

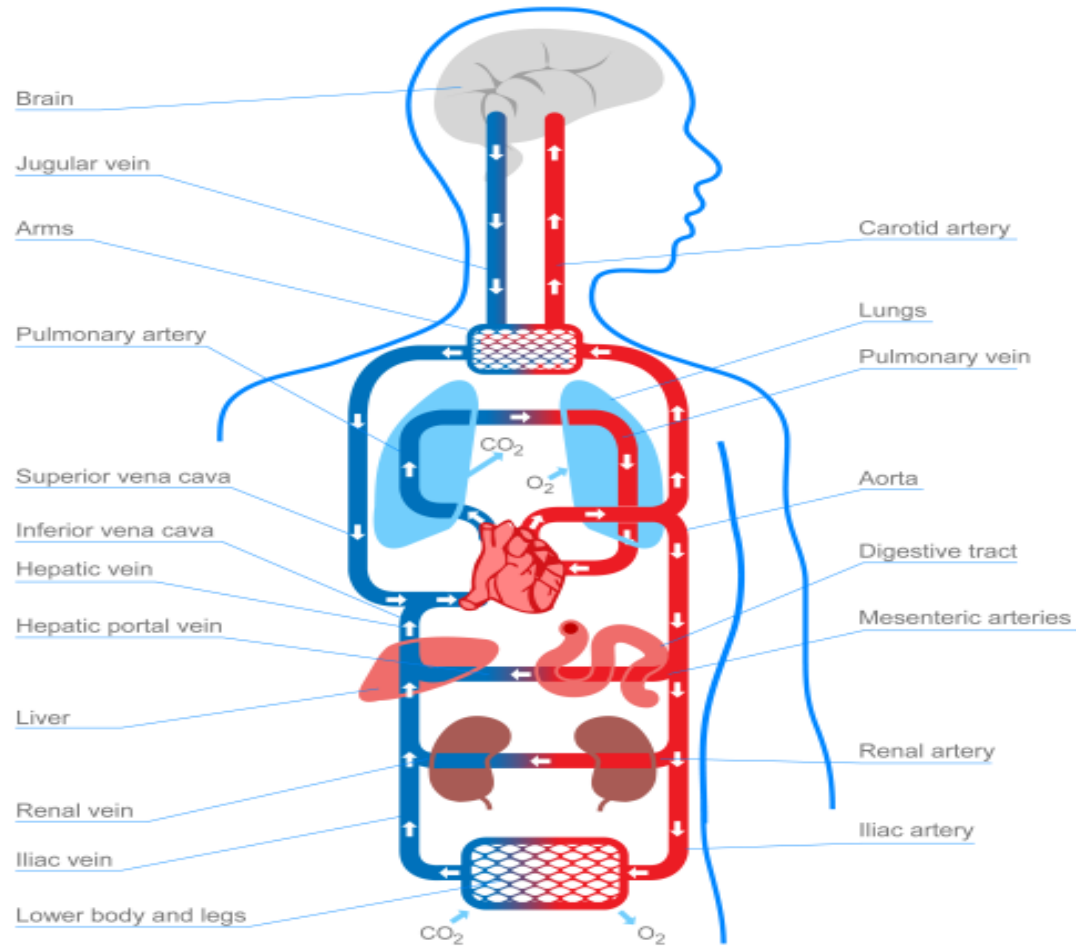


# Modeling of the Human Respiratory System

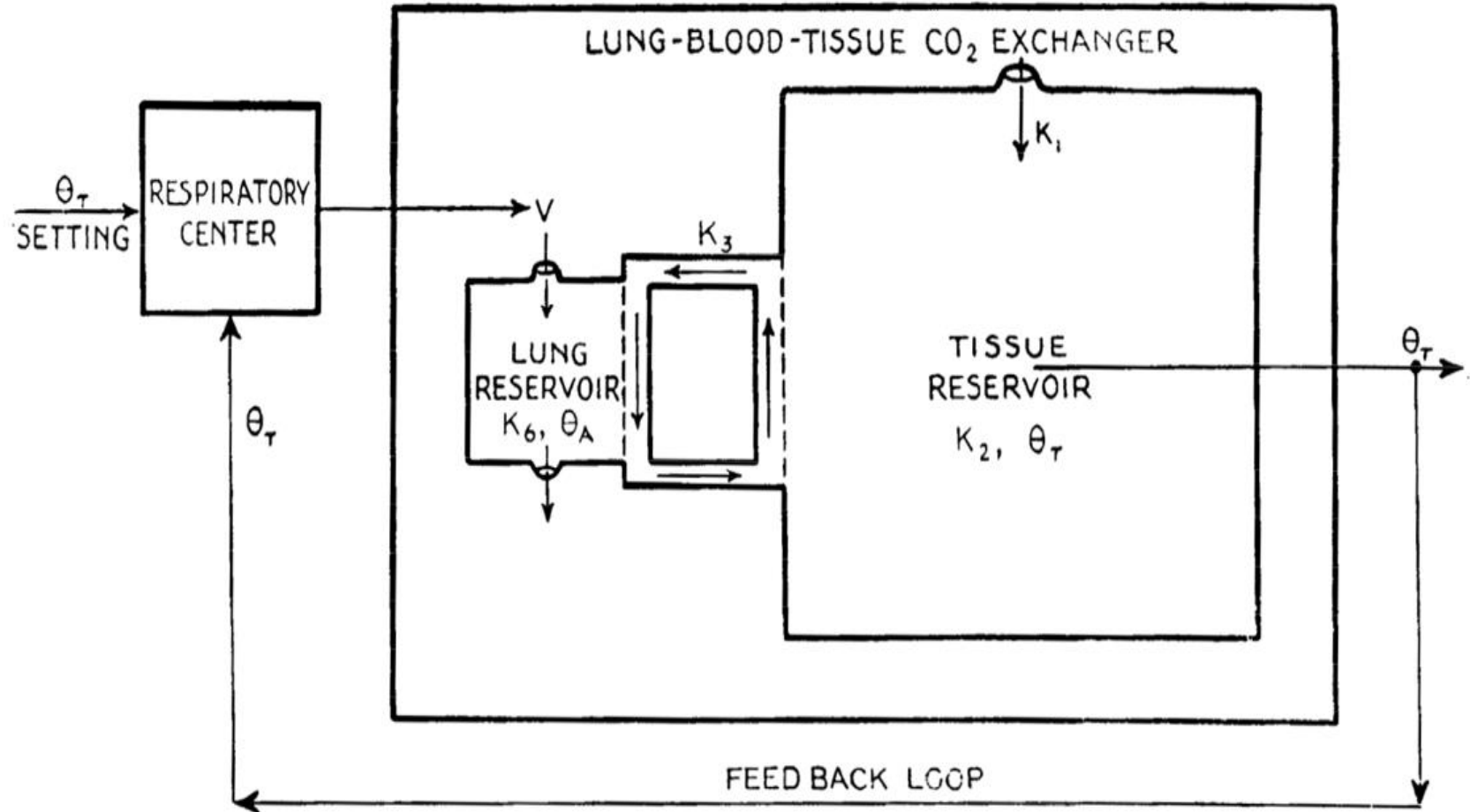
Jurandir Itizo Yanagihara

# Human Respiratory System

## THE CIRCULATORY SYSTEM



# Respiratory System Model (Grodins et al. , 1954)



# Albuquerque et al. (2008)

11 compartments:

Alveolar air space compartment

Pulmonary capillaries compartment

Body is divided into two regions:

a) Cerebral region:

