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Microeconomia II
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## Seção 14.B - Moral Hazard

**Exercise 1.** Consider the principal-agent problem from section 14.B of MWG (1994). Let $E = \{0, 1\}$ be the set of possible effort levels the agent has and $\Pi = [0, 1]$ the set of possible project's profit (which is observable), and the cost of effort given by $g(0) = 0$ and $g(1) = k > 0$. Suppose the agent is risk neutral with $v(w) = w$ and that the profit distribution, conditional on effort, is given by $f(\pi|0) = 1$, $\forall \pi \in \Pi$, e $f(\pi|1) = \pi + 1/2$, $\forall \pi \in \Pi$. The agent's opportunity cost is 0.

(a) Verify that the profit distribution with high effort first order stochastically dominates the profit distribution with low effort.

(b) If effort is observable, which condition the wage should satisfy in order to implement $e = 0$ optimally? And to implement $e = 1$ optimally?

(c) Assume further that effort is observable. For which values of $k$ is the optimal contract given by $e = 1$?

(d) Suppose the optimal contract when effort is observable is with $e = 1$. Which wage should be set in order to implement the optimal contract when effort is not observable, but the agent can make payments to the principal?

**Exercise 2 (MGW 14.B.2).** Derive the first-order condition characterizing the optimal compensation scheme for the two-effort-level hidden action model studied in section 14.B when the principal is strictly risk averse.

**Exercise 3 (MGW 14.B.3).** Consider a hidden action model in which the owner is risk neutral while the manager has preferences defined over the mean and the variance of his income $w$ and his effort level $e$ as follows: Expected utility $= E[w] - \phi Var(w) - g(e)$, where $g'(0) = 0$, $(g'(e), g''(e), g'''(e)) \gg 0$ for $e > 0$, and $\lim_{e\to\infty} g'(e) = \infty$. Possible effort choices are $e \in \mathbb{R}_+$. Conditional on effort level $e$, the realization of profit is normally distributed with mean $e$ and variance $\sigma^2$.

(a) Restrict attention to linear compensation schemes $w(\pi) = \alpha + \beta\pi$. Show that the manager's expected utility given $w(\pi)$, $e$ and $\sigma^2$ is given by $\alpha + \beta e - \phi\beta^2\sigma^2 - g(e)$.

(b) Derive the optimal contract when $e$ is observable.

(c) Derive the optimal linear compensation scheme when $e$ is not observable. What effects do changes in $\phi$ and $\sigma^2$ have?

**Exercise 4 (MGW 14.B.4).** Consider the following hidden action model with three possible actions $E = \{e_1, e_2, e_3\}$. There are two possible profit outcomes: $\pi_H = 10$ and $\pi_L = 0$. The probabilities of $\pi_H$ conditional on the three effort levels are $f(\pi_H|e_1) = \frac{2}{3}$, $f(\pi_H|e_2) = \frac{1}{2}$, and $f(\pi_H|e_3) = \frac{1}{3}$. The agent's effort cost function has $g(e_1) = \frac{5}{3}$, $g(2) = \frac{8}{5}$, $g(e_3) = \frac{4}{3}$. Finally, $v(w) = \sqrt{w}$, and the manager's reservation utility is $\bar{u} = 0$.

(a) What is the optimal contract when effort is observable?

(b) Show that if effort is not observable, then $e_2$ is not implementable. For what levels of $g(e_2)$ would $e_2$ be implementable? [Hint: Focus on the utility levels the manager will get for the two outcomes, $v_1$ and $v_2$, rather than on the wage payments themselves.]

(c) What is the optimal contract when effort is not observable?

(d) Suppose, instead, that $g(e_1) = \sqrt{8}$, and let $f(\pi_H|e_1) = x \in (0, 1)$. What is the optimal contract if effort is observable? As $x$ approaches 1, is the level of effort implemented higher or lower when effort is not observable than when it is observable?