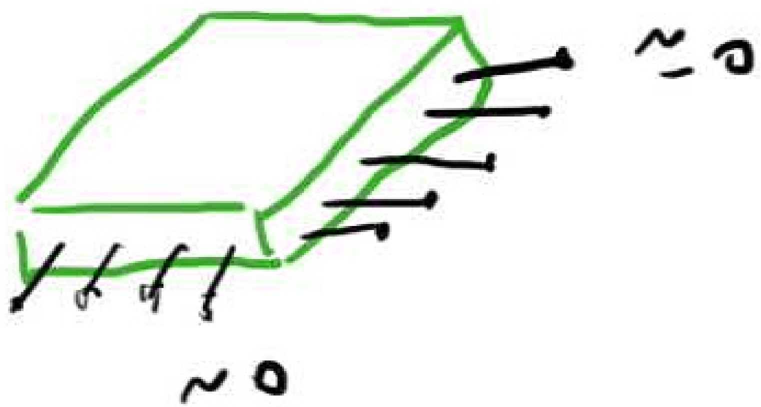
4) FLEXÃO DE PLACAS

HIPÓTESES BÁSICAS

- MATERIAL LINEAR-ELÁSTICO, HOMOGÊNEO E ISOTRÓPICO
- PEQUENOS DESLOCAMENTOS w

$$w < t$$

- TENSÕES MEMBRANA NULAS

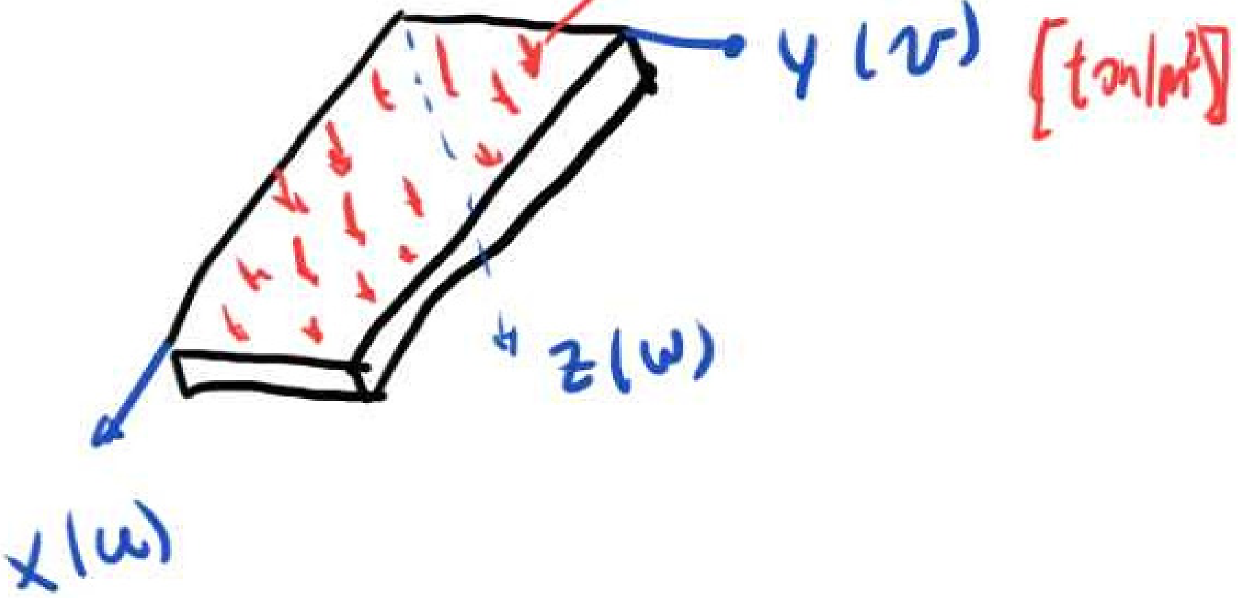


HIPÓTESE KIRCHOFF

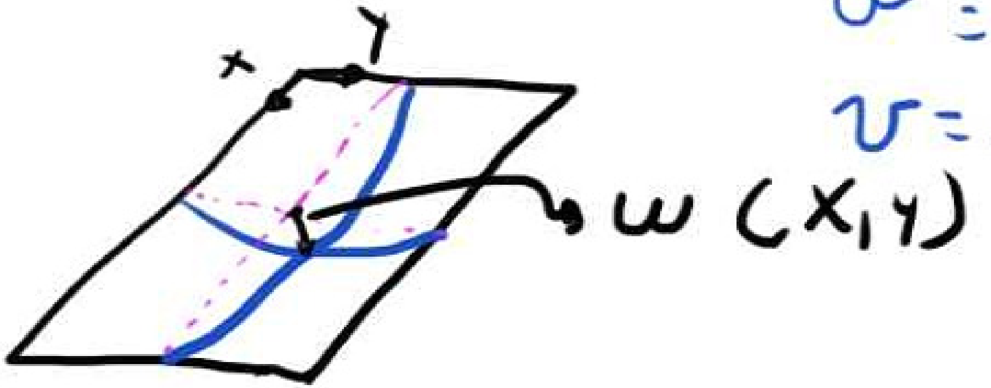
- SEGMENTO DE RETA NORMAL SUP. MÉDIA INICIAL (SEM CARGA) SE CONSERVA NORMAL A SUP MÉDIA APÓS A DEFORMAÇÃO, PERMANECENDO RETA E COM O MESMO COMPRIMENTO

- CARGAS EXTERNAS

$q(x,y) \longrightarrow w(x,y)$ [mm]