Arterial compartment

Tissue compartment

Capillaries tissue compartment

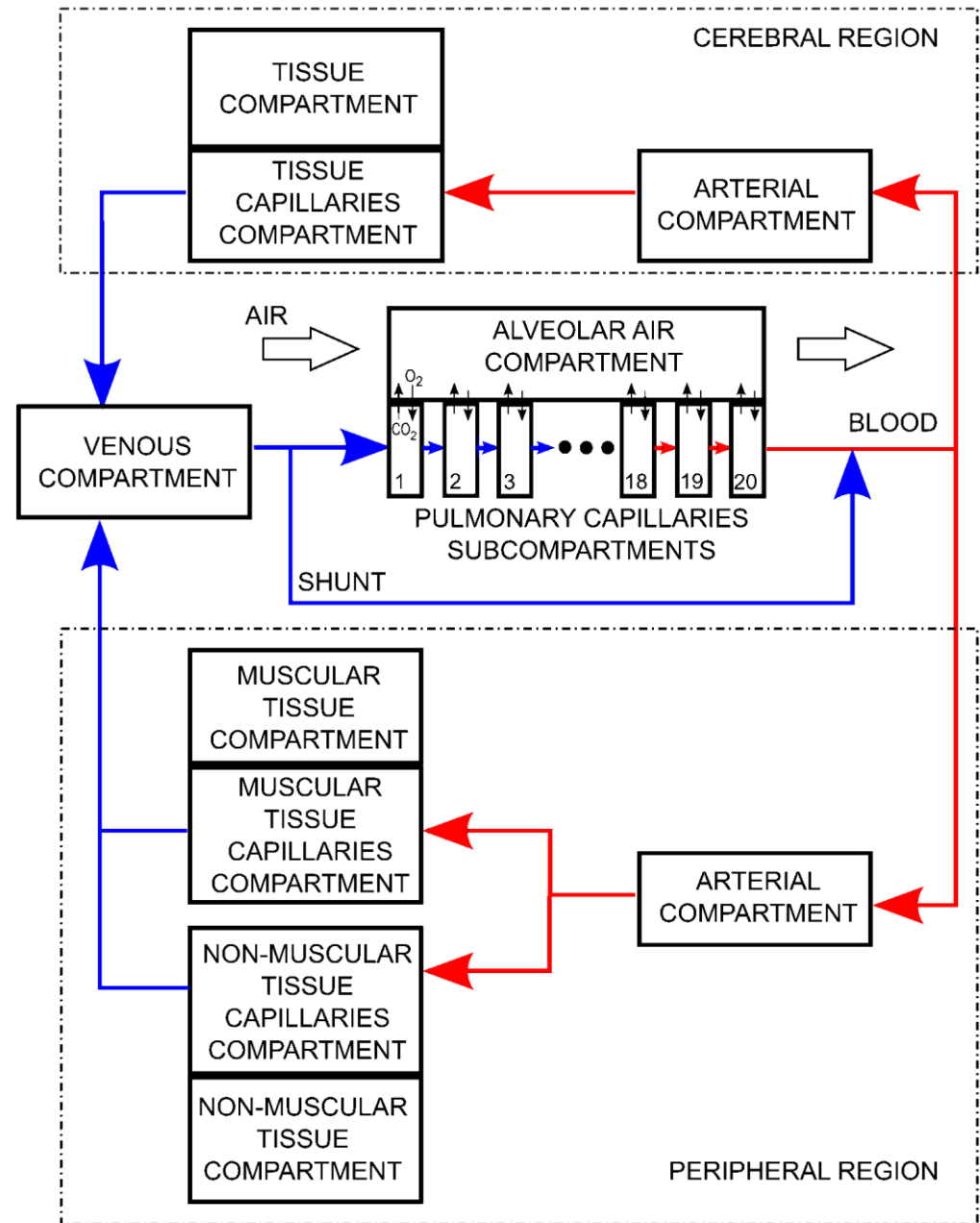
b) Peripheral region

Arterial compartment

Two subregions:

