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ARTIFICIAL NEURAL NETWORKS FOR THE PREDICTION OF MECHANICAL BEHAVIOR OF METAL MATRIX COMPOSITES

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Abstract—In this paper we demonstrate the power of artificial neural networks in predicting strengthening in the transverse direction of metal matrix composites by regularly arranged strong fibers. A neural network is trained in different ways based on a numerical study in which the fiber volume fraction and the matrix hardening ability was studied systematically for fibers in a hexagonal arrangement loaded at 0 and 30° transverse direction and for a square arrangement of fibers loaded at 0 and 45° transverse directions. Strengthening predictions are then made for hardening cases of both fiber arrangements which were not covered by the finite element calculations as well as for arbitrary loading directions not achievable by simple finite element unit cell calculations in the case of square fiber arrangements.

INTRODUCTION

Transverse strengthening of metal matrix composites (MMC) by strong fibers was the focus of a great deal of systematic work in the past few years. Researchers investigated the influence of residual stresses on the mechanical behavior of particle reinforced MMCs with regularly arranged particles [1], the influence of 3D fiber arrangements [2] as well as the influence of fiber staggering on the overall composite behavior [3, 4].

Recently, a study came up in which for regularly arranged fibers with circular cross-section the transverse mechanical behavior was analyzed [5]. The finite element method was used to solve a few cases of loading direction, fiber orientation and matrix hardening. As a new model needs to be set up for each situation, it is very complicated and time consuming to generalize this method for any volume fraction, material behavior or loading direction. For the convenience of macromechanical studies the composite strengthening was summarized in an empirical expression for a limited set of parameters. The development of an empirical expression for a strengthening model requires repeated trials with a number of parameters. The degree of generalization achieved by such an expression beyond the investigated parameter field is difficult to measure.

In this paper we attempt an alternative method of machine learning using artificial neural networks (ANN) to model the mechanical behavior of MMCs. In this method the machine automatically gathers the knowledge embodied in the examples presented to it. Therefore, no parameter needs to be set by trial and error. For the present study the results of the finite element analysis presented in [5] have been used. The artificial neural networks are able to generalize and apply the knowledge to a new problem for which they has not been trained. Therefore, we have used the artificial neural network to predict the strengthening of MMCs for the loading directions and matrix hardening for which analytical and experimental solutions are not yet available.

The questions solved in this paper are as follows:

(1) Would an ANN be able to capture the strengthening behavior of MMCs from a pool of examples,

(2) Would an ANN be able to predict strengthening effects in the right manner for cases which are within the range of available results but which have not yet been computed or have been made available otherwise,

(3) What degree of strengthening could be expected for square fiber arrangements when the loading direction is gradually varied? At present, this question is not simple to solve by other means. Thus this paper deals with a new application of ANN in materials science.

NEURAL NETWORKS

Artificial neural networks are developed in the model of human brain. The concept of neural networks is discussed in detail elsewhere [6, 7]. A brief description is included in the following section.

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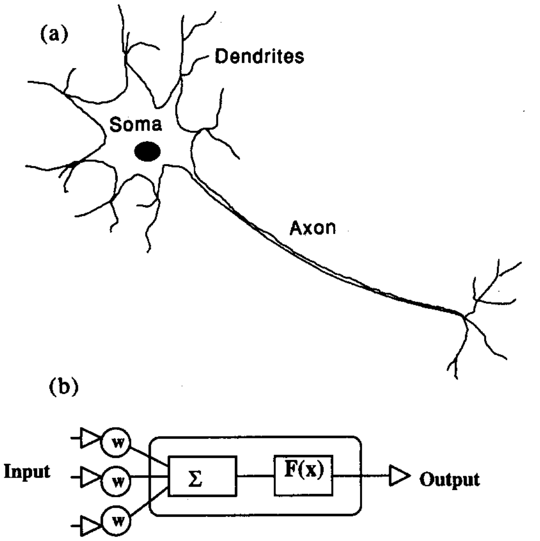

Fig. 1. Biological and artificial neurons: (a) a biological neuron; (b) an artificial neuron.

Biological neurons

The structure and the functioning of the human brain has been studied by many neurophysiologists. However, only an overview of it is available at present. Basically the brain functions with a very dense network of neurons. Figure 1(a) shows a typical biological neuron. The brain contains as many as 10^{11} neurons connected to each other by as many as 10^{15} interconnections among them. A neuron consists mainly of the following parts:

1. The cell body
2. The axon
3. The dendrite.

The dendrite is responsible for carrying the signals from various other neurons to the neuron of which it is a part. These dendrites are spread in a branched form to carry complex electro-chemical signals. On the other hand, an axon carries the signal from the cell body to various other neurons. When many dendrites carry signals to the cell body they are essentially accumulated there. After a sufficient time a signal is generated by the cell body and the same is sent down by the axon if the accumulation exceeds a threshold. The biological neural network also demonstrates various other behaviors which is very difficult to simulate using presently available hardware and software. Hence, the neural units in the artificial neural network are developed as a very approximate model of the biological neurons.