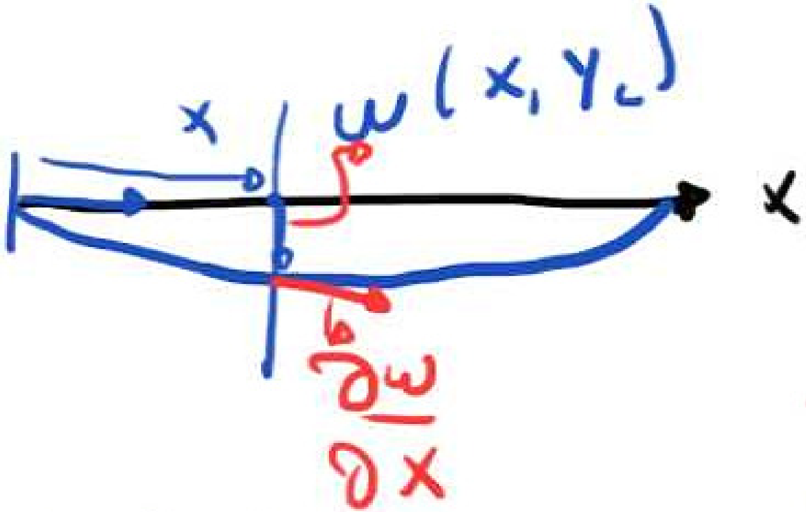
• CAMPO DE DESLOCAMENTOS



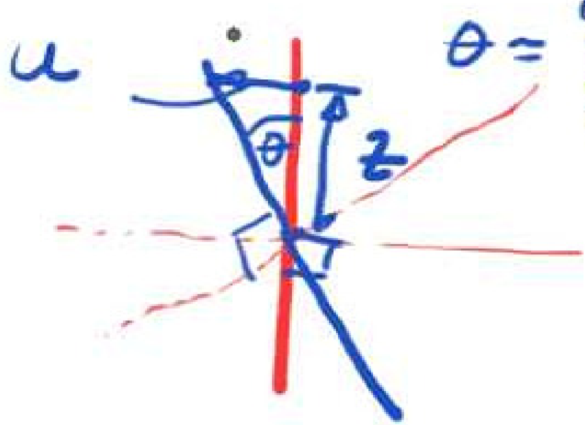
$$u = f(x, y, z)$$

$$v = f(x, y, z)$$

$$\boxed{u = -z \frac{\partial w}{\partial x}} \quad (1)$$

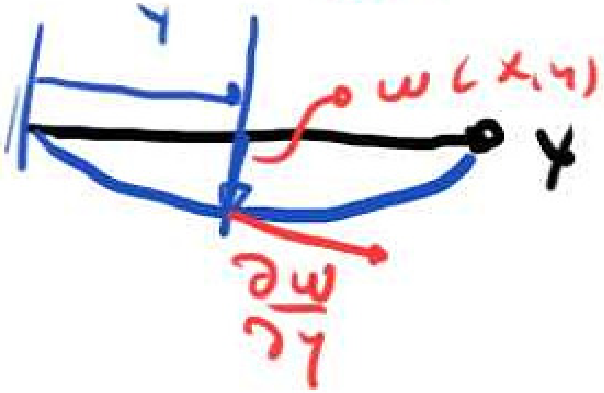


$$u = -z\,\theta$$



$$\theta = \frac{\partial w}{\partial x}$$

$\rightarrow u$



$\rightarrow v$

$$\boxed{v = -z \frac{\partial w}{\partial y}} \quad (2)$$

· Relação ε-deslocamentos

Def

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y}$$

$$\varepsilon_x = -z \frac{\partial^2 w}{\partial x^2} \quad (4) \qquad \varepsilon_y = -z \frac{\partial^2 w}{\partial y^2} \quad (5)$$

Deformação angular

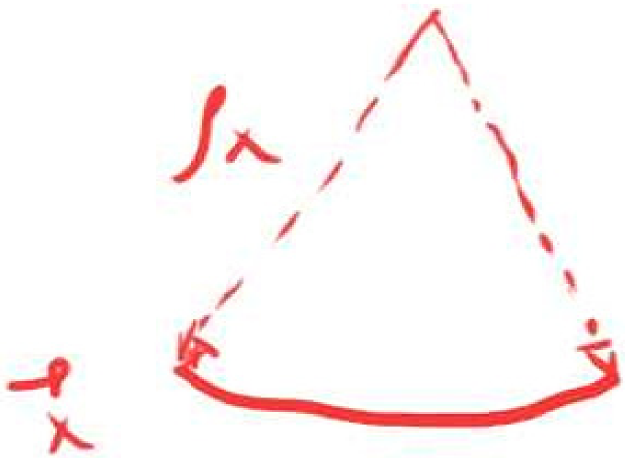
$$\gamma_{xy} = \gamma_1 + \gamma_2$$

$$\gamma_{xy} = \partial u + \partial v$$

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y} \qquad (6)$$

- CURVATURA (RAIO DE CURVATURA ρ)

K



$$\varepsilon_x = -\frac{z}{\rho_x} \qquad \varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{\rho_x} = K_x$$

$\varepsilon_x = -z K_x \qquad \varepsilon_y = -z K_y \qquad \gamma_{xy} = -2z K_{xy}$

Eq. constitutiva

$\varepsilon_{ij} = C \sigma_{ij}$

(ESTADO PLANO TENSÃO)

$\sigma_z = 0$

$\leftarrow t \ll a, b$

$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu \sigma_y]$

$\rightarrow \sigma \propto \varepsilon$

$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu \sigma_x]$

$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu (\sigma_x + \sigma_y)] \rightarrow \varepsilon_z = -\frac{\nu}{E} (\sigma_x + \sigma_y)$

INVERTENDO AS RELAÇÕES $\varepsilon - \sigma$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\,\varepsilon_y\right) \qquad \sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\,\varepsilon_x\right)$$

? $w = f(x,y)$

$$\sigma_x = -\frac{zE}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad (7) \qquad \sigma_y = -\frac{zE}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad (8)$$

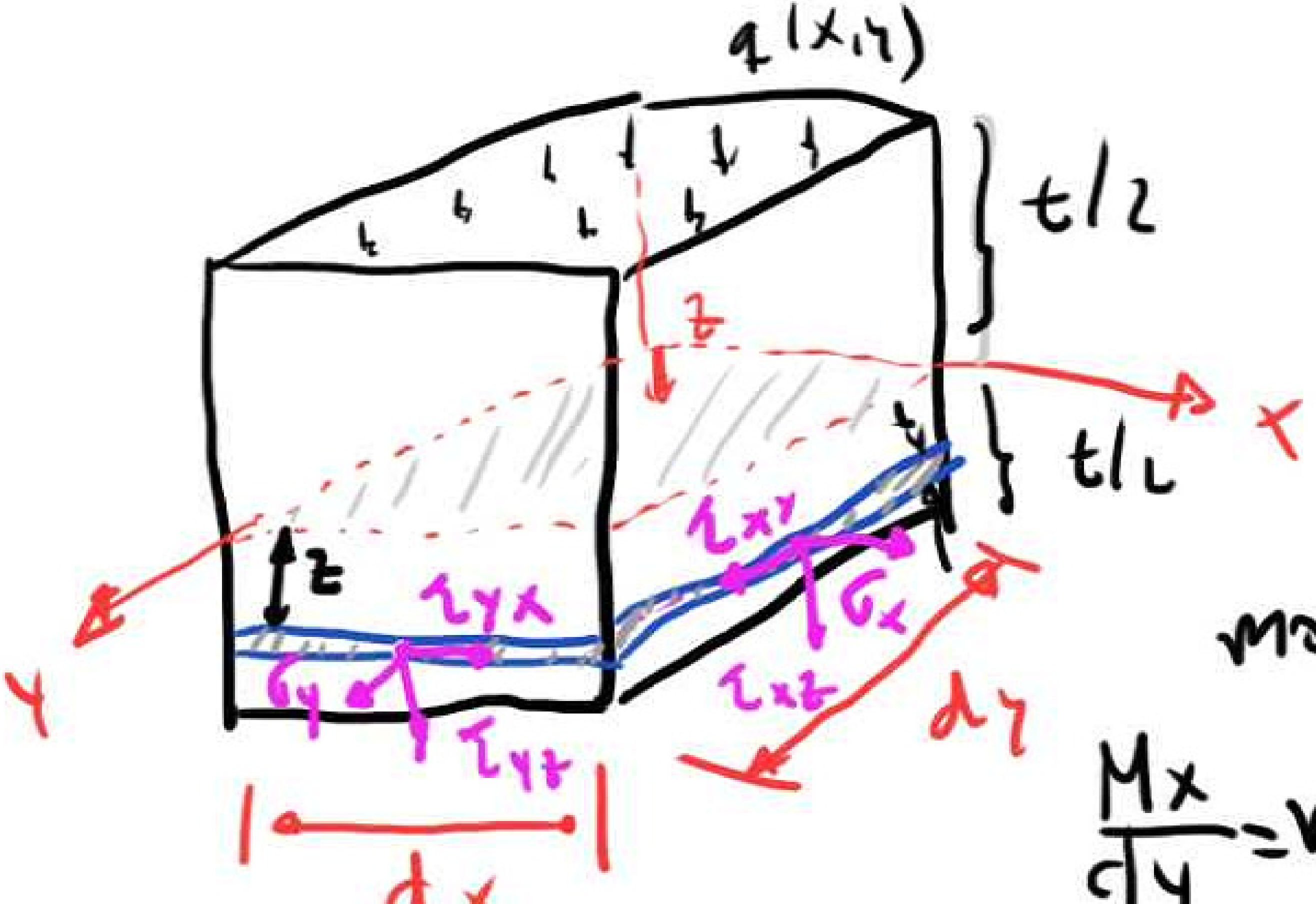
$$\tau_{xy} = G\,\gamma_{xy} = \frac{E}{2(1+\nu)}\left(-2z\frac{\partial^2 w}{\partial x\partial y}\right)$$