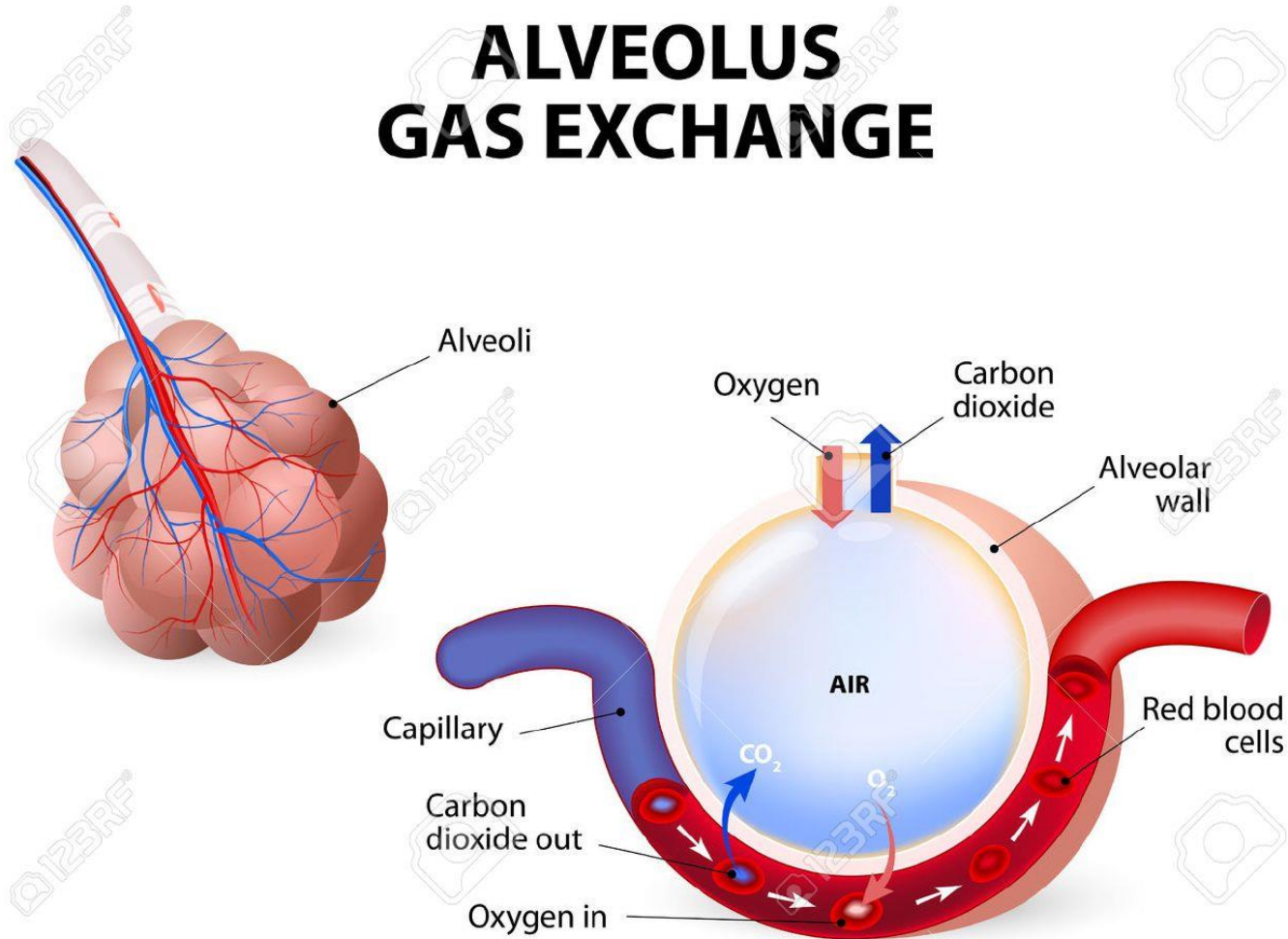
b.1) muscular tissues

b.2) non-muscular tissues

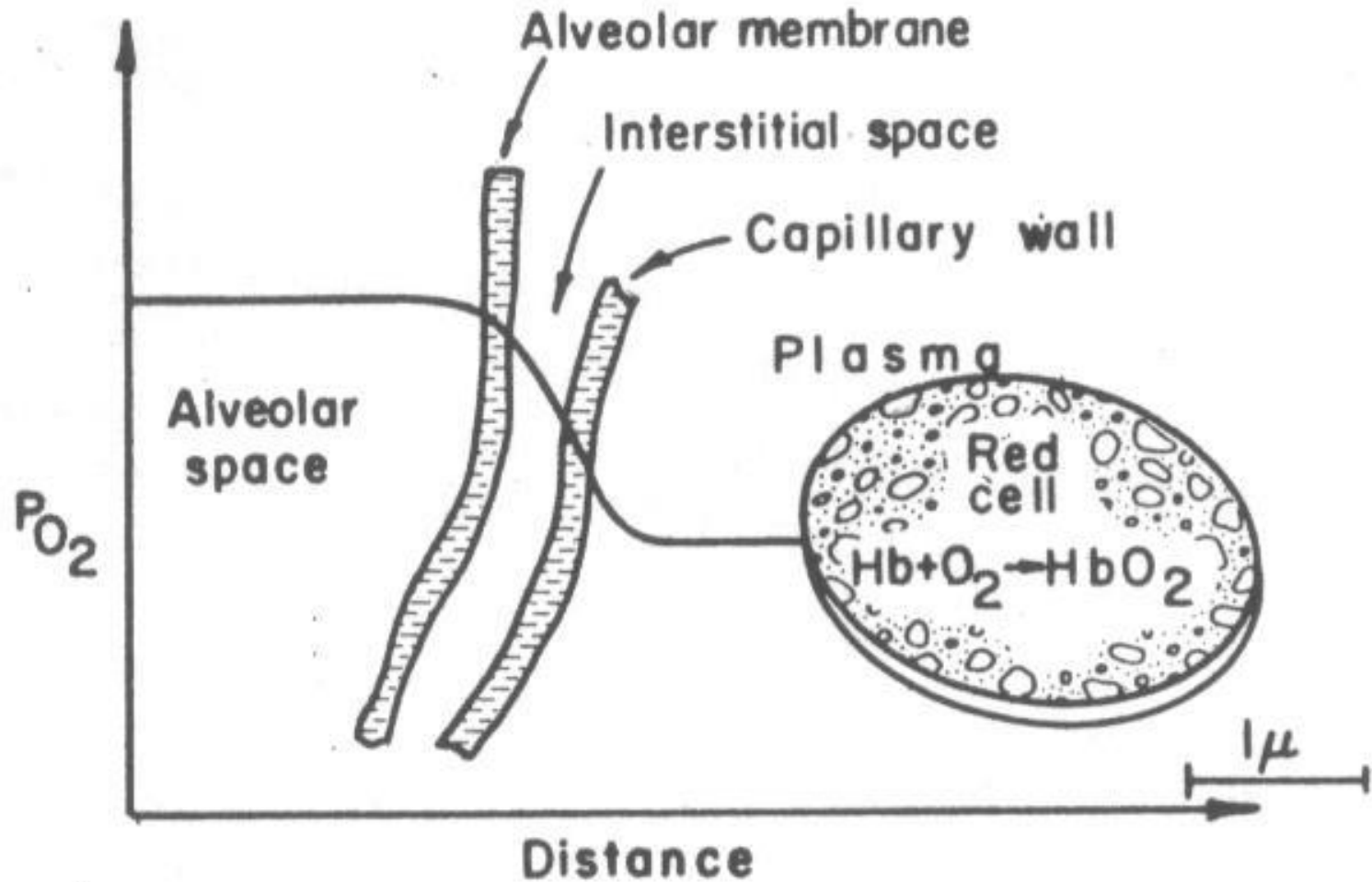


# Alveolus Gas Exchange

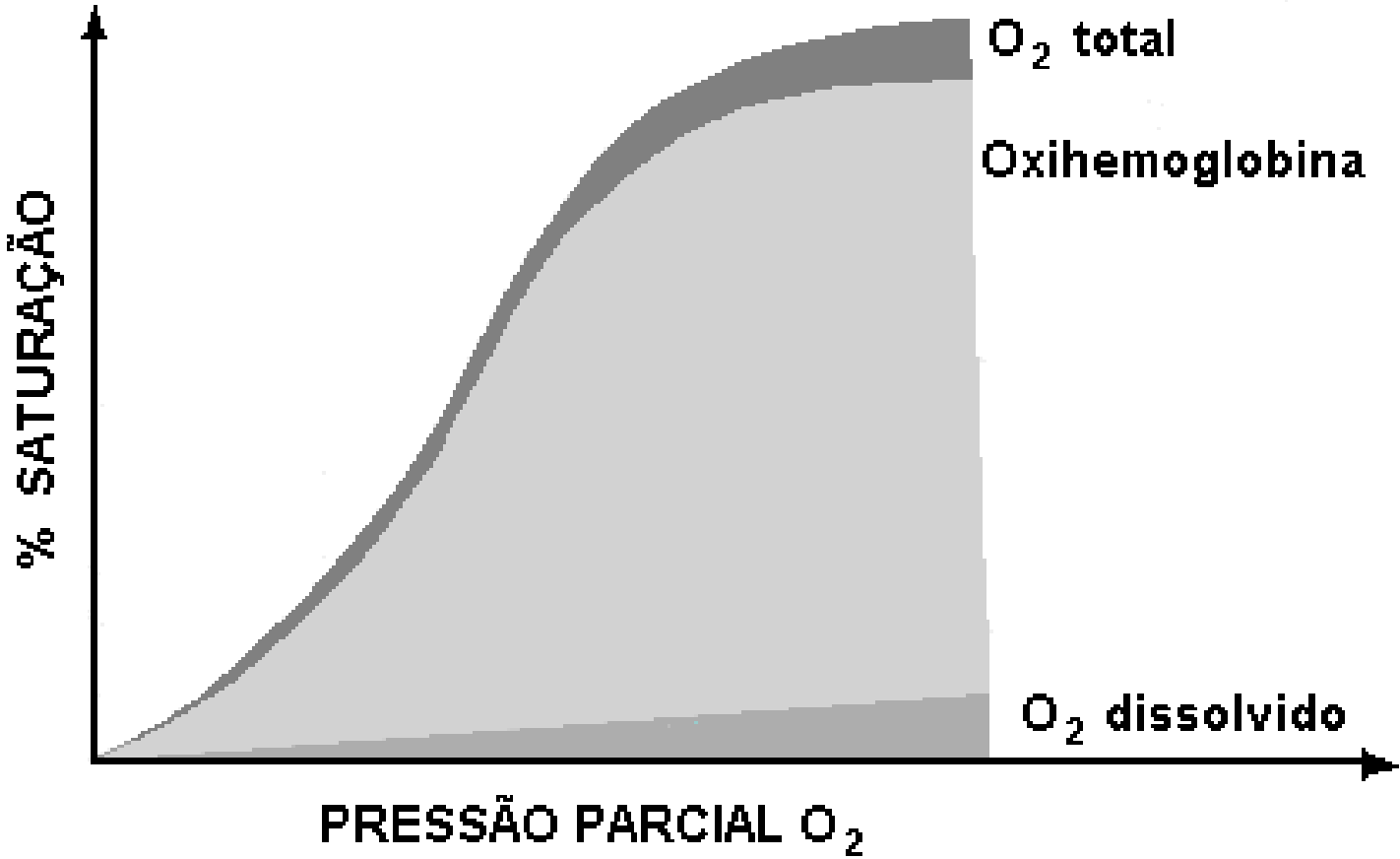
## ALVEOLUS GAS EXCHANGE



# Oxygen Diffusion in the Alveolus



# Transport of O<sub>2</sub>



# Transport of O<sub>2</sub>

Douglas et al (1988):

