lambda}} \right) \underbrace{e^{\lambda x_1}}_{>0}$$

$$a(x) + \frac{b_1(x)}{\lambda} \geq \underbrace{\min_{x \in \bar{U}} a(x)}_{\geq c > 0} - \underbrace{\frac{\max_{x \in \bar{U}} |b_1(x)|}{\lambda}}_{\lambda \to \infty \ < \frac{c}{2}} \overset{\to 0}{} > \frac{c}{2}$$

, PARA λ GRANDE

c) $\dfrac{\partial u}{\partial t} = a\Delta u + b\cdot\nabla u.$

$\max_{U_T} u = \max_{\Gamma_T} u$

$v = u + \varepsilon w.$

$$\begin{cases} \dfrac{\partial v}{\partial t} = \dfrac{\partial u}{\partial t} + \varepsilon \dfrac{\partial w}{\partial t} = \dfrac{\partial u}{\partial t} \\ a\Delta v + b\cdot\nabla v = a\Delta u + b\cdot\nabla u + \varepsilon(a\Delta w + b\cdot\nabla w) \end{cases}$$

$$\frac{\partial v}{\partial t} = \frac{\partial u}{\partial t} = a\Delta u + b\cdot\nabla u = a\Delta v + b\cdot\nabla v \underbrace{- \varepsilon(a\Delta w + b\cdot\nabla w)}_{g}.$$

$$\frac{\partial v}{\partial t} = a\Delta v + b\cdot\nabla v + g \quad , \quad g < 0. \qquad g = \underbrace{-\varepsilon\underbrace{(a\Delta w + b\cdot\nabla w)}_{\geq 0,\ b)}}_{<0.}$$

PELO ITEM a) $\max_{U_T} v_\varepsilon = \max_{\Gamma_T} v_\varepsilon$

TOMANDO LIMITE $\varepsilon \to 0$ $\max_{U_T} u = \max_{\Gamma_T} u.$

FAZENDO O MESMO P/ $-u \Rightarrow \underbrace{\max_{U_T}(-u) = \max_{\Gamma_T}(-u)}$

$\min_{U_T}(u) = \min_{\Gamma_T}(u)$

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + b\cdot\nabla u + g \\ u = 0 \quad \text{em } \partial U \\ u = g \quad t = 0 \end{cases}$$

SE u E $\tilde{u}$ SÃO SOLUÇÕES, ENTÃO $w = u - \tilde{u}$, É T.Q.

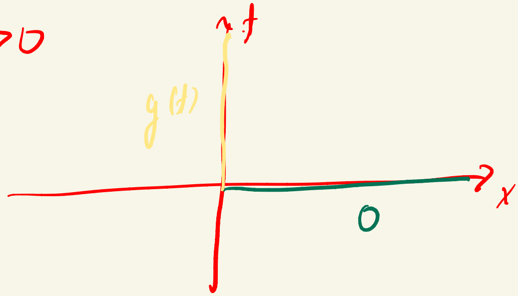
$$\left.\begin{cases} \dfrac{\partial w}{\partial t} = \Delta w + b\cdot\nabla w \\ w = 0 \quad \text{em } \partial U \\ w = 0 \quad t = 0 \end{cases}\right\} \max w = \min w = 0$$

↳ CORRESPONDE A Γ_T

Ex: 247

$u(x,t) = v(x,t) + g(t)$

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, & t > 0,\ x > 0 \\ u(x,0) = 0, & x > 0 \\ u(0,t) = g(t), & t \geq 0 \end{cases}$$



$$v(x,t) = \begin{cases} u(x,t) - g(t), & x > 0 \\ -u(-x,t) + g(t), & x < 0 \end{cases}$$

$$\frac{\partial v}{\partial t}(x,t) = \begin{cases} \dfrac{\partial u}{\partial t} - g', & x > 0 \\ -\dfrac{\partial u}{\partial t} + g', & x < 0 \end{cases}$$

$$\frac{\partial^2 v}{\partial x^2}(x,t) = \begin{cases} \dfrac{\partial^2 u}{\partial x^2}(x,t), & x > 0 \\ -\dfrac{\partial^2 u}{\partial x^2}(-x,t), & x < 0 \end{cases}$$

Logo $\dfrac{\partial v}{\partial t}(x,t) = \dfrac{\partial^2 v}{\partial x^2}(x,t) + G(t)$

$$G(t) = \begin{cases} -g'(t), & x > 0 \\ g'(t), & x < 0 \end{cases}$$

$$\frac{\partial v}{\partial t} = \frac{\partial u}{\partial t} - g' = \frac{\partial^2 u}{\partial x^2} - g'$$

$$\boxed{\begin{cases} \dfrac{\partial v}{\partial t}(x,t) = \dfrac{\partial^2 v}{\partial x^2}(x,t) + G(t), & x \in \mathbb{R},\ t > 0 \\ v(x,0) = 0, & x \in \mathbb{R}. \end{cases}}$$

$$v(x,t) = \int_0^t \int_{\mathbb{R}^n} K(x-y, t-s)\, G(s)\, ds\, dy$$

$$u(x,t) = \int_0^t \int_{\mathbb{R}^n} K(x-y, t-s)\, G(s)\, ds\, dy + \underbrace{g(t)}_{\int_0^t \int_{\mathbb{R}^n} K(x-y,\, t-s)\, g'(s)\, ds\, dy}$$

$$\boxed{g(t) = \int_0^t g'(s)\, ds}$$

$$\boxed{\int_{\mathbb{R}^n} K(x-y, t-s)\, dy = 1} \qquad \left(\int_{\mathbb{R}^n} K(x,t)\, dx = 1 \right)$$

PROVA! 6ª

REGRAS NO SITE

1) CONVENCIONAL

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~~POUCO 7~~

2) 7-8 . DEIXAREI PROVA ÀS 7.

3) A MÃO FOTO / ESCANEADA

4) SEM ATRASO

5) E-MAIL PPLOPES@[illegible]