$$\tau_{xy} = -\frac{zE}{(1+\nu)}\frac{\partial^2 w}{\partial x\partial y} \quad (9)$$

· EQUILÍBRIO $\sum F_x \quad \sum F_y \quad \sum F_z = 0$

$\sum M_x \quad \sum M_y \quad \sum M_z = 0$

· TENSÕES ATUANTES → ESFORÇOS INTERNOS



tensão σ_x

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, dy \, dz \, z$$

momento

$$\frac{M_x}{dy} = m_x = \int_{-t/2}^{t/2} z \, \sigma_x \, dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ M_{yx} \end{Bmatrix} = \int_{-\frac{t}{2}}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yx} \end{Bmatrix} z\,dz$$

$w = f(x,y)$

INTEGRAÇÃO

$$M_x = \int_{-t/2}^{t/2} \overbrace{-\left(\frac{Ez}{1-\nu^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \partial\,\frac{\partial^2 w}{\partial y^2}\right)}^{\sigma_x} z\,dz$$

$$m_x = -\frac{E}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right)\underbrace{\int_{-t/2}^{t/2} z^2\,dz}_{\frac{t^3}{12}}$$

$$\frac{z^3}{3}\Bigg|_{-t/2}^{t/2}$$

$$\frac{1}{3}\left[\frac{t^3}{8} - \left(-\frac{t^3}{8}\right)\right]$$

$$\frac{t^3}{12}$$

$$m_x = \boxed{-\frac{Et^3}{12(1-\nu^2)}}\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right)$$

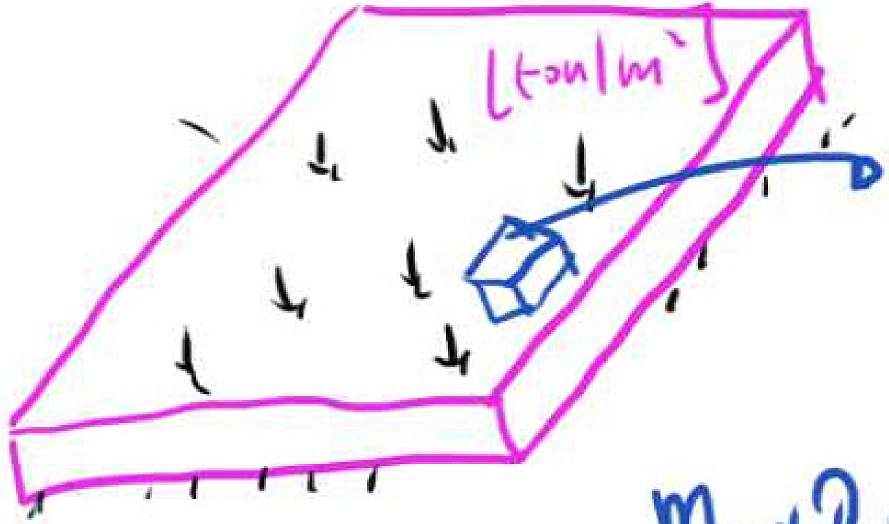
↳ D = RIGIDEZ FLEXIONAL DA PLACA

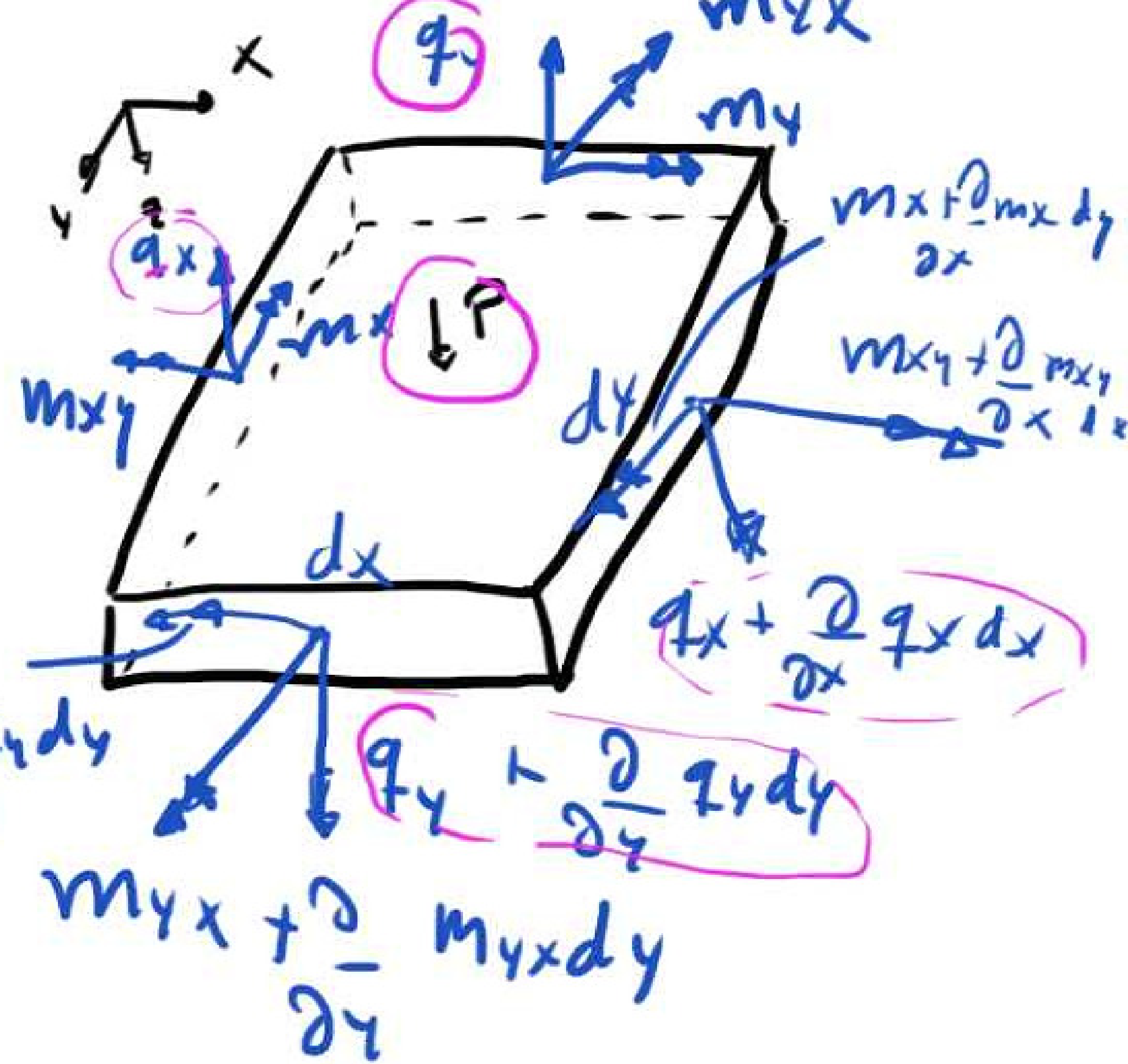
$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \qquad 9.1$$

