L3

6. $\lim \frac{5n^2}{n^2+1} = 5$

Dado $\varepsilon > 0$ arbitrário

$n_0 = \frac{\varepsilon}{2}$

$a_n = \frac{5n^2}{n^2+1}$, $(a_n)_{n\in\mathbb{N}}$ $\lim_{n\to\infty} a_n = 5$

$n \geq n_0 \Rightarrow |a_n - 5| < \varepsilon$

$\forall \varepsilon > 0$, $\exists n_0 \in \mathbb{N}$, tal que $\underbrace{|a_n - 5| < \varepsilon, \ \forall n \geq n_0.}_{(*)}$

distância

$|a-b|$ distância entre a e b

$|a_n - L| < \varepsilon$

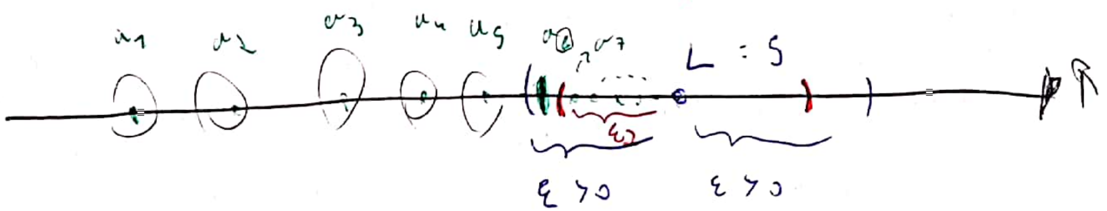
$\Updownarrow$

$L - \varepsilon < a_n < L + \varepsilon$

$\Updownarrow$

$a_n \in (L-\varepsilon, L+\varepsilon)$

1.



$n \geq 6 \Rightarrow |a_n - L| < \varepsilon$
$\quad \overset{\parallel}{n_0}$

$n_0 = 7$ $n \geq 7 \Rightarrow |a_n - L| < \varepsilon_2$

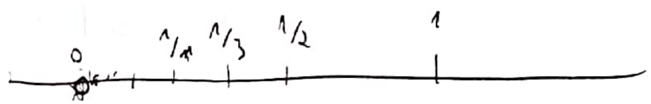
$a_1 \quad a_2 \quad \boxed{a_3 \quad a_4 \quad \dots}$

x é uma cauda da sequência

$a_1 \quad a_2 \quad \dots \quad \boxed{a_{525} \quad a_{526} \quad a_{527} \quad \dots}$

x é uma cauda

$a_n = \frac{1}{n}$



$n^2 \geq n \quad \forall n \in \mathbb{N}$

$n^2 + 1 \geq n$

$\Downarrow$

$\frac{1}{n^2+1} < \frac{1}{n}$

$\frac{x}{n^2+1}$

$\frac{x}{n^2+1} < \frac{x}{n}$

$\frac{1}{n} \xrightarrow[n\to\infty]{} 0$: $\forall \varepsilon > 0$, $\exists n_0 \in \mathbb{N}$ t.q.:

$n \geq n_0 \Rightarrow \left|\frac{1}{n} - 0\right| < \varepsilon$

$\left|\frac{1}{n} - 0\right| = \left|\frac{1}{n}\right| = \frac{1}{n} < \varepsilon$ $\qquad \varepsilon > 0 \Rightarrow \frac{1}{n} < \varepsilon$

Prop. Arquimediana: $\varepsilon > 0 \Rightarrow \exists n_0 \in \mathbb{N}$ t.q. $\frac{1}{n_0} < \varepsilon$

$n \geq n_0 \Rightarrow \frac{1}{n} \leq \frac{1}{n_0} < \varepsilon \Rightarrow \frac{1}{n} < \varepsilon \Rightarrow \left|\frac{1}{n} - 0\right| < \varepsilon$

$\frac{1}{n} > 0 \Rightarrow \frac{1}{n} = \left|\frac{1}{n}\right| = \left|\frac{1}{n} - 0\right|$

$\Rightarrow \lim \frac{1}{n} = 0$

$$\left|\frac{5n^2}{n^2+1} - 5\right| = \left|\frac{\cancel{5n^2} - \cancel{5n^2} - 5}{n^2+1}\right| = \left|\frac{-5}{n^2+1}\right| = \frac{5}{n^2+1}$$

$$\frac{1}{n^2+1} < \frac{1}{n}$$

$\alpha > 0$

$\varepsilon > 0$, $\frac{1}{n_0} < \varepsilon$

$\frac{1}{n_0} < \alpha\varepsilon$

$\frac{1}{n_0} < \frac{\varepsilon}{\alpha}$

$L = \lim a_n$

11. $(a_n)_{n\in\mathbb{N}}$ t.q. $a_n < 1$.