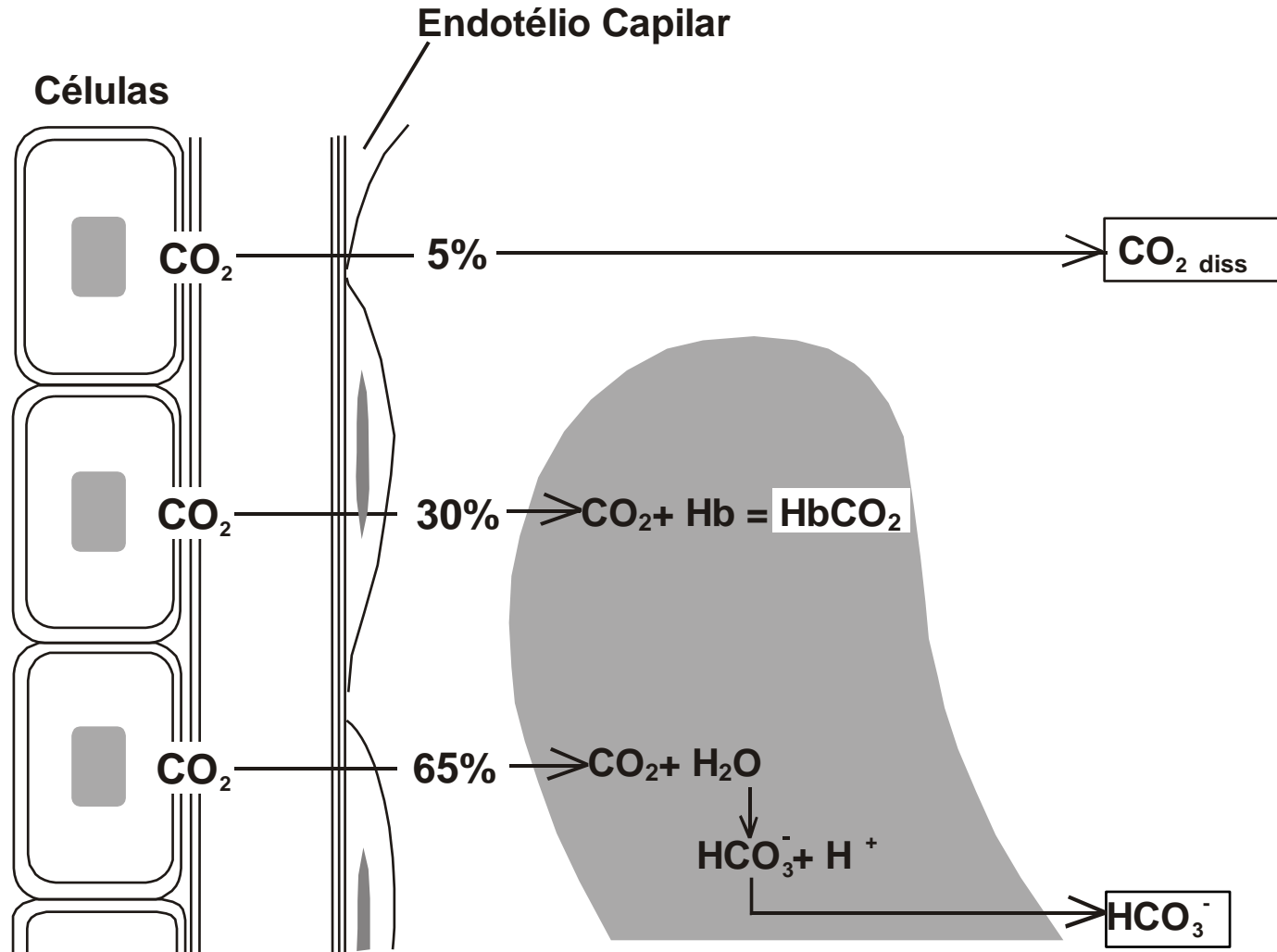
$$x_{O_2,s} = (0,003105 \times P_{O_2} + 1,34 \times [Hb] \times \frac{S_{O_2}}{100}) \times \frac{1}{100}$$

$$S_{O_2} = \frac{N^4 - 15N^3 + 2045N^2 + 2000N}{N^4 - 15N^3 + 2400N^2 - 31100N + 2,4 \times 10^6}$$

$$N = P_{O_2} \cdot 10^{(0,48 \cdot (pH - 7,4) - 0,024 \cdot (T - 37) - 0,0013 \cdot BE)}$$



# Transport of CO<sub>2</sub>



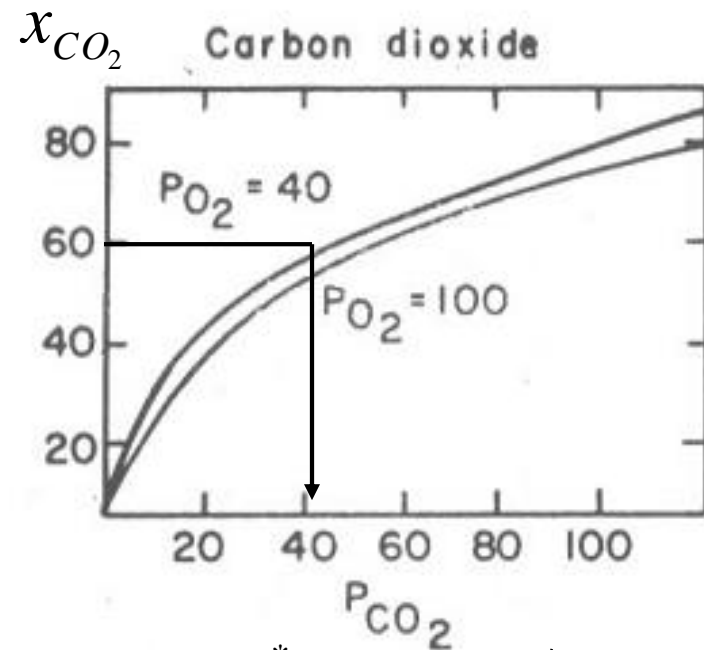
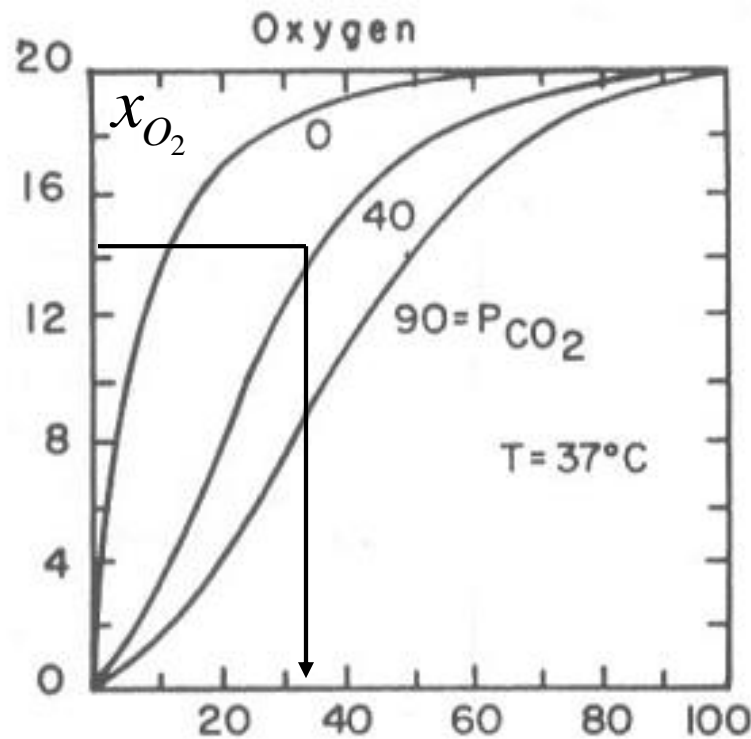
# Transport of CO<sub>2</sub>