$$m_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \qquad 9.2$$

$$m_{xy} = -D\,(1-\nu)\,\frac{\partial^2 w}{\partial x \partial y} \qquad 9.3$$

EQUILIBRIO





- $\sum F_z = 0 \; (\downarrow +)$

$$-q_x\, dy + \left(q_x + \frac{\partial}{\partial x} q_x\, dx\right) dy - q_y\, dx + \left(q_y + \frac{\partial}{\partial y} q_y\, dy\right) dx$$
$$+ P\, dx\, dy = 0$$

$$\frac{\partial}{\partial x} q_x \, dx \, dy + \frac{\partial}{\partial y} q_y \cdot dx \, dy + p \, dx \, dy = 0$$

$$dx \, dy \neq 0$$

$$\Downarrow$$

$$\sum F_z = 0$$

$$\Downarrow$$

$$\frac{\partial}{\partial x} q_x + \frac{\partial}{\partial y} q_y + p = 0 \qquad (10)$$

$$\sum M_x = 0$$

$$(p\, dx\, dy)\frac{dy}{2} - q_x\, dy \cdot \frac{dy}{2} + \left(q_x + \frac{\partial}{\partial x} q_x dx\right) dy \cdot \frac{dy}{2}$$

$$+ \left(q_y + \frac{\partial}{\partial y} q_y\, dy\right) dx \frac{dy}{2} + m_y\, dx - \left(m_y + \frac{\partial}{\partial y} m_y\, dy\right) dx$$

$$- \left(m_{xy} + \frac{\partial}{\partial x} m_{xy}\, dx\right) dy + m_{xy}\, dy = 0$$

& simplificand

$$(dy)^2 \simeq 0$$

$$q_y\, dx\, dy - \frac{\partial}{\partial y} m_y\, dy\, dx - \frac{\partial}{\partial x} m_{xy}\, dx\, dy = 0$$

$$q_y = \frac{\partial}{\partial y} m_y + \frac{\partial}{\partial x} m_{xy} \qquad (11)$$

$$\sum M_y = 0 \; (\cancel{\circlearrowleft}+)$$

$$q_x = \frac{\partial}{\partial x} m_x + \frac{\partial}{\partial y} m_{yx} \qquad (12)$$

Eq. 10:

$$\frac{\partial}{\partial x} q_x + \frac{\partial}{\partial y} q_y + p = 0$$

Eq. 11

Eq. 12

$$\frac{\partial^2}{\partial x \partial y} m_x + 2 \frac{\partial^2}{\partial x \partial y} m_{xy} + \frac{\partial^2}{\partial y^2} m_y = -p \quad (13)$$

$$m_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \leftarrow \text{Eq. 9.1}$$

$$\frac{\partial}{\partial x} m_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \nu \frac{\partial}{\partial x} \frac{\partial^2 w}{\partial y^2}\right)$$

$$\frac{\partial^2}{\partial x^2} m_x = -D\left(\frac{\partial^4 w}{\partial x^4} + \nu \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2}\right)\right) = -D\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial \ldots}\right.$$

$$\boxed{\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D}}$$

Eq. LAGRANGE

$$\nabla^4 w = P/D \rightarrow \left.\begin{matrix} M_x \\ M_y \\ M_{xy} \\ q_x \\ q_y \end{matrix}\right\} \rightarrow \begin{matrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{matrix}$$

Governa flexão de placas finas

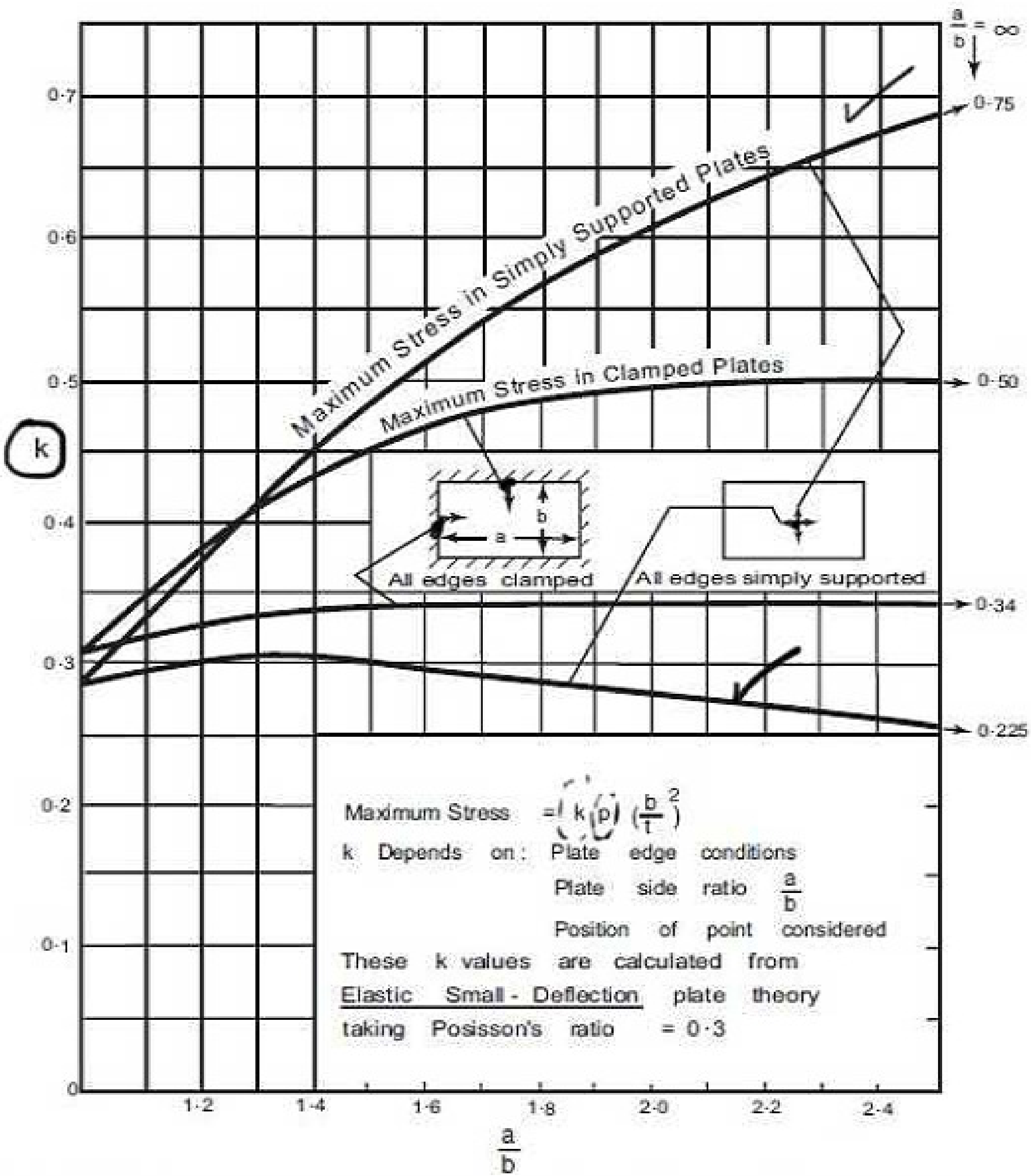
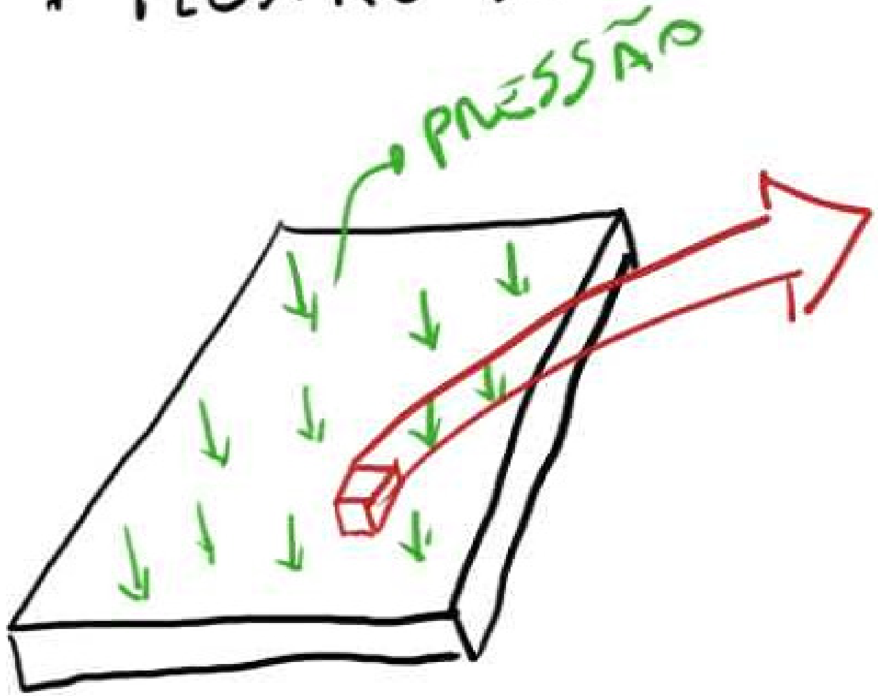


