$$S_{fi} = \langle f | S | i \rangle = \delta_{if} + (2\pi)^4 \delta^{(4)}\left(\sum_f p_f' - \sum_i p_i\right)$$

$$* \prod_i \left(\frac{1}{2VE_i}\right)^{1/2} \prod_f \left(\frac{1}{2VE_f}\right)^{1/2} \prod_l (2m_l)^{1/2} \quad M$$

$$\left(\frac{m}{VE}\right)^{1/2} \qquad \left(\frac{1}{2V\omega_k}\right)^{1/2}$$

$$\hbar\omega_k = E$$

$V \qquad T$

$$|W_{fi}|^2$$

$$(2\pi)^4 \delta^{(4)}\left(\sum_f p_f' - \sum_i p_i\right) = \lim_{\substack{T \to \infty \\ V \to \infty}} \delta_{TV}\left(\sum p_f' - \sum p_i\right)$$

$$= \lim_{T \to \infty, V \to \infty} \int_{-T/2}^{T/2} dt \int_V d^3x \; e^{ix(\sum p_f' - \sum p_i)}$$

$$\left(\delta_{T\nu}\left(\sum p_f' - \sum p_i\right)\right)^2 = (2\pi)^4 \delta^{(4)}\left(\sum p_f' - \sum p_i\right) *$$

$$\int_{-T/2}^{T/2} dt \int_V d^3\vec{x}\, \underbrace{e^{ix(\sum p_f' - \sum p_i)}}_{1}$$

$$= T\, V\, (2\pi)^4 \delta^{(4)}\left(\sum p_f' - \sum p_i\right)$$

$$W = \frac{|W_{fi}|^2}{T} = V (2\pi)^4 \delta^{(4)}\left(\sum p_f' - \sum p_i\right) \prod_i \frac{1}{2VE_i} \prod_f \frac{1}{2VE_f} * \prod_\ell (2m_\ell)\, |M|^2$$

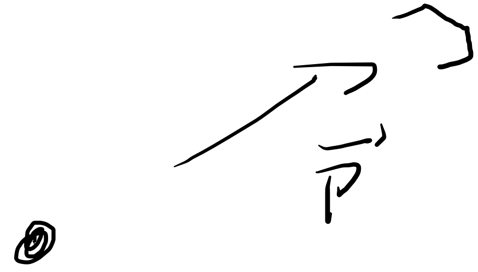
$$P'_f \quad , \quad P'_f + \delta P'_f$$

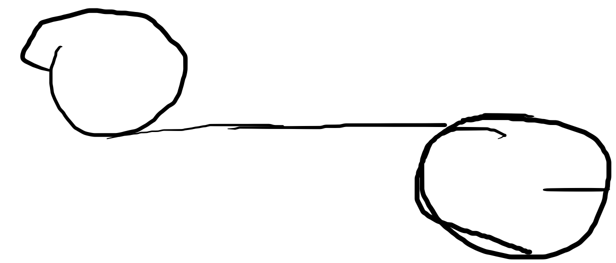
$$\frac{L}{\lambda} = n \qquad \frac{2\pi}{\lambda} = k$$

$$\vec{p} = \left(n_x \frac{2\pi}{L_x} \,,\; n_y \frac{2\pi}{L_y} \,,\; n_z \frac{2\pi}{L_z} \right) \qquad \frac{(2\pi)^3}{V}$$

$$d^3p \, \frac{V}{(2\pi)^3} \qquad \vec{p} \;,\; \vec{p} + d\vec{p}$$

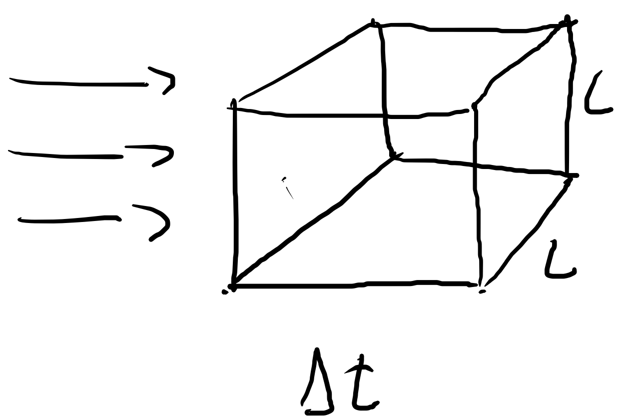
$$\boxed{\prod_f \frac{V \, d^3 p'_f}{(2\pi)^3}} \longleftarrow$$