Artificial neurons

An artificial neuron can carry out a simple mathematical operation and/or can compare two values. Figure 1(b) describes an artificial neuron. An artificial neuron gets input from other neurons or directly from the environment. The path connecting two neurons is associated with a certain variable weight which represents the synaptic strength of the connection. The input to a neuron from another neuron is obtained by multiplying the output of the connected neuron by the synaptic strength of the connection between them. The artificial neuron then sums up all the weighted inputs coming to it

$$x_j = \sum_{i=1}^{m} w_{ij} o_i \tag{1}$$

where x_j = summation of all the inputs for neuron j, w_{ij} = synaptic strength between neuron i and neuron j, o_i = output of neuron i, m = total number of neurons sending input to neuron j.

Each neuron is associated with a threshold value and a squashing function. The squashing function is used to compare the weighted sum of inputs and the threshold value of that neuron. If the threshold value is exceeded by the weighted sum the neuron goes to a higher state, i.e. the output of the neuron becomes *high*. Many different squashing functions are used in different applications. In the present work a back-propagation learning algorithm has been used. This algorithm necessitates the use of a continuous, differentiable weighting function. Therefore, a sigmoidal squashing function has been used here which is as follows

$$o_j = \frac{1}{1 + e^{-\alpha(x_j - \theta_j)}} \tag{2}$$

where o_j = output of the neuron j, x_j = summation of all the weighted sum of the inputs for neuron j, θ_j = threshold value of the neuron j, α = is a parameter which controls the slope of the squashing function.

Figure 2 presents the squashing function for different values of α.

The output of the neuron for a given input can be controlled to a desired value by adjusting the synaptic strengths and the threshold values of the neuron. In an artificial neural network (ANN) several neurons can be connected in a variety of ways. Many different types of neural nets have already been developed [8]. The network architecture has to be selected keeping the problem at hand in mind. The present work requires training of a set of examples in a supervised manner. Therefore, a feedforward network is most suitable. A brief description of the feedforward network follows.

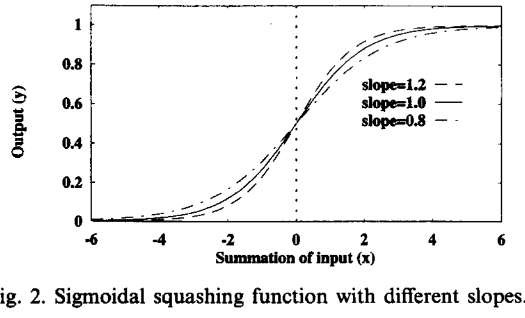

Fig. 2. Sigmoidal squashing function with different slopes.

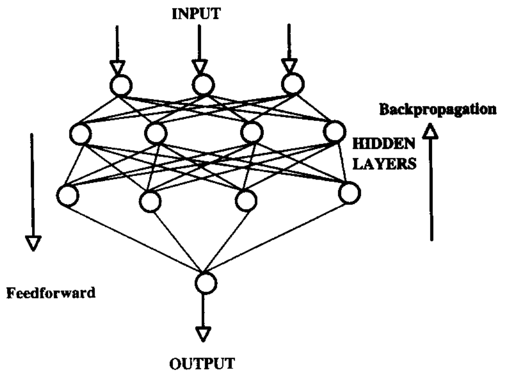

Fig. 3. A feedforward network.

Feedforward networks

In a feedforward network the neural units are classified into different layers. The network consists of one input layer, one or two hidden layers and one output layer of neurons. Figure 3 presents a typical feedforward network. It may be noted that all the neurons between two successive layers are fully connected, i.e. each neuron of a layer is connected to each neuron of the neighboring layers. However, there is no connection between neurons of the same layer or the neurons which are not in successive layers. The input layer receives input information and passes it onto the neurons of the hidden layer(s), which in turn pass the information to the output layer. The output from the output layer is the prediction of the net for the corresponding input supplied at the input nodes. Each neuron in the network behaves in the same way as discussed in equations (1) and (2). There is no reliable method for deciding the number of neural units required for a particular problem. This is decided based on experience and a few trials are required to determine the best configuration of the net.

In a feedforward network the knowledge (e.g. transverse strengthening of MMCs) is stored in a distributed manner, in the form of synaptic strengths and thresholds. Thus it can be generalized, i.e. it may be used for the situations for which the net has not been trained. Initially, the synaptic strengths and the threshold values are allocated randomly. To train the network for a specific knowledge a set of training examples is prepared. A training example consists of a set of values for the input neurons and the corresponding values for the output neurons. Several of such input–output pairs are to be prepared carefully to reflect all the aspects that the net needs to learn. All the training examples together form the training set. In the beginning of the training process, as the synaptic strengths and thresholds are selected randomly the output predicted by the net for a particular input and the output supplied in the corresponding training examples may not match. However, the synaptic strengths and the thresholds can be adjusted so that the net predicts the output correctly. As several examples are to be learnt by the net there must be a sufficient number of neural units in the net. The adjustments in the synaptic strengths and thresholds are carried out following a *learning algorithm*. The back propagation algorithm has been used in the present work for this purpose.

