

# SEM5950 - SEM0586

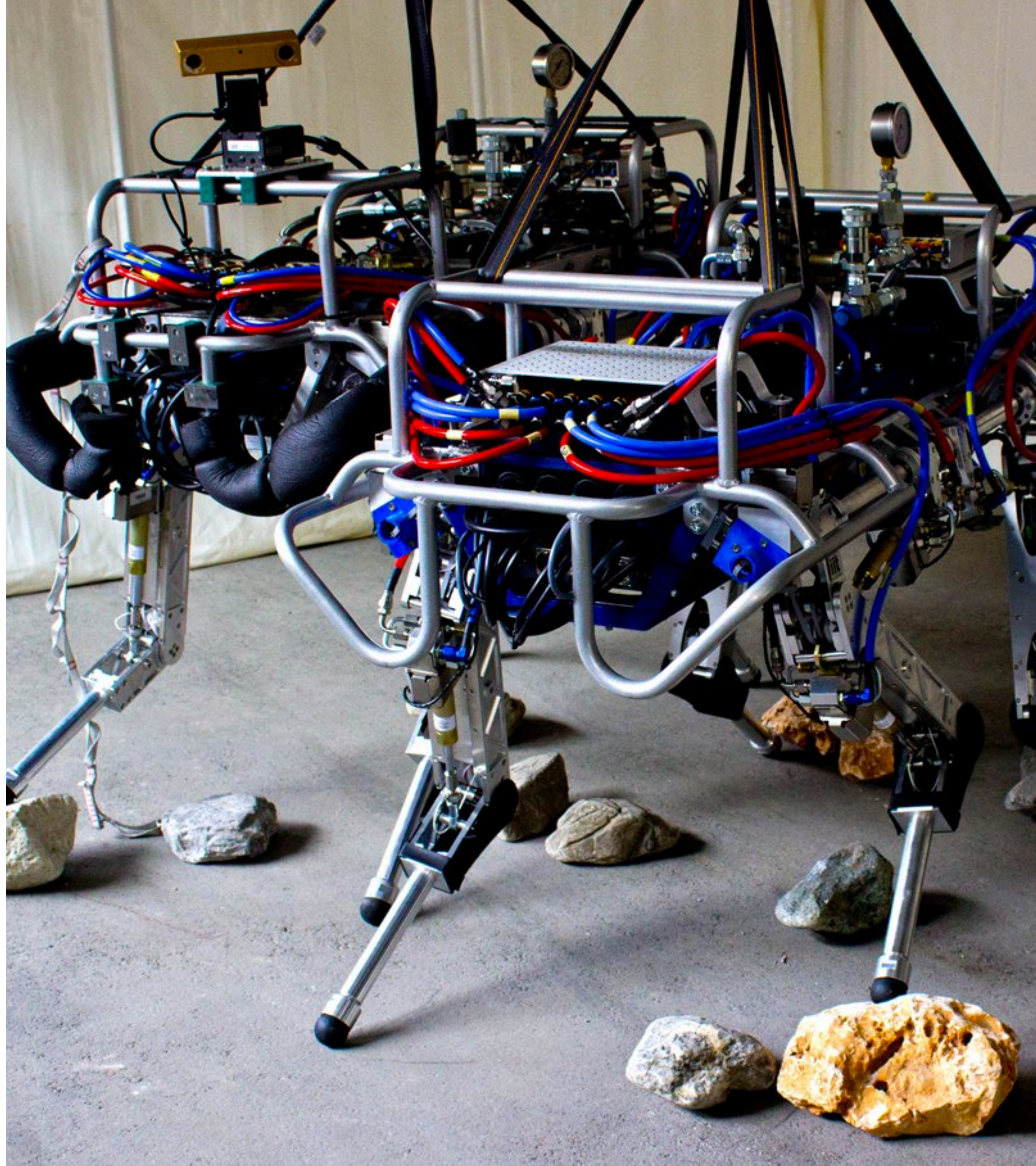
## Legged Robots

Aula #10: Estabilidade e passividade  
de controladores de impedância

**Prof. Dr. Thiago Boaventura**  
[tboaventura@usp.br](mailto:tboaventura@usp.br)



São Carlos, 09/11/20

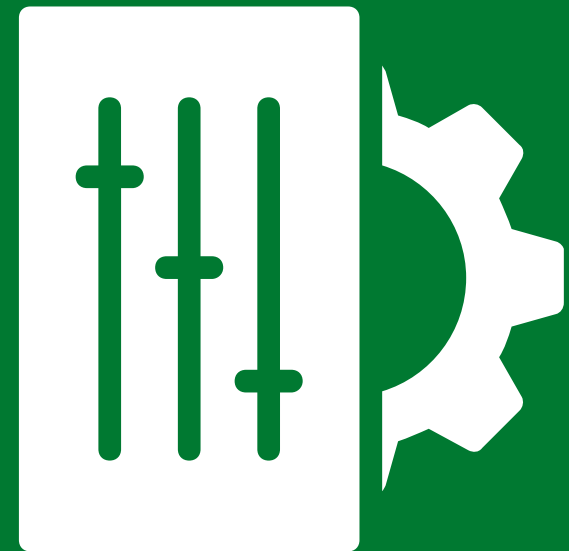


# Conteúdo



- Controlador de impedância
- Controlador de força

Revisão



- Estabilidade da interação
- Passividade
- Z-width

Estabilidade



- Take-home messages
- Bibliografia

Conclusão

# Conteúdo

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- Controlador de impedância
- Controlador de força

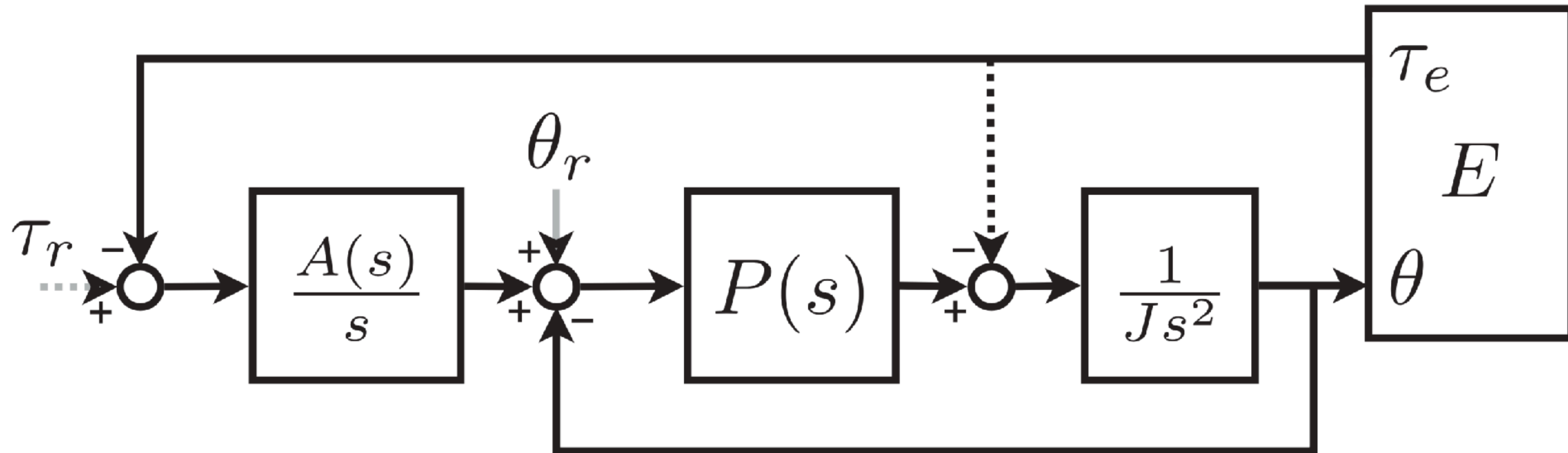
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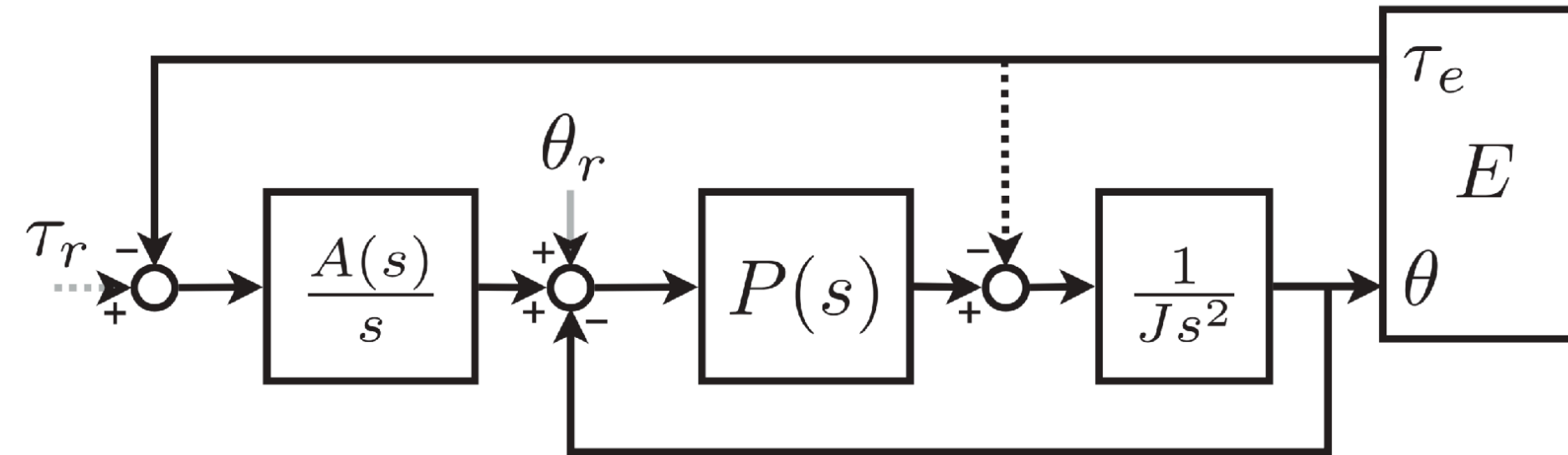
# Controlador de admitância

**“Controlador de impedância baseado em posição”**



# Controlador de admitância

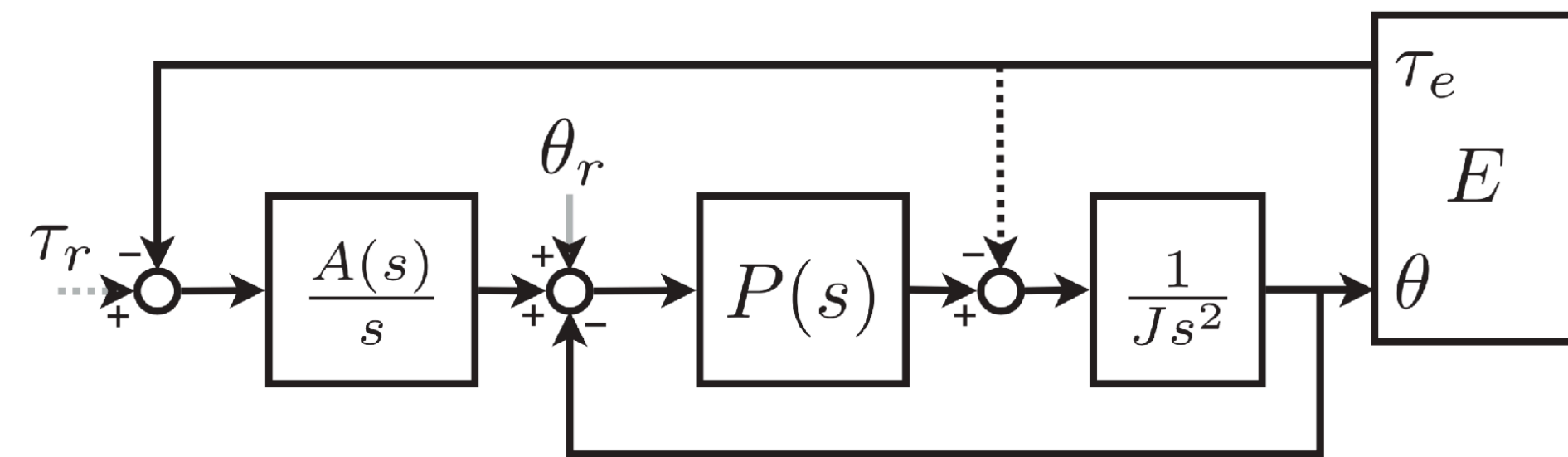
“Controlador de impedância baseado em posição”



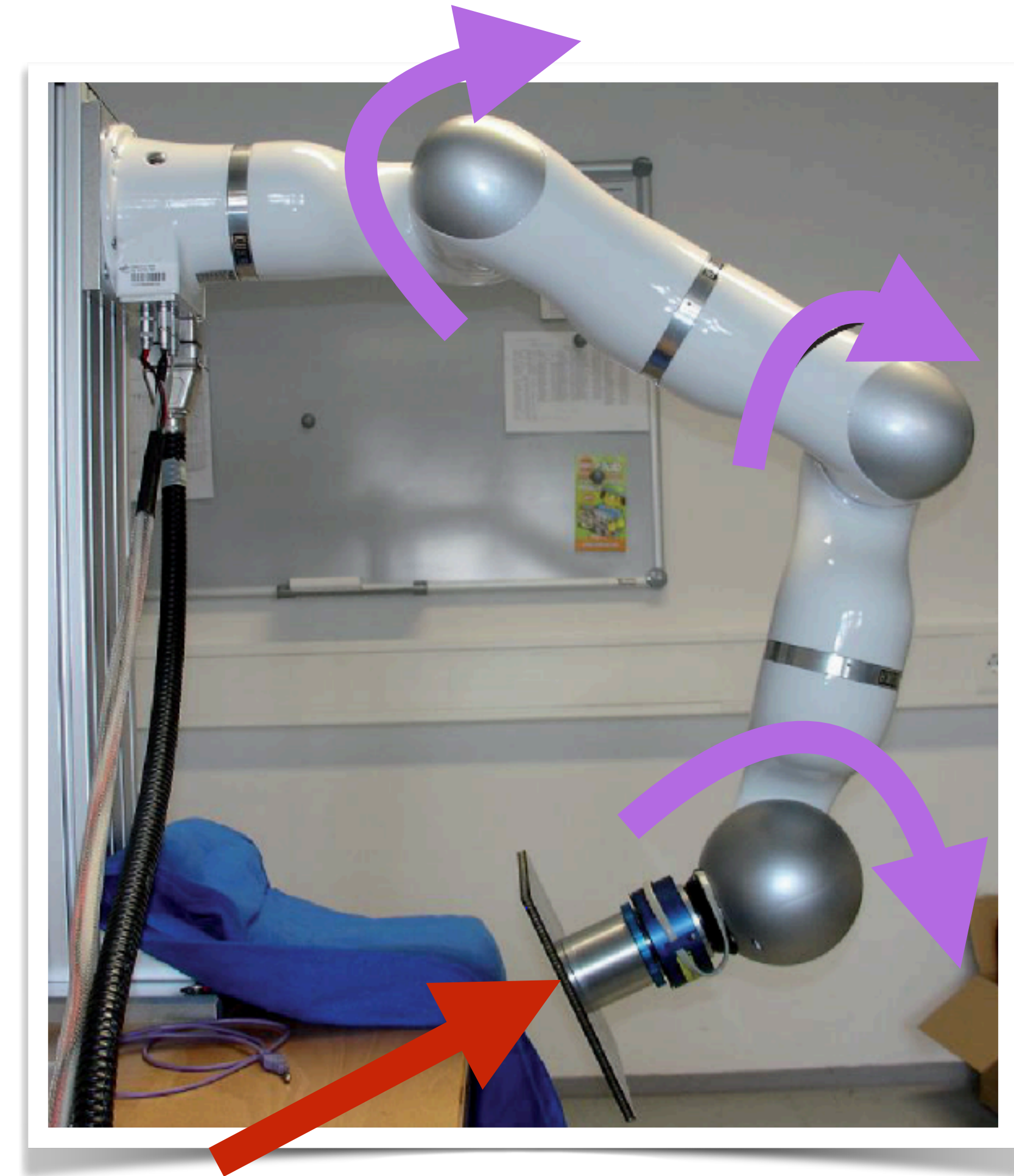
	Impedance	Admittance
Transfer function definition	$z(s) = \frac{f(s)}{v(s)}$	$a(s) = \frac{1}{z(s)} = \frac{v(s)}{f(s)}$
Spring	$k/s$	$s/k$
Mass	$ms$	$1/ms$
Damper	$b$	$1/b$
Spring-mass-damper	$ms + b + k/s$	$\frac{1}{ms + b + k/s}$

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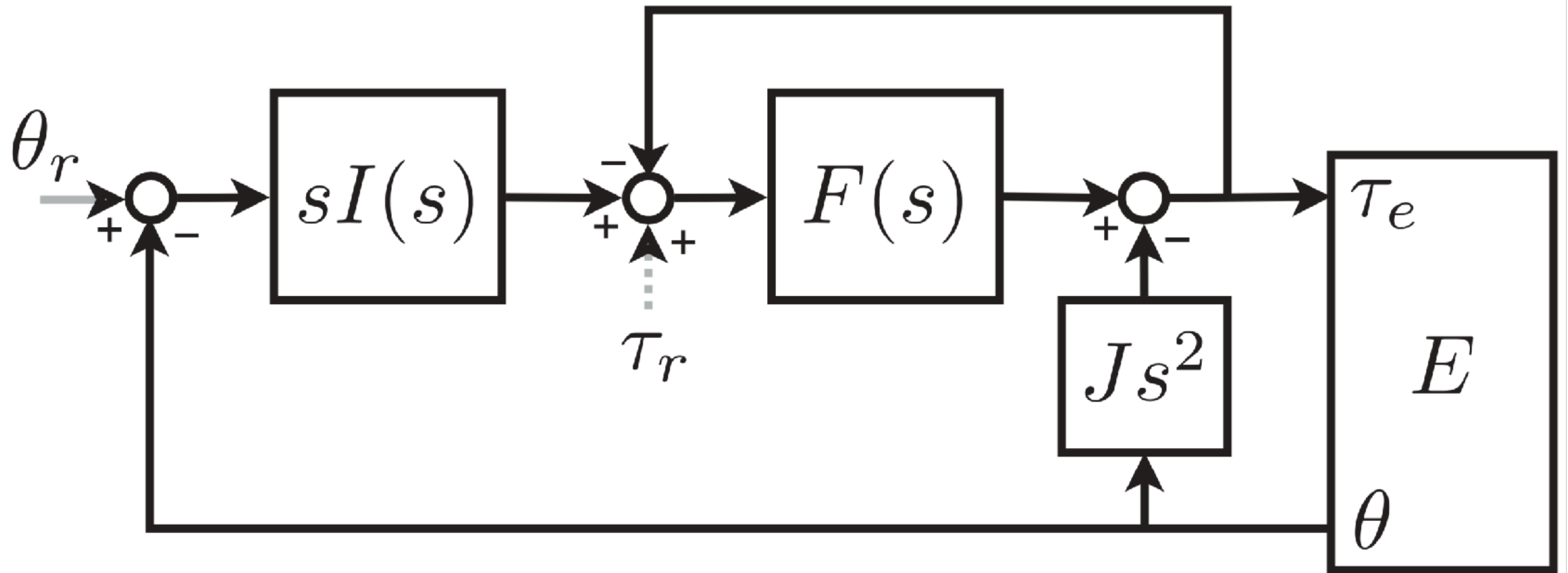


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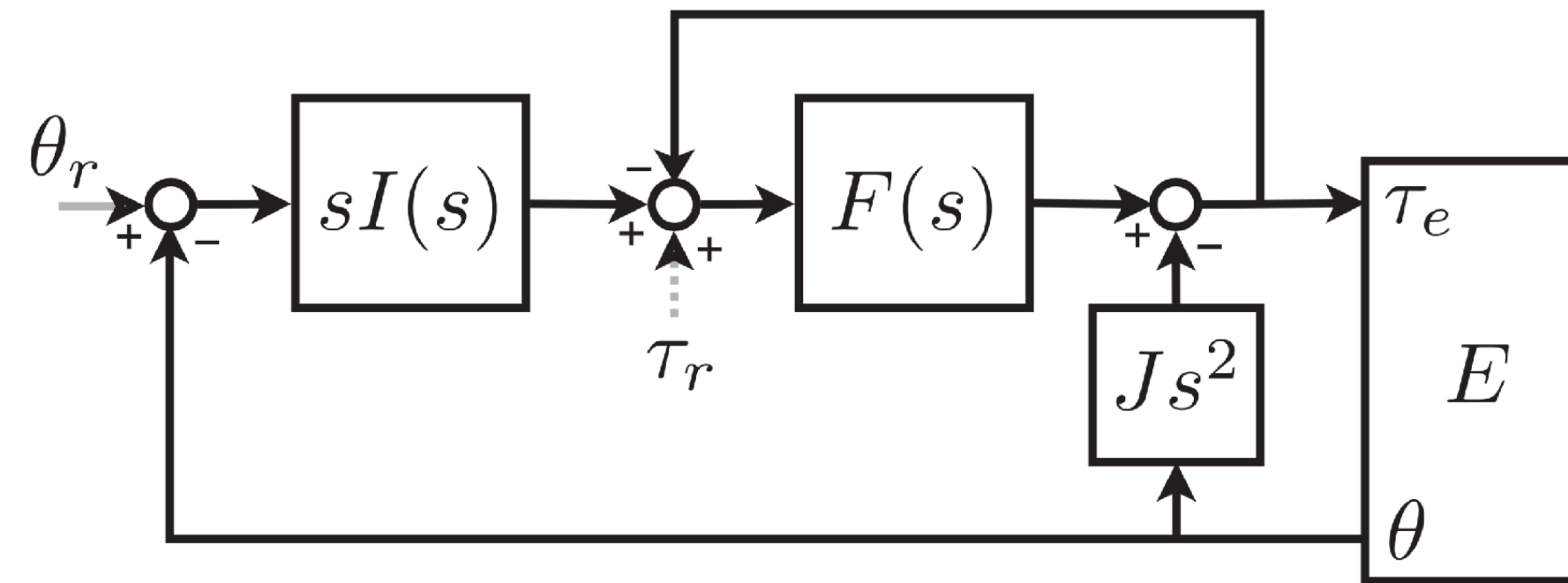
# Controlador de impedância

“Controlador de impedância baseado em força”



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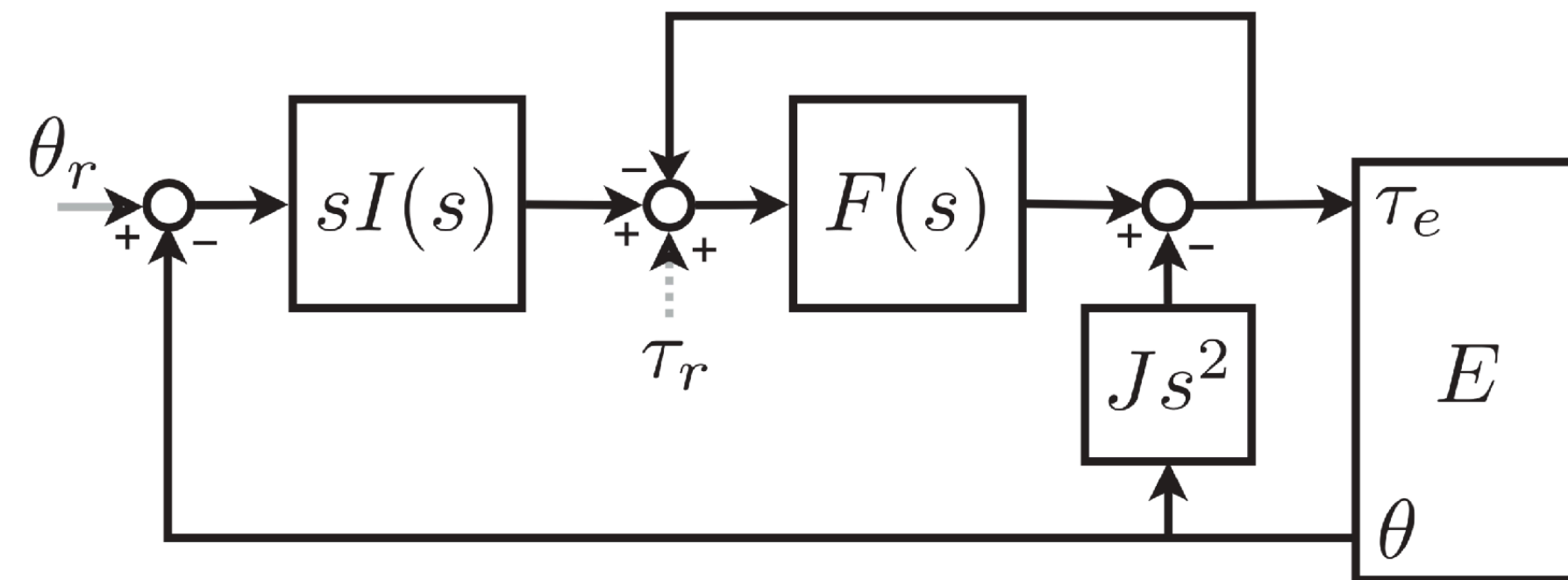