Kelman (1966):

$$x_{CO_2,pl} = 0,02568 \cdot 0,0301 \cdot P_{CO_2} \cdot (1 + 10^{(pH-6,1)})$$

$$x_{CO_2,s} = x_{CO_2,pl} \cdot \left( 1 - \frac{0,0289 \cdot Hb}{(3,352 - 0,456 \cdot S_{O_2} / 100) \cdot (8,142 - pH)} \right)$$

# Equilibrium Concentration for O<sub>2</sub> and CO<sub>2</sub>



$$P_{O_2} \longrightarrow y_{O_2}^* = P_{O_2} / P_B \quad y_{CO_2}^* = P_{CO_2} / P_B$$

$$[HCO_3] = 24 + \frac{BE - (9,5 + 1,63 \cdot Hb)(pH - 7,4)}{1 - 0,014 \cdot Hb}$$

$$[HCO_3] = 0,0301 \cdot P_{CO_2} \cdot 10^{(pH-6,1)}$$

$$x_{CO_2,pl} = 0,02226 \cdot 0,0301 \cdot P_{CO_2} \cdot (1 + 10^{(pH-6,1)})$$

$$N = P_{O_2} \cdot 10^{(0,48 \cdot (pH-7,4) - 0,024 \cdot (T-37) - 0,0013 \cdot BE)}$$

$$S_{O_2} = \frac{N^4 - 15N^3 + 2045N^2 + 2000N}{N^4 - 15N^3 + 2400N^2 - 31100N + 2,4 \times 10^6}$$

$$x_{CO_2,s} = x_{CO_2,pl} \cdot \left( 1 - \frac{0,0289 \cdot Hb}{(3,352 - 0,456 \cdot S_{O_2} / 100) \cdot (8,142 - pH)} \right)$$

$P^*_{CO_2,s,\infty}$

$pH$

$HCO_3^-$

$x_{CO_2,pl}$

$N$

$P^*_{O_2,s,\infty}$

$S_{O_2}$

$$[HCO_3] = 24 + \frac{BE - (9,5 + 1,63 \cdot Hb)(pH - 7,4)}{1 - 0,014 \cdot Hb}$$

$$[HCO_3] = 0,0301 \cdot P_{CO_2} \cdot 10^{(pH-6,1)}$$

$$N = P_{O_2} \cdot 10^{(0,48 \cdot (pH-7,4) - 0,024 \cdot (T-37) - 0,0013 \cdot BE)}$$

$$S_{O_2} = \frac{N^4 - 15N^3 + 2045N^2 + 2000N}{N^4 - 15N^3 + 2400N^2 - 31100N + 2,4 \times 10^6}$$

$$x_{O_2,s} = (0,003105 \times P_{O_2} + 1,34 \times [Hb]) \times \frac{S_{O_2}}{100} \times \frac{1}{100}$$