πa^2

$$\vec{V}_1 - \vec{V}_2 = \vec{v}_{rel}$$



$v_{rel} L^2 \times$ densidade de part

= nº part que passaram

por L^2 em Δt

densidade $= \dfrac{1}{V}$

fluxo $= v_{rel} \dfrac{1}{V} = \dfrac{v_{rel}}{V}$

$$W \prod_f \frac{V d^3 p'_f}{(2\pi)^3} = \text{nº de partículas espalhadas}$$

$\vec{p}\,'_f$ e $\vec{p}\,'_f + d\vec{p}\,'_f$

$$d\sigma \text{ fluxo} = \boxed{d\sigma \frac{v_{rel}}{V} = W \prod_f \frac{V d^3 p'_f}{(2\pi)^3}}$$

$$d\sigma = W \frac{V}{v_{rel}} \prod_f V \frac{d^3 p'_f}{(2\pi)^3}$$

$$= \frac{V}{v_{rel}} \prod_f V \frac{d^3 p'_f}{(2\pi)^3} V (2\pi)^4 \delta^{(4)}\left(\sum p'_f - \sum p_i\right) \times$$

$$\times \prod_{i=1}^{2} \frac{1}{2VE_i} \prod_f \frac{1}{2VE_f} \prod_e 2m_e |M|^2$$

$$\frac{1}{64\pi^2}$$

$$d\sigma = (2\pi)^4 \delta^{(4)}\left(\sum p'_f - \sum p_i\right) \frac{1}{4E_1 E_2 v_{rel}} \prod_{f=1}^{2} \frac{d^3\vec{p}'_f}{(2\pi)^3 2E'_f} \prod_e 2m_e |M|^2$$

$$\vec{p} = \gamma m \vec{v} \qquad E = \gamma m \qquad \gamma = \frac{1}{\sqrt{1-v^2}} \quad (c=1)$$

$$\vec{v} = \frac{\vec{p}}{E}$$

$$\vec{v}_{rel} = \vec{v}_1 - \vec{v}_2 = \frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2}$$

$$\varepsilon = \pm 1$$

$$v_{rel}^2 = \frac{\vec{p}_1^{\,2}}{E_1^2} + \frac{\vec{p}_2^{\,2}}{E_2^2} - \frac{2|\vec{p}_1||\vec{p}_2|\,\varepsilon}{E_1 E_2}$$

$$= \frac{|\vec{p}_1|^2 E_2^2 + \vec{p}_2^{\,2} E_1^2 - 2\varepsilon|\vec{p}_1||\vec{p}_2| E_1 E_2}{(E_1 E_2)^2}$$

$$p_1 \cdot p_2 = E_1 E_2 - |\vec{p}_1||\vec{p}_2|\varepsilon$$

$$(p_1 \cdot p_2)^2 = E_1^2 (|p_2|^2 + m_2^2) + |\vec{p}_1|^2 (E_2^2 - m_2^2)$$

$$- 2\varepsilon |\vec{p}_1||\vec{p}_2| E_1 E_2$$

$$= (E_1 E_2)^2 v_{rel}^2 + \underbrace{E_1^2 m_2^2 - |\vec{p}_1|^2 m_2^2}_{m_1^2 m_2^2}$$

$$\boxed{(E_1 E_2)^2 v_{rel}^2 = (p_1 \cdot p_2)^2 - m_1^2 m_2^2}$$ inv. Lorentz

$$d\sigma = f(p_1', p_2')\,\delta^{(4)}(p_1 + p_2 - p_1' - p_2')\,d^3p_1'\,d^3p_2'$$

$$f = \frac{1}{64\pi^2 v_{rel} E_1 E_2 E_1' E_2'} \prod_i (2m_i)\,|M|^2$$

Integrar em $\vec{p}_2'$

$$d\sigma = f(p_1', p_2')\,\delta^{(1)}(E_1' + E_2' - E_1 - E_2)\,|\vec{p}_1'|^2\,d|\vec{p}_1'|\,d\Omega_1' \qquad \frac{d^3p_1'}{}$$

$$\vec{p}_2' = \vec{p}_1 + \vec{p}_2 - \vec{p}_1'$$

$$\frac{d(E_1' + E_2')}{\left(\frac{\partial}{\partial}\right)}$$

$$E_1' = \sqrt{|\vec{p}_1'|^2 + m_1'^2} \qquad E_2' = \sqrt{(\vec{p}_1 + \vec{p}_2 - \vec{p}_1')^2 + m_2'^2}$$

$$d(E_1' + E_2')\Big|_{\substack{\theta \\ \varphi \text{ fixed}}} = \frac{\partial (E_1' + E_2')}{\partial |\vec{p}_1'|} \, d|\vec{p}_1'|$$

$$\boxed{\underline{d\sigma} = f \, |\vec{p}_1'|^2 \underline{d\Omega_1'} \, \frac{1}{\dfrac{\partial (E_1' + E_2')}{\partial |\vec{p}_1'|}}}$$