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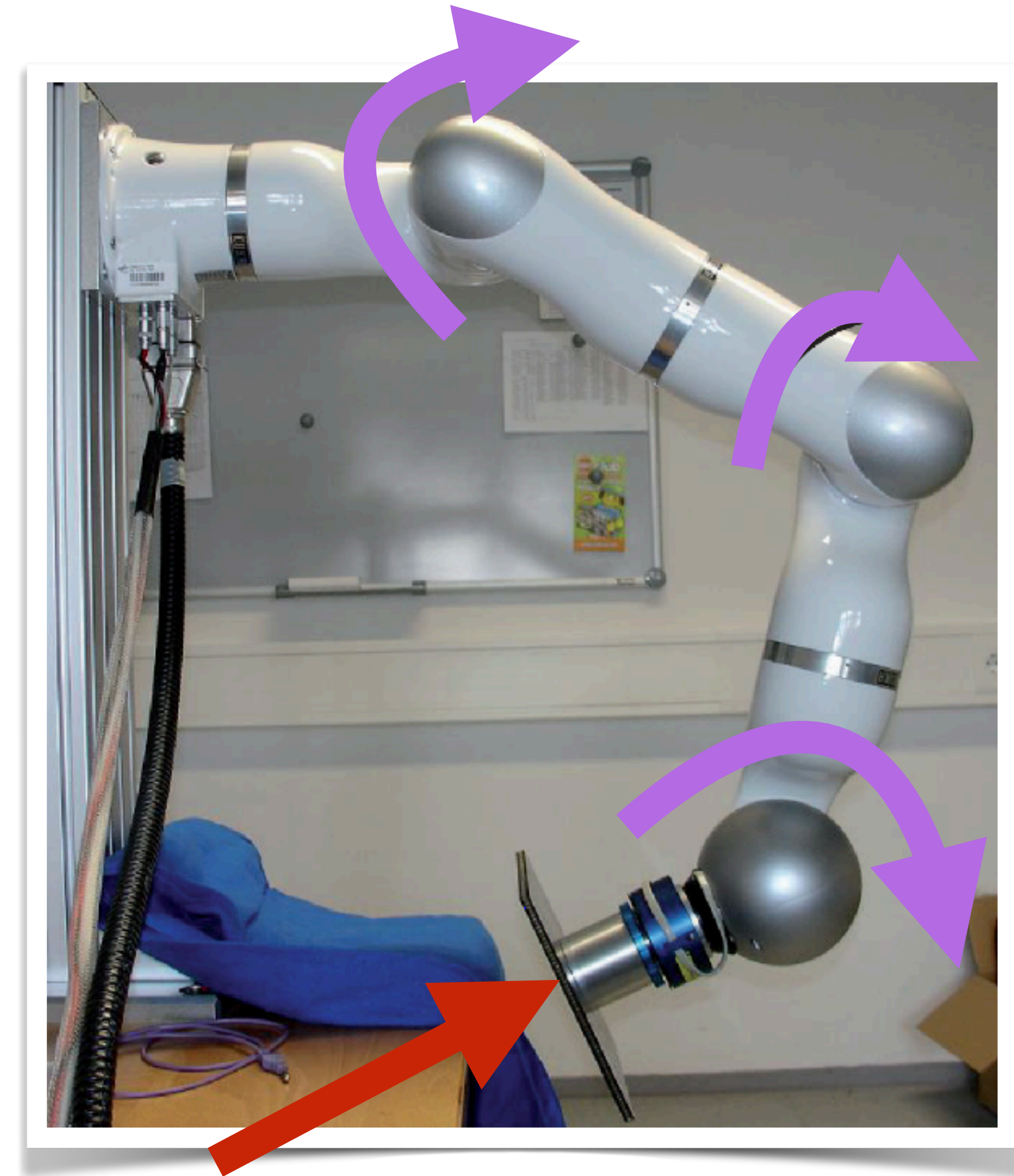


# Controlador de impedância

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# Controlador de impedância

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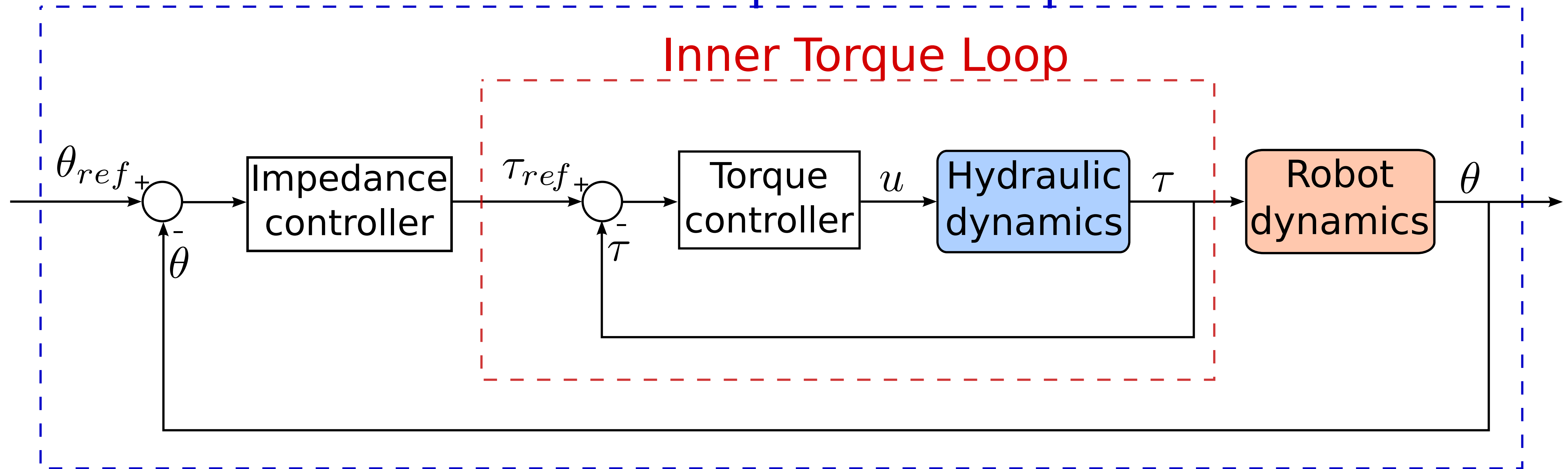
$$\begin{aligned} \mathbf{Tact} = & I(\theta)\mathbf{J}^{-1}(\theta)\mathbf{M}^{-1}\mathbf{K}[\mathbf{X}_0 - \mathbf{L}(\theta)] + S(\theta) \text{ (position terms)} \\ & + I(\theta)\mathbf{J}^{-1}(\theta)\mathbf{M}^{-1}\mathbf{B}[\mathbf{V}_0 - \mathbf{J}(\theta)\boldsymbol{\omega}] + V(\boldsymbol{\omega}) \text{ (velocity terms)} \\ & + I(\theta)\mathbf{J}^{-1}(\theta)\mathbf{M}^{-1}\mathbf{Fint} - \mathbf{J}^T(\theta)\mathbf{Fint} \text{ (force terms)} \\ & - I(\theta)\mathbf{J}^{-1}(\theta)\mathbf{G}(\theta, \boldsymbol{\omega}) + C(\theta, \boldsymbol{\omega}) \text{ (inertial coupling terms)} \end{aligned}$$

**Hogan**, N. (1985). Impedance control: An approach to manipulation: Part II— Implementation. *Journal of dynamic systems, measurement, and control*, 107(1), 8-16.

# Arquitetura clássica

## Outer Impedance Loop

## Inner Torque Loop

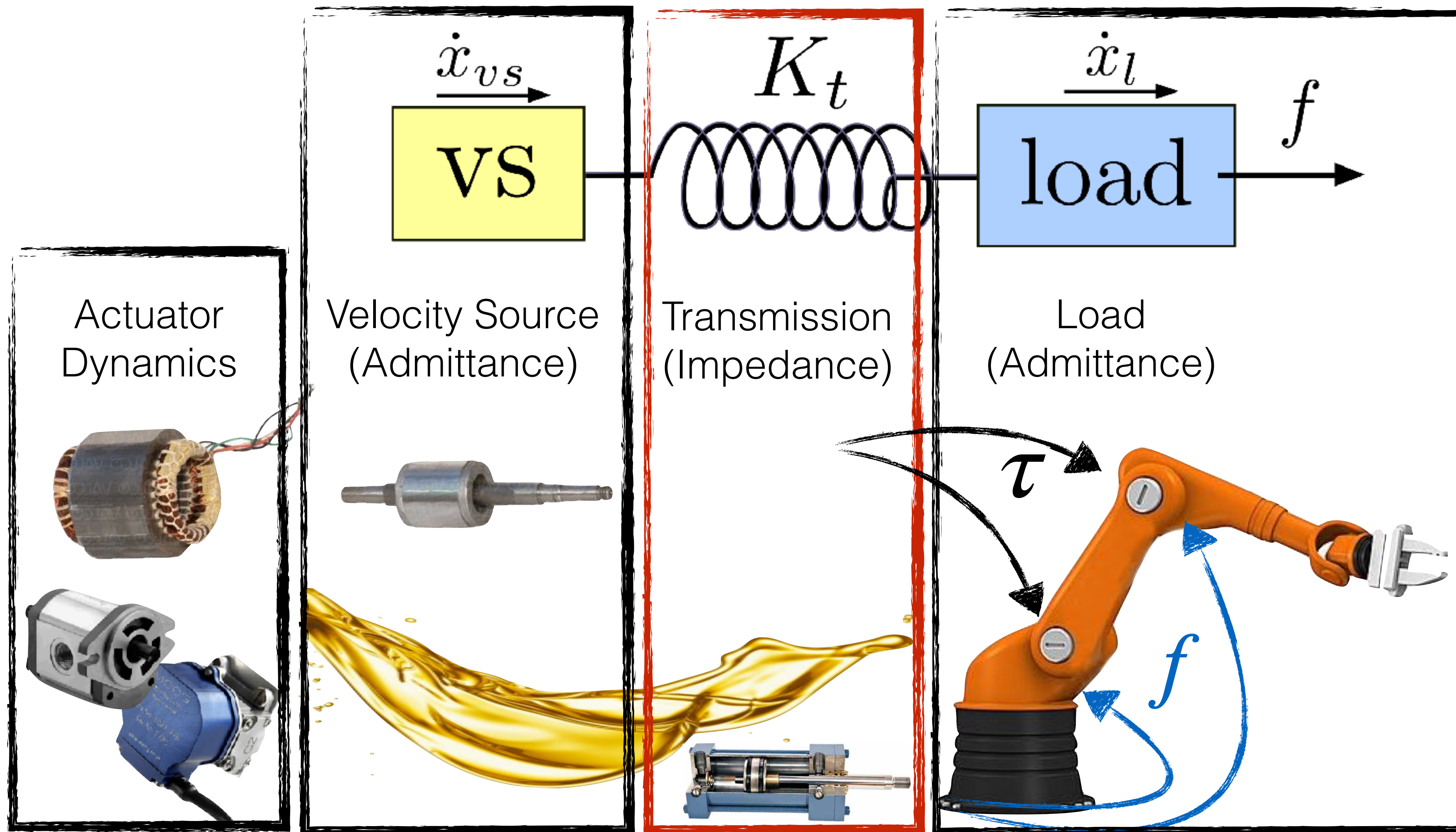


# Cadeia de atuação

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Estabilidade

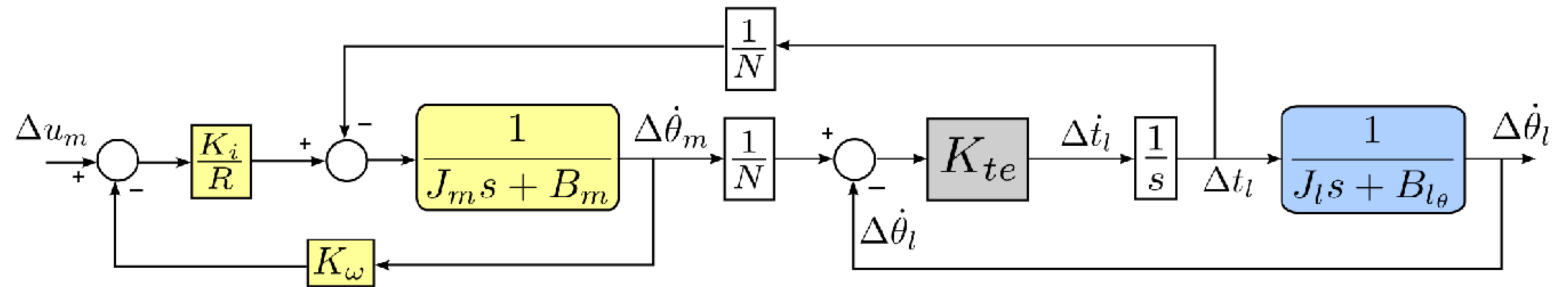
Conclusão



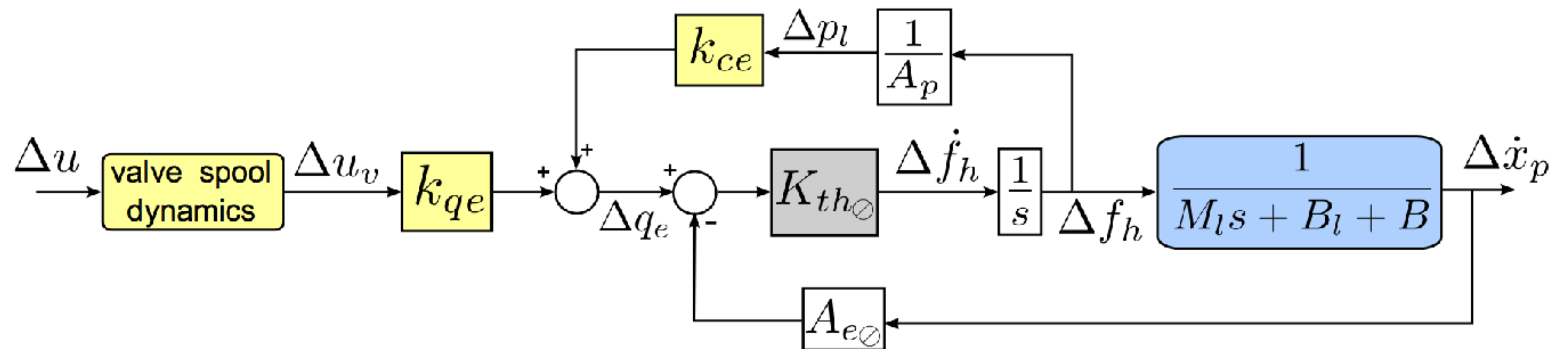
# Realimentação intrínseca da velocidade

## Open-loop torque/force dynamics

Electric actuation



Hydraulic actuation



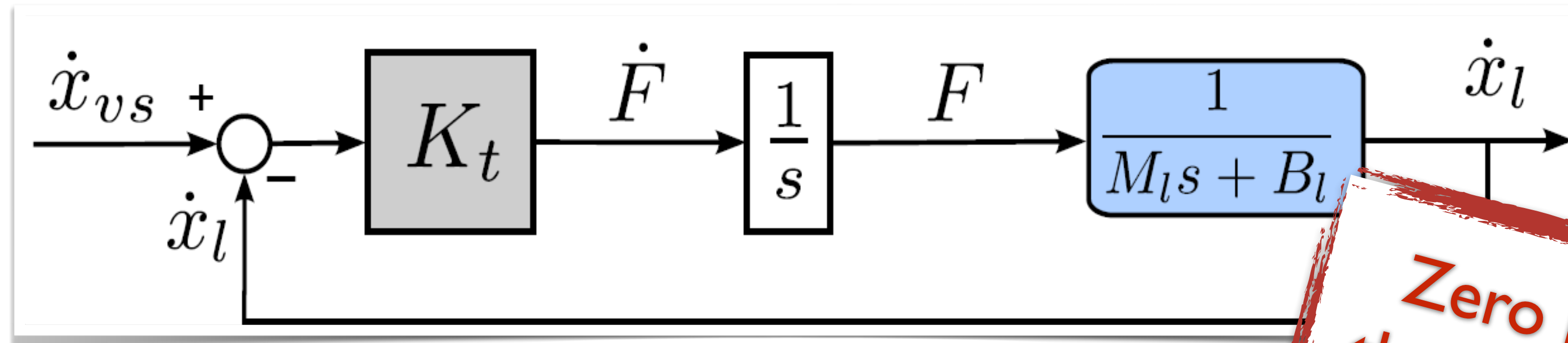
Boaventura, T., et al. On the role of load motion compensation in high-performance force control. IROS, 2012

Revisão

Estabilidade

Conclusão

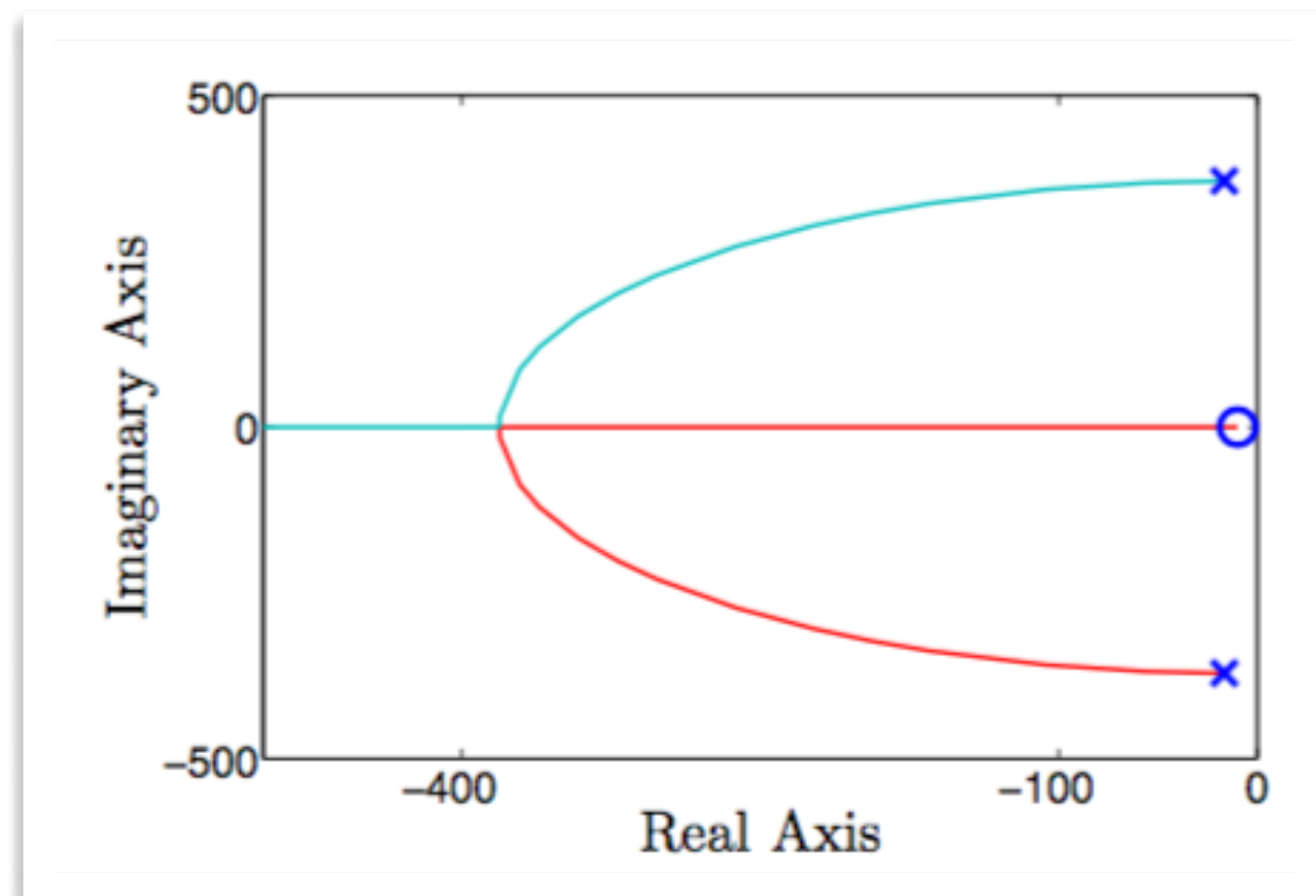
# Realimentação intrínseca da velocidade



Zero limits the response.

$$\frac{f(s)}{\dot{x}_{vs}(s)} = \frac{K_t (M_l s + B_l)}{s (M_l s + B_l) + K_t}$$

**It does not depend on the actuation**



of load motion compensation in high-performance force control. IROS, 2012

Revisão

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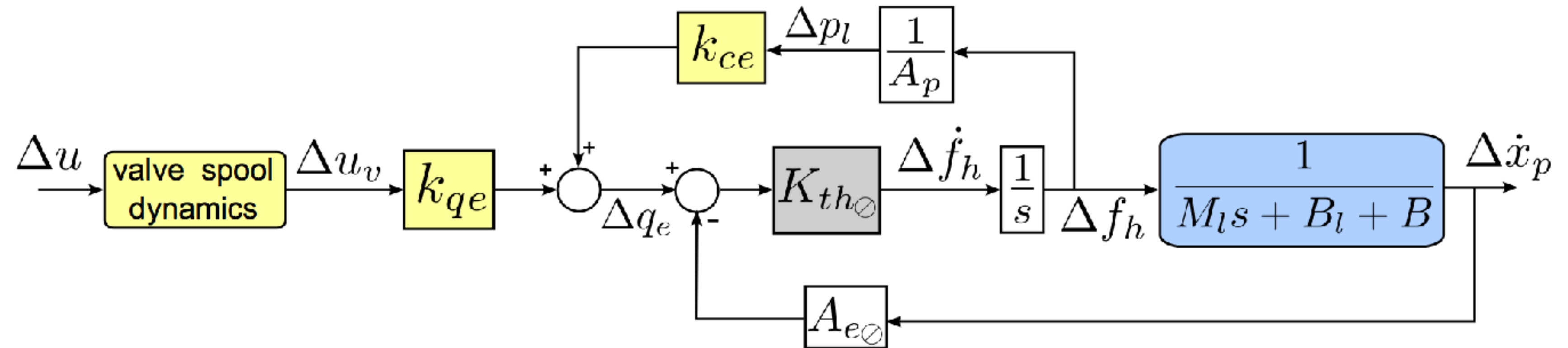
# Compensação de velocidade baseada em modelo