Tese: $\lim a_n \leq 1$

Por absurdo, suponha que $1 < \lim a_n = L$

$\boxed{L - 1 > 0}$

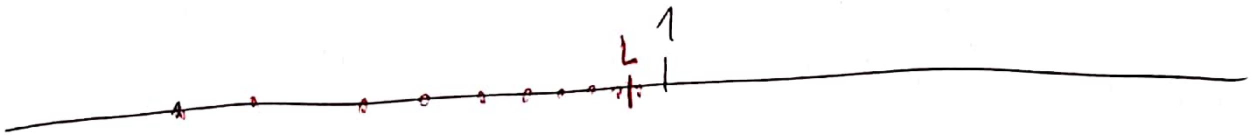
Def. de limite: $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}$ t.q.: $n_0 \leq n \Rightarrow |a_n - L| < \varepsilon$

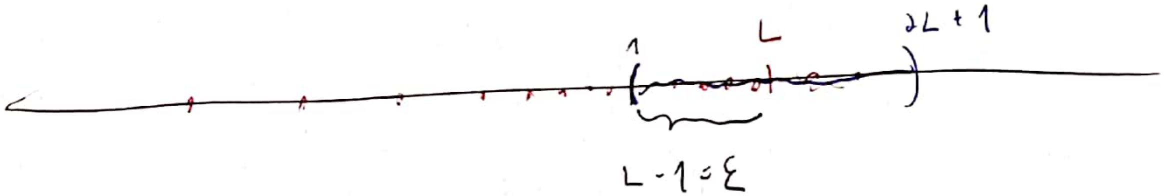
$\varepsilon = L - 1 > 0$. $\exists n_0 \in \mathbb{N}$ t.q.:

$n \geq n_0 \Rightarrow |a_n - L| < L - 1 \Leftrightarrow L - (L-1) < a_n < L + L - 1$

$\Rightarrow 1 < a_n < \cancel{2L} - 1$

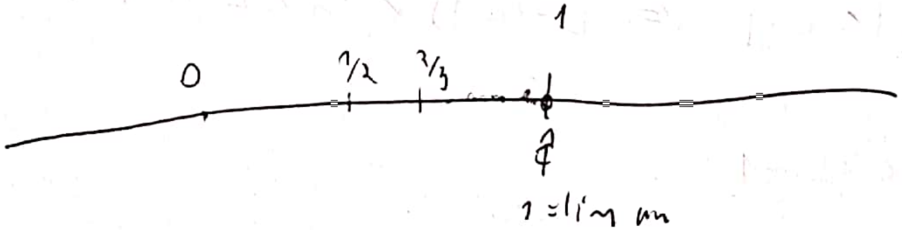
$a_n < 1 \quad \forall n \in \mathbb{N}$ $\Rightarrow \lim a_n \leq 1$





$a_n < 1 \quad \forall n \quad \overset{?}{\Rightarrow} \quad \lim a_n \leq 1$

$a_n = 1 - \frac{1}{n}$



11, b)

$$f(x) = \sqrt{x}\left(\sqrt{x+5} - \sqrt{x}\right)$$

$$= \sqrt{x}\left(\sqrt{x\left(1+\frac{5}{x}\right)} - \sqrt{x}\right)$$

$$= \sqrt{x}\left[\sqrt{x}\cdot\sqrt{1+\frac{5}{x}} - \sqrt{x}\right]$$

$$= x\left(\sqrt{1+\frac{5}{x}} - 1\right)$$

$$= \frac{\sqrt{1+5/x} - 1}{1/x} \quad \text{do tipo } \frac{0}{0}$$

$\sqrt{1+5/x} - 1 \to 0$, $5/x \to 0$

$1/x \to 0 \quad x \to +\infty$

$$\lim f(x) = \frac{\frac{1}{2}\cdot\frac{1}{\sqrt{1+5/x}}\cdot 5\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \frac{5}{2}\cdot\frac{1}{\sqrt{1+5/x}} = \frac{5}{2}$$

$1 \to 1$, $\sqrt{1+5/x} \to 1$, $5/x \to 0$