*pH*

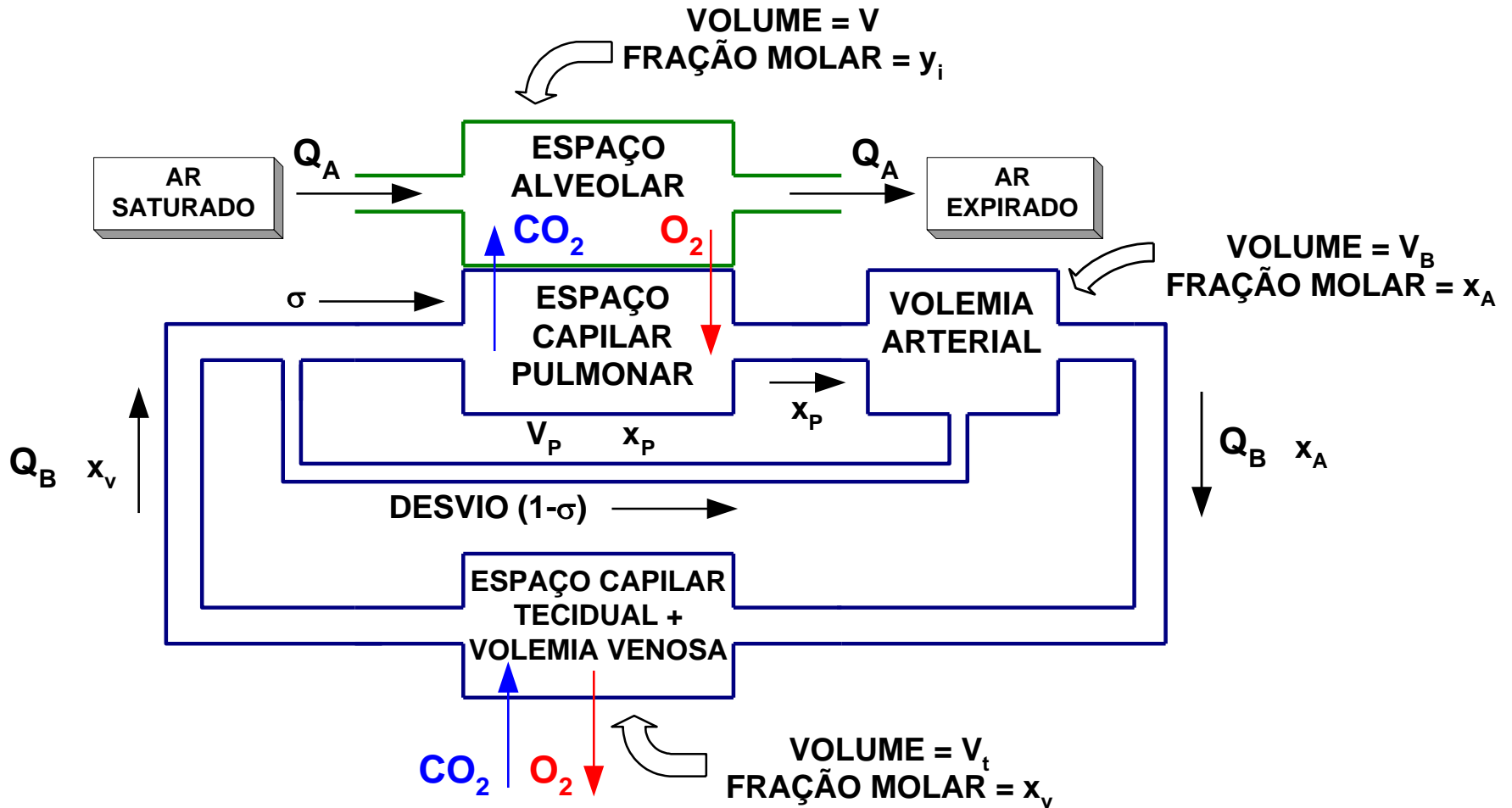
*HCO3-*

*N*

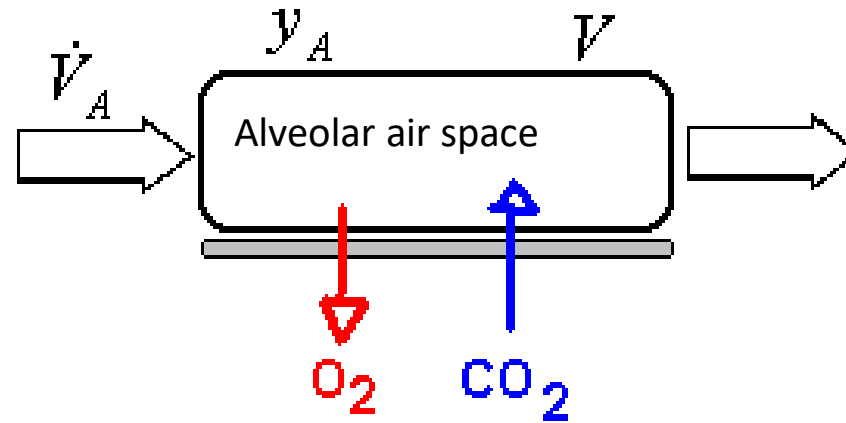
*P\*<sub>O<sub>2</sub>,s,∞</sub>*

*S<sub>O<sub>2</sub></sub>*

# Cardio-Respiratory Model



# Alveolar Air Space Mass Conservation Analysis



Model  
Perturbation

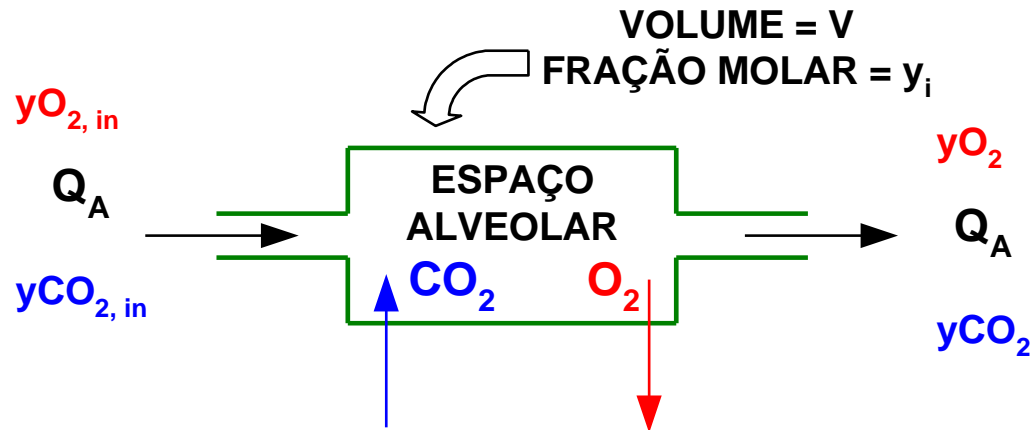
$$\frac{dy_{O_2}}{dt} = \frac{P_B}{760} \frac{273 \dot{V}_A}{310 V} \cdot (y_{O_2, in} - y_{O_2}) - \sum_{i=1}^{20} \frac{D_{LO_2(i)} P_B}{20 V} (y_{O_2} - y_{O_2, i}^*)$$

Net mass balance

Convective mass  
transport balance

Diffusive mass  
transport balance

# Mass Balance in the Alveolar Space



**OXIGÊNIO:** 
$$\frac{dy_{O_2}}{dt} = \frac{Q_A}{V} \cdot (y_{O_2, in} - y_{O_2}) - k_{O_2} \cdot a \cdot (y_{O_2} - y_{O_2}^*)$$