Revisão

Estabilidade

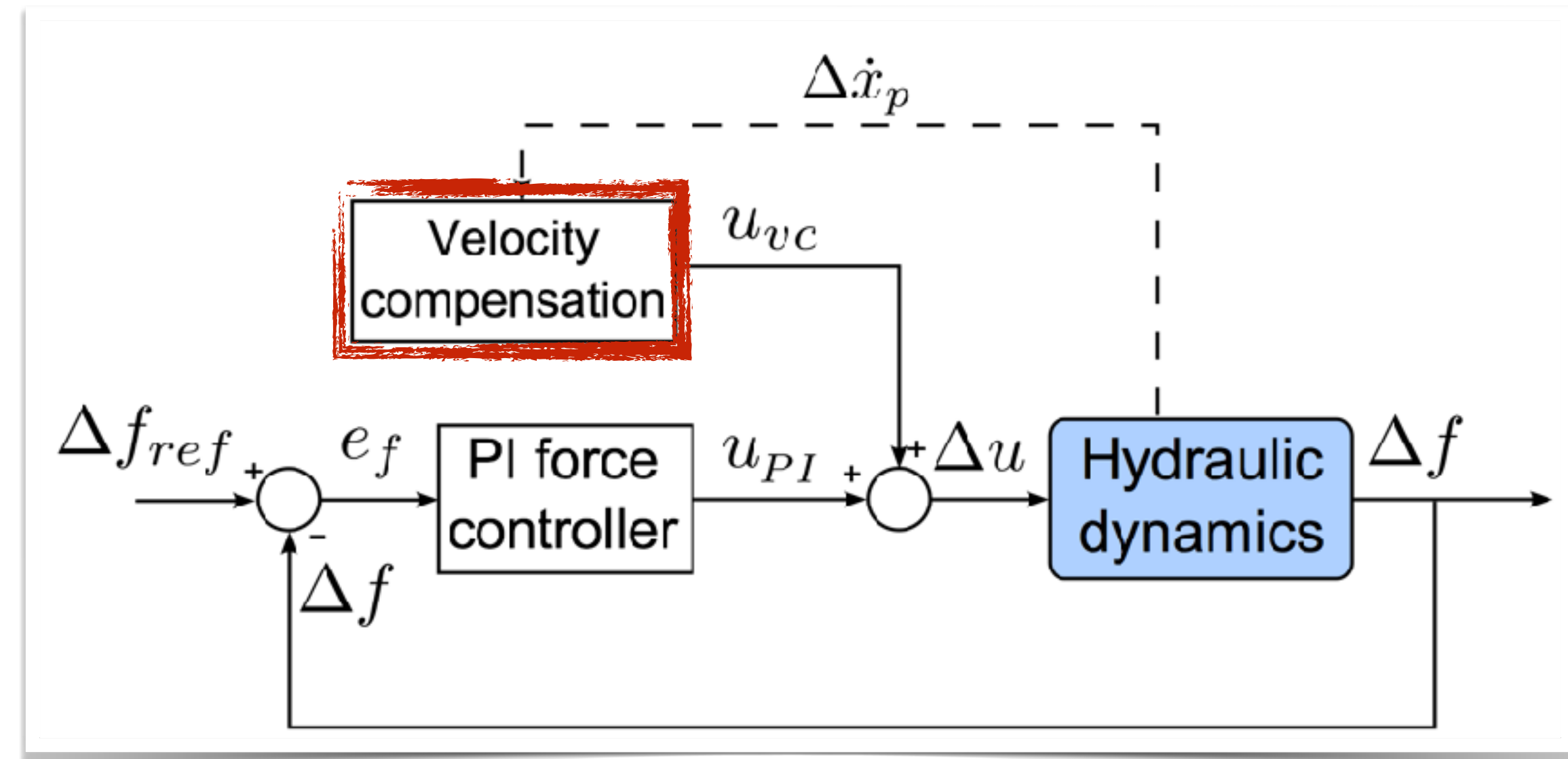
Conclusão

## Hydraulic actuation



Feedforward command:

$$u_{vc} = \frac{A_{e\phi} \Delta \dot{x}_p}{K_{qe}}$$



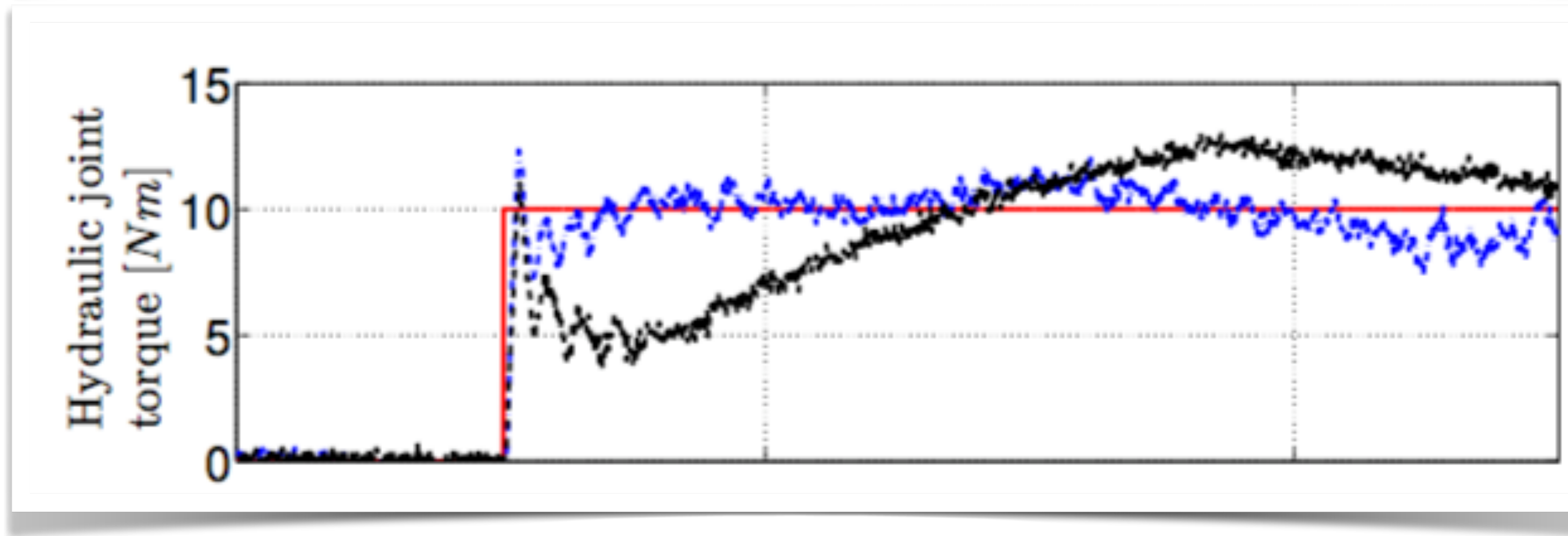
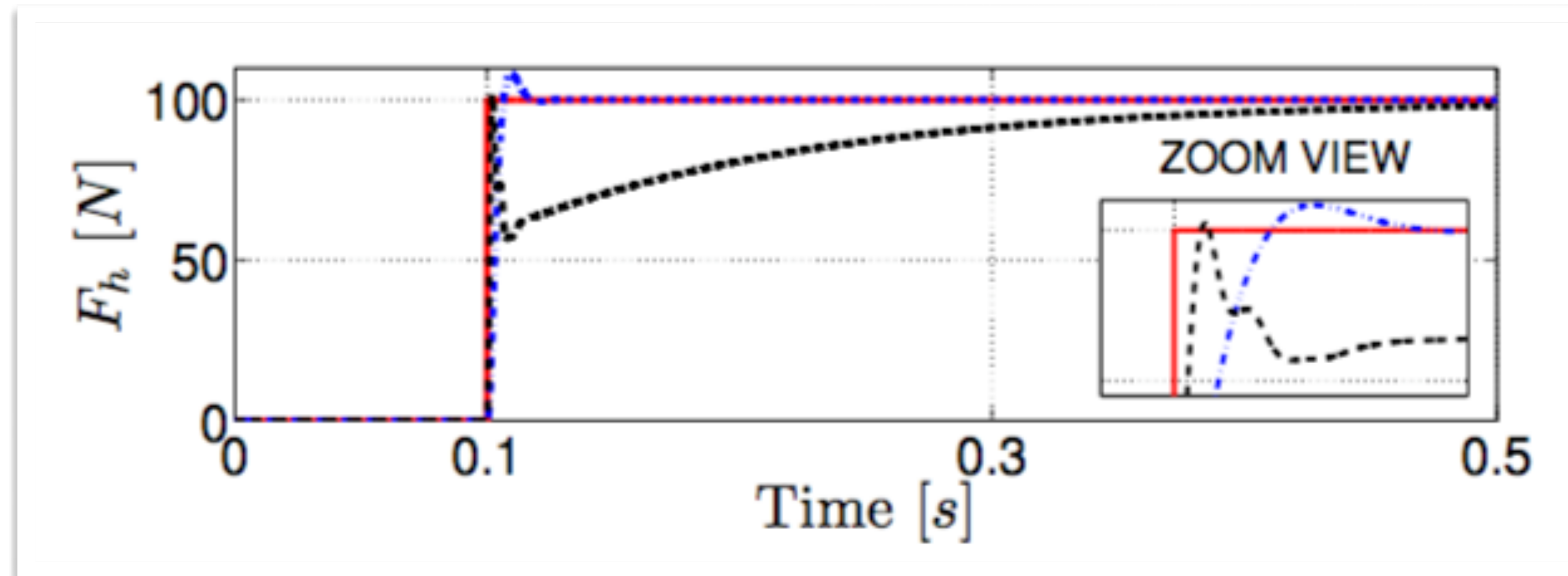
Boaventura, T., et al. On the role of load motion compensation in high-performance force control. IROS, 2012

# Compensação de velocidade baseada em modelo

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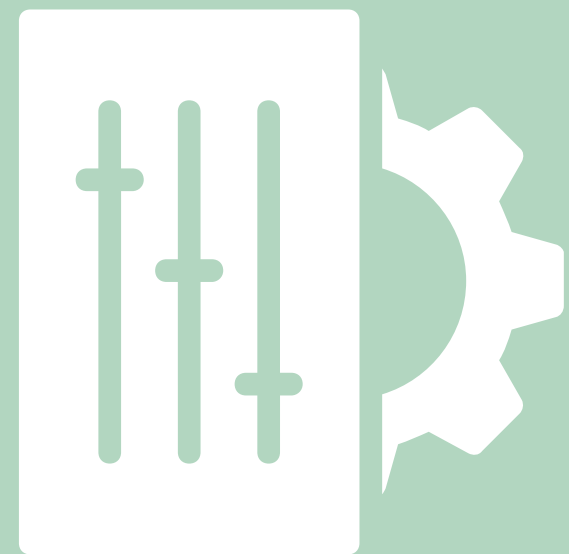


Boaventura, T., et al. On the role of load motion compensation in high-performance force control. IROS, 2012



# Conteúdo

Revisão



- Estabilidade da interação
- Passividade
- Z-width

Estabilidade

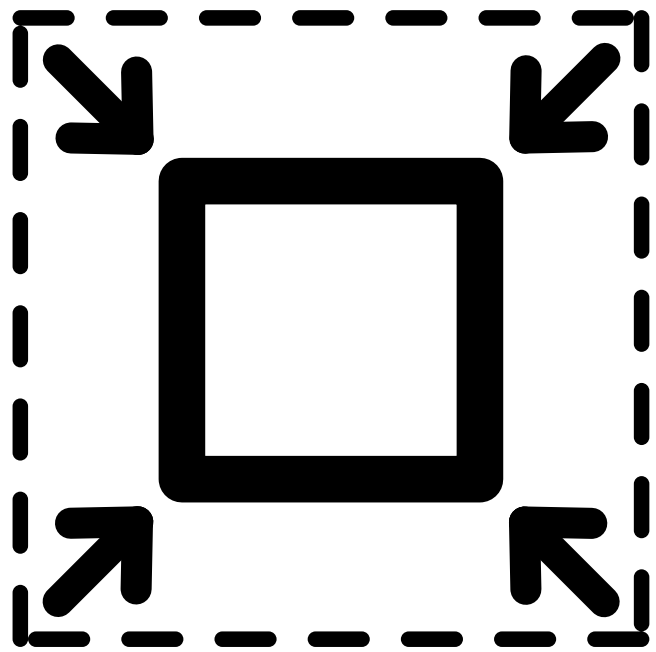
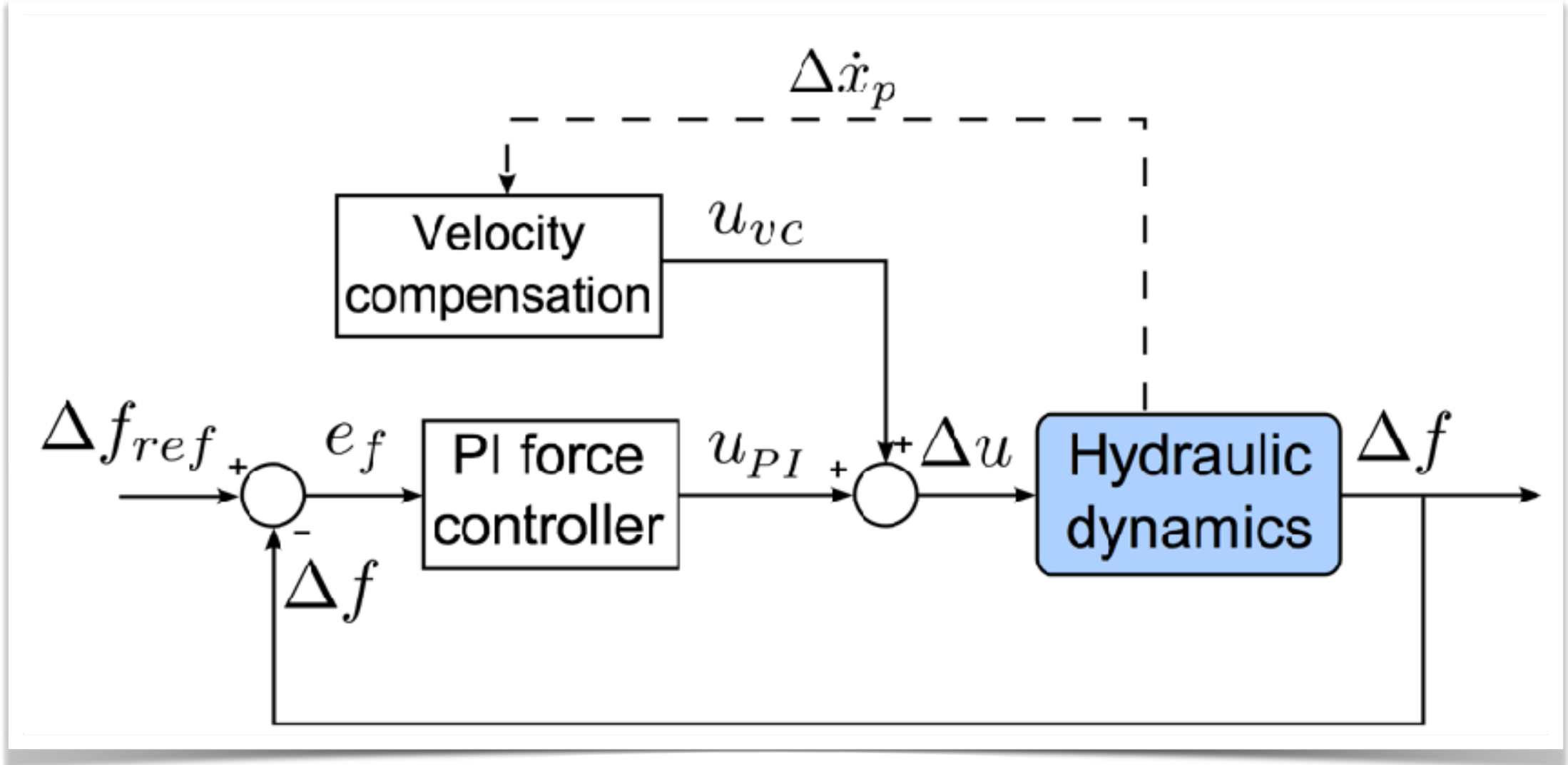
Conclusão

