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Microeconomia II
Programa de Pós-Graduação em Economia
FEARP - USP

Seção 23.B - Mechanism Design

Exercise 1. Considere a Função de Escolha Social (FES) $f : \Theta \mapsto \mathbb{X}$ tal que

$$f(\theta) = [k(\theta), t_1(\theta), \cdots, t_I(\theta)],$$

para cada $\theta \in \Theta$ para cada um dos casos a seguir:

(a) Projeto Público (exemplo 23.B.3):

$$\mathbb{X} = \{(k, t_1, t_2, \cdots, t_I) : k \in \{0,1\}, t_i \in \mathbb{R}, \text{ and } \sum_i t_i \leqslant -ck\}$$

Mostre que $f(\cdot)$ é *ex post* eficiente se $\forall \theta \in \Theta$,

(i) $k(\theta)$ satisfaz

$$k(\theta) = \begin{cases} 1 & \text{se } \sum_i \theta_i \geqslant c \\ 0 & \text{caso contrário} \end{cases} \tag{23.B.1}$$

(ii) $t_i(\theta)$ satisfaz

$$\sum_i t_i(\theta) = -ck(\theta) \tag{23.B.2}$$

(b) Alocação de bem privado indivisível (exemplo 23.B.4):

$$\mathbb{X} = \left\{(y_1, \cdots, y_I, t_1, \cdots, t_I) : y_i \in \{0,1\} \text{ and } t_i \in \mathbb{R}, \forall i, \sum_{i=1}^{I} y_i = 1 \text{ and } \sum_{i=1}^{I} t_i \leqslant 0\right\}$$

Mostre que $f(\cdot)$ é *ex post* eficiente se $\forall \theta \in \Theta$,

$$y_i(\theta)\left[\theta_i - \max_j\{\theta_j\}\right] = 0, \quad \forall i$$

$$\sum_{i=1}^{I} t_i(\theta) = 0$$

Exercise 2. Suppose that two agents collectively choose from $X = \{x, y, z\}$. Each agent can be of two types, so $\Theta_1 = \{\theta_1', \theta_1''\}$ and $\Theta_2 = \{\theta_2', \theta_2''\}$. Preferences are given by:

$$x \succ_1^{\theta_1'} y \succ_1^{\theta_1'} z \qquad y \succ_1^{\theta_1''} z \succ_1^{\theta_1''} x$$

$$z \succ_2^{\theta_2'} x \succ_2^{\theta_2'} y \qquad y \succ_2^{\theta_2''} x \succ_2^{\theta_2''} z$$

Find all ex-post efficient social choice function. Which of those are truthfully implementable?

Exercise 3 (MWG 23.B.2). . Consider a bilateral trade setting (see Examplr 23.B.4) in which both the seller's (agent 1) and the buyer's (agent 2) types are drawn independently from the uniform distribution on [0,1]. Suppose that we try to implement the social choice function $f(.) = (y_1(.)y_2(.), t_1(.), t_2(.))$ such that

$$\begin{aligned} y_1(\theta) &= \begin{cases} 1 & \text{if } \theta_1 \geqslant \theta_2 \\ 0 & \text{if } \theta_1 < \theta_2 \end{cases} \\ y_2(\theta) &= \begin{cases} 1 & \text{if } \theta_1 < \theta_2 \\ 0 & \text{if } \theta_1 \geqslant \theta_2 \end{cases} \\ t_1(\theta) &= \frac{1}{2}(\theta_1 + \theta_2) y_2(\theta_1, \theta_2) \\ t_2(\theta) &= -\frac{1}{2}(\theta_1 + \theta_2) y_2(\theta_1, \theta_2) \end{aligned}$$

Suppose that the seller truthfully reveals his type for all $\theta_1 \in [0,1]$. Will the buyer find it worthwhile to reveal his type? Interpret.

Exercise 4 (MWG 23.B.3). .Show that $b_i(\theta_i) = \theta_i \ forall \ \theta_i \in [0,1]$ is weakly dominant strategy for each agent i in the second-price sealed-bid auction.

Exercise 5 (MWG 23.B.4). .Consider a bilateral trade setting (see Example 23.B.4) in which both the seller's and the buyer's types are drawn independently from the uniform distribution on [0,1].

(a) Consider the double auction mechanis in which the seller (agent 1) and buyer (agent 2) each subimit a sealed bid, $b_i \geqslant 0$. If $b_1 \geqslant b_2$, the seller keeps the good and no monetary transfer is made; while if $b_2 > b_1$, the buyer gets the good and pays the seller the amount $\frac{1}{2}(b_1 + b_2)$. (The interpretation is that the seller's bid is his minimum acceptable price, while the buyer's is his maximum acceptable price splits the difference between these amounts.) Solve for a Bayesian Nash equilibrium of this game in which each agent $i's$ strategy takes the form $b_i(\theta_i) = \alpha_i + \beta_i \theta_i$. What social choice function does this equilibrium of this mechanism implement? Is it ex post efficient?

(b) Show that the social choice function derived in **(a)** is incentive compatible; that is, that it can be truthfully implemented in Bayesian Nash equilibrium.

Exercise 6. Consider the allocation model of a single unit of an indivisible private good among two agents. One of them, the seller, possesses the good. The other agent is the buyer. The seller can have two valuations for the good, c_0 and c_1, with equal probabilities ($c_1 > c_0 > 0$). When the seller has valuation c_i, the buyer has valuation v_i, with $v_i > c_i$ for $i \in \{0, 1\}$. The seller knows his and the buyer's type as well. The buyer does not know his type (or only knows it after the mechanism is defined). Both are risk neutral with relation to monetary values and the good. So, given a probability x of the buyer getting the good, and a transfer t made by him, the utilities of a buyer and a seller of an specific type are, respectively:

$$v_i - t$$

$$t - c_i$$

Consider that $\frac{v_1+v_0}{2} < c_1$. Show that incentive compatibility and individual rationality are not consistent with the ex-post efficiency of the mechanism.

Exercise 7. Considere a FES $\tilde{f}(.)$ tal que $\tilde{f}(\theta) = (\tilde{y}_0(\theta), \tilde{y}_1(\theta), ..., \tilde{y}_I(\theta), \tilde{t}_0(\theta), \tilde{t}_1(\theta), ..., \tilde{t}_I(\theta))$ e

$$\begin{aligned}
\tilde{y}_1(\theta) &= \begin{cases} 1 & \text{se } \theta_1 \geqslant \theta_2 \\ 0 & \text{se } \theta_1 < \theta_2 \end{cases} \\
\tilde{y}_2(\theta) &= \begin{cases} 1 & \text{se } \theta_1 < \theta_2 \\ 0 & \text{se } \theta_1 \geqslant \theta_2 \end{cases} \\
\tilde{y}_0(\theta) &= 0, \quad \forall \theta \\
\tilde{t}_1(\theta) &= -\theta_2 \tilde{y}_1(\theta) \\
\tilde{t}_2(\theta) &= -\theta_1 \tilde{y}_2(\theta) \\
\tilde{t}_0(\theta) &= -[\tilde{t}_1(\theta) + \tilde{t}_2(\theta)]
\end{aligned}$$

Em que o ganhador do leilão paga a "segunda maior valoração" se recebe o bem. Neste caso o jogador $i = 1$ fala a verdade sob $\tilde{f}(.)$?