$$\underline{\sigma_{r,x} E_1 E_2} \, \underline{E_1' E_2'}$$

CM

$\vec{p}_1' = -\vec{p}_2'$

$$E_1' = \sqrt{|\vec{p}_1'|^2 + m_1'^2} \qquad E_2' = \sqrt{\underbrace{(\vec{p}_1 + \vec{p}_2 - \vec{p}_1')^2}_{|\vec{p}_1'|^2} + m_2'^2}$$

$$\frac{\partial E_1'}{\partial |\vec{p}_1'|} = \frac{1}{2}\frac{2|\vec{p}_1'|}{E_1'} \qquad \frac{\partial E_2'}{\partial |\vec{p}_1'|} = \frac{1}{2}\frac{2|\vec{p}_1'|}{E_2'}$$

$$\frac{\partial (E_1' + E_2')}{\partial |\vec{p}_1'|} = |\vec{p}_1'| \frac{E_1' + E_2'}{E_1' E_2'}$$

CM

CM $\vec{p}_1 = -\vec{p}_2$ $\qquad \vec{p}_1{}' = -\vec{p}_2{}'$

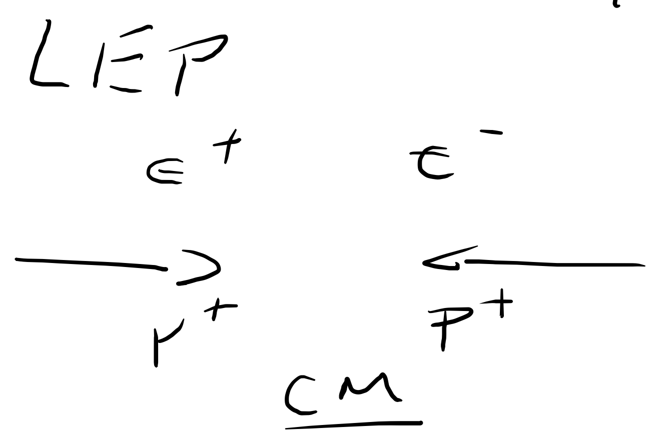
$$\vec{v} = \frac{\vec{p}}{E} \qquad v_{rel} = \frac{|\vec{p}_1|}{E_1} + \frac{|\vec{p}_2|}{E_2}$$

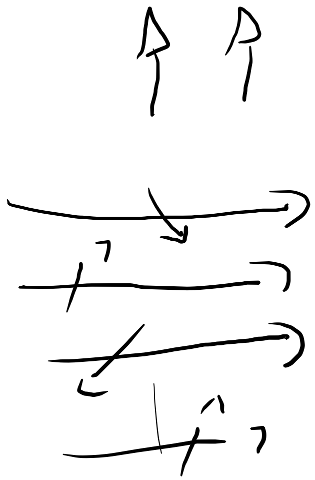
$$= |p_1| \frac{E_1 + E_2}{E_1 E_2}$$

$$v_{rel} E_1 E_2 = |p_1| \left(\underline{E_1 + E_2} \right)$$

$$\left(\frac{d\sigma}{d\Omega_1'}\right)_{CM} = \frac{1}{64\pi^2 (E_1+E_2)^2} \frac{|\vec{p}_1'|}{|\vec{p}_1|} \prod_{l} (2m_l)\, \underline{|M|^2}$$

$$\sigma^{Total}_{CM} = \underset{\text{identical}}{\frac{1}{2}} \int_{-1}^{1} d(\cos\theta_1') \int_0^{2\pi} d\phi_1' \left(\frac{d\sigma}{d\Omega_1'}\right)_{CM}$$



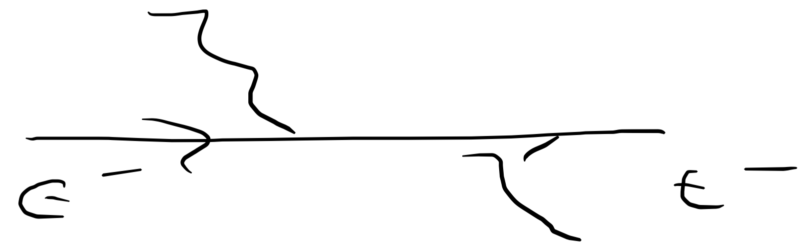


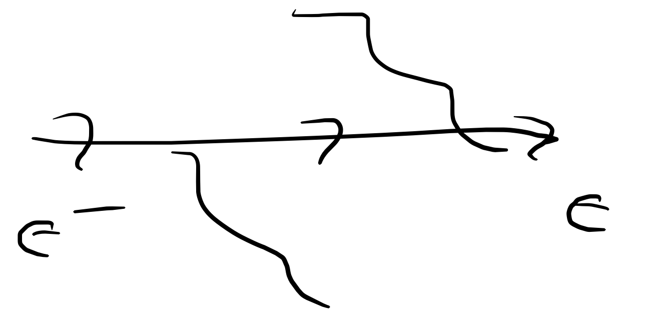
$$\frac{1}{2}\sum_{r=1}^{2} u_r(p)$$

$$\sum_{s=1}^{2}$$

$\frac{d\sigma}{d\Omega}$ não polarizada = média pol iniciais soma das pol. finais $\left(\frac{d\sigma}{d\Omega}\right)_{pol}$