# Realimentação de força

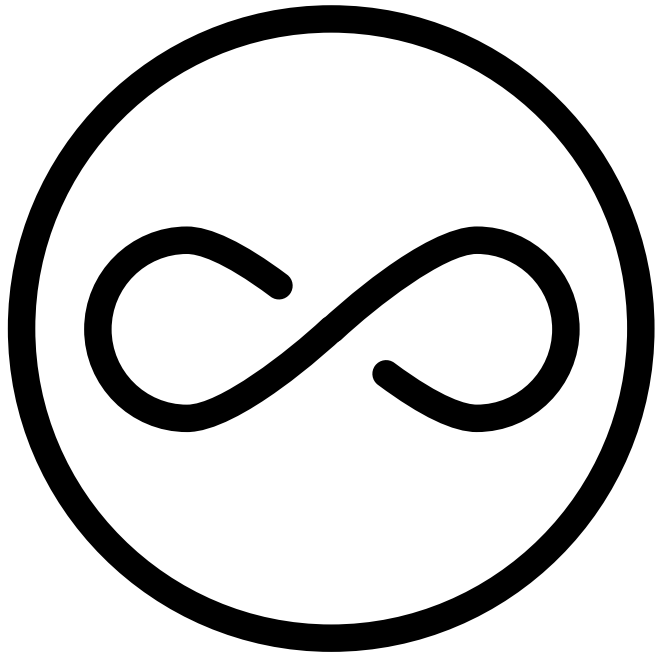
Revisão

Estabilidade

Conclusão



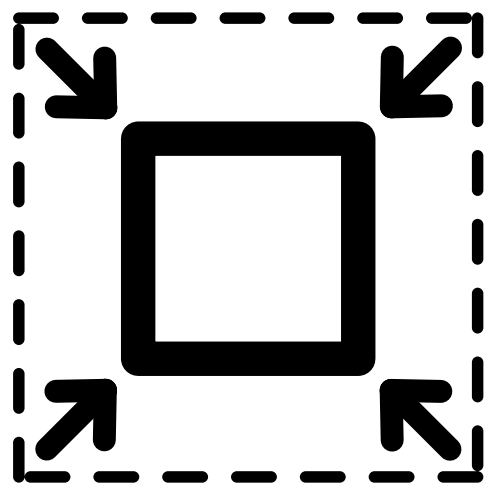
Reduzir inércia aparente



Problemas de estabilidade

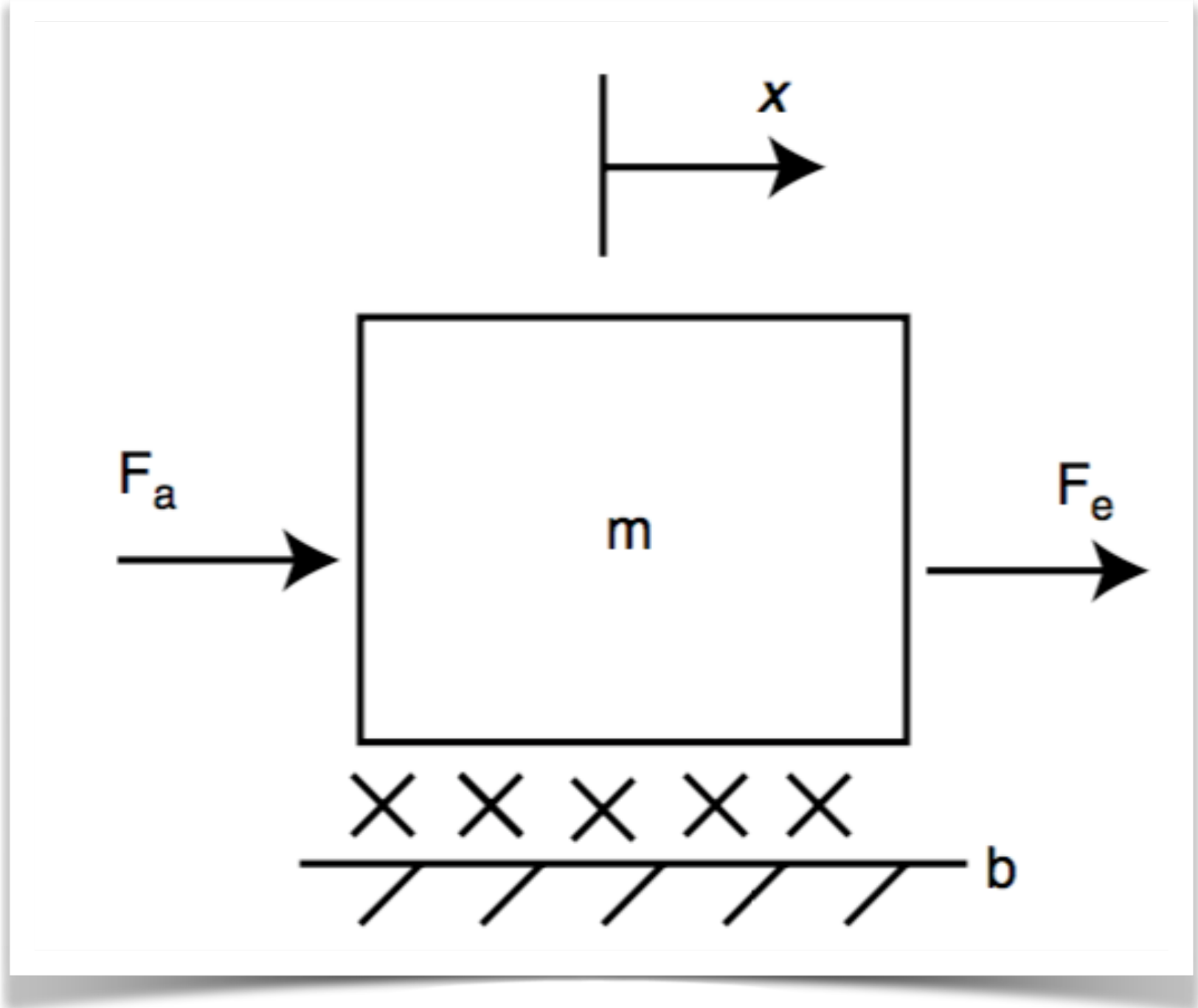
# Realimentação de força

Revisão



Reduzir inércia aparente

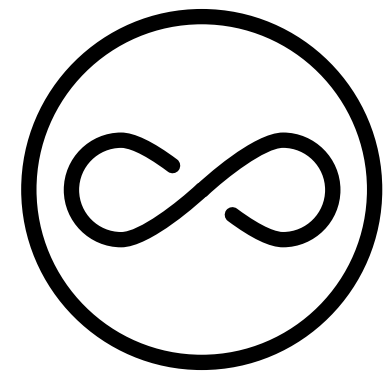
$$(ms^2 + bs)X = F_a + F_e$$



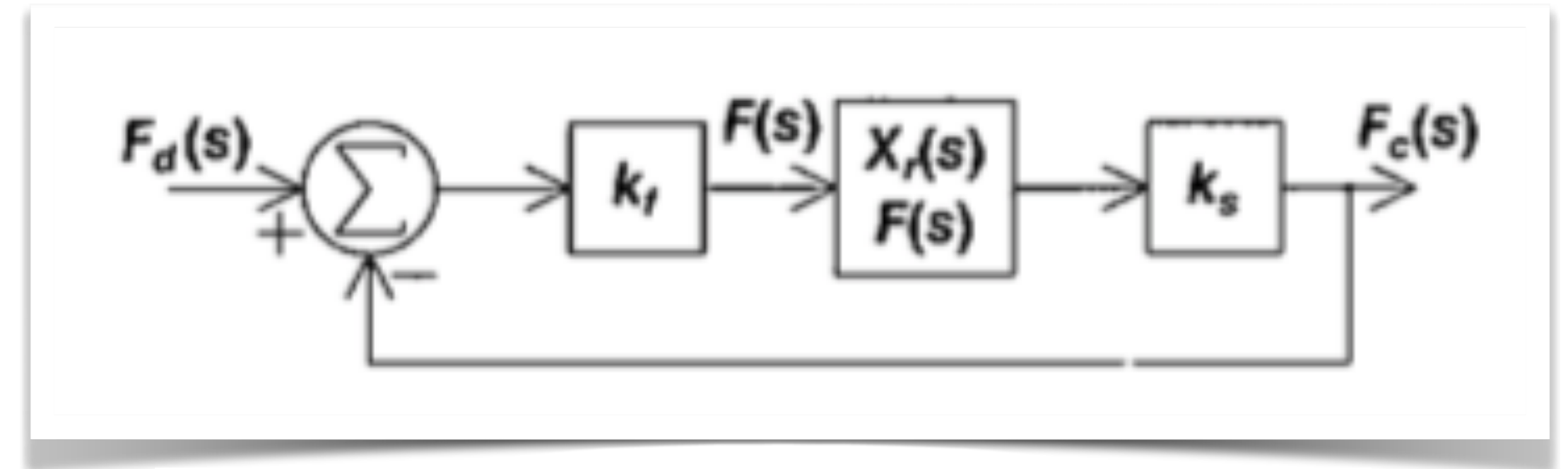
Estabilidade

Conclusão

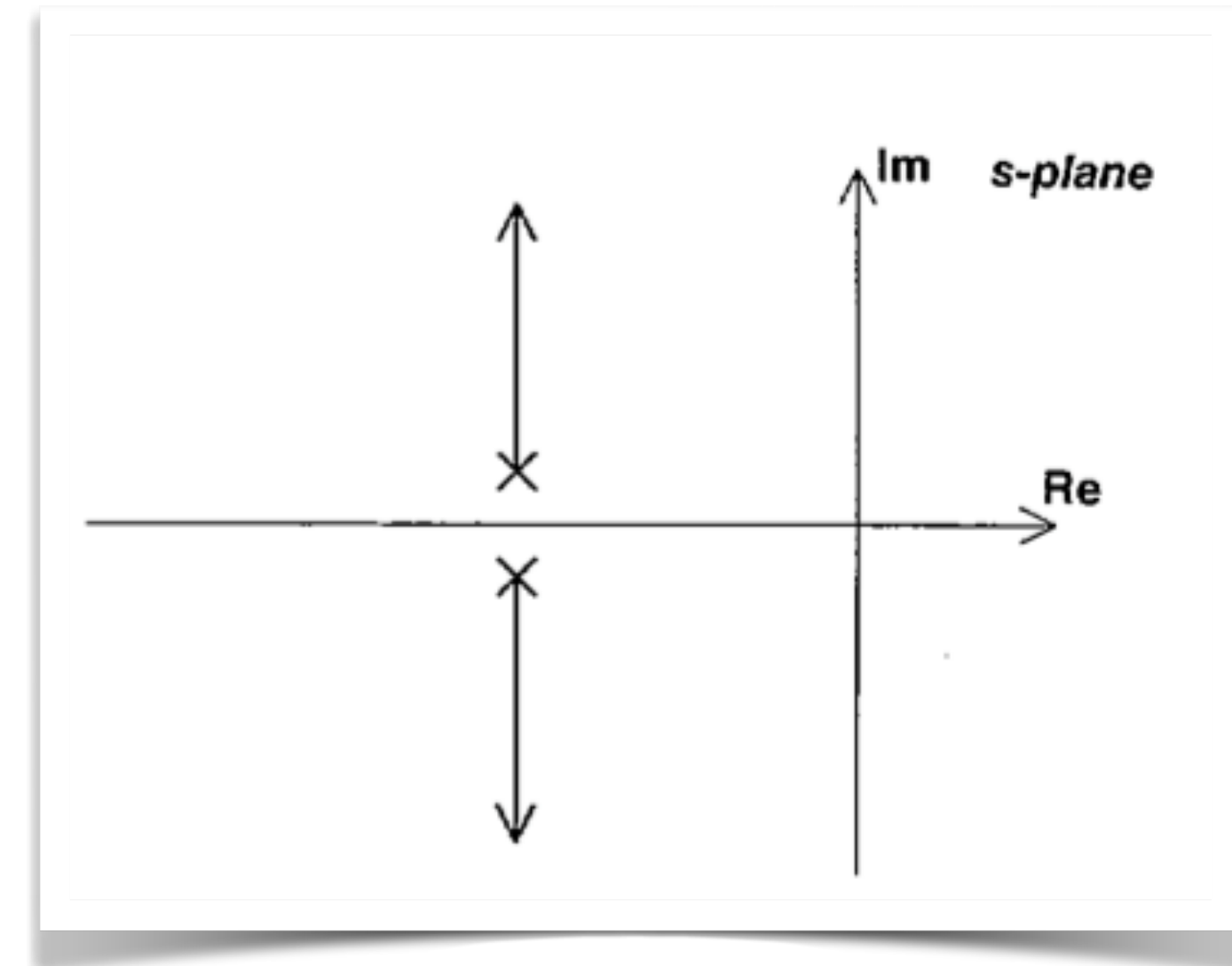
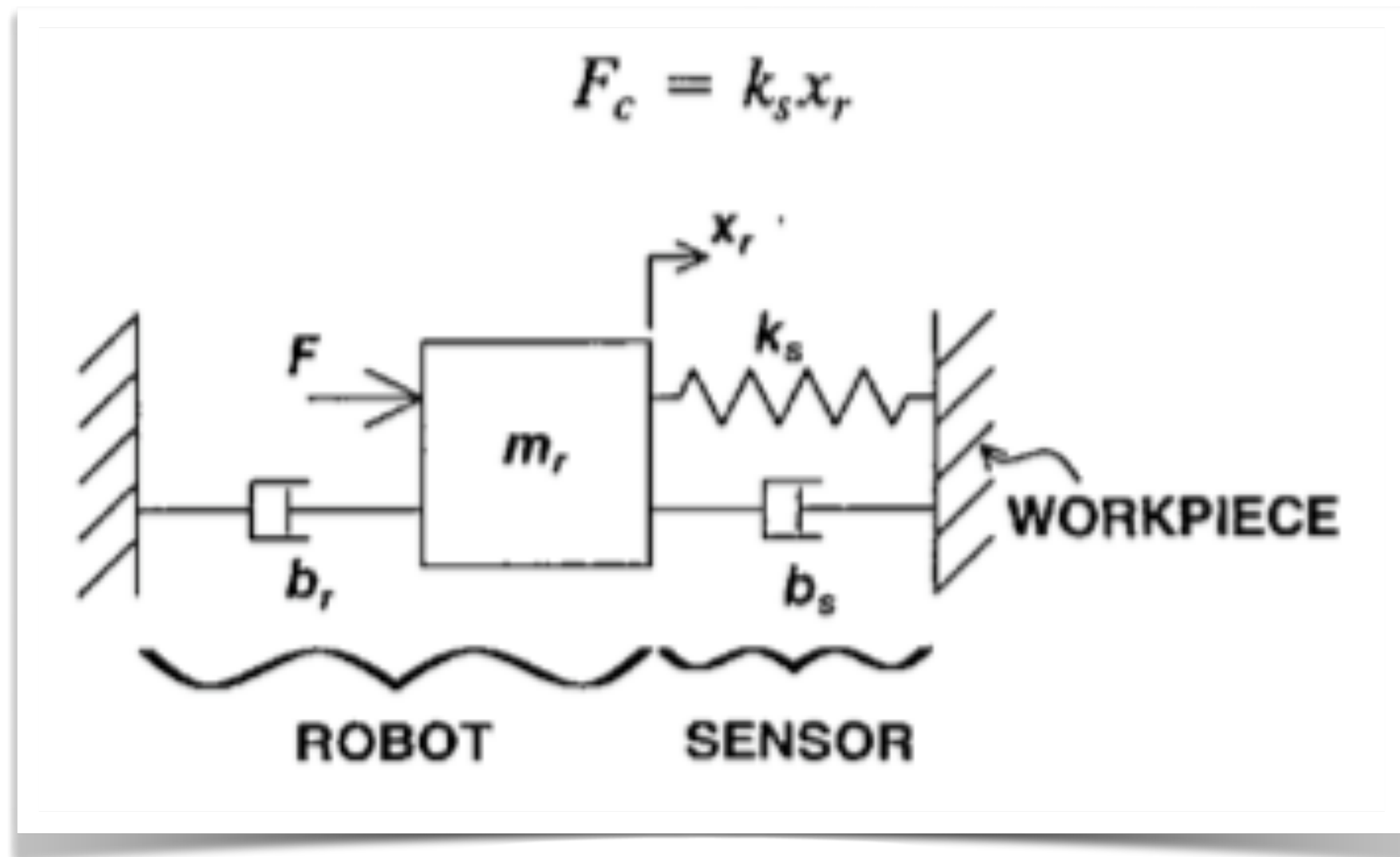
# Realimentação de força



## Problemas de estabilidade



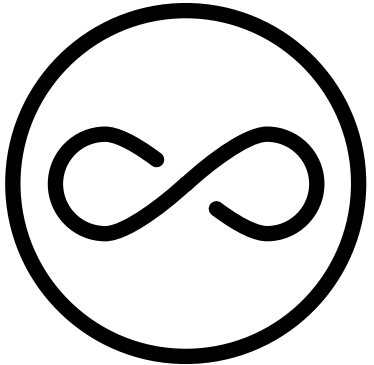
$$\frac{F_c(s)}{F_d(s)} = \frac{k_f k_s}{m_r s^2 + (b_r + b_s)s + k_s(1 + k_f)}$$



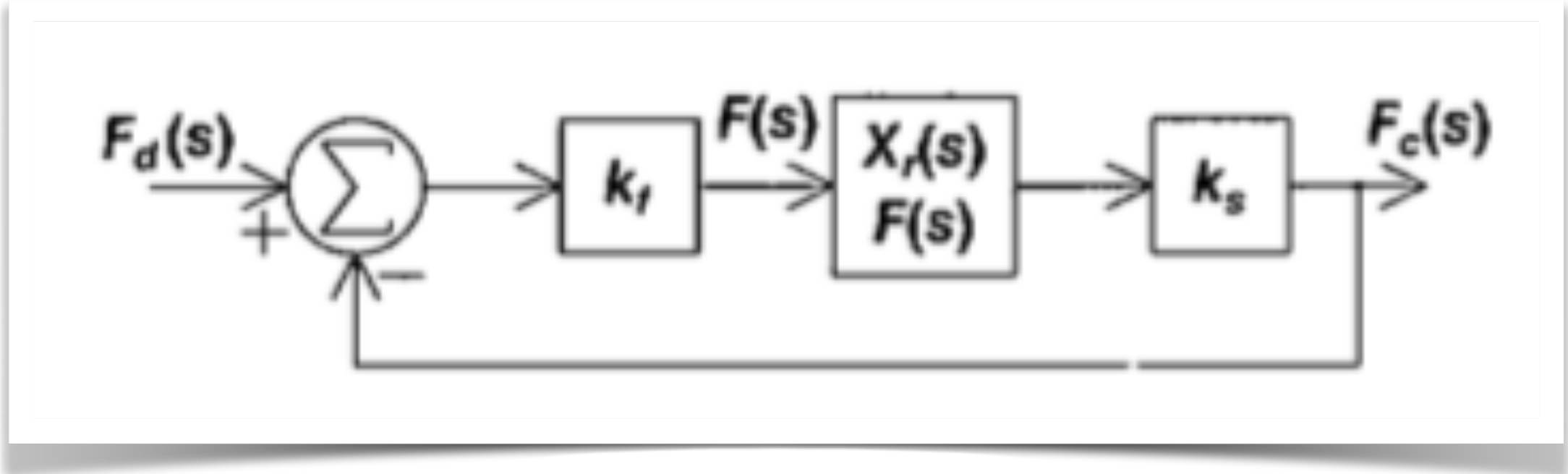
Eppinger & Seering, "Introduction to dynamic models for robot force control", 1987

# Realimentação de força

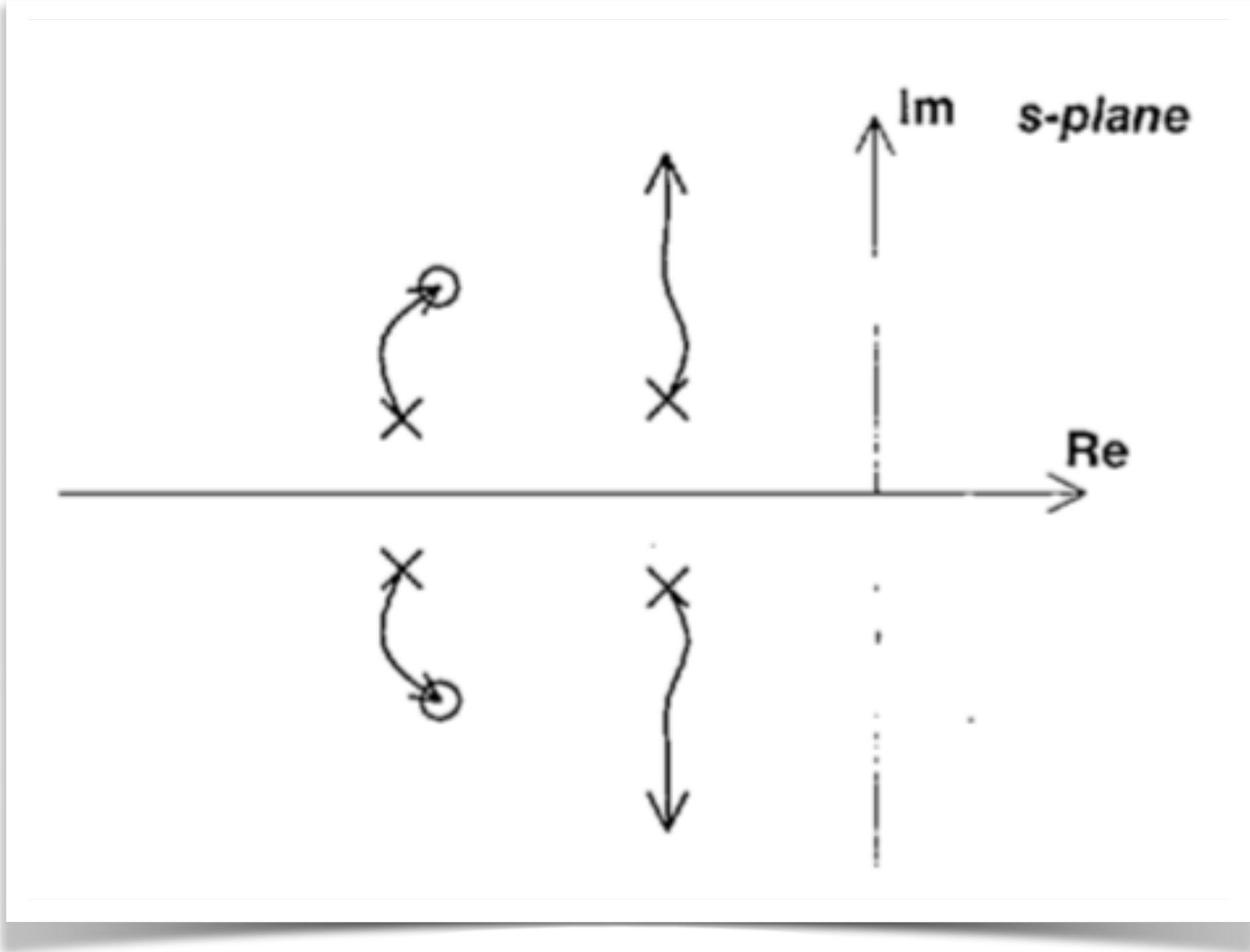
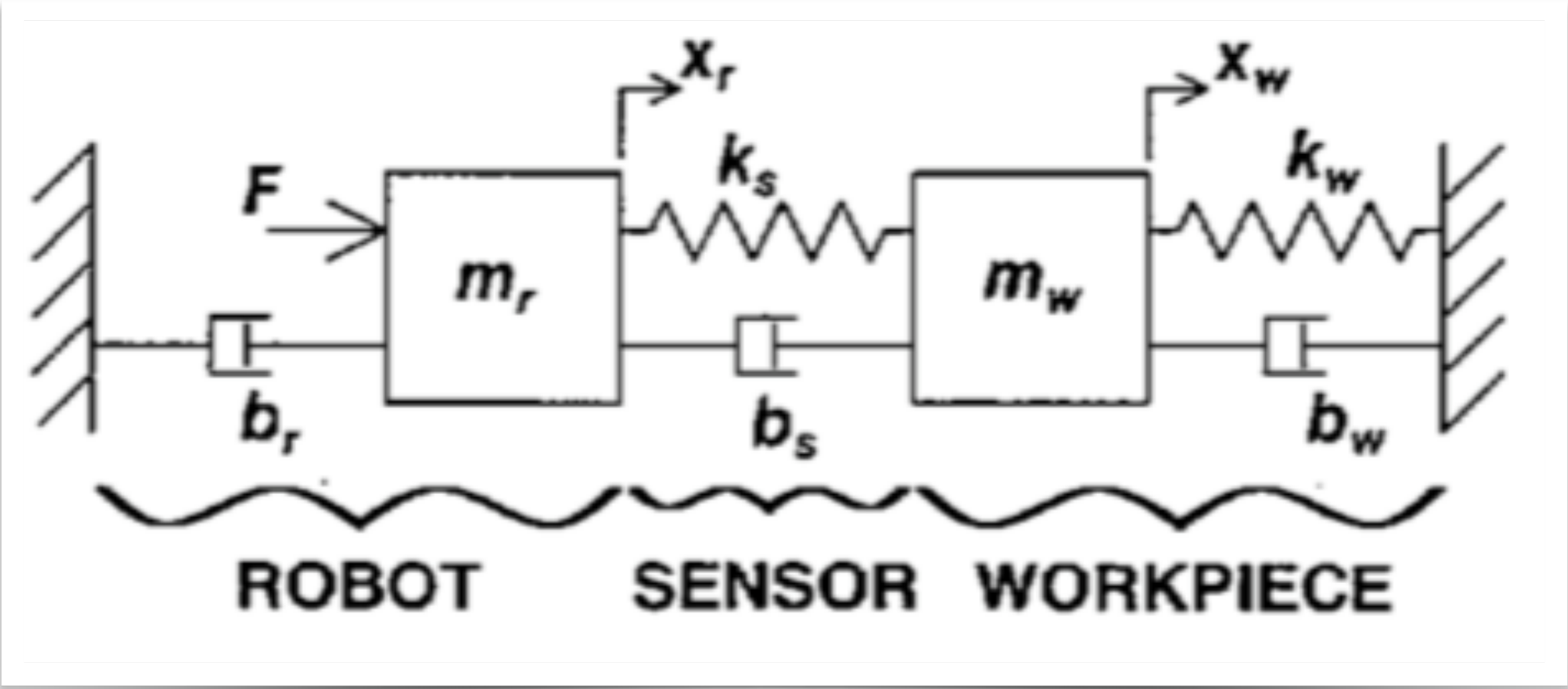
Revisão



## Problemas de estabilidade



Estabilidade

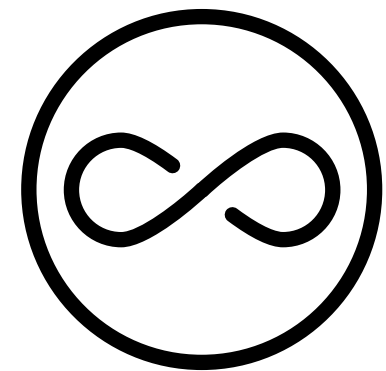


Conclusão

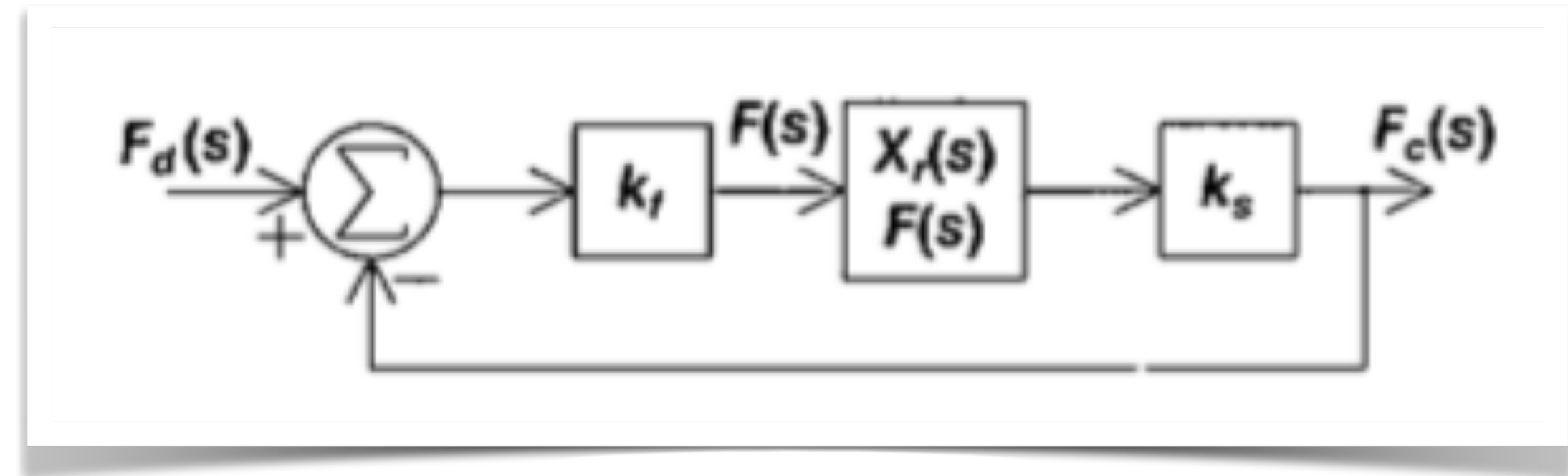
Eppinger & Seering, "Introduction to dynamic models for robot force control", 1987

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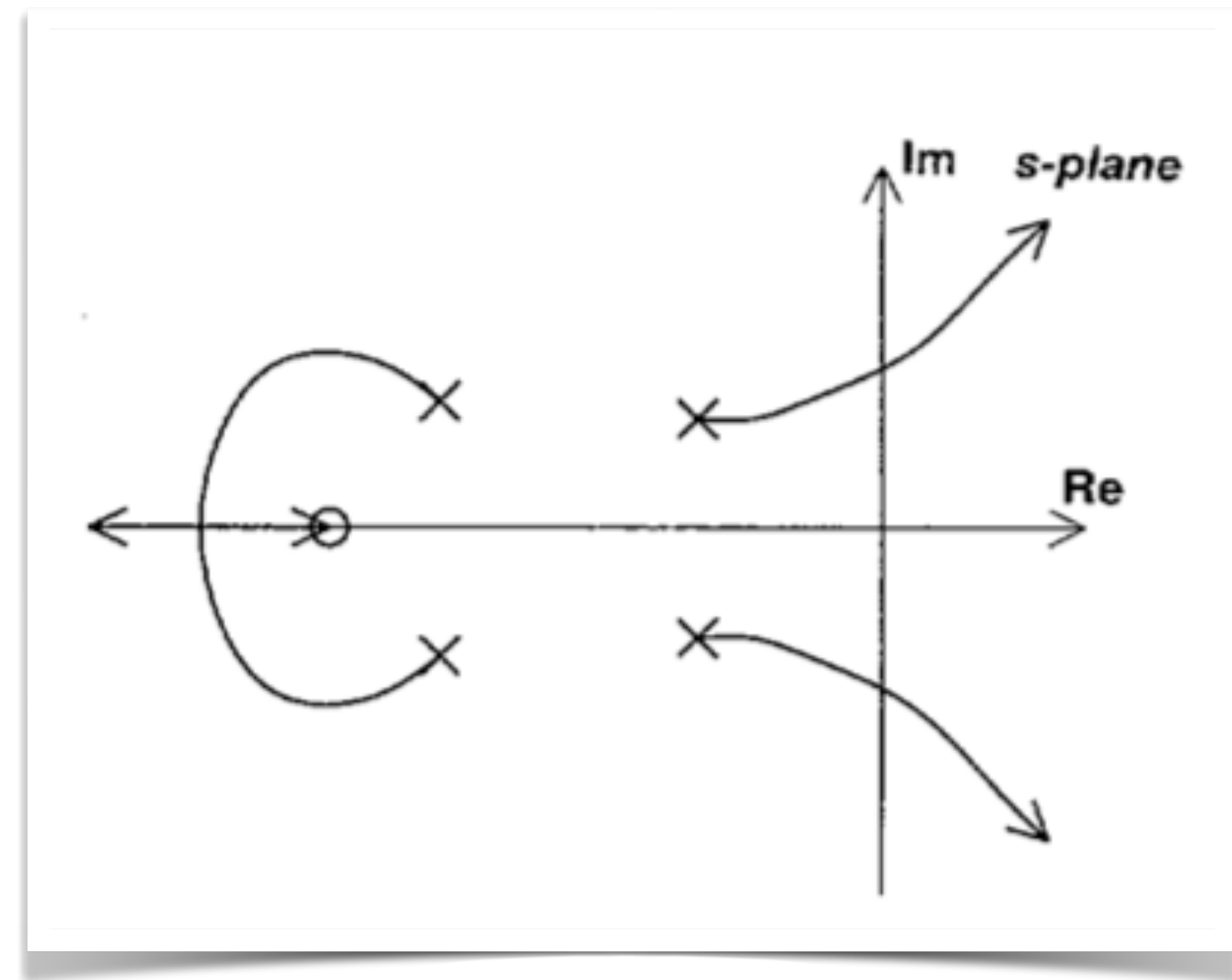
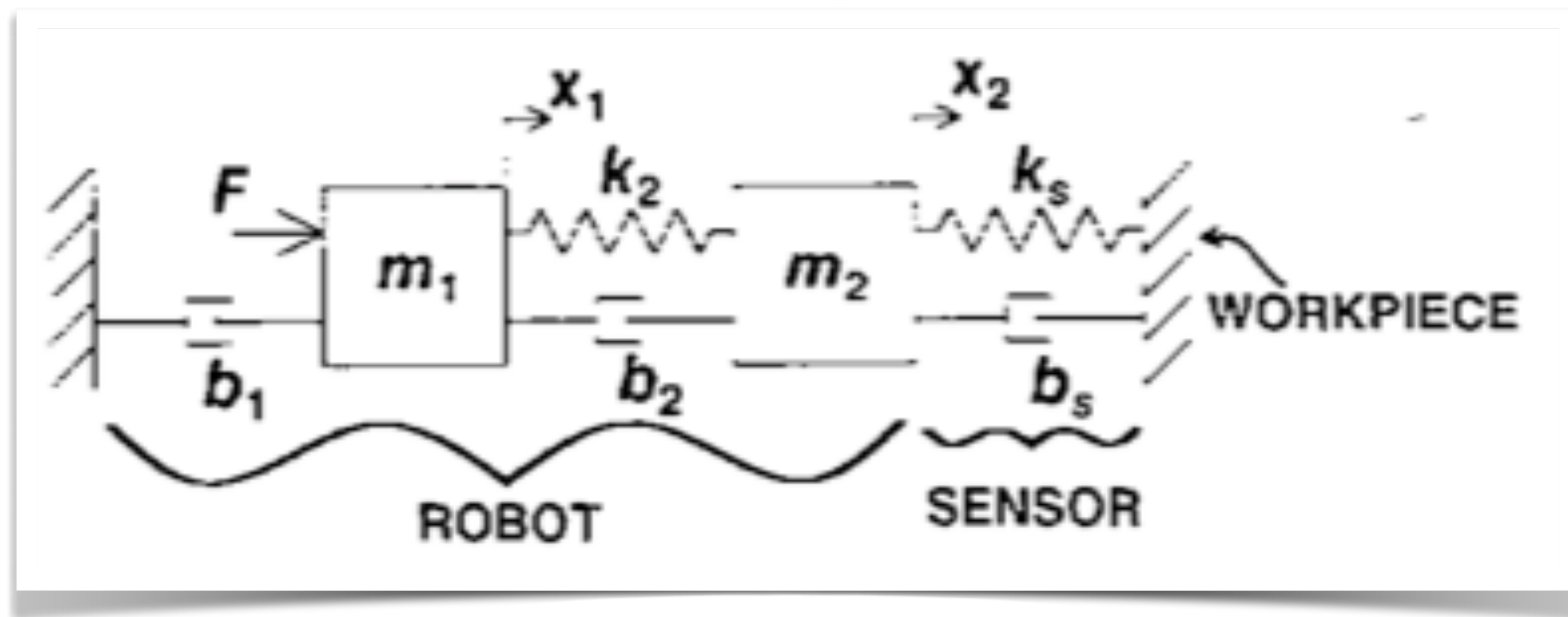
Revisão