Figure 9.6 Maximum stresses in rectangular plates under uniform lateral pressure.

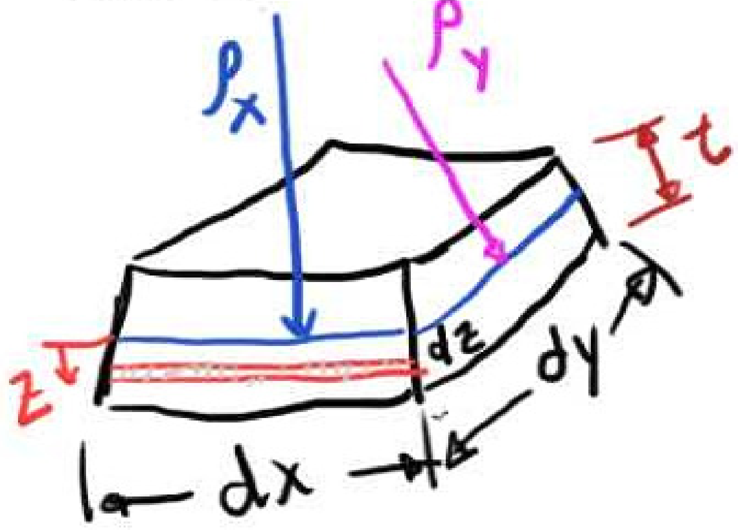
MAX. TENSÕES
NORMAIS

$k = f(a/b)$ | contorno
posição

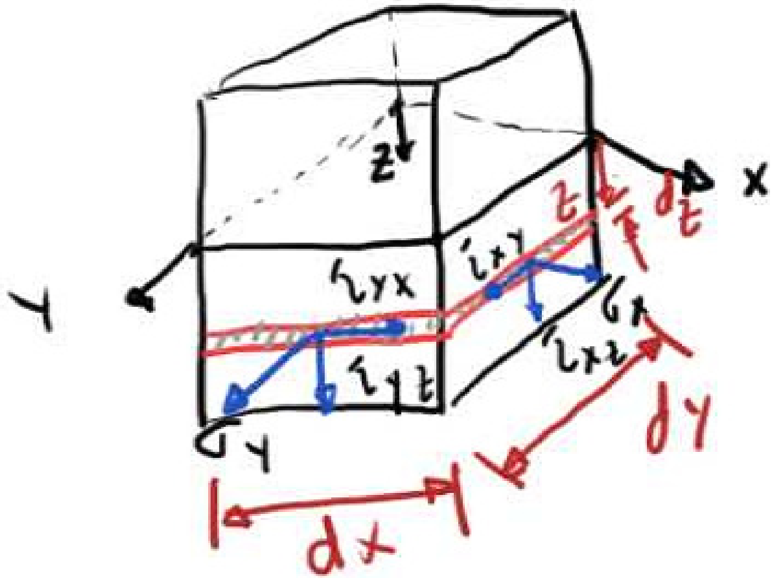
FLEXÃO DE PLACAS



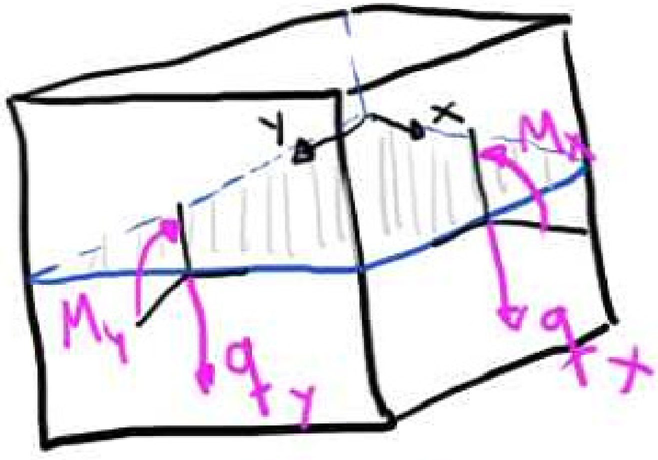
ELEMENTO DIFERENCIAL

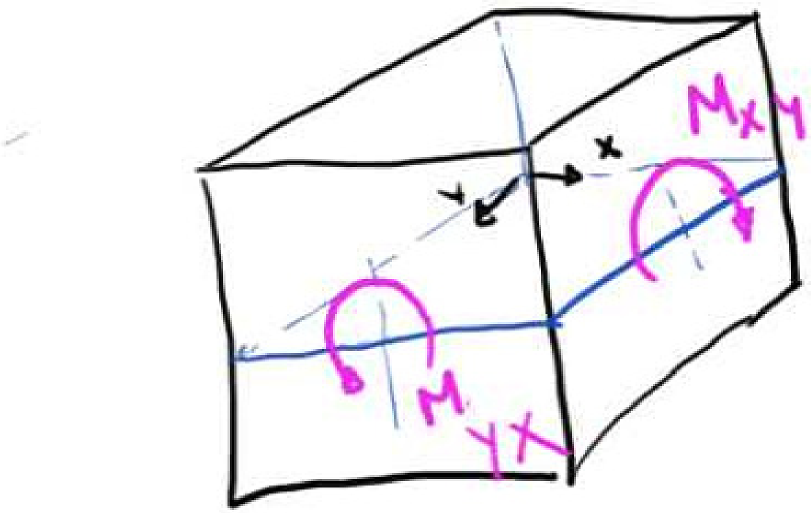


TENSÕES

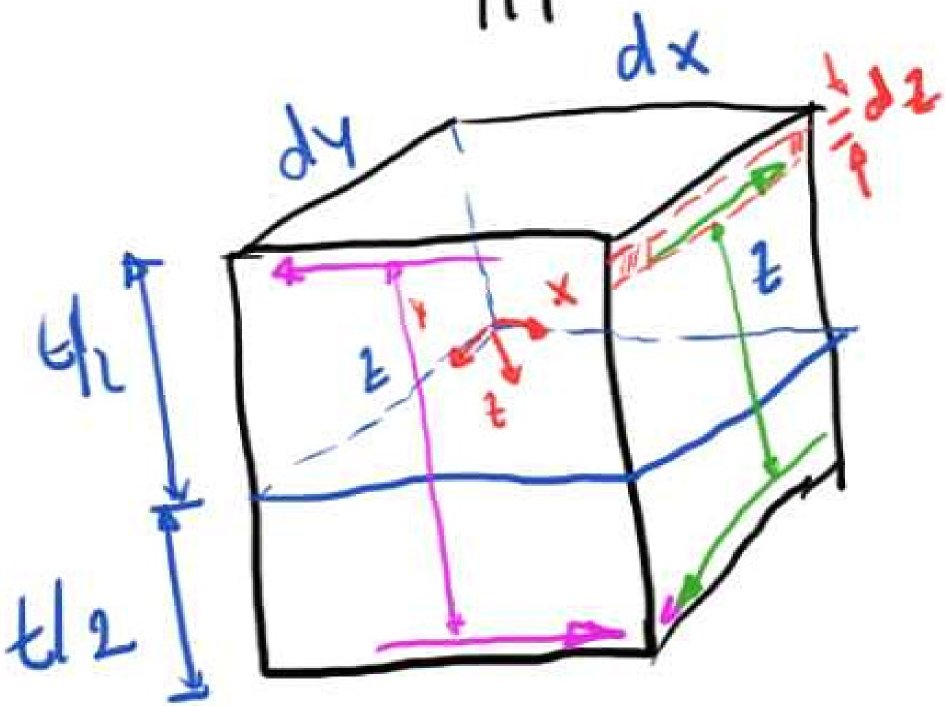


ESFORÇOS





$\equiv$



$M_{xy} \equiv$ momento torsor

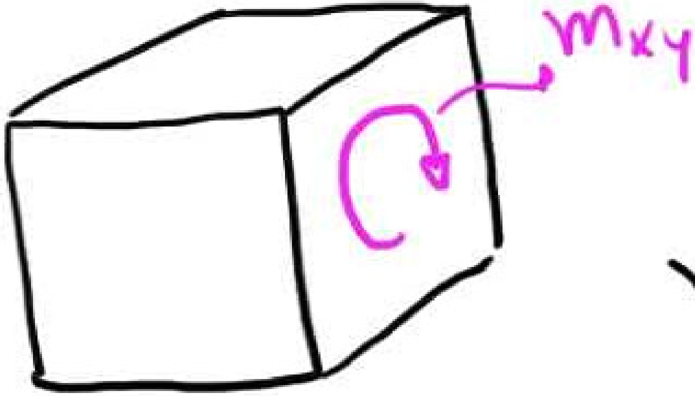
$$M_{xy} = \int_{-t/2}^{t/2} \underbrace{z}_{\text{braço}} \cdot \underbrace{\tau_{xy} \overbrace{dy\, dz}^{dA}}_{dF}$$

$$\downarrow$$

$$\frac{M_{xy}}{dy} = m_{xy}$$

$$m_{xy} = \int_{-t/2}^{t/2} z\, \tau_{xy}\, dz$$

$$\tau_{xy} = -\frac{E z}{(1+\nu)} \frac{\partial^2 \omega}{\partial x \partial y}$$



$$m_{xy} = -\frac{E}{(1+\nu)} \frac{\partial^2 w}{\partial x \partial y} \int_{-t/2}^{t/2} z^2 \, dz$$

$$\downarrow$$