The back propagation algorithm

The back propagation algorithm is a generalized form of the least mean square training algorithm for perceptron learning [9, 10]. It uses the gradient search method to minimize the error function which is the mean square difference between the desired and the predicted output. The error for the pth example is given by

$$E_p = \sum_j (d_j - o_j)^2 \tag{3}$$

where d_j = the output desired at neuron j and o_j = the actual output of neuron j. As presented in equations (1) and (2) the output o_j is a function of synaptic strengths and outputs of the previous layer.

$$o_j = f(\beta_j) = f\left(\sum_i w_{ij} o_i\right). \tag{4}$$

The error can be minimized by moving along the steepest descent direction on the error surface

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial E}{\partial \beta_j}\frac{\partial \beta_j}{\partial w_{ij}} = \frac{\partial E}{\partial \beta_j} o_i = \delta_j o_i \tag{5}$$

where δ_j for a neuron is

$$\delta_j = f'(\beta_j) \sum_k \delta_k w_{kj} \tag{6}$$

and f' indicates the first order derivative of the function and k indicates a neuron in the layer which is successive to the layer which contains neuron j. Therefore, the weight matrix can be adjusted recursively for each example

$$w_{ij}(t+1) = w_{ij}(t) + \eta \delta_j x_i \tag{7}$$

where η is an adjustable gain term which controls the rate of convergence.

The above operation is repeated for each example and for all the neurons until a satisfactory convergence is achieved for all the examples present in the training set.

NEURAL NETWORKS IN MATERIAL SCIENCE

The feedforward neural networks have been applied to the solution of various engineering problems such as design of equipment and structures, fault detection, management of manufacturing and construction, etc. They have also been effective in computer implementation of natural processes such as natural language understanding, speech recognition, pattern recognition, etc. This tool can be utilized very effectively in the solution of problems of material science. The materials are either available in nature or are the product of engineering. The behavior of the

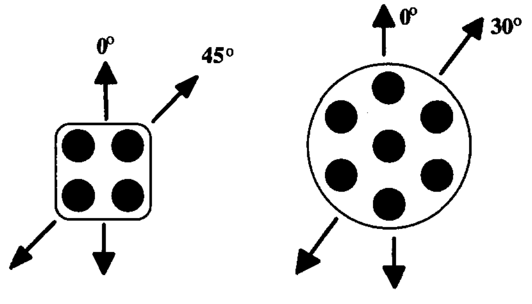

Fig. 4. Arrangements of fiber with primary loading directions: (a) rectangular arrangement; (b) hexagonal arrangement.

material is best understood by carrying out experiments. Conventionally, the experimentally observed behavior of a material is modeled analytically using simple algebraic expressions. The analytical expression should predict the material behavior which agrees closely with the experimental observations. However, it may not always be possible to capture every material behavior by means of a simple expression. The development of such expressions can be extremely difficult and time consuming. Moreover, the behavior of modern materials is becoming more and more complicated and they demand a more detailed study. The feedforward neural networks can be extremely helpful in capturing the experimentally observed material behavior directly which precludes the necessity of developing analytical expressions. The neural networks generalize on their own. Therefore, they are also effective in predicting the behavior of a new material before the material is produced in the laboratory. This may reduce the cost of expensive experiments. The authors, however, have not come across any application of feedforward neural networks in the field of material science. The present work demonstrates the effectiveness of the method in the prediction of strengthening of metal matrix composites.

ARTIFICIAL NEURAL NETWORKS FOR PREDICTION OF STRENGTHENING

The purpose of the present investigation is to study the transverse mechanical behavior of continuous fiber metal matrix composites. Study of the effects of fiber arrangement, volume fraction and matrix hardening which has hitherto not been possible by other methods has been emphasized here. Only fibers with circular cross-section will be studied here. The focus of this paper is limited to the fully developed plastic flow of two phase composites. The fibers are well bonded to the matrix so that no debonding or sliding is permitted at the interface. Figure 4 presents the fiber arrangements considered in this work. A square arrangement of fibers is shown in Fig. 4(a), with the loading directions at 0 and 45° indicated. Similarly, Fig. 4(b) represents a hexagonal arrangement of fibers, with loading directions at 0 and 30° shown.

A continuum mechanics approach is used to model the composite behavior, thus eliminating the influence of size from the calculations. The ultimate stress–strain behavior is characterized by

$$\sigma = \sigma_0\left(\frac{\epsilon}{\epsilon_0}\right) = \epsilon E \quad \epsilon \leqslant \epsilon_0 \tag{8}$$

$$\sigma = \sigma_0\left(\frac{\epsilon}{\epsilon_0}\right)^N \quad \epsilon > \epsilon_0$$

where σ = axial stress, ϵ = axial strain, σ_0 = yield stress in tension, $\epsilon_0 = \sigma_0/E$, E = Young's modulus and N = strain hardening exponent.