## Problemas de estabilidade

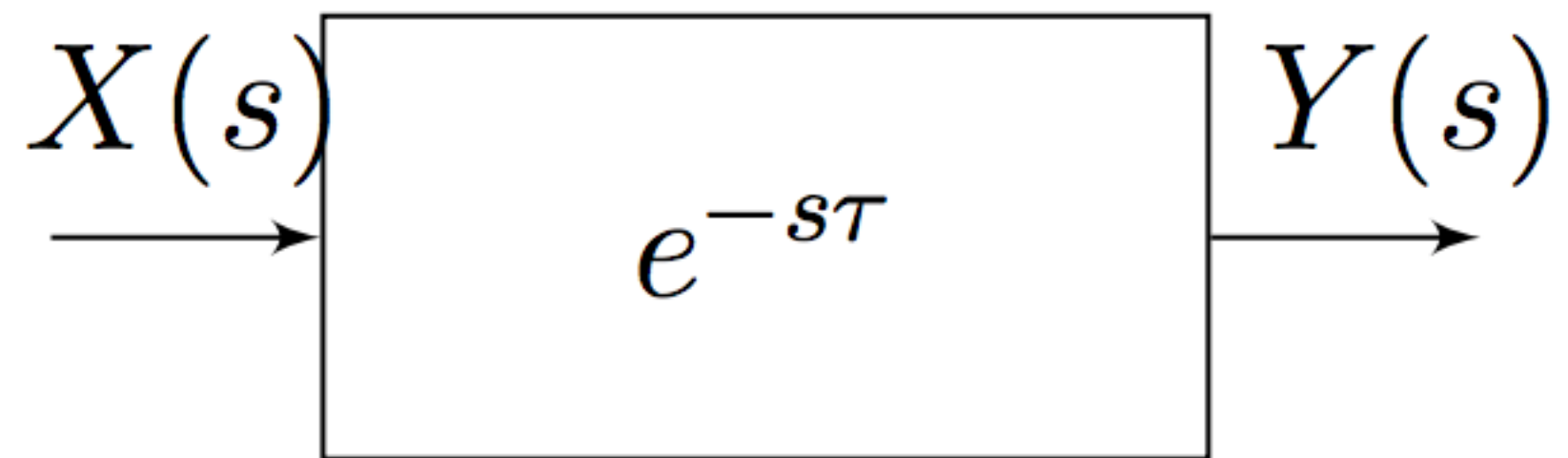


Estabilidade



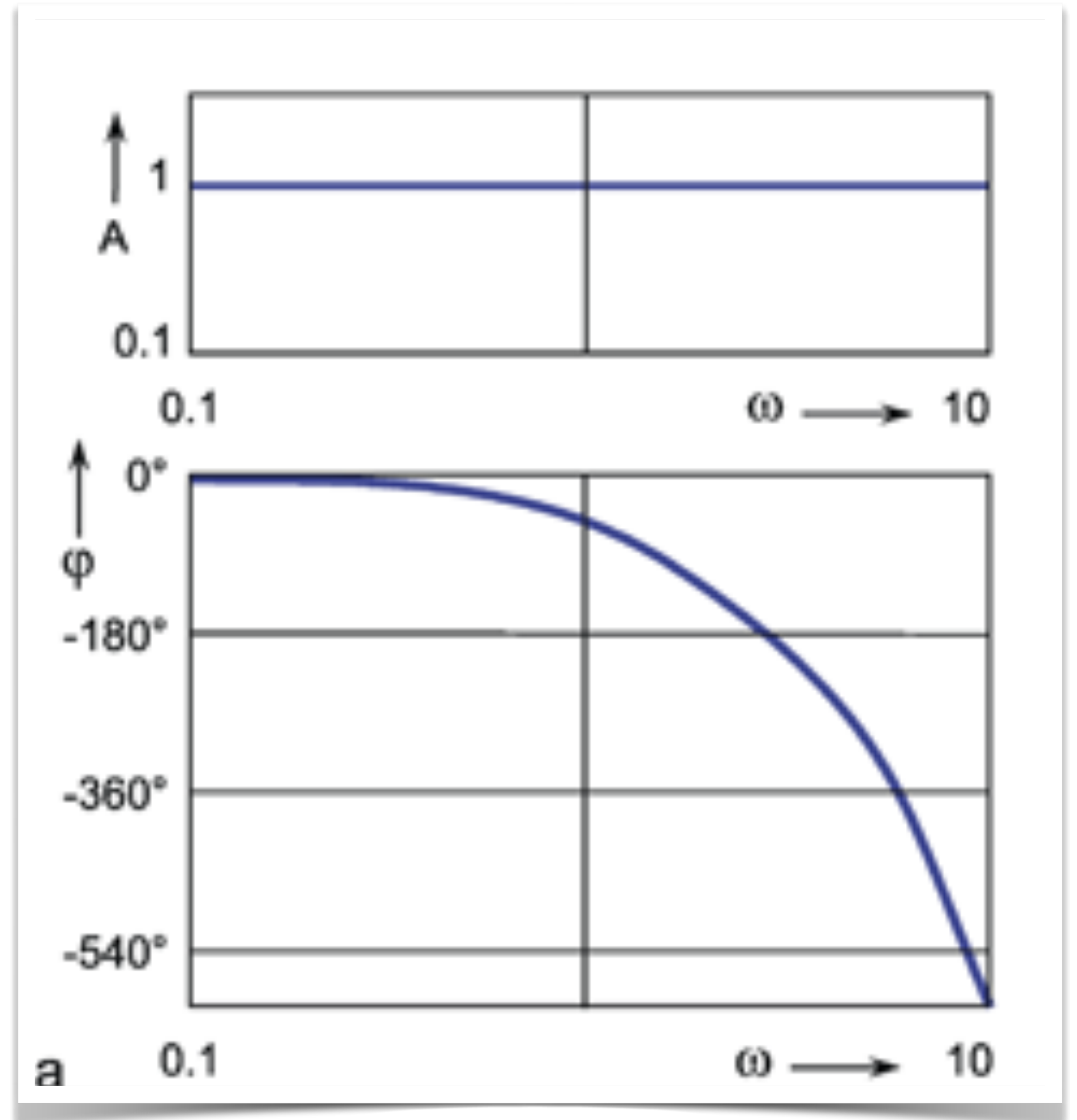
Eppinger & Seering, "Introduction to dynamic models for robot force control", 1987

# Atraso no sistema



$$|e^{-i\omega T}| = 1$$

$$\angle e^{-i\omega T} = \omega T$$



# Non-collocation

Revisão



Means that the actuator and sensor are not at the same physical location, but are separated by dynamic elements.



Estabilidade



Conclusão

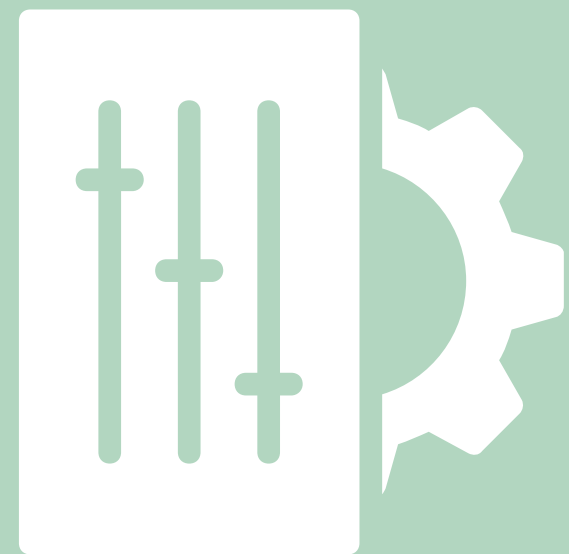
Colgate & Hogan, 1989

Eppinger & Seering, 1987



# Conteúdo

Revisão



- Estabilidade da interação
- **Passividade**
- Z-width

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# Passividade

Reafirmação do princípio de  
**conservação de energia**

**Um sistema passivo  
não pode armazenar  
mais energia do que a  
que foi fornecida  
exteriormente**



# Passividade

$$\Sigma : \begin{cases} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{cases}$$

$$\underbrace{H[x(t)] - H[x(0)]}_{\text{stored energy}} \leq \underbrace{\int_0^t u^\top(s)h(x(s))ds}_{\text{supplied energy}}$$

# Passividade de sistemas lineares

$$y(s) = h(s)u(s)$$

Teorema de Parseval:

$$\int_0^T y(t)u(t)dt = \int_{-\infty}^{\infty} y(t)u_T(t)dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} y(j\omega)u_T^*(j\omega)d\omega$$

$$\mathbf{Re}[h(j\omega)] \geq 0 \text{ for all } \omega$$

Lozano, Rogelio, et al. Dissipative systems analysis and control: theory and applications. Springer Science & Business Media, 2013.

# Passividade de sistemas lineares

$$y(s) = h(s)u(s)$$

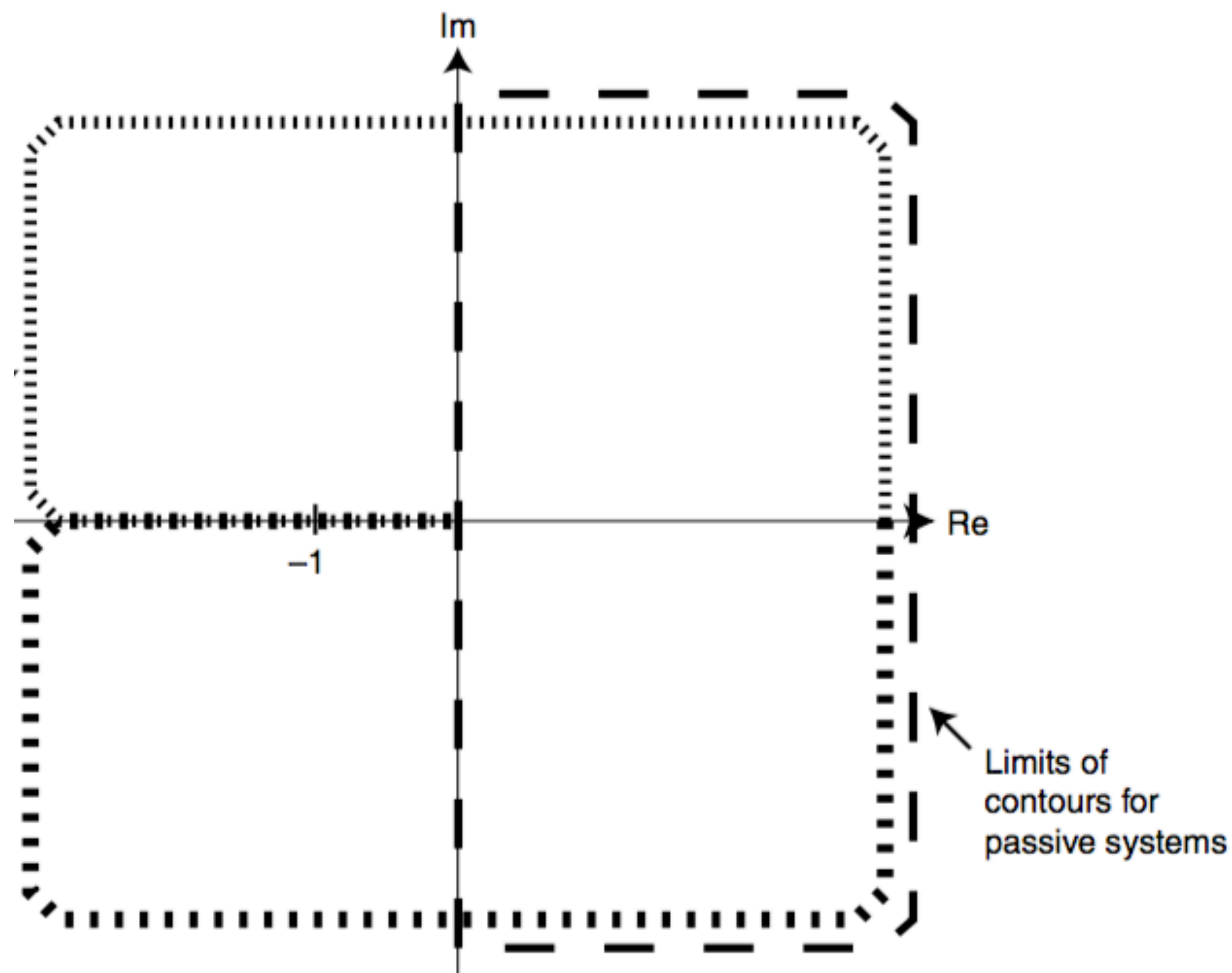
  

Condições para passividade:

1.  $Z(s)$  é estável
2.  $\text{Re}[Z(i\omega)] \geq 0 \quad \forall \omega \in \mathbb{R}^+$

# Passividade de sistemas lineares

## 2. $\text{Re}[Z(i\omega)] \geq 0 \quad \forall \omega \in \mathbb{R}^+$



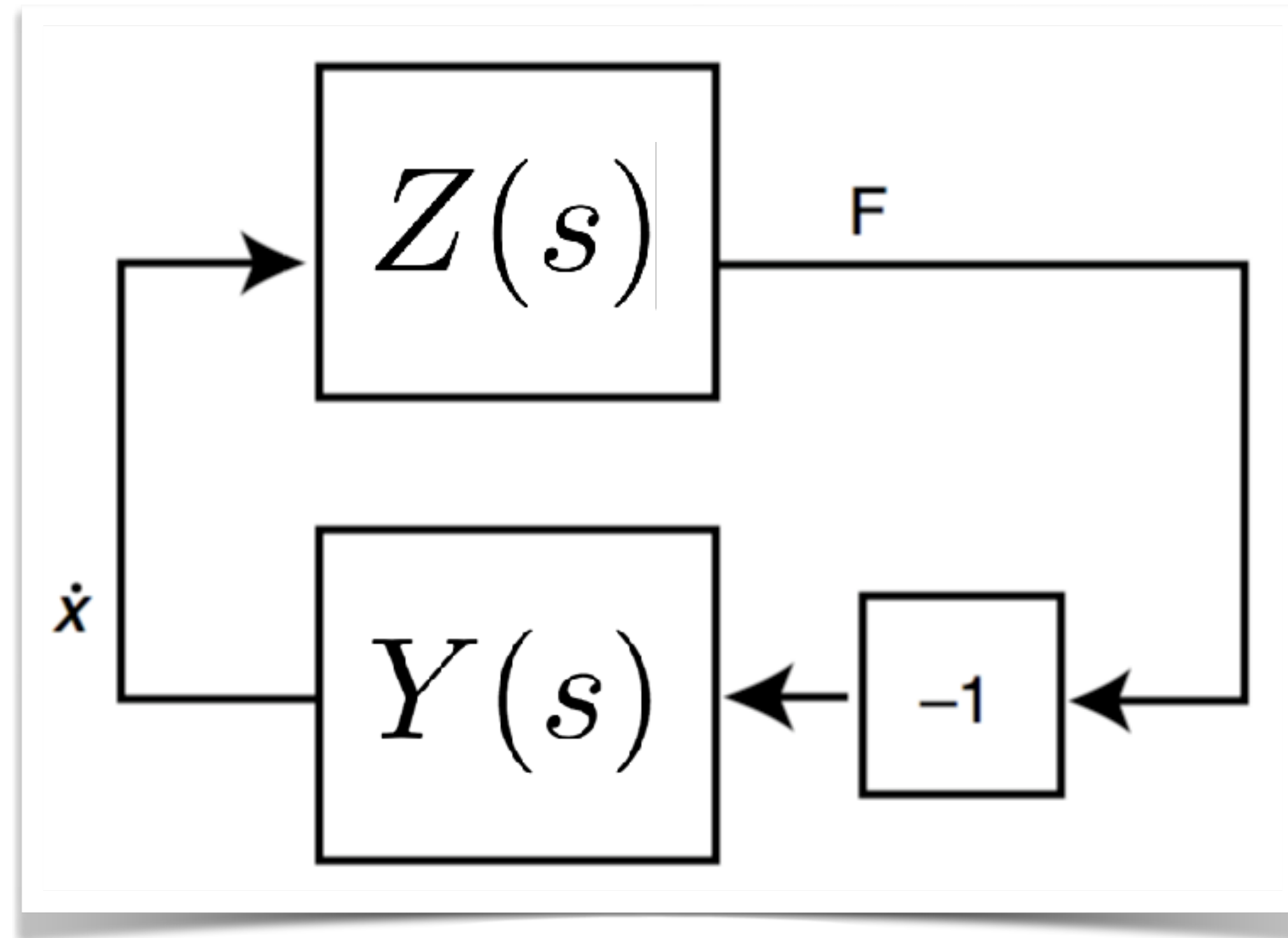
$$|\angle h(j\omega)| \leq 90^\circ$$

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# Estabilidade acoplada via passividade



Revisão

Estabilidade

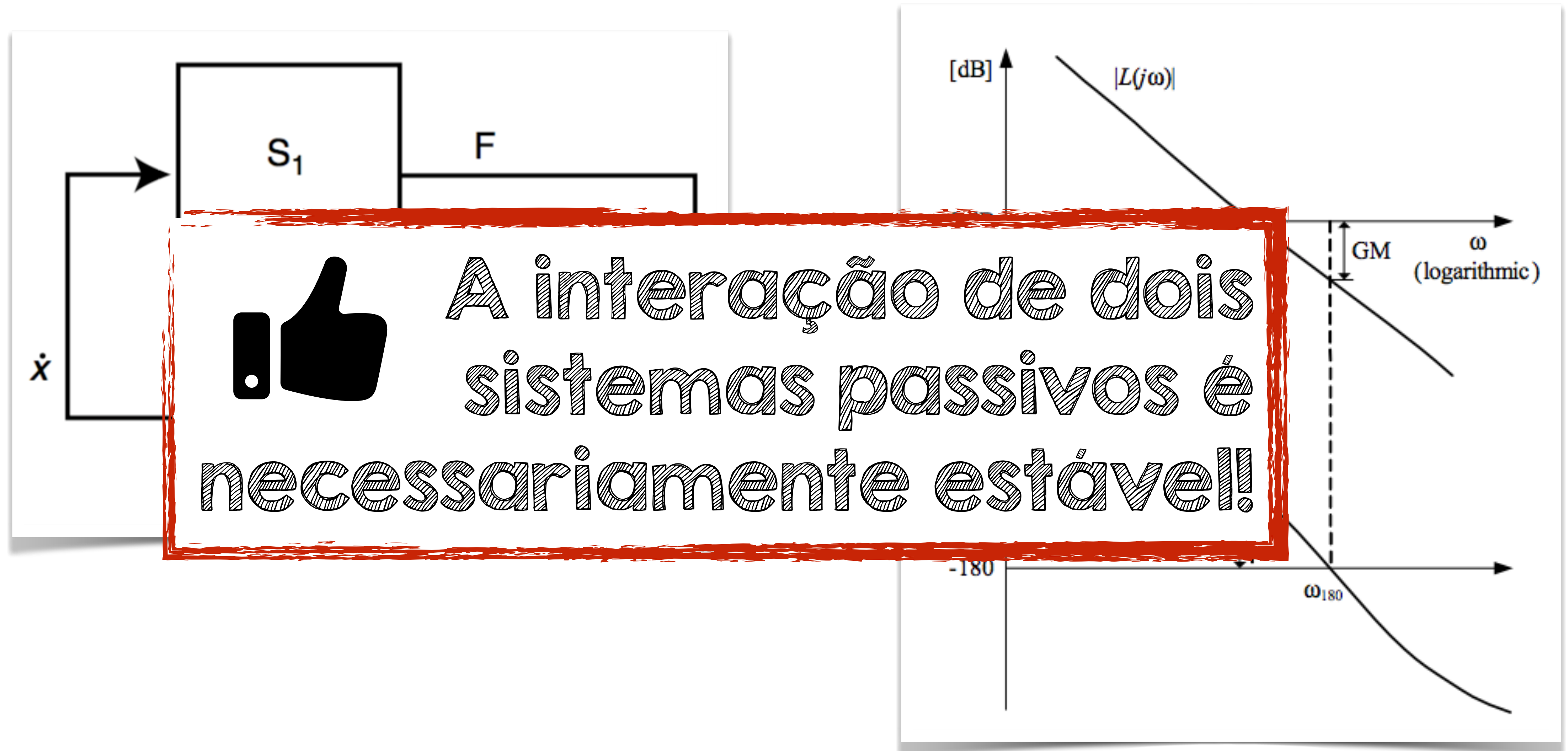
Conclusão

# Estabilidade acoplada via passividade

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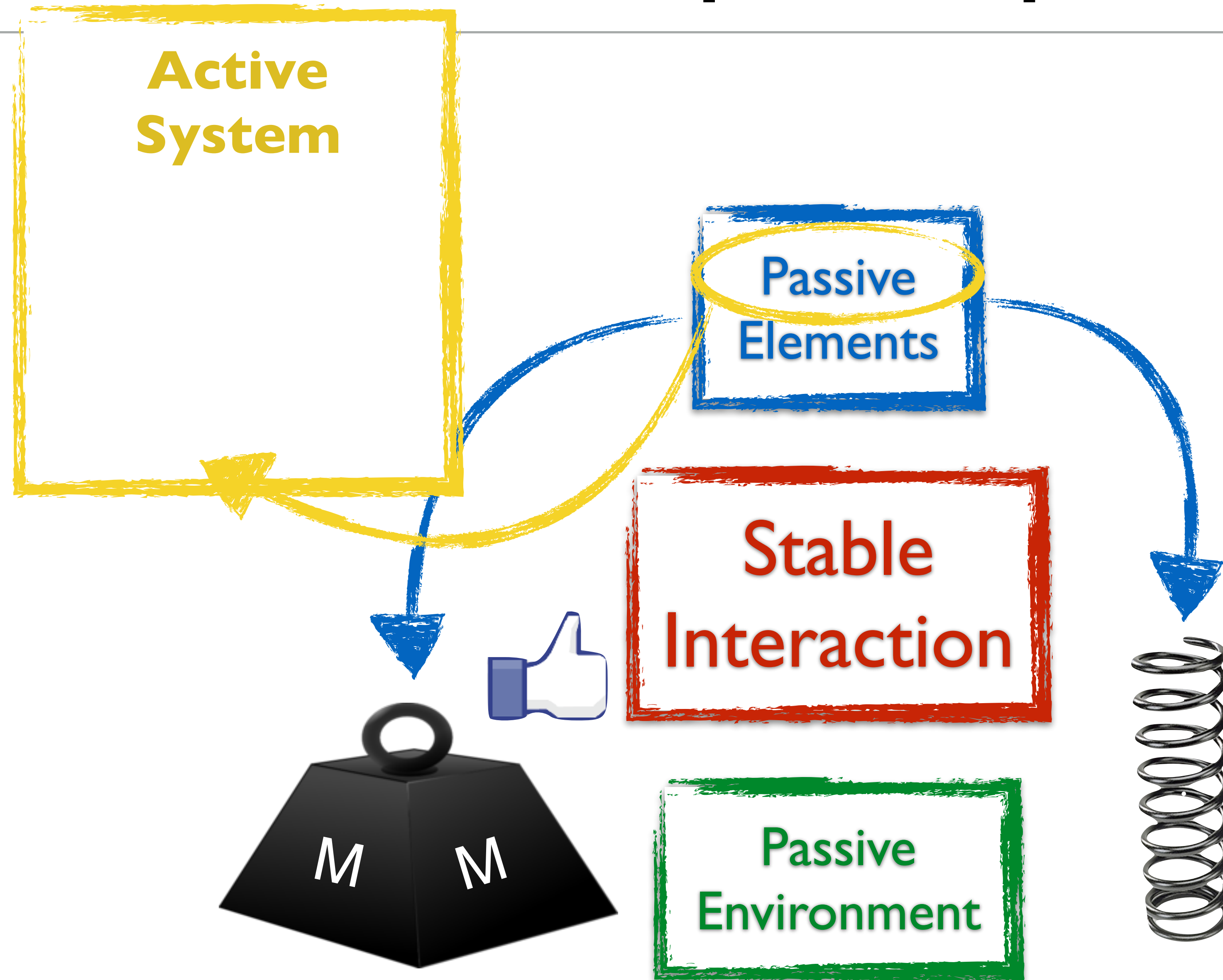