$$\frac{M_{xy}}{dy} = m_{xy} = -\frac{E}{(1+\nu)} \frac{\partial^2 w}{\partial x \partial y} \left. \frac{z^3}{3} \right|_{-t/2}^{t/2}$$

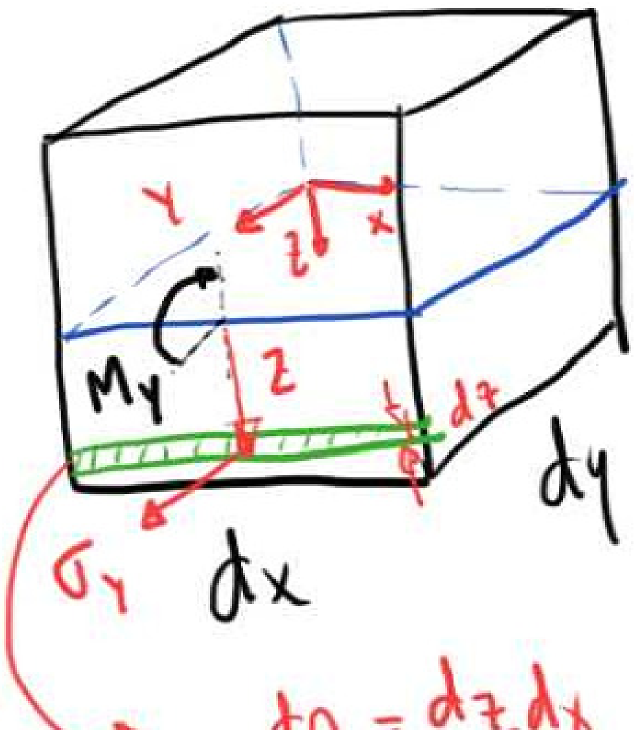
$$m_{xy} = -\frac{E}{(1+\nu)} \frac{\partial^2 w}{\partial x \partial y} \frac{1}{3} \left[\frac{t^3}{8} - \left(-\frac{t^3}{8} \right) \right]$$

$$m_{xy} = -\frac{E}{(1+\nu)} \frac{\partial^2 w}{\partial x \partial y} \cdot \frac{t^3}{12} \times \left(\frac{1-\nu}{1-\nu} \right)$$

$$m_{xy} = -\frac{Et^3}{(1-\nu^2)} \cdot (1-\nu) \frac{\partial^2 w}{\partial x \partial y}$$

$$m_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \qquad (9.3)$$

· FLEXÃO EM TORNO DO EIXO "X"
(MOMENTO M_y)



$$dM_y = \overbrace{dF}^{\sigma_y\, dA} \cdot \overbrace{\text{braço}}^{z}$$

$$dM_y = \sigma_y\, dA \cdot z$$

$$dM_y = \sigma_y \cdot dx\,dz \cdot z$$

$$\frac{dM_y}{dx} = m_y = \int^{t/2} \sigma_y \cdot dx\, z$$

$$m_y = \int_{-t/2}^{t/2} \underbrace{-\frac{Ez}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)} z\, dz$$