Figure 5 presents the features of the overall stress–strain curves of primary concern in this work. For the case of fibers perfectly bonded to the matrix, the composite will necessarily harden with the same strain hardening exponent, N, as the matrix, when strains are in the regime of fully developed flow. At sufficiently large strains (1–5%) the composite behavior is then described by

$$\bar{\sigma} = \bar{\sigma}_N\left(\frac{\bar{\epsilon}}{\epsilon_0}\right)^N \tag{9}$$

where $\bar{\sigma}$ = overall stress, $\bar{\epsilon}$ = overall strain and $\bar{\sigma}_N$ = asymptotic reference stress.

The asymptotic reference stress can be determined by normalizing the composite stress by the stress in the matrix alone at the same overall strain, as indicated in Fig. 5.

A detailed study on the asymptotic reference stress and its effect on fiber volume fraction and matrix hardening has been reported in Ref. [4]. The finite element method has been used to model the fiber–matrix system very accurately. It was attempted to summarize the results of the finite element analysis in simple empirical expressions. Many polynomial and other forms of algebraic expressions have been attempted to fit the results accurately. The final expressions are as follows:

$$\bar{\sigma}_0 = \sigma_0^* \quad f \leqslant f^* \tag{10}$$

$$\bar{\sigma}_0 = C_1(f - f^*)^2 + \sigma_0^* \quad f > f^*. \tag{11}$$

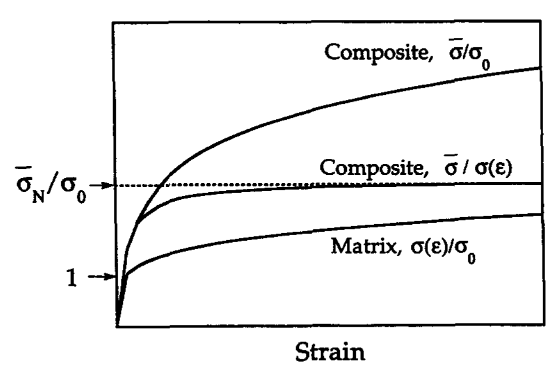

Fig. 5. Asymptotic reference stress $\bar{\sigma}_N$, for work hardening matrices.

Table 1. Constants for equations (10) and (11)

	σ_0^*	C_1	f^*
Square arrangement 0° loading	$\frac{2}{\sqrt{3}}\sigma_0$	14.2	0.345
Square arrangement 45° loading	$\frac{2}{\sqrt{3}}\sigma_0$	0.0	N/A
Hexagonal arrangement	$\frac{2}{\sqrt{3}}\sigma_0(1+0.26f)$	27.2	0.634

The values of the above parameters for different fiber arrangements have been presented in Table 1.

For the matrix with strain hardening another simple empirical relationship has been attempted

$$\bar{\sigma}_N = \bar{\sigma}_0 \exp(C_2 N f^{C_3}). \tag{12}$$

The arbitrary constants C_2 and C_3 have been presented in Table 2.

Equation (12) in conjunction with equations (10) and (11) predicts the composite asymptotic reference stresses for the fiber arrangements and volume fractions considered here. These empirical relationships were arrived at after a considerable amount of trial and error with the form and the values of arbitrary coefficients.

Here instead of an empirical expression we attempt to train an ANN with the results obtained by the finite element analysis. The asymptotic reference stress for different volume fractions (f), hardening exponents (N), fiber arrangements (square and hexagonal) and loading directions are available in Ref. [5]. The square fiber arrangement has been analyzed for two loading directions, 0 and 45°. In case of hexagonal fiber arrangement there is marginal difference between the results for loading at 0 and 30° angles. Therefore, the following cases have been included in the present net

(i) Square arrangement of fibers loaded at 0°,

(ii) Square arrangement of fibers loaded at 45°,

(iii) Hexagonal arrangement of fibers loaded at any angle.

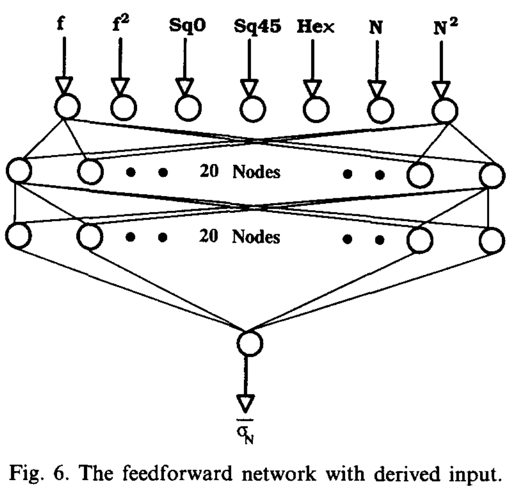

Fig. 6. The feedforward network with derived input.