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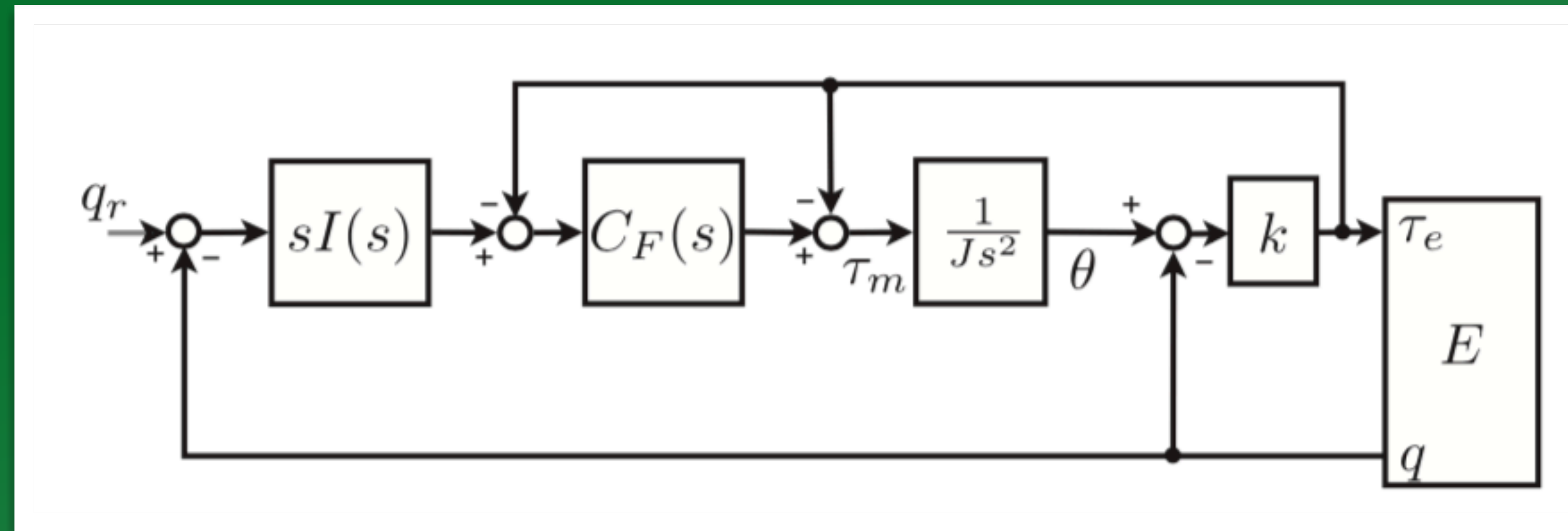


# Estabilidade acoplada via passividade

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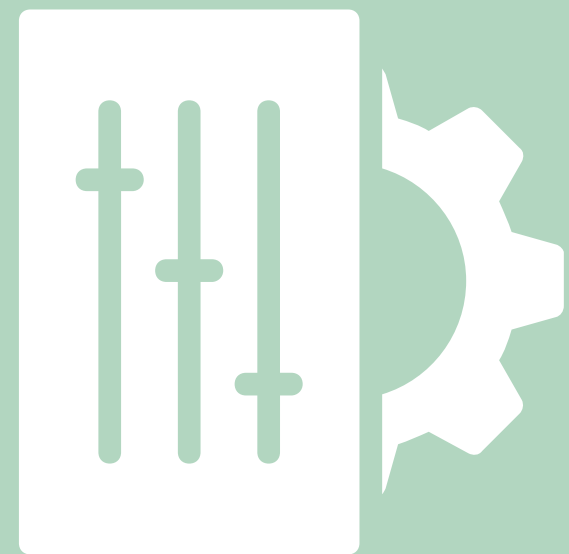
$$C_F(s) = P + sD$$

$$-\tau_e = \frac{C_F(s)I(s) + Js}{C_F(s) + \frac{J}{k}s^2 + 1} \dot{q}$$



# Conteúdo

Revisão



- Estabilidade da interação
- Passividade
- Z-width**

Estabilidade

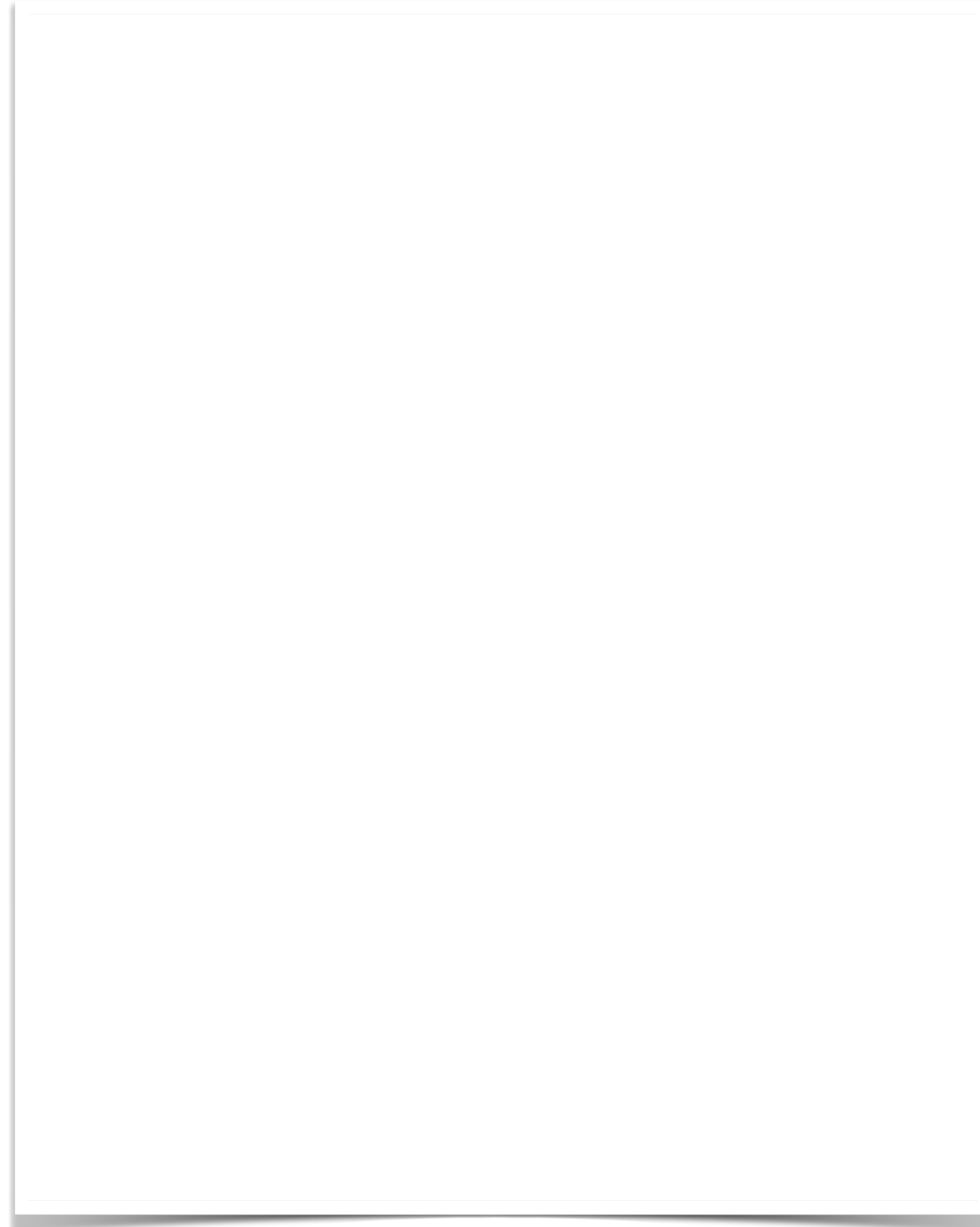
Conclusão

# Z-Width

Revisão

Estabilidade

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Range of **stiffness** and **damping** that keeps the system **passive**

[Colgate, J. E. and Brown, J. M. (1994). "Factors affecting the z-width of a haptic display"]

# Z-Width

Revisão

Estabilidade

Conclusão



- Passive range
- Stable, but not passive range
- Unstable range

Boaventura, T., et al. Stability and performance of the compliance controller of the quadruped robot HyQ. IROS, 2013

# Z-Width

$$\frac{\Delta f(s)}{\Delta u(s)} = \left( \frac{1}{\frac{1}{\omega_v^2} s^2 + \frac{2D_v}{\omega_v} s + 1} \right) \left( \frac{K_{u_v}(M_l s + B_l)}{(s - K_{f_h})(M_l s + B_l + B) - K_{\dot{x}_p}} \right)$$

$$\omega_v = 2\pi F_v$$

The higher the actuator bandwidth, the larger the Z-width!

Boaventura

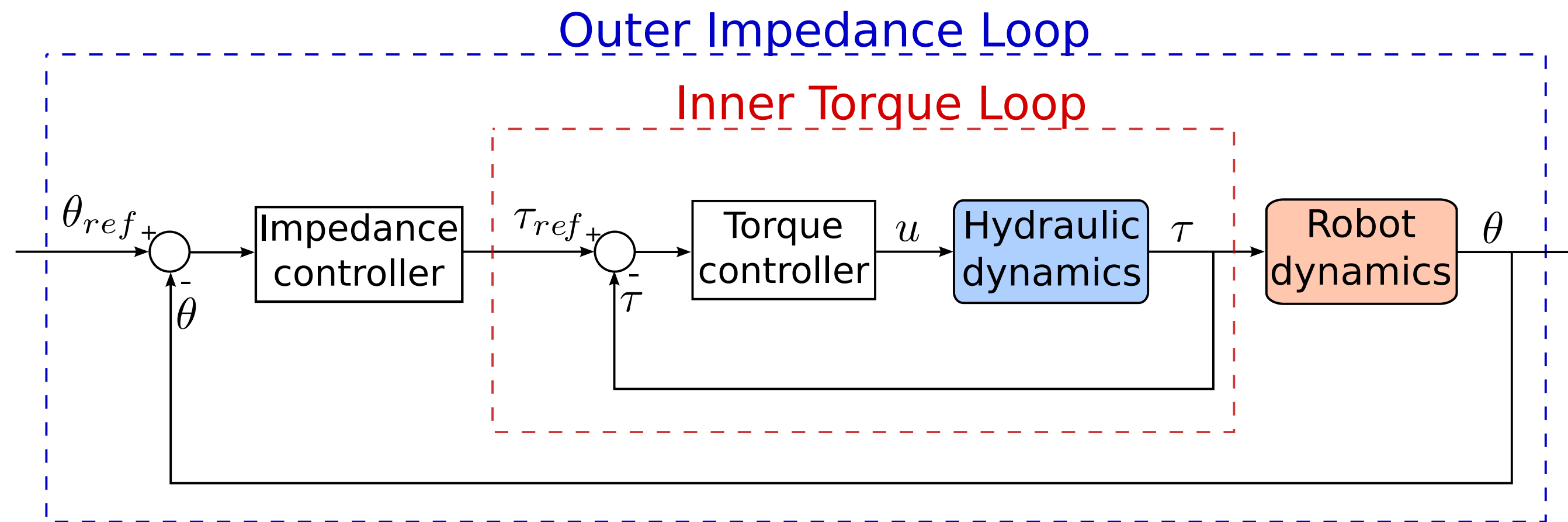
OS, 2013

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# Z-Width

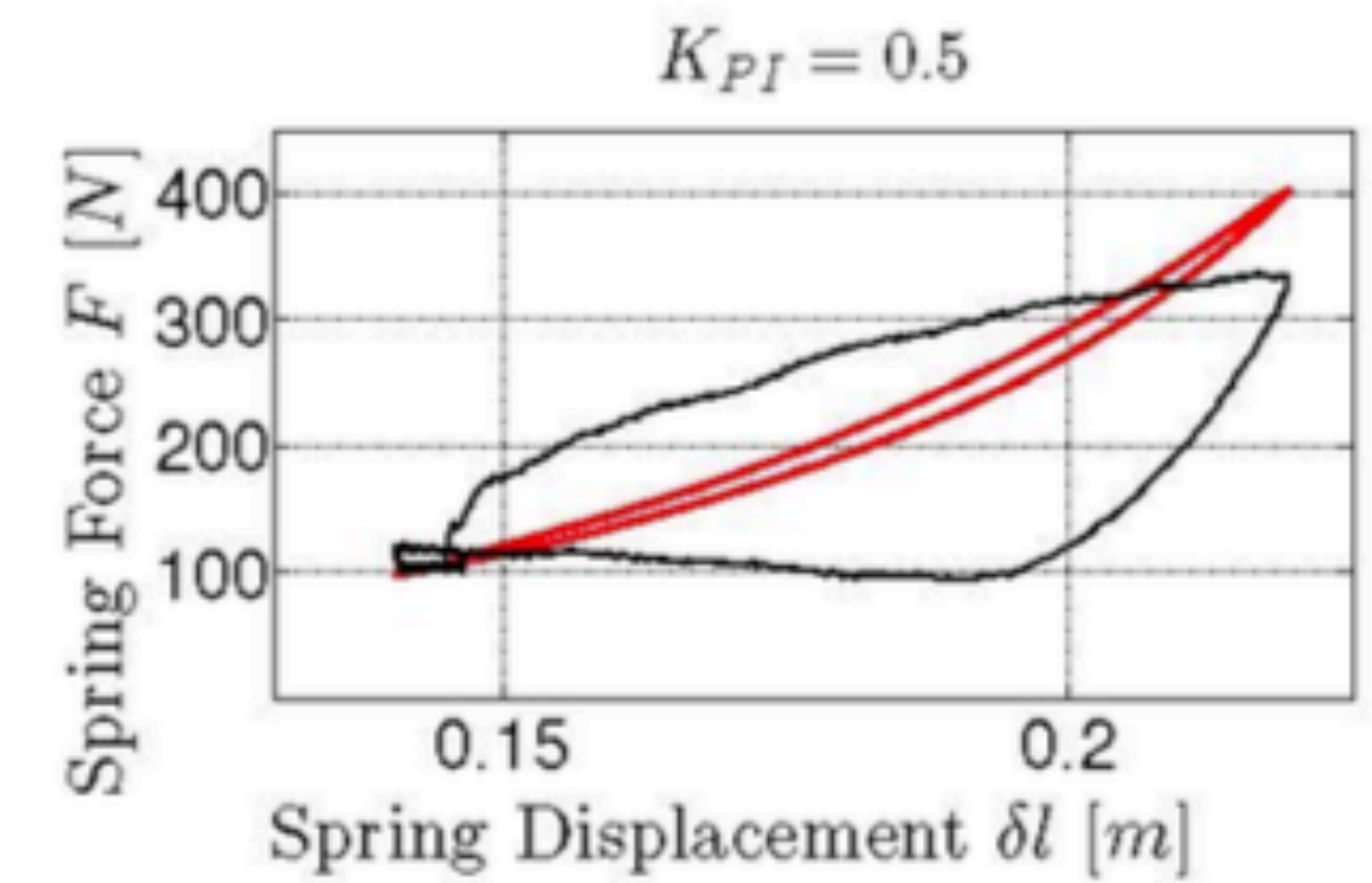
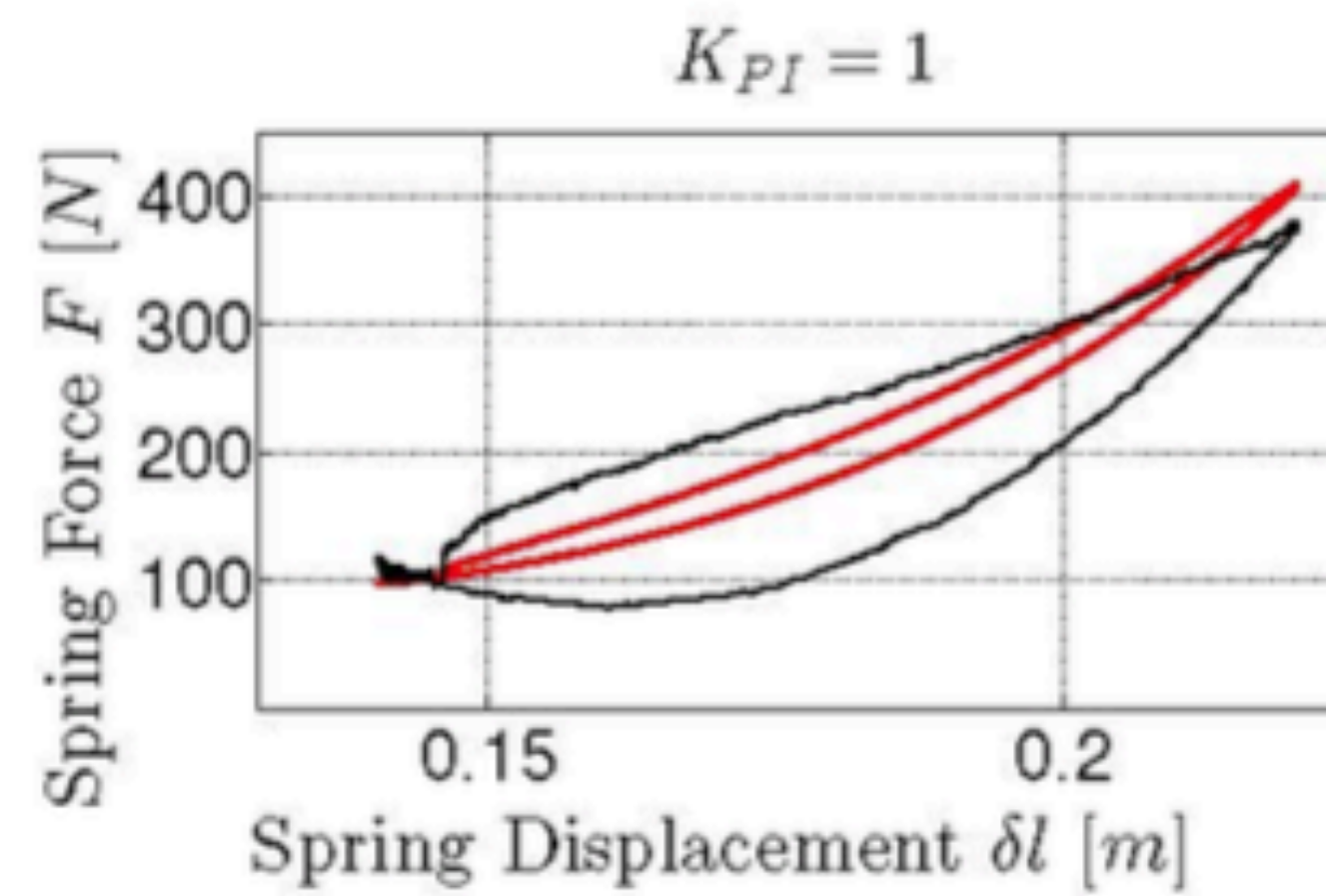
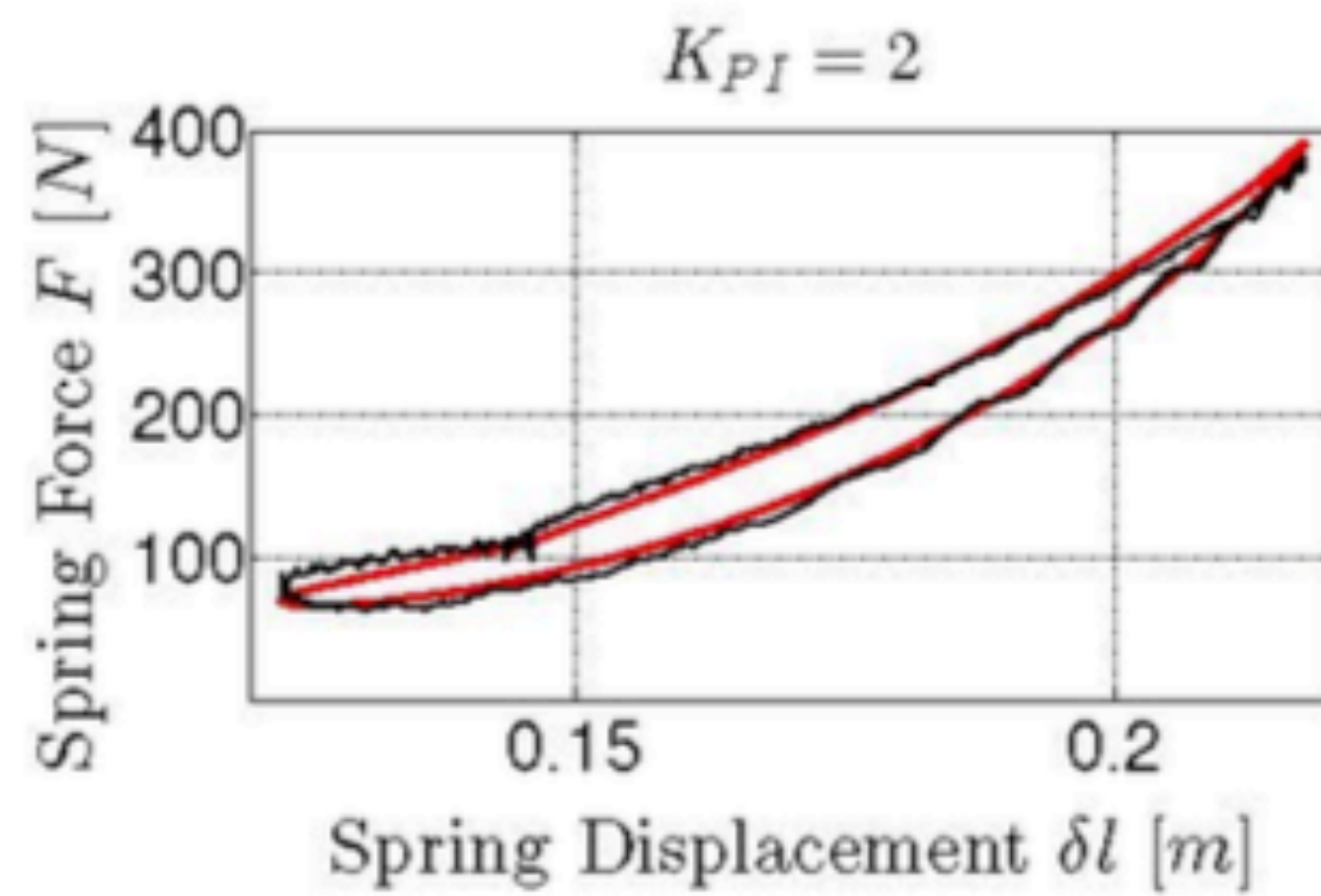


$$C = K_{PI} \frac{0.03838 (z - 0.9953)}{z - 1}$$

Boaventura, T., et al. Stability and performance of the compliance controller of the quadruped robot HyQ. IROS, 2013