LEI DE HOOKE (ESTADO PLANO TENSÃO)

$$\downarrow$$

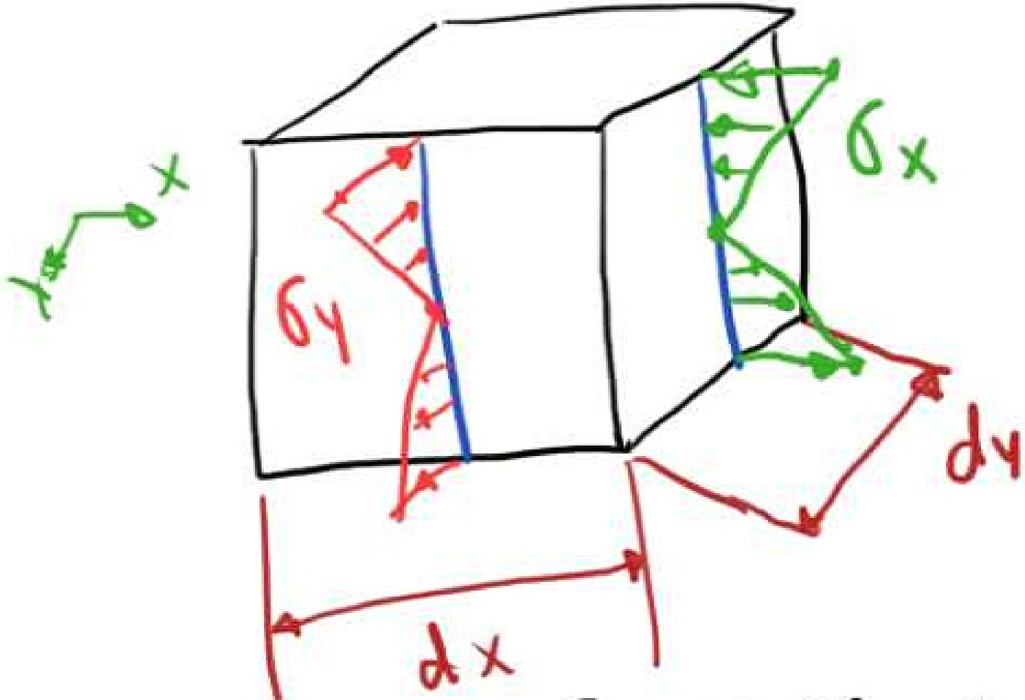
$$m_y = -\frac{E}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \underbrace{\int_{-t/2}^{t/2} z^2\, dz}_{\frac{t^3}{12}}$$

$$\downarrow$$

$$m_y = -\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \leftarrow \frac{t^3}{12}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

· TENSÕES NA ESPESSURA



$$\sigma_y = -\frac{zE}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

$\sigma_y \propto z$ VARIAÇÃO LINEAR

$$\sigma_y = -\frac{zE}{(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \times \frac{t^3}{t^3} \cdot \frac{12}{12}$$

$$\sigma_y = \frac{12\, M_y}{t^3} \cdot z$$

$$\sigma_x = \frac{12\, m_x}{t^3} \cdot z \qquad \tau_{xy} = \frac{12\, m_{xy}}{t^3} \cdot z$$

RESUMINDO

ESFORÇOS INTERNOS

$$m_x = -D\,(K_x + \nu K_y) \quad m_y = -D\,(K_y + \nu K_x) \quad m_{xy} = \frac{-D}{(1-\nu)^{-1}} K_{xy}$$

$$D = \frac{E t^3}{12(1-\nu^2)}$$

Curvaturas $K_x = \frac{\partial^2 w}{\partial x^2}$ $K_y = \frac{\partial^2 w}{\partial y^2}$ $K_{xy} = \frac{\partial^2 w}{\partial x \partial y}$

TENSÕES $\sigma_x = \frac{12\, m_x}{t^3} z$ $\sigma_y = \frac{12\, m_y}{t^3} \cdot z$ $\tau_{xy} = \frac{12\, m_{xy}}{t^3} z$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D}$$

* $w(x,y) <<$ small

$w(x,y) < t$!

· BORDAS SUPORTADAS

$$p(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin m\pi \frac{y}{a} \sin n\pi \frac{x}{a}$$

↳ pressão uniforme p_0

$$A_{mn} = \frac{16\, p_0}{\pi^2 mn}$$

Resposta

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin m\pi \frac{y}{a} \sin n\pi \frac{x}{a}$$

↳ substituir na eq. dif $\nabla^4 w = P/D$

↓

$$B_{mn} \left(\pi^4 \frac{m^4}{a^4} + 2\pi^4 \frac{m^2 n^2}{a^2 b^2} + \pi^4 \frac{n^4}{b^4} \right) = \frac{16 P_0}{\pi^2 mn D}$$

↓

$$B_{mn} = \frac{16 P_0}{\pi^6 mn D \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2}$$

$$w(x,y) = \sum \sum \frac{16P_0}{\pi^6 mnD\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \sin m\pi\frac{y}{a} \sin n\pi\frac{x}{b}$$