The results of the finite element analysis have been presented to the net in the form of examples. Each example consists of volume fraction, fiber arrangement, loading direction and hardening exponent as input information and the corresponding asymptotic stress as output information. Figure 6 presents the neural net developed. It may be noted that along with the volume fraction (f) and hardening exponent (N) the squares of them f^2 and N^2 have also been provided in the input vector. This has facilitated the learning of the net. Three separate nodes have been provided for three different cases considered here—square fiber arrangement with 0° loading, square fiber arrangement with 45° loading and hexagonal arrangement. The inputs corresponding to these nodes are either 1 or 0. When an example for a particular case is being presented the input for that node is 1 and the input into nodes for all other cases is 0. All other parameters f, N, f^2, N^2 and $\bar{\sigma}_N$ have also been scaled down between 0 and 1 by multiplying them with suitable scaling factors.

As results of finite element analysis were available for four hardening exponents (N), 0.0, 0.1, 0.2, 0.5, the net has been trained for these four values of N only. The training set consisted of 31 and 32 examples for square fiber arrangement with loading at 0 and 45° respectively and 35 examples for the hexagonal fiber arrangement. That made a total of 98 input–output pairs. Two hidden layers have been used with 20 neurons in each hidden layer. The training session consisted of repeated presentation of the training set to the net and adjusting the synaptic strengths and thresholds using the back propagation algorithm. A software in C_{++} language on an IBM PC/486 computer has been developed for this purpose. After approx. 1000 cycles of training the average root mean square of difference between the finite element results and the prediction of the net came down to 0.0003. The predictions of the net along with the results of the finite element analysis and the prediction of equation (12) have been presented in Figs 7(a), (b) and (c). It may be noted that the agreement in results between the artificial neural network and the finite element analysis is very good. Therefore, it may be concluded that the artificial neural network has been able to capture the behavior of asymptotic reference stress very accurately from the examples presented to it. The empirical expression of equation (12) has predicted the behavior accurately in lower hardening exponents, up to $N = 0.2$. However, it overpredicted the asymptotic reference stress

Table 2. Constants for equation (12)

	C_2	C_3
Square arrangement 0° loading	4.53	1.21
Square arrangement 45° loading	2.88	1.50
Hexagonal arrangement	4.65	1.78

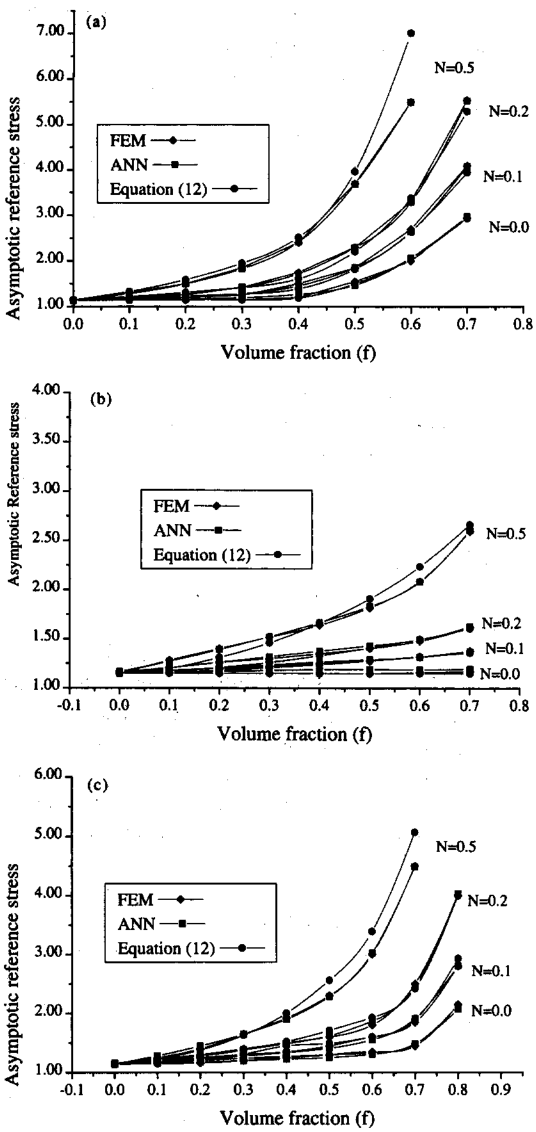

Fig. 7. Asymptotic reference stress, $\bar{\sigma}_N$, for (a) square arrangement for loading at 0°; (b) square arrangement for loading at 45°; (c) hexagonal arrangement.