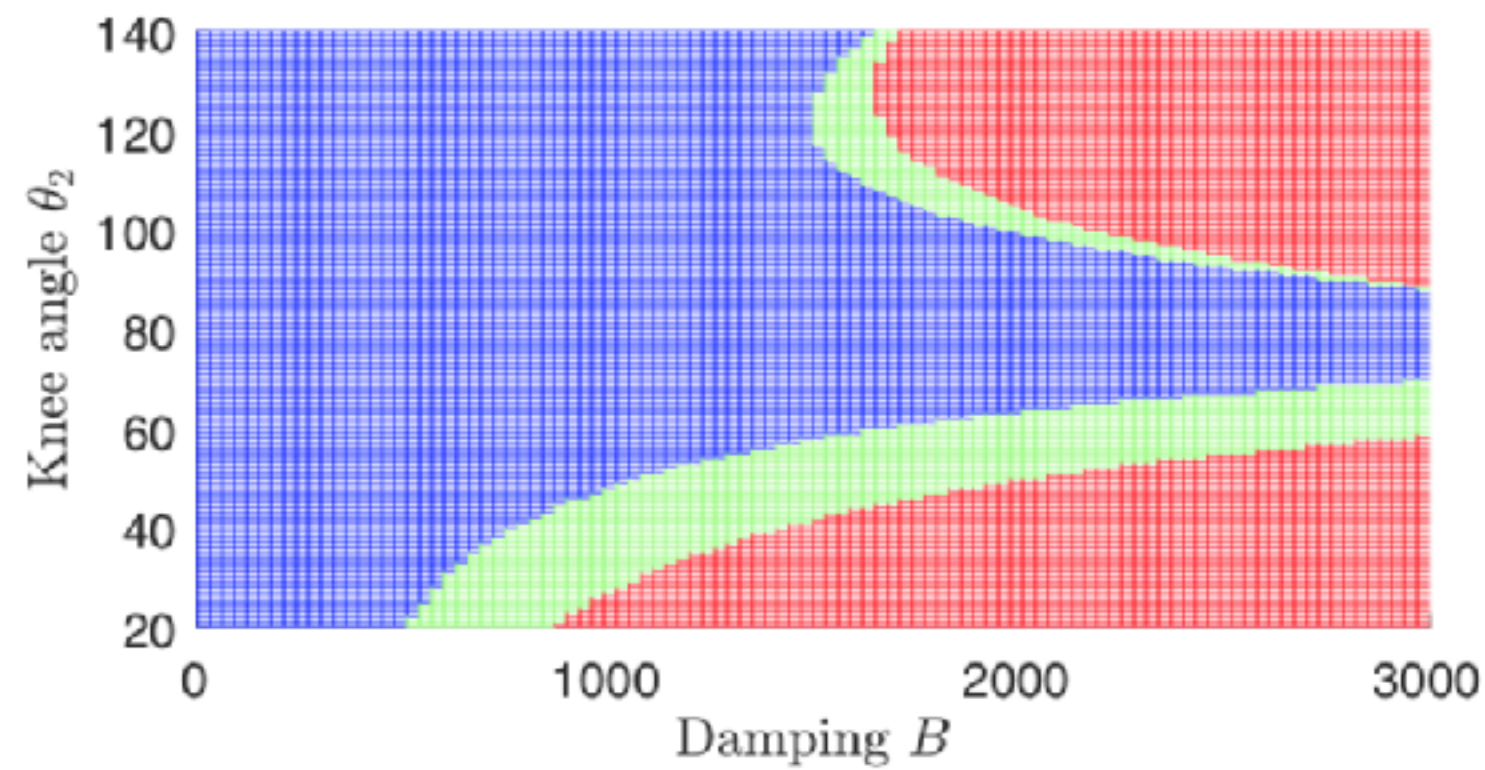
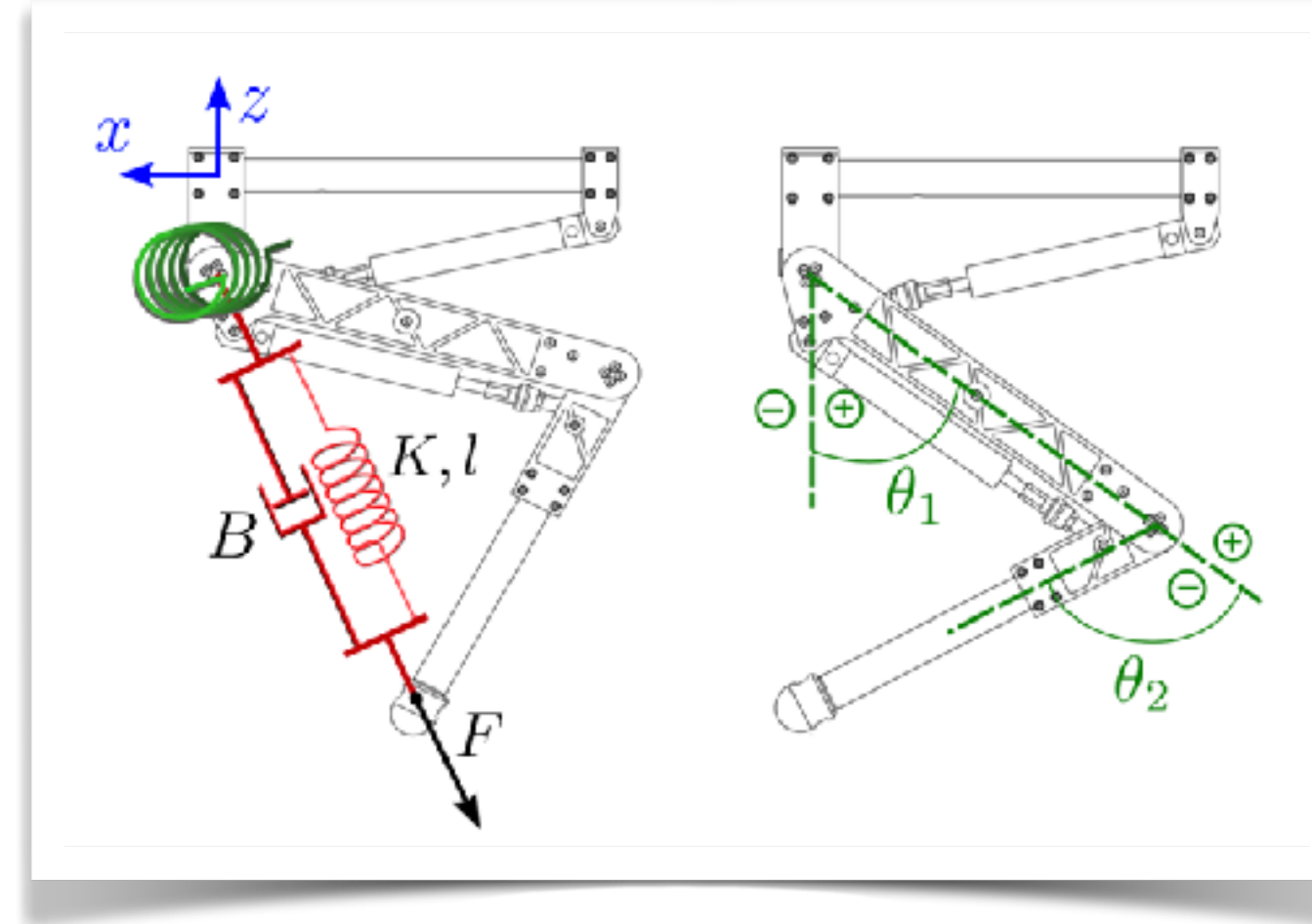


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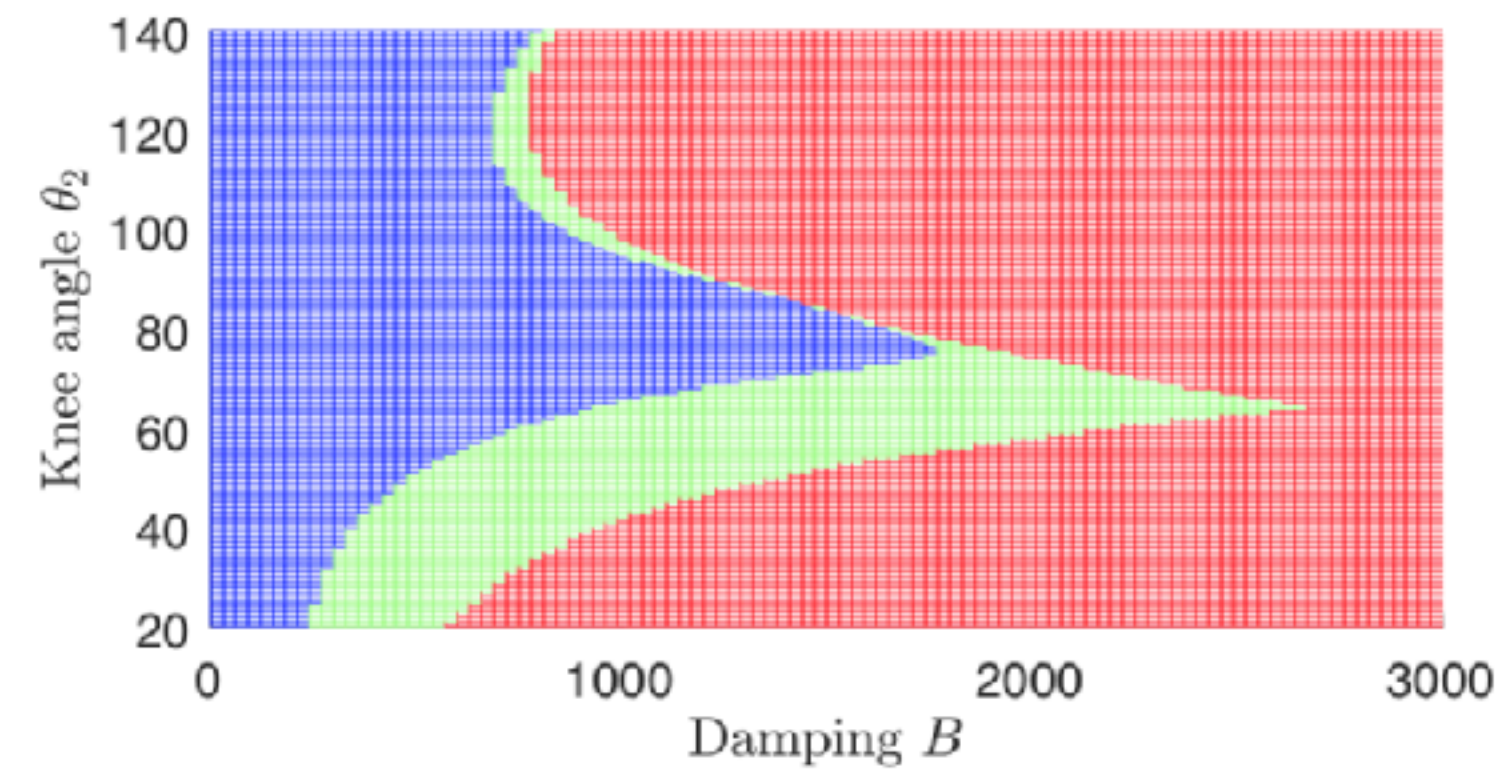


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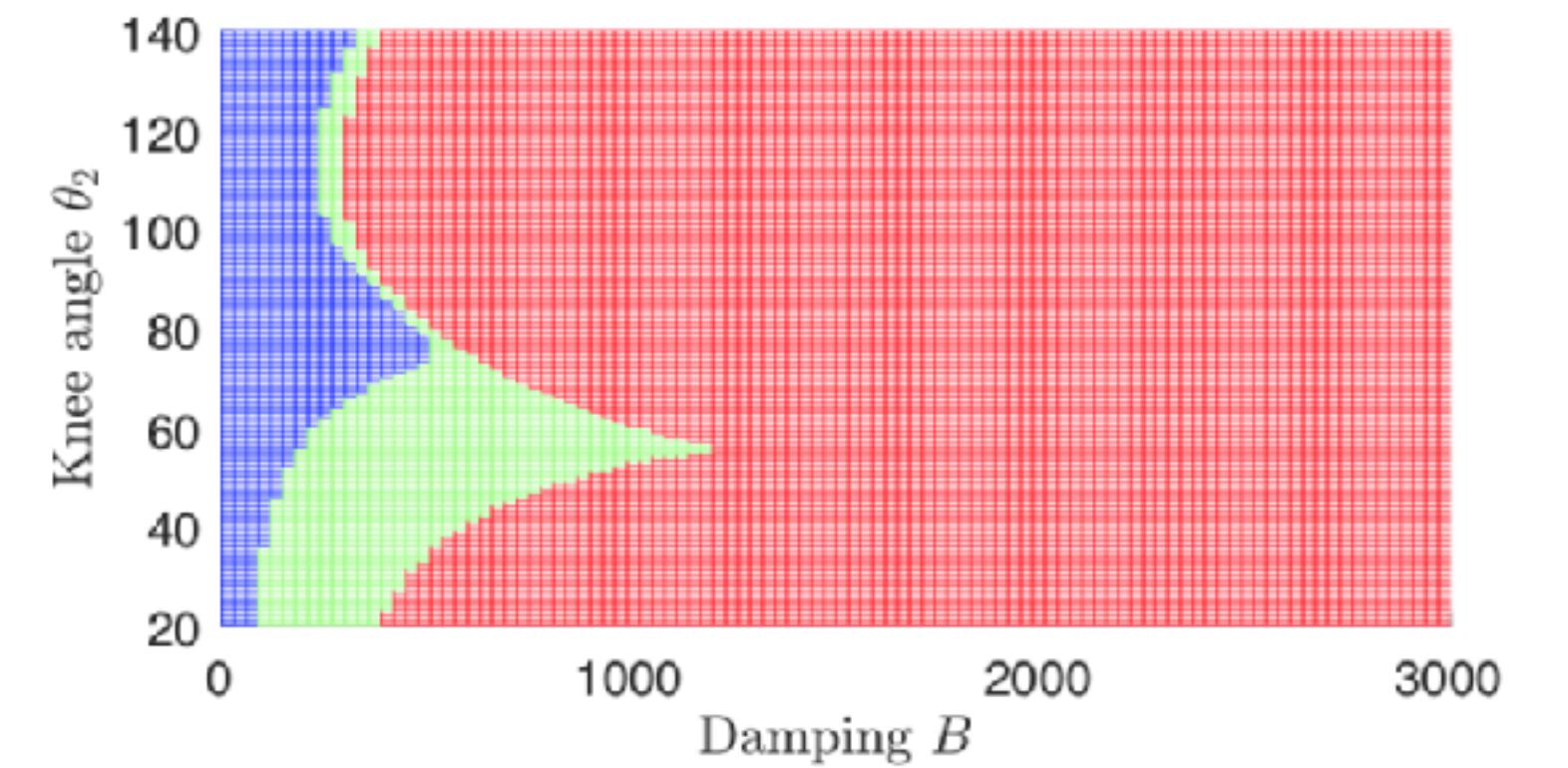
# Z-Width



(a)  $K_{PI} = 0.5$



(b)  $K_{PI} = 1.0$



(c)  $K_{PI} = 2.0$

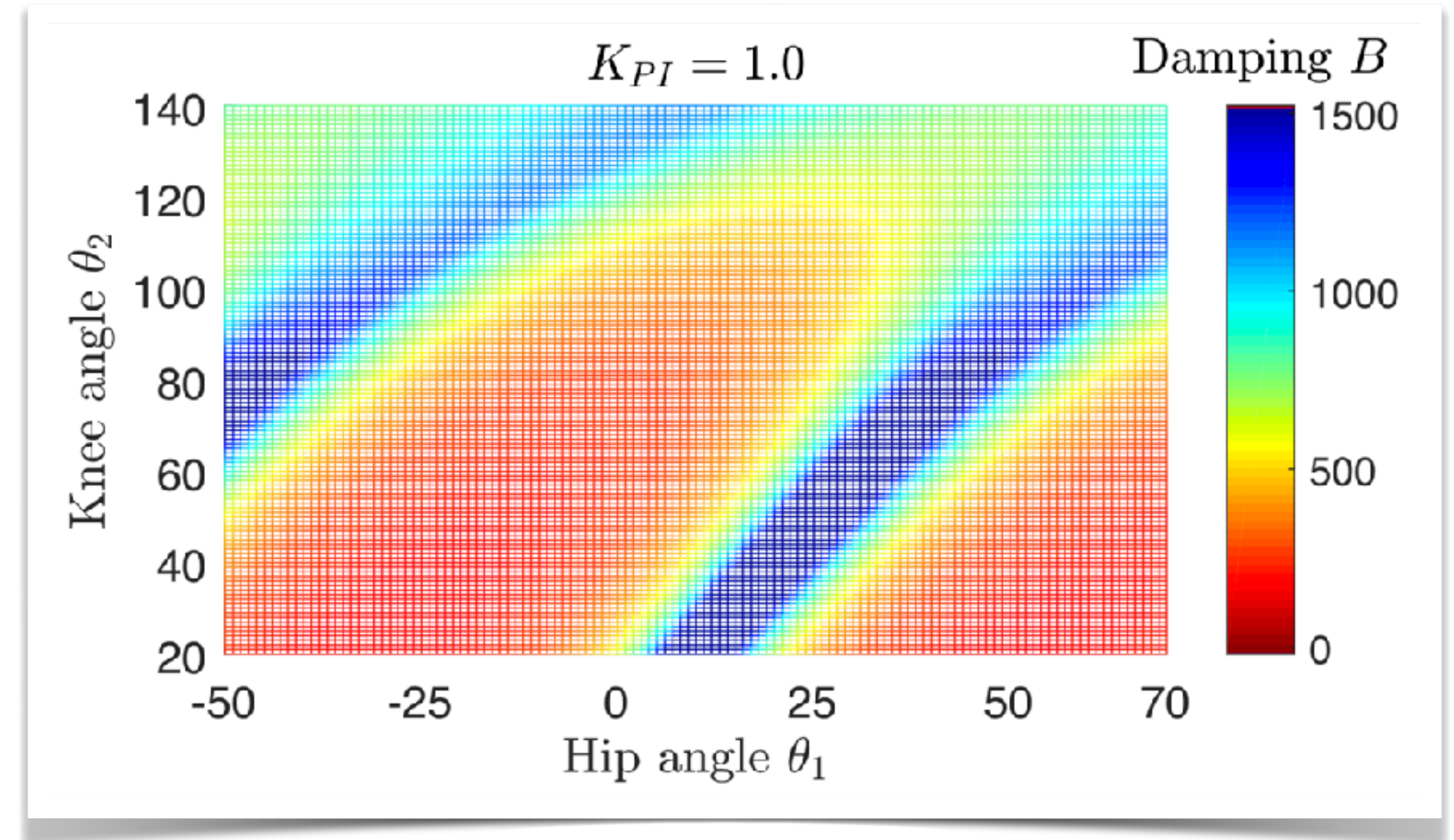
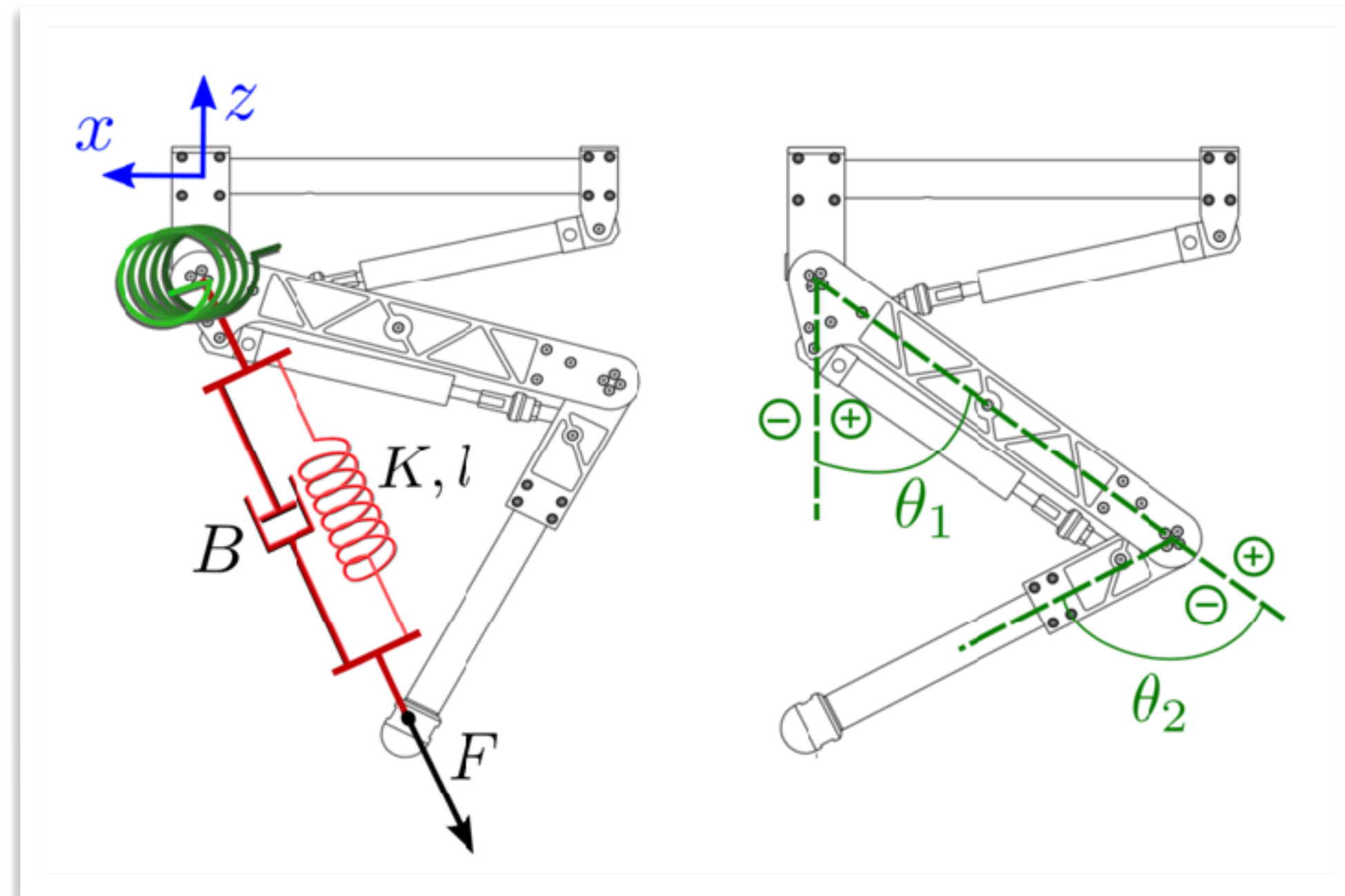
Higa, F. Y. G., et al. Joint kinematic configuration influence on the passivity of an impedance-controlled robotic leg. ICRA, 2019

# Z-Width

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Higa, F. Y. G., et al. Joint kinematic configuration influence on the passivity of an impedance-controlled robotic leg. ICRA, 2019