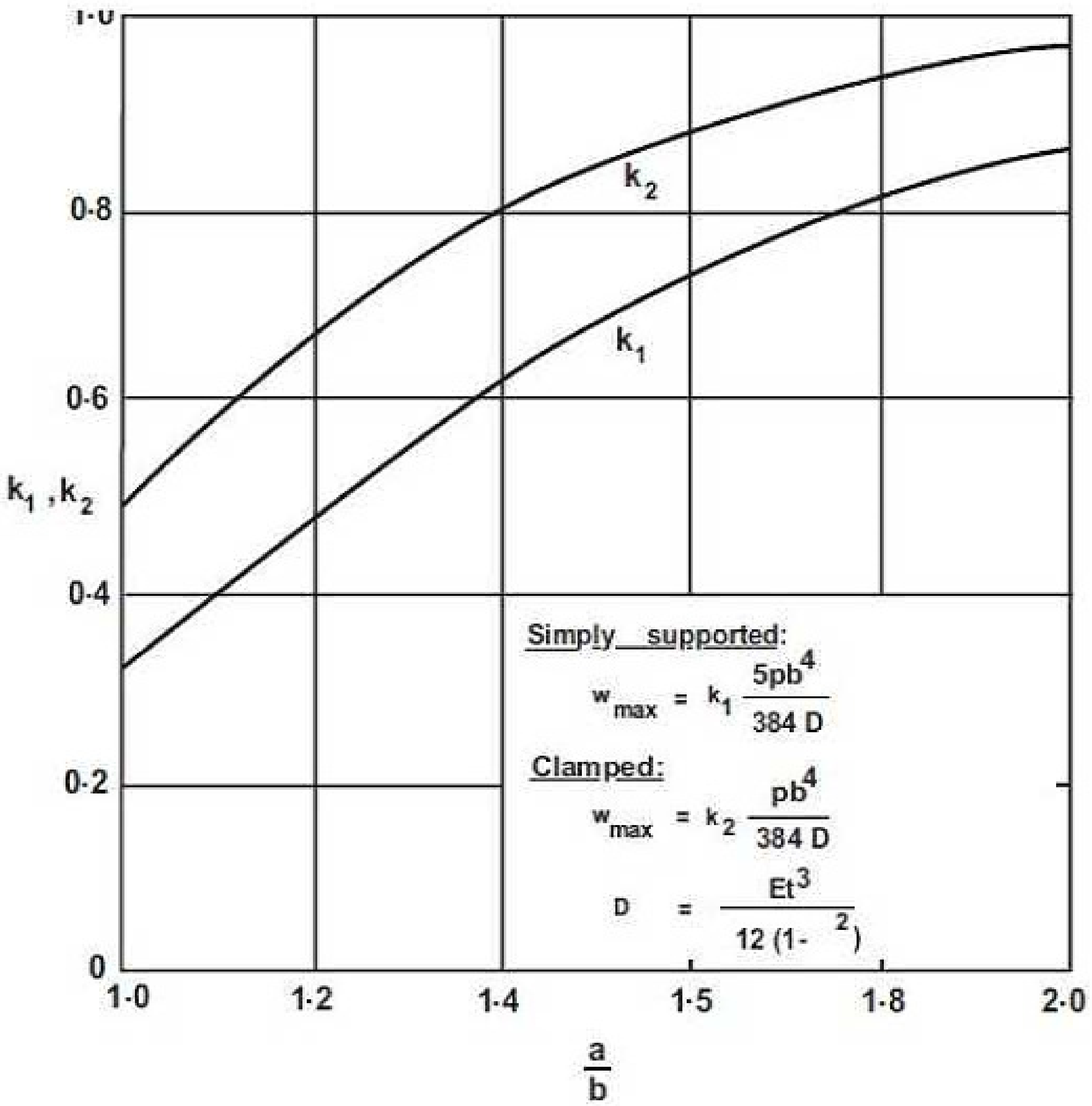


Figure 9.5 Maximum deflection of rectangular plates under

TENSÕES

curvatura → momento fletor maior na direção do menor span

↓

b → lado menor
a → lado maior

→ $\frac{a}{b} > 1$

↓

CURVAS DE PROJETO

TENSÕES MÁXIMAS EM PLACAS $(t \ll a, b)$
$w < t$

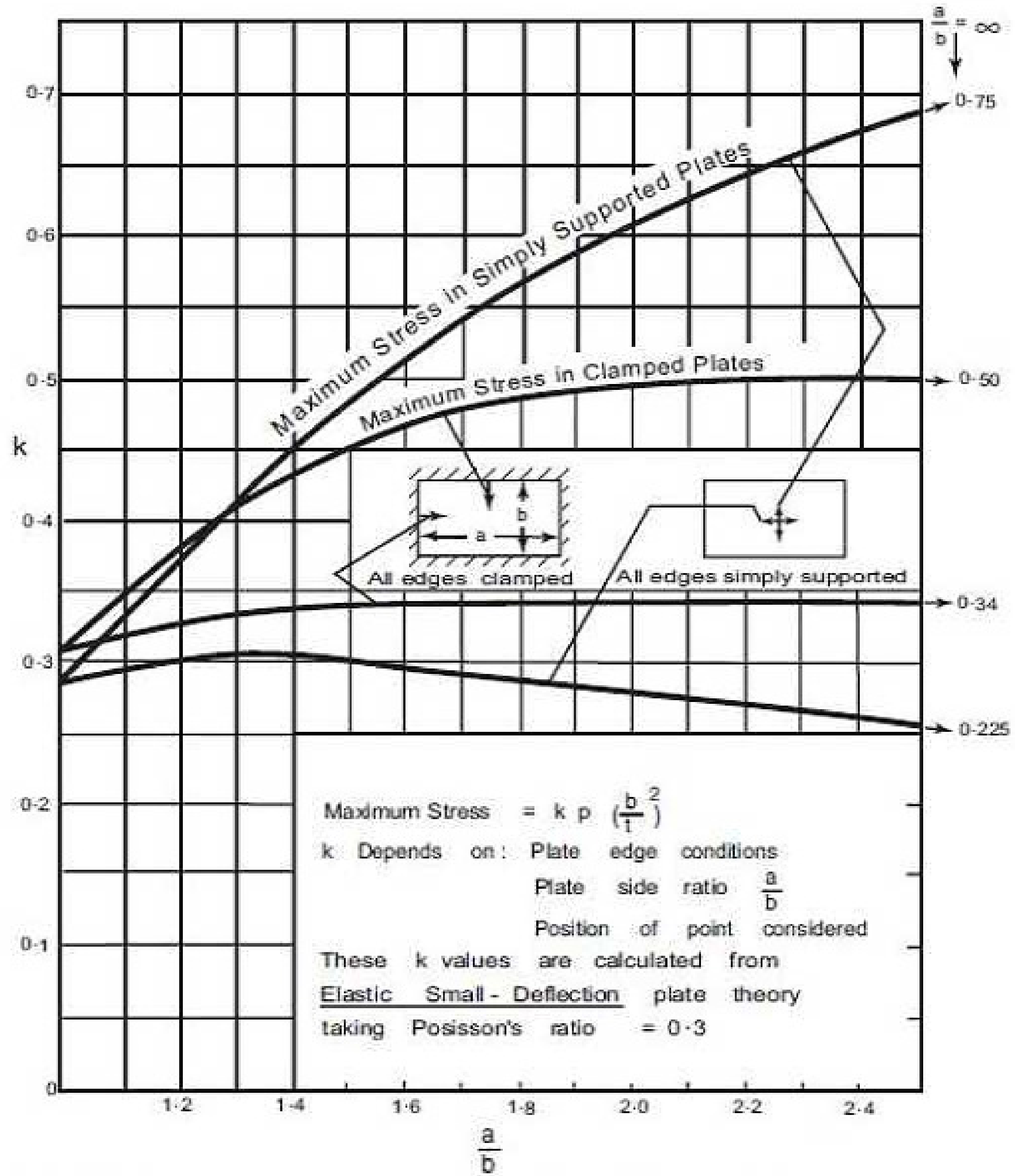


Figure 9.6 Maximum stresses in rectangular plates under uniform lateral pressure.

Parte do painel do fundo de uma embarcação, submetido a pressão constante p, é mostrado na figura. As dimensões da menor unidade de chapeamento são a x b (separação entre reforços).

Sendo o engenheiro responsável pela construção desse painel, escolha de forma racional as dimensões a e b de tal forma de limitar a tensão efetiva no centro do painel a metade da tensão de escoamento do material. Considere MPa, pressão equivalente a uma coluna de água de 15 m, espessura do chapeamento t=25 mm

$\frac{a}{b} = ?$