$$\mathcal{M} = \underset{4\times 1}{\bar{u}_s(p')}\ \underset{4\times 4}{\Gamma}\ \underset{1\times 4}{u_r(p)}$$





$$X = \frac{1}{2} \sum_{r=1}^{2} \sum_{s=1}^{2} |\mathcal{M}|^2$$

$$\tilde{\Gamma} = \gamma^0 \Gamma^{\dagger} \gamma^0$$

$$\mathcal{M}^{\dagger} = u_r^{\dagger}(p)\, \Gamma^{\dagger} \gamma_0\, u_s(p') = \bar{u}_r(p)\, \tilde{\Gamma}\, u_s(p')$$

$$X = \frac{1}{2} \sum_{r,s=1}^{2} \underbrace{\left(\bar{u}_{s_\alpha}(p') \Gamma_{\alpha\beta} u_{r_\beta}(p)\right)}_{M} \underbrace{\left(\bar{u}_{r_\gamma}(p) \tilde{\Gamma}_{\gamma\delta} u_{s_\delta}(p')\right)}_{M^+}$$

$$= \frac{1}{2} \sum_{s} u_{s\delta}(p') \bar{u}_{s\alpha}(p') \Gamma_{\alpha\beta} \sum_{r} u_{r\beta}(p) \bar{u}_{r\gamma}(p) \tilde{\Gamma}_{\gamma\delta}$$

$$\Lambda^+_{\alpha\beta} = \sum_{r=1}^{2} u_{r\alpha}(p) \bar{u}_{r\beta}(p) = \left(\frac{\not{p} + m}{2m}\right)_{\alpha\beta}$$

$$\boxed{X = \frac{1}{2} \mathrm{Tr}\left(\Lambda^+(p') \Gamma \Lambda^+(p) \tilde{\Gamma}\right)}$$

$$M = \bar{v}_s(p')\,\Gamma\,v_r(p)$$

$$\rightarrow \quad M = \bar{u}_s(p')\,\Gamma\,v_r(p) \qquad \tilde{\Gamma} = \gamma^0 \Gamma^+ \gamma^0$$

$$M = \underline{\bar{v}_s(p')}\,\Gamma\,\underline{u_r(p)}$$

$$\Lambda^-_{\alpha\beta} = -\sum_{r=1}^{2} v_{r\alpha}(p)\,\bar{v}_{r\beta}(p) = -\left(\frac{\not{p} - m}{2m}\right)_{\alpha\beta}$$

$$\frac{1}{2}\sum_{r,s} \underbrace{\bar{u}_{s\alpha}(p')\hat{T}_{\alpha\beta}v_{r\beta}}_{M} \quad \underbrace{\bar{v}_{r\gamma}(p)\tilde{T}_{\gamma\delta}u_{s\delta}(p')}_{M^+}$$

$$= \frac{1}{2}\, \overbrace{u_{s\delta}(p')\bar{u}_{s\alpha}}\, \hat{T}_{\alpha\beta}\, v_{r\beta}(p)\bar{v}_{r\gamma}(p)\, \tilde{T}_{\gamma\delta}$$

$$= \frac{1}{2}\operatorname{Tr}\left(\Lambda^+(p')\,\hat{T}\,\bar{\Lambda}(p)\,\tilde{T}\right)$$

$$Y = \left| \bar{u}_2(p') \Gamma u_1(p) \right|^2$$

$\longrightarrow$ 1 $\qquad \longrightarrow$ 2

$$= \bar{u}_2(p') \Gamma u_1(p) \bar{u}_1(p) \tilde{\Gamma} u_2(p')$$

$$\Pi^{\pm}(p) = \frac{1}{2}(1 \pm \sigma_p) \qquad \sigma_p = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

$$\Pi^{\pm}(p) u_r(p) = \delta_{1r,\,2r} u_r(p)$$

$$Y = \sum_{s,r} (\bar{u}_{2s}(p') \Gamma \Pi^+(p) u_{1r}(p))(\bar{u}_{1r}(p) \tilde{\Gamma} \Pi^-(p') u_{2s}(p'))$$

$$Y = T_r\left(\Lambda^+(p') \Gamma \Pi^+(p) \Lambda^+(p) \tilde{\Gamma} \Pi^-(p')\right)$$

$$M = e^2 M^{(2)} + e^{(4)} M^{(4)} + \ldots$$

$$\pi^{\pm} \rightarrow \frac{1}{2}\left(1 \pm \gamma^{5}\right)$$

altas energias