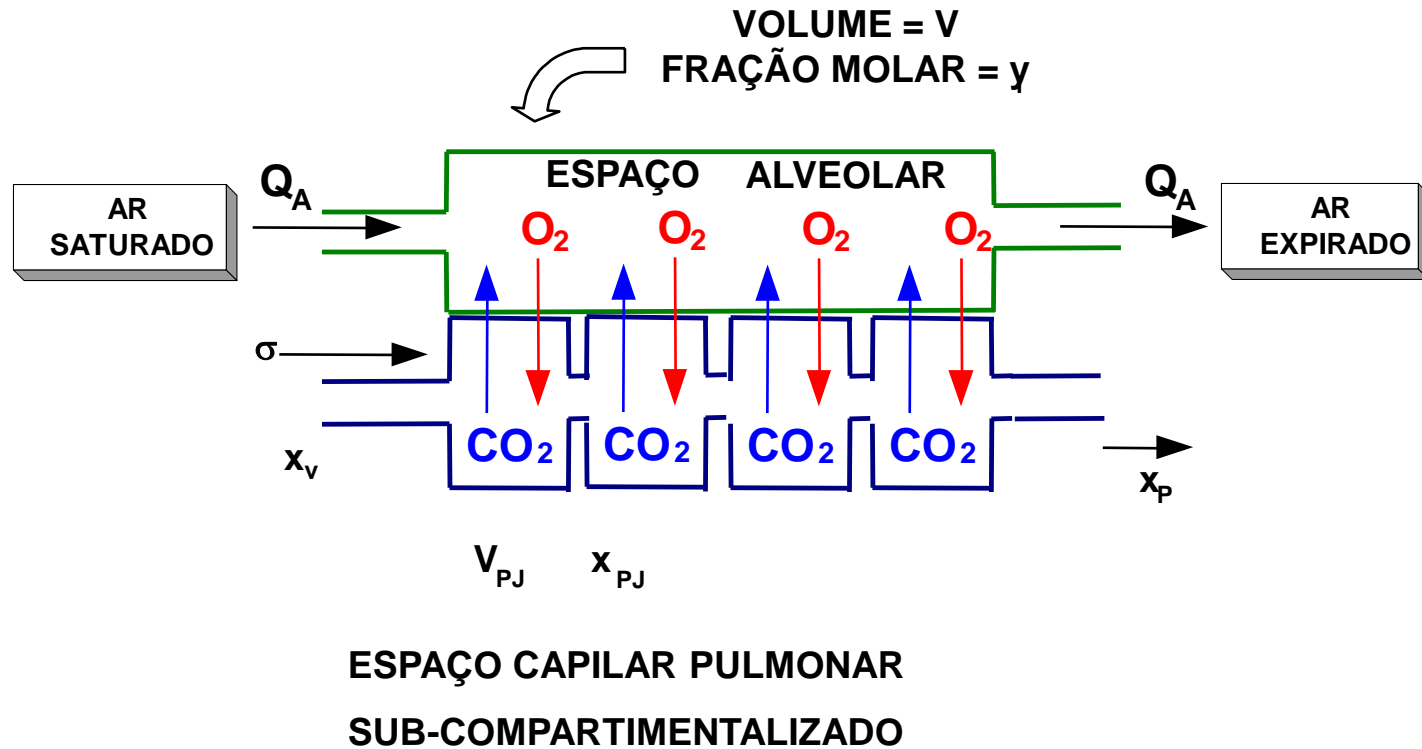
RESPOSTA DO SISTEMA DE CONTROLE

COEFICIENTES DE DIFUSÃO

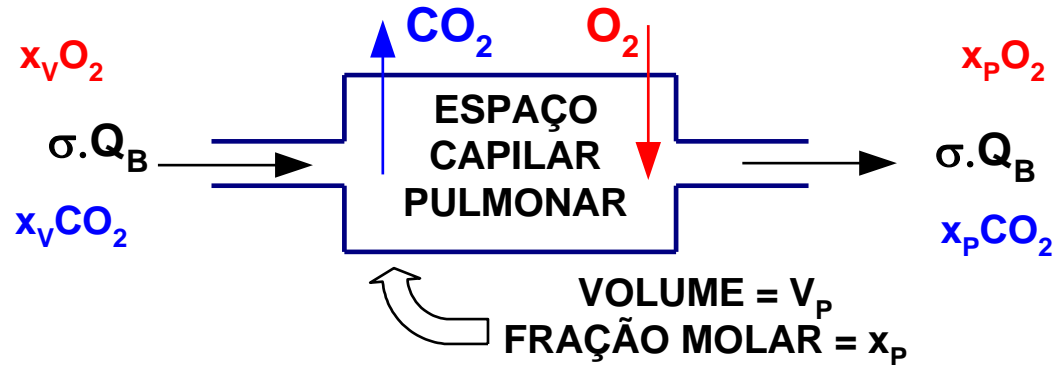
**GÁS CARBÔNICO:** 
$$\frac{dy_{CO_2}}{dt} = \frac{Q_A}{V} \cdot (y_{CO_2, in} - y_{CO_2}) + k_{CO_2} \cdot a \cdot (y_{CO_2}^* - y_{CO_2})$$



# Gas Transport in the Alveolus Use of Local Coefficients



# Mass Balance in the Lung Blood Space

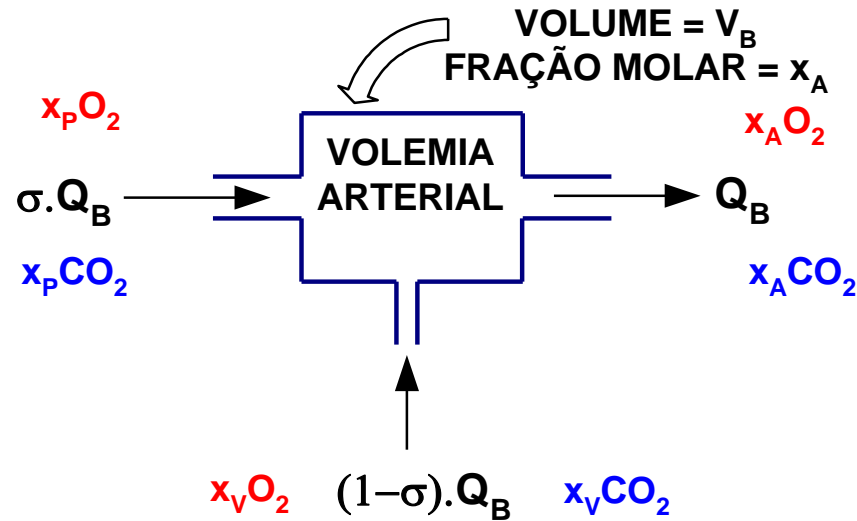


**OXIGÊNIO:** 
$$V_p \cdot \frac{dx_{p,O_2}}{dt} = Q_B \cdot \sigma \cdot (x_{v,O_2} - x_{p,O_2}) + k_{O_2} \cdot a \cdot (y_{O_2} - y_{O_2}^*) \cdot V$$

**GÁS CARBÔNICO:** 
$$V_p \cdot \frac{dx_{p,CO_2}}{dt} = Q_B \cdot \sigma \cdot (x_{v,CO_2} - x_{p,CO_2}) - k_{CO_2} \cdot a \cdot (y_{CO_2}^* - y_{CO_2})$$

RESPOSTA DO SISTEMA DE CONTROLE

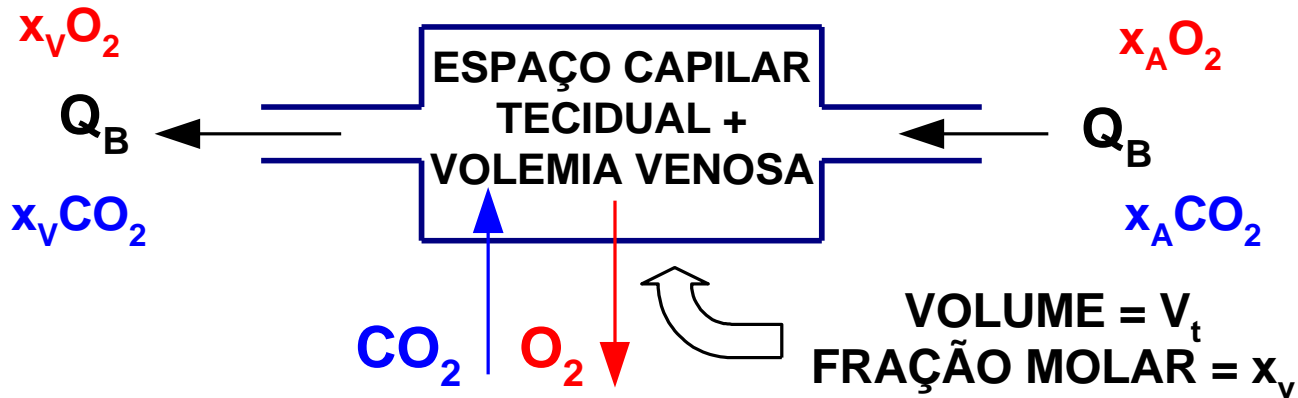
# Mass Balance in the Blood Space



**OXIGÊNIO:** 
$$V_b \cdot \frac{dx_{A,O_2}}{dt} = Q_B \cdot \left[ (1-\sigma) \cdot x_{v,O_2} + \sigma \cdot x_{p,O_2} - x_{A,O_2} \right]$$

**GÁS CARBÔNICO:** 
$$V_b \cdot \frac{dx_{A,CO_2}}{dt} = Q_B \cdot \left[ (1-\sigma) \cdot x_{v,CO_2} + \sigma \cdot x_{p,CO_2} - x_{A,CO_2} \right]$$

# Mass Balance in the Tissue Space



OXIGÊNIO:

CONSUMO DE OXIGÊNIO

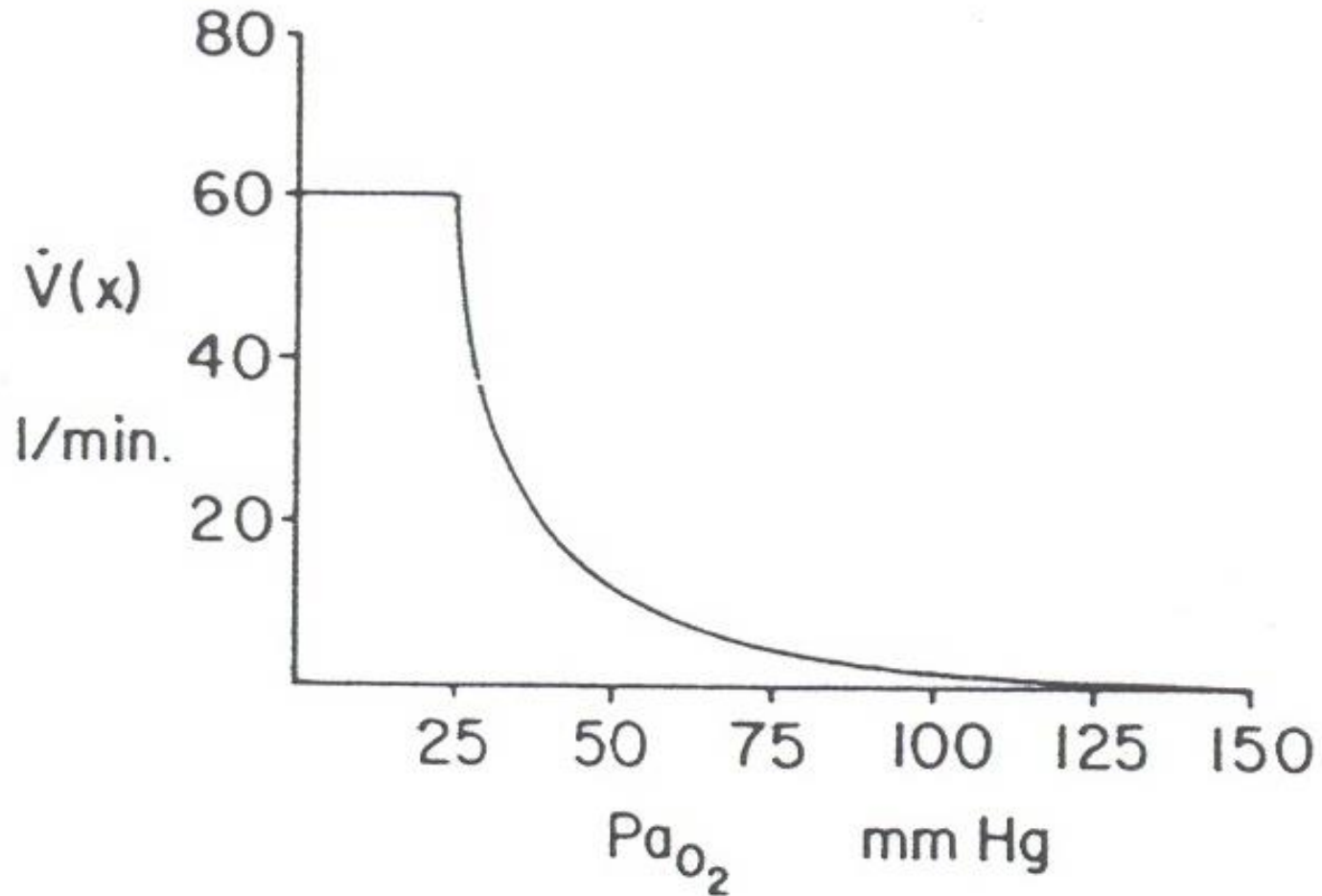
$$V_t \cdot \frac{dx_{v,O_2}}{dt} = Q_B \cdot (x_{A,O_2} - x_{v,O_2}) - M_{O_2}$$

GÁS CARBÔNICO:

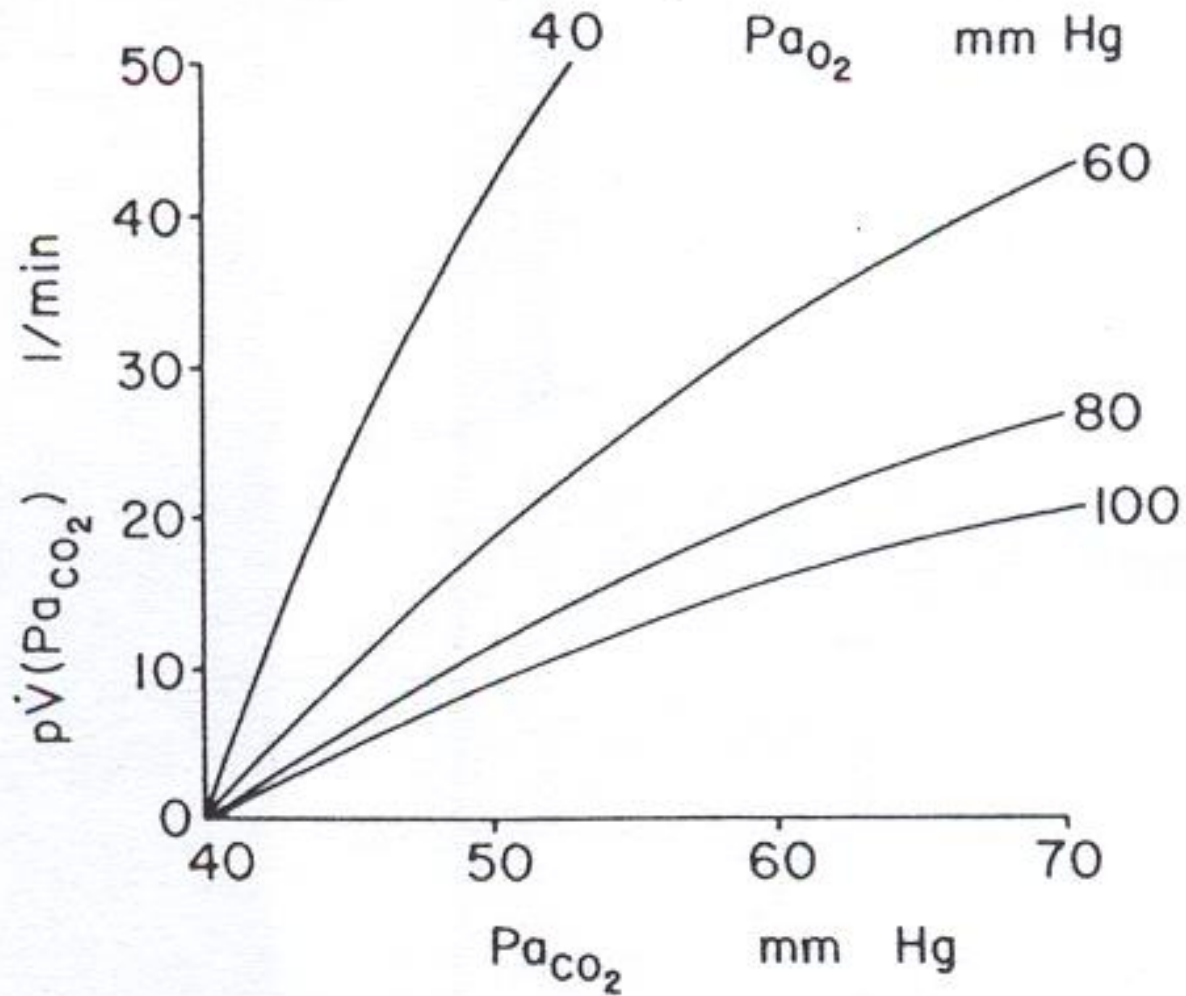
PRODUÇÃO DE GÁS CARBÔNICO

$$V_t \cdot \frac{dx_{v,CO_2}}{dt} = Q_B \cdot (x_{A,CO_2} - x_{v,CO_2}) + R \cdot M_{O_2}$$

# Alveolar Ventilation – Effect of $PO_{2a}$



# Alveolar Ventilation – Effect of Peripheral $PCO_{2a}$



# Alveolar Ventilation – Effect of Central $PCO_2$