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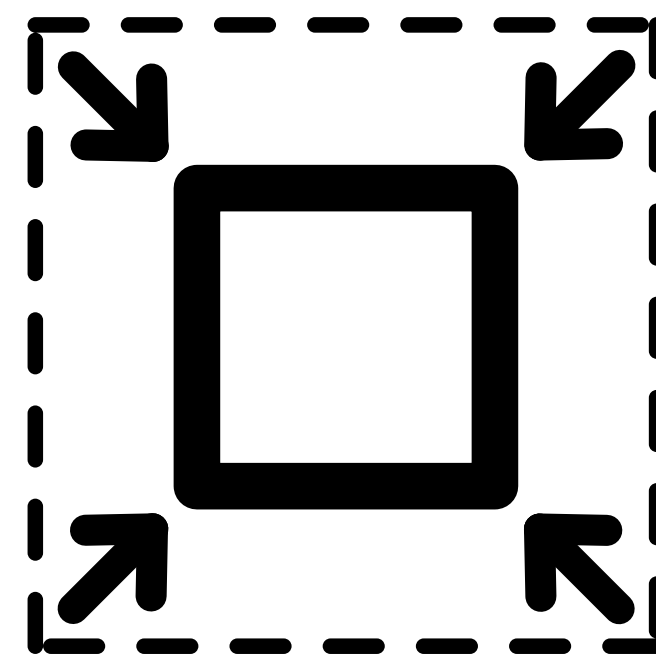
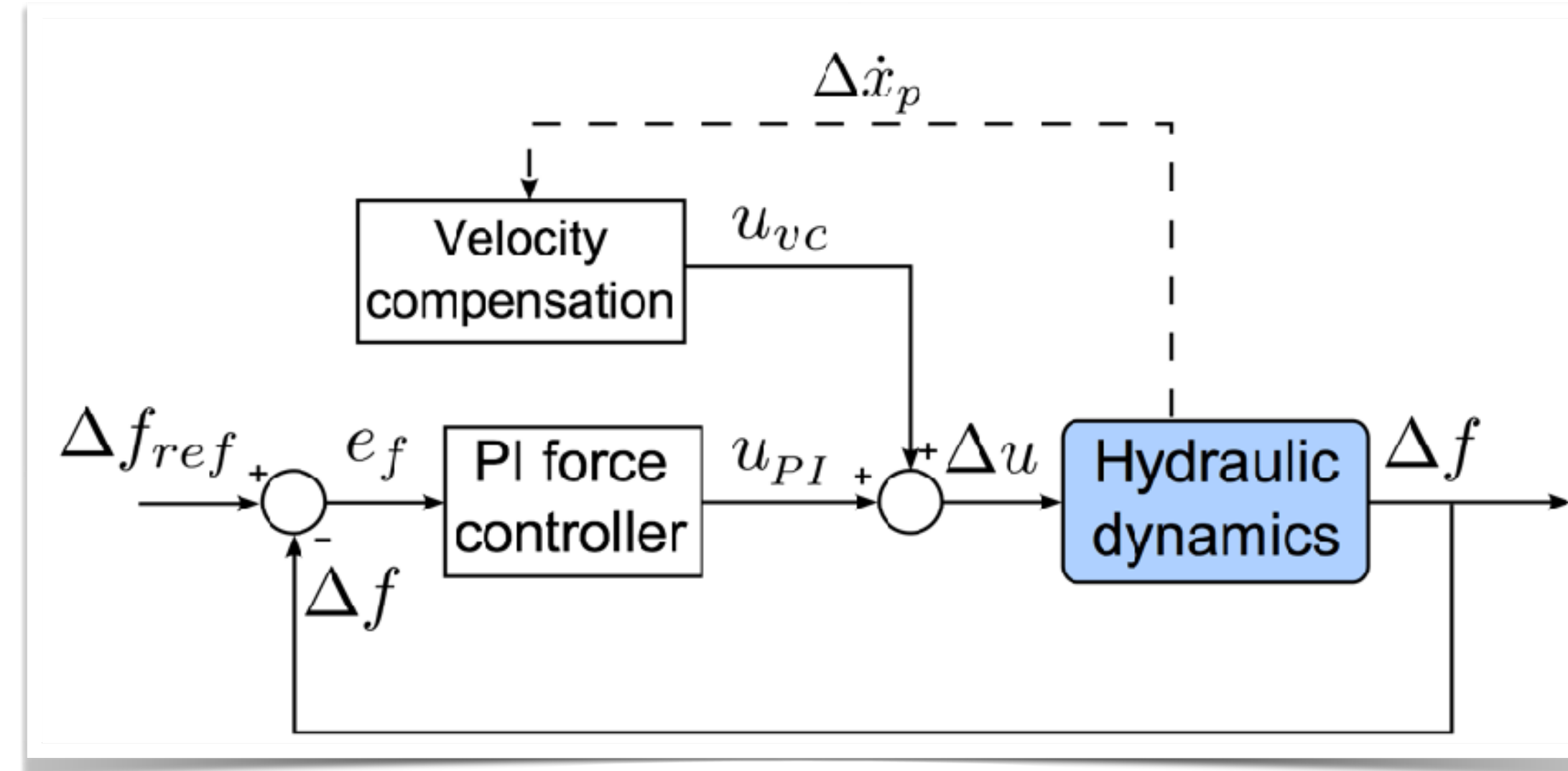
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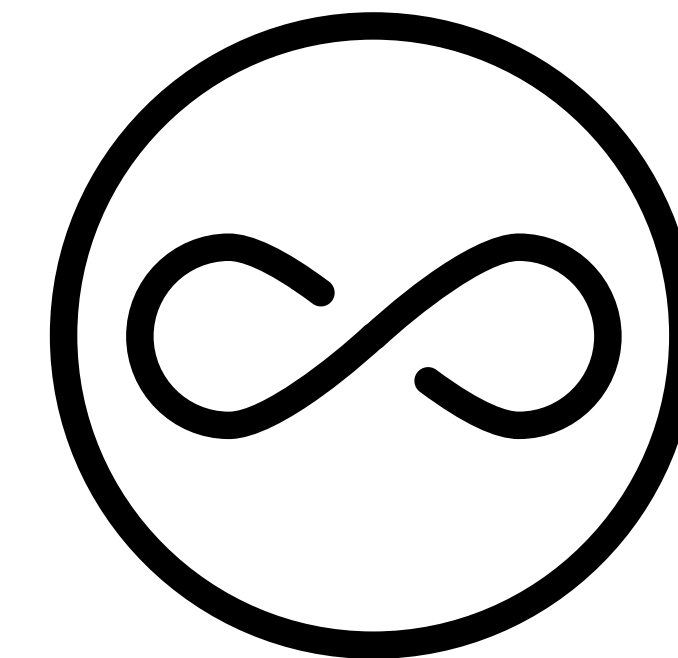
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Reduzir inércia aparente



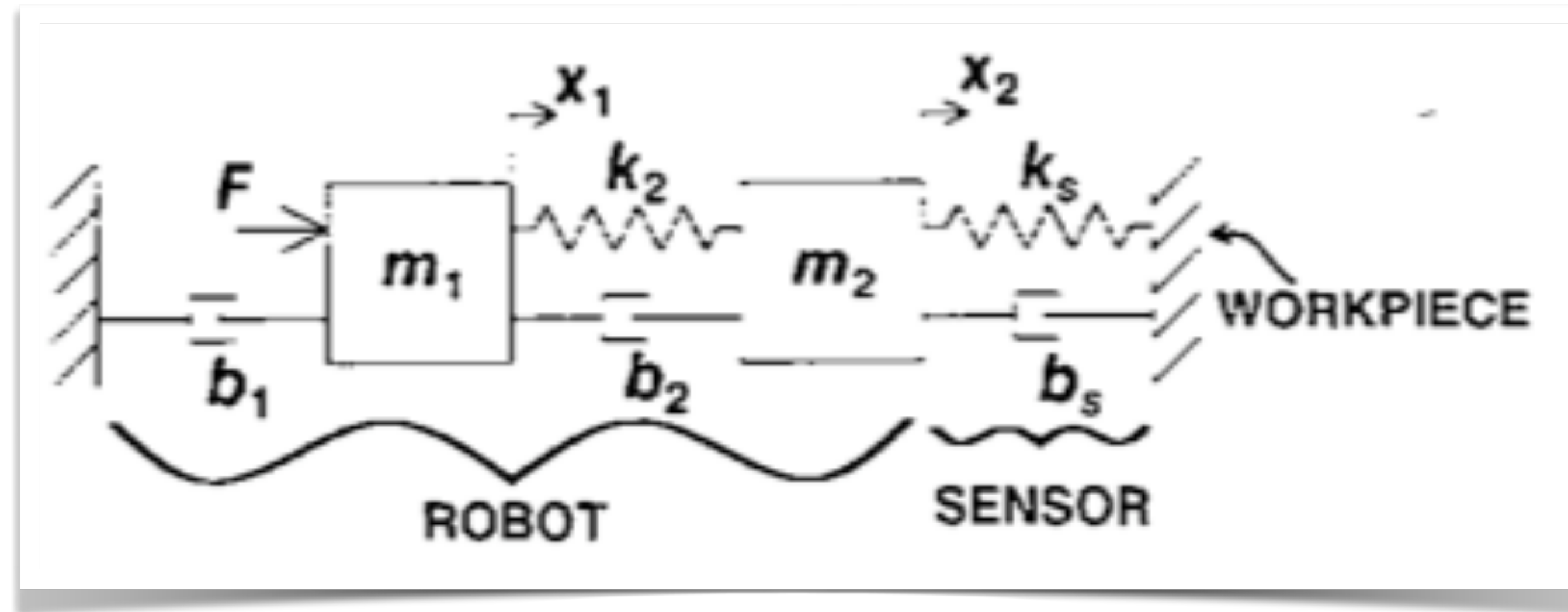
Problemas de estabilidade

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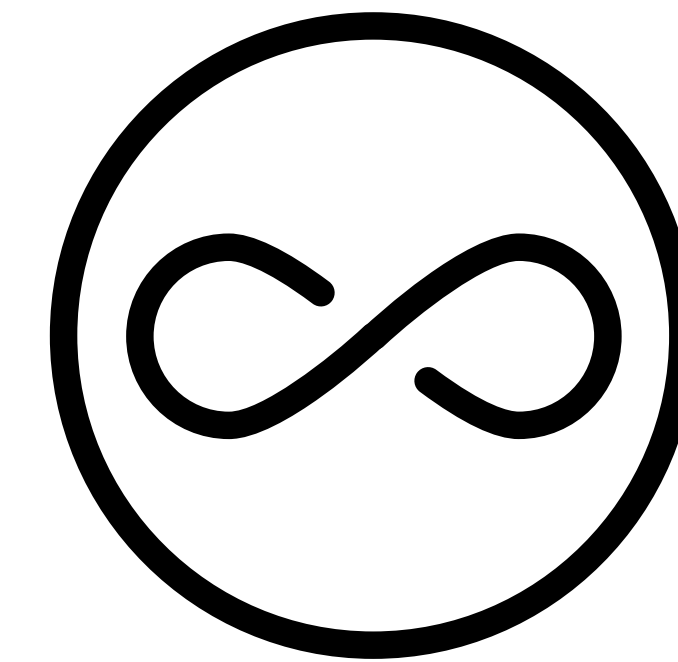
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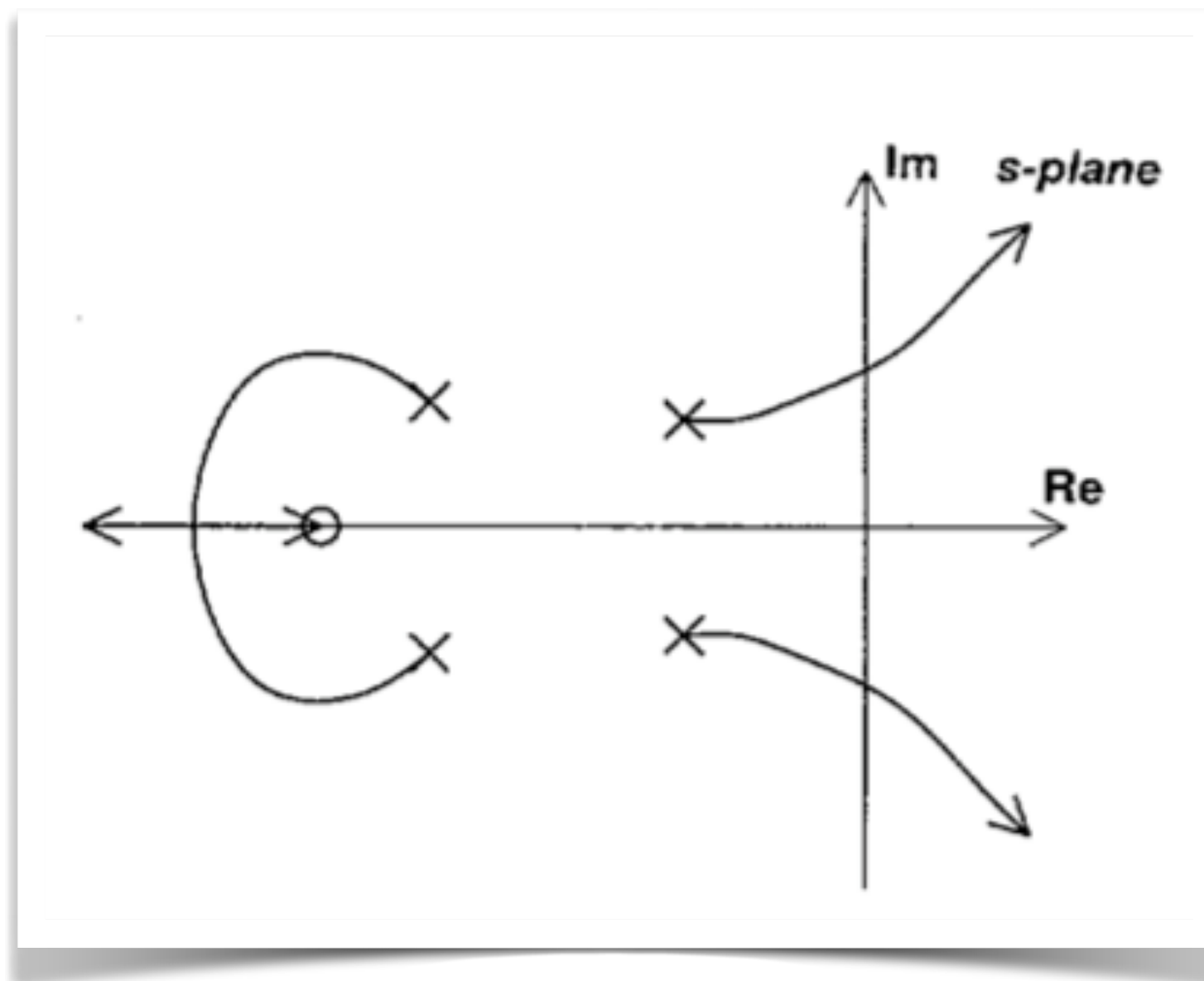
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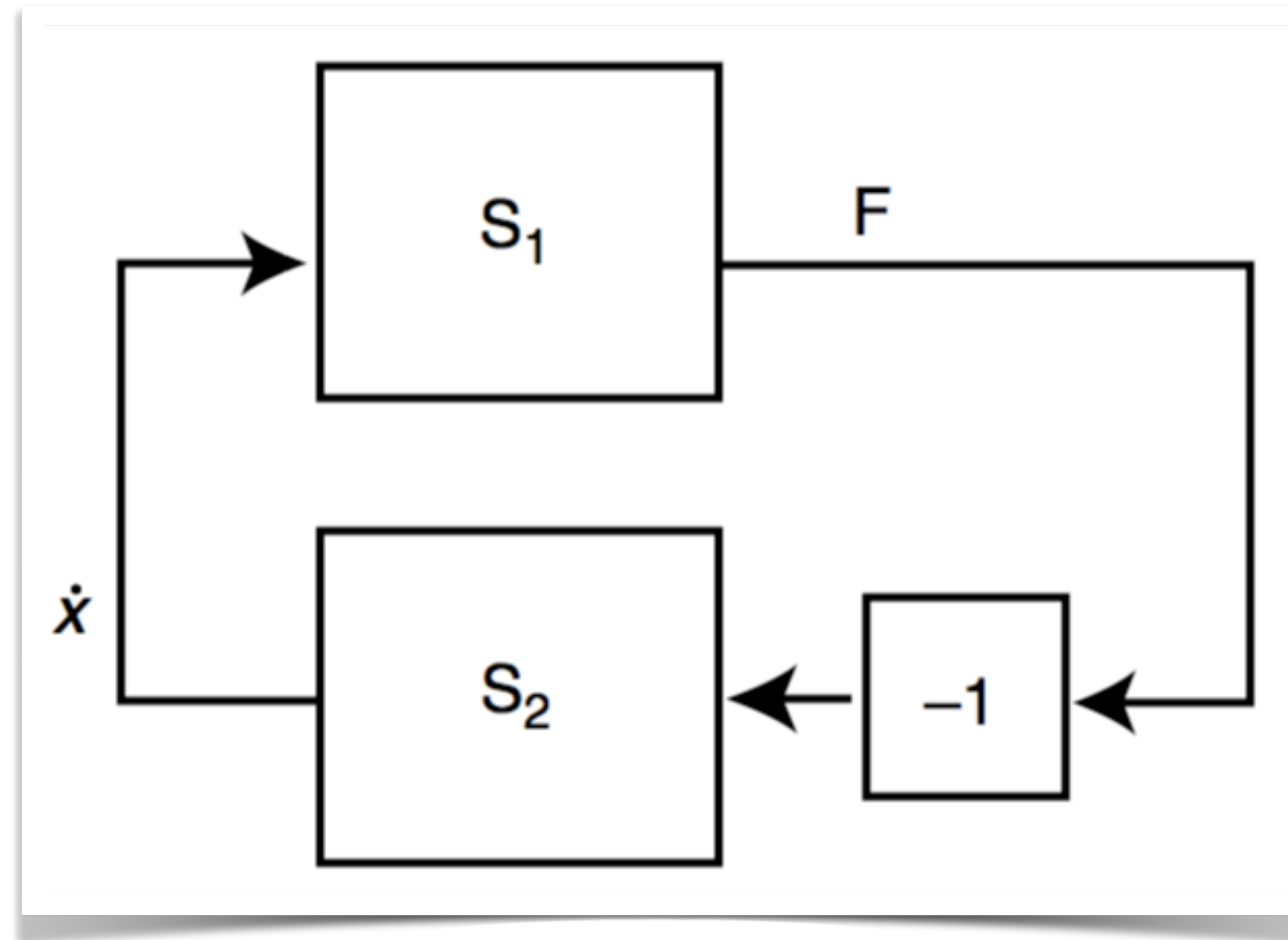
## Non-collocation



Problemas de estabilidade



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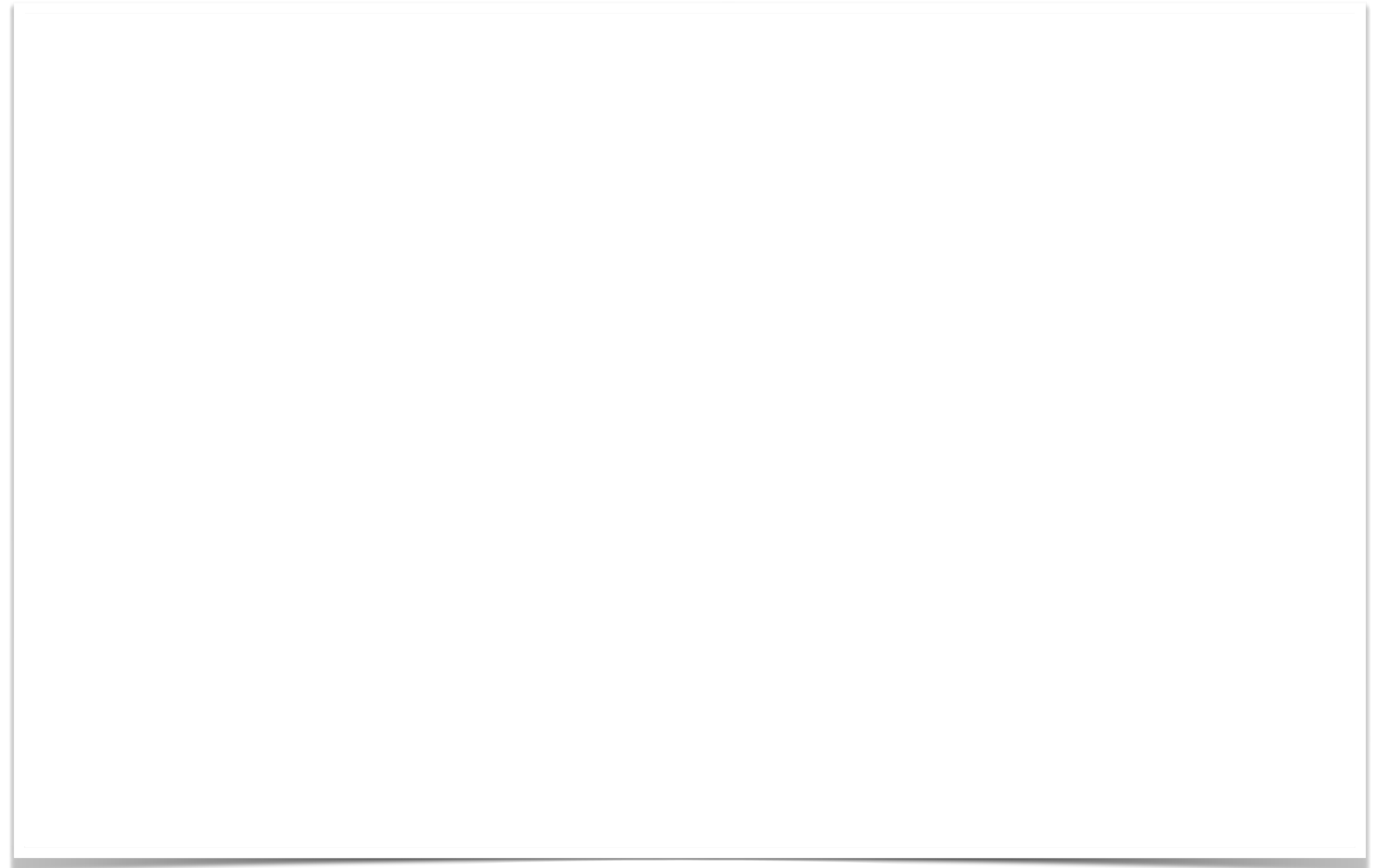
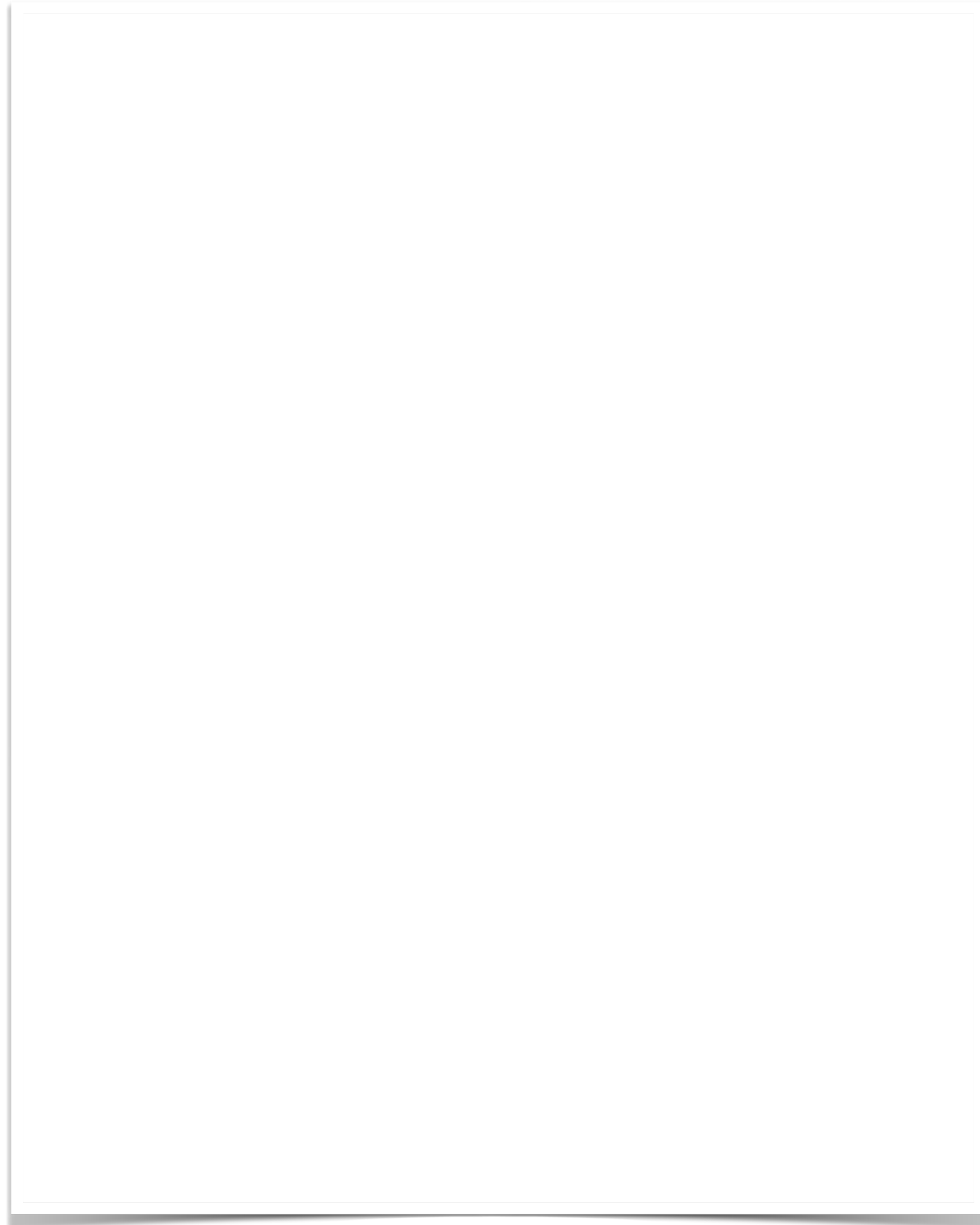
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*That's all Folks!*