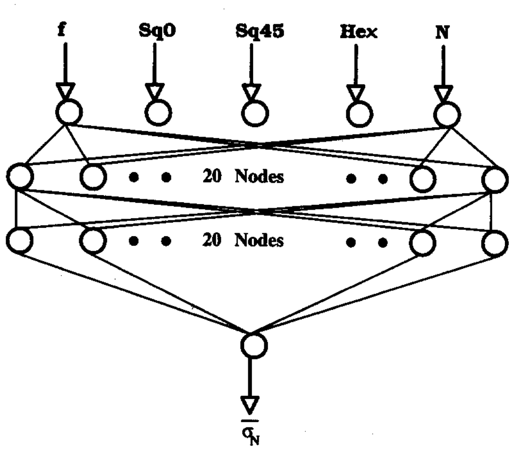

Fig. 8. The feedforward network without derived input.

in the case of $N = 0.5$, especially for square fiber loaded at 0°. This emphasizes the utility of the simple empirical expression for lower hardening exponents. However, the overall performance of the artificial neural network was superior to that of the empirical relationship. The empirical expression has been developed by experts after many trials and errors [5]. The neural network, on the other hand, has learnt the relationship *intuitively* without any help from experts. This underlines the learning ability of the artificial neural networks even for complex relationships.

The derived inputs

It may be noted that along with fiber volume fraction (f) and hardening exponent (N) the squares of these values have been presented to the net. These

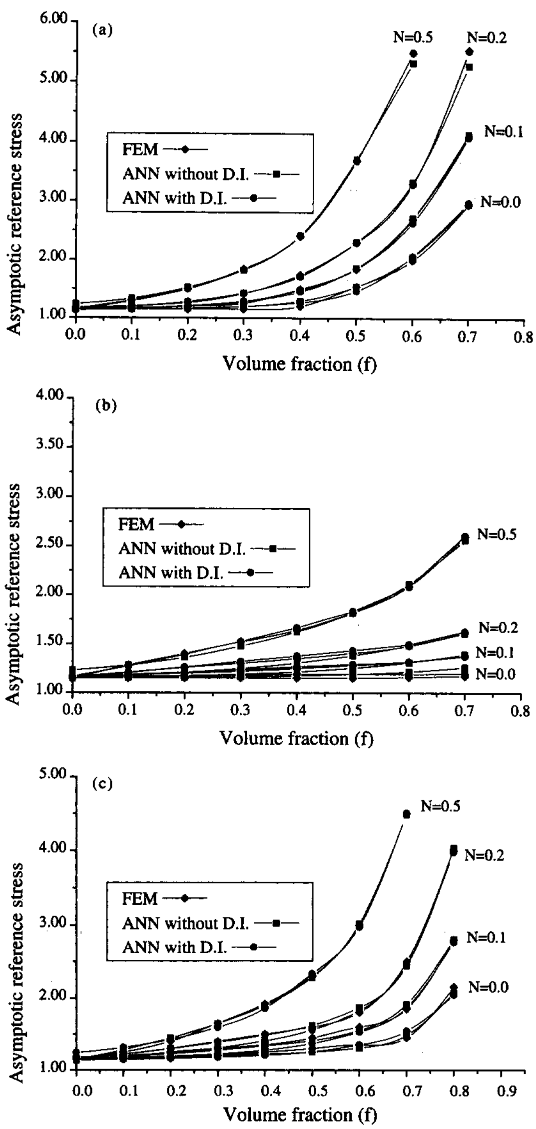

Fig. 9. Performance of derived input, asymptotic reference stress, $\bar{\sigma}_N$, for (a) square arrangement for loading at 0°; (b) square arrangement for loading at 45°; (c) hexagonal arrangement.

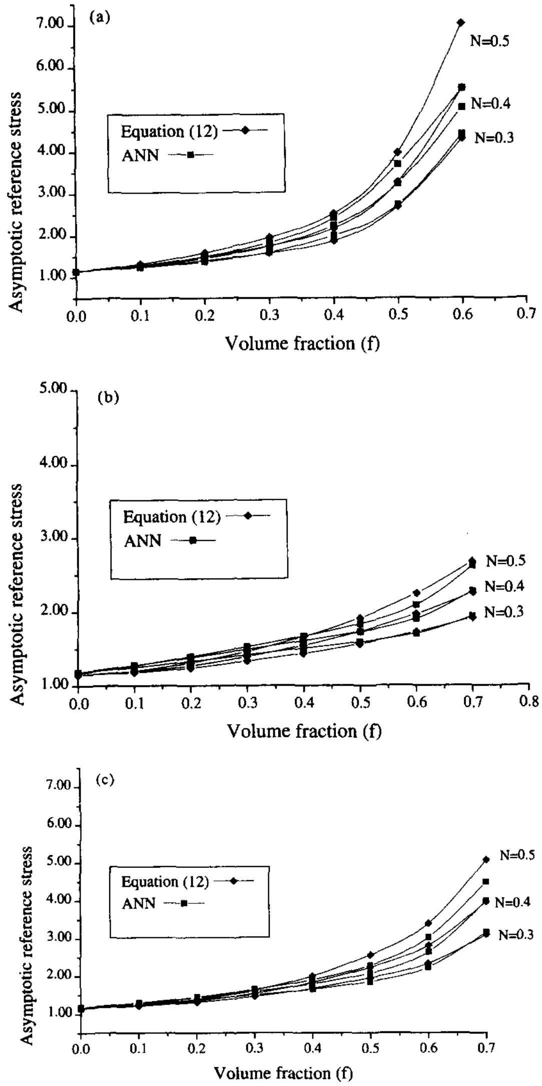

Fig. 10. New hardening exponents, asymptotic reference stress, $\bar{\sigma}_N$, for (a) square arrangement for loading at 0°; (b) square arrangement for loading at 45°; (c) hexagonal arrangement.

extra input parameters have been *derived* from the set of *natural* input parameters. This has been done as it was evident from the finite element results that the relationship the net is asked to learn is not linear with either f or N. The derived inputs when selected judiciously are known to accelerate the convergence. However, wrong selection of the derived inputs may adversely affect the performance of the net. Therefore, it is necessary to examine the suitability of the derived inputs. Moreover, it may be difficult, especially for a beginner, to select the correct derived inputs. Therefore, to test the efficiency of the derived inputs another net has been trained without providing the derived inputs. The net is shown in Fig. 8. The net has been trained in the same manner as discussed in the previous section. It has been observed that this net took longer time to train for the same relationships. Moreover, the average root mean square difference at the end of 2000 cycles has been observed to be 0.0008, marginally higher than the net with derived inputs. This is evident in Figs 9(a), (b) and (c) where the prediction of asymptotic reference stress from the two nets along with that of the finite element analysis is presented. The net with derived input shows marginally better agreement. This emphasizes the efficacy of the derived input. In all following examples the net with derived input has been used.

Predictions for new hardening exponents

It has already been mentioned that the artificial neural networks store the knowledge in a distributed manner. Therefore, the knowledge can be utilized to solve a new problem which it has not been trained for. Now we test the generalization capability of the net by presenting it with new problems with hardening exponents of 0.3 and 0.4 which was not a part of the training set. To obtain the output from the net only the input for the new problem is presented, and the prediction of the net is tested against reliable values. However, FEM results for these hardening exponents are not presently available. Therefore, the empirical relationship of equation (12) has been used for a *qualitative* testing of the net. The prediction of the net for hardening exponents of 0.3, 0.4 and 0.5 along with the results of the empirical expression have been presented in Figs 10(a), (b) and (c). It has been stated earlier that the empirical equation was very accurate for lower hardening exponents and it overpredicted in the case of higher hardening

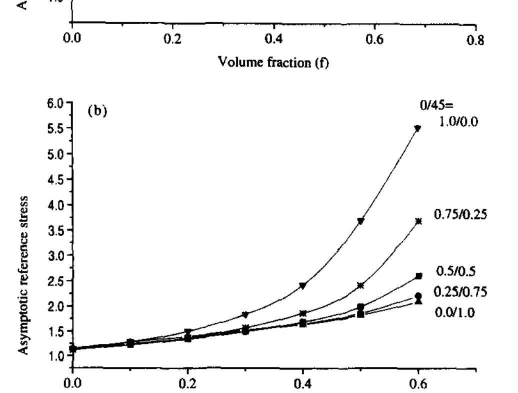

Fig. 11. Mixture of 0 and 45° loading, asymptotic reference stress, $\bar{\sigma}_N$, for (a) hardening exponent $N = 0.0$; (b) hardening exponent $N = 0.5$.

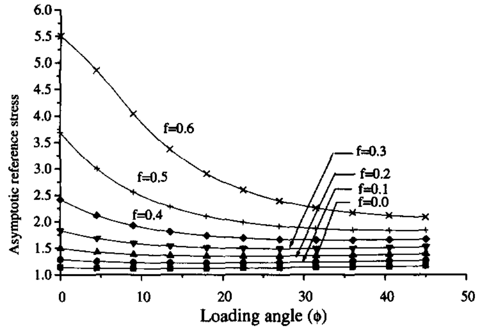

Fig. 12. Asymptotic reference stress, $\bar{\sigma}_N$, for various loading angles and varying volume fraction, hardening exponent $N = 0.0$.

exponents. The same trend is observed for the new cases as well. The prediction of the two methods agree very well for the hardening exponent of 0.3. The discrepancy between the two results increases progressively along with the increase in the hardening exponent. The predictions of the net were generally lower than that of the empirical expression. The expression is known to overpredict the asymptotic stresses. Therefore, the prediction of the net for the new cases is reasonable.

Prediction for new loading angles

The finite element analysis has been carried out for only two loading angles, 0 and 45°. It is not easy to carry out the finite element calculations for other loading angles as the provision of boundary conditions becomes extremely difficult. As a result, no numerical result exists for loading angles other than the above two angles. Experimental results are also not available for this case. Here we investigate whether the artificial neural network can be applied to this new situation. As results for only two loading angles were available it was envisaged that the input information may not be sufficient for the net to generalize for all intermediate angles. However, as two different input nodes have been provided for two cases of loading (e.g. Fig. 6) it was possible to mix the two loading angles in different proportions which roughly simulates different loading angles. To mix the

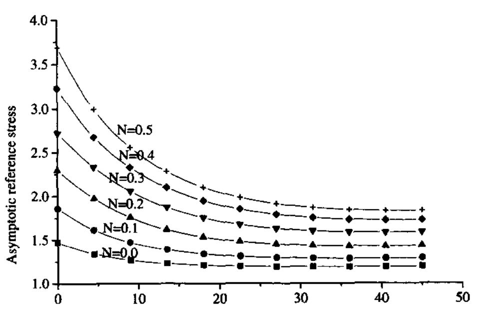

Fig. 13. Asymptotic reference stress, $\bar{\sigma}_N$, for various loading angles and varying hardening exponent, volume fraction $f = 0.5$.

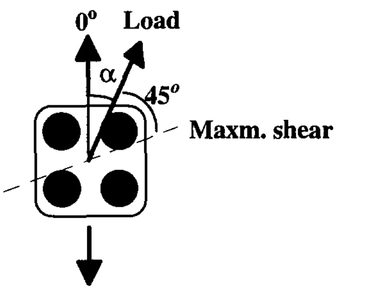

Fig. 14. Critical angle for strengthening for arbitrary loading direction.

two loading angles a value between 0 and 1 has been input in the nodes corresponding to 0 and 45° loading directions. The summation of the two entries is always 1. Hence, an entry of 1 in the node for 0° loading and 0 for the node for 45° loading signifies that all the fibers are loaded at 0° and vice versa. By changing the above two inputs the two directions of loading can be mixed in different proportions to simulate other loading directions. In Fig. 11(a) the results of this mixing in different proportions for a nonhardening matrix have been presented. The results for a hardening exponent of 0.5 have been presented in Fig. 11(b). It can be seen that the asymptotic stress is extremely sensitive to angle changes near 0° loading. Near 45° it is not sensitive at all. The asymptotic stress has changed approximately exponentially with the change in angle. The sensitivity increases when the hardening exponent or the fiber volume fraction is high. To study the effects of varying f and N two graphs have been plotted (Figs 12 and 13). They show that the strengthening reduces gradually as the loading angle is changed from 0 to 45°. At lower volume fractions there is little strengthening even when the loading angle is changed slightly from 0°. Figure 14 shows that at lower volume fractions the maximum shear plane which is at 45° with the loading direction goes entirely through the matrix without impinging any fiber. Therefore, there is no strengthening due to the presence of fiber. At high volume fractions (e.g. $f = 0.6$) there is a considerable strengthening even when the loading angle is away from 0°. The critical loading angle (α) after which there will be no interference of the fiber in case of $f = 0.6$ is calculated as 37°. In Fig. 12, it may be noticed that the strengthening continued up to a loading angle of approx. 35°. Therefore, the predictions of the net seem to be realistic. It may be noted in Fig. 12 that for $f = 0.6$ a change in curvature at about 5° loading direction has taken place. This is, however, expected as the curves are symmetric to 0° loading angle. This signals that at high volume fractions the strengthening is relatively stable for small changes in loading directions. At high volume fractions the relative diameter of a single fiber is larger. Therefore, for small changes in loading angle the maximum shear plane hits the fiber at approximately the same distance, resulting in higher strengthening. Thus the behavior of the net seems to be sensible. In the absence of any reliable experimental or analytical results, the behavior of the net could be explained only by logical reasoning. However, a test with a reliable experimental or analytical investigation will be most desirable.

CLOSURE

The power of artificial neural networks in predicting the strengthening of metal matrix composites under transverse loading has been demonstrated in this paper. An artificial neural network has been accordingly trained based on an accurate numerical study on these materials. The network was able to learn the behavior from the examples presented to it. The trained network has been used in prediction for the cases for which no other results, analytical or experimental, are available. The predictions of the network for hardening exponents for which it has not been trained were reasonable. The network has been used to predict the mechanical behavior of composites with square arrangement of fibers loaded at angles other than 0 or 45° where at present no other reliable predictions are available. The net predicted a high sensitivity for loading angles near 0° and a very flat behavior in the case of loading in the vicinity of 45°. Thus high strength values are restricted to a very narrow regime of loading directions. Although the predictions of the net seem reasonable it is desirable to compare the predictions of the net with a reliable analytical or experimental investigation.

The present work emphasizes the usefulness of this new technique of machine learning in solving difficult problems of materials science. The complicated material behavior obtained through experimental or numerical work can be directly captured into an artificial neural net and the net can later be used for cases for which experiments have not been performed. This will be attempted in a future work.

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