von Mises

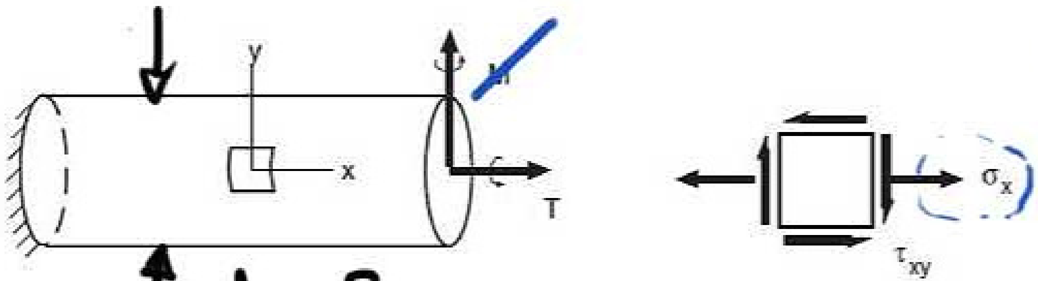
A solid circular shaft subjected to pure torsion must be designed to avoid yielding, with a safety factor X. Find the required diameter as a function of the torque T and the yield strength σo, using (a) the maximum shear stress criterion, and (b) the octahedral shear stress criterion. How much do these two sizes differ?

TRESCA

$\boxed{M=0}$

$\sigma_x = \sigma_y = 0$

$$I = \frac{\pi d^4}{64}$$



$d = ?$

$$\sigma_{1,2} = \cancel{\frac{\sigma_x + \sigma_y}{2}} \pm \sqrt{\left(\cancel{\frac{\sigma_x - \sigma_y}{2}}\right)^2 + \tau_{xy}^2}$$

0

$$\sigma_{1,2} = \pm\sqrt{\tau_{xy}^2}$$

$$\sigma_{1,2} = \pm \tau_{xy}$$

· CONDIÇÃO

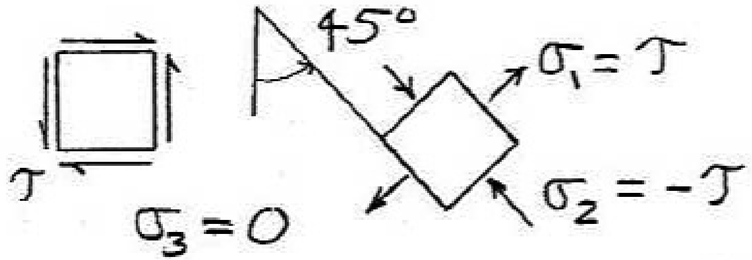
$$\bar{\sigma}_{ef} \leq \frac{\sigma_0}{X}$$

$\sigma_0 \rightarrow$ TENSÃO ESCOAMENTO

$X \rightarrow$ FATOR SEGURANÇA > 1

TENSÃO EFETIVA (TRESCA, von Mises)

- **TRESCA**



$$\bar{\sigma}_{ef} = \max\left(|\sigma_1 - \sigma_2|;\ |\sigma_2 - \sigma_3|;\ |\sigma_3 - \sigma_1|\right)$$

$$= \max\left(|\tau - (-\tau)|;\ |-\tau|;\ |\tau|\right)$$

$$= \max\left(|2\tau|,\ \tau;\ \tau\right)$$

$$\boxed{\bar{\sigma}_{ef} = 2\tau} \rightarrow \sigma_{ef} = \frac{\sigma_0}{X} = 2\tau$$

$$\tau = \frac{T}{J} \cdot r = \frac{T}{\frac{\pi r^4}{2}} \cdot r \rightarrow \tau = \frac{2T}{\pi r^3}$$

$$\sigma_{et} = 2\tau \qquad \tau = \frac{2T}{\pi r^3} \qquad r = \frac{d}{2}$$

$$\tau = \frac{16T}{\pi d^3}$$

$$\sigma_{et} = 2\left(\frac{16T}{\pi d^3}\right)$$

ESFORZO

F.S

$$\frac{\sigma_0}{X} = \frac{32T}{\pi d^3} \rightarrow d_{TRESCA} = \left(\frac{32\,T\cdot X}{\pi\,\sigma_0}\right)^{1/3}$$

GEOMETRIA

Resist. Material

. von Mises

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

$$\sigma_1 = \tau \qquad \sigma_2 = -\tau \qquad \sigma_3 = 0$$

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\left[4\tau^2+\tau^2+\tau^2\right]$$

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\sqrt{6\tau^2} = \sqrt{3}\,\tau \rightarrow \bar{\sigma}_{ef} \leq \frac{\sigma_0}{X}$$

$$\sqrt{3}\left(\frac{16\,T}{\pi d^3}\right) = \frac{\sigma_0}{X}$$

$$d_{Mises} = \left(\frac{16\sqrt{3}\, T x}{\pi \sigma_0} \right)^{1/3}$$

vs

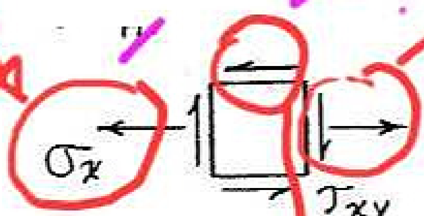
$$d_{TRESCA} = \left(\frac{32\, T x}{\pi \sigma_0} \right)^{1/3}$$

$$\frac{d_{TRESCA}}{d_{Mises}} = \left(\frac{32}{16\sqrt{3}} \right)^{1/3} = \left(\frac{2}{\sqrt{3}} \right)^{1/3}$$

$$\frac{d_{TRESCA}}{d_{MISES}} = 1{,}049 \quad \sim 5\%$$

$M \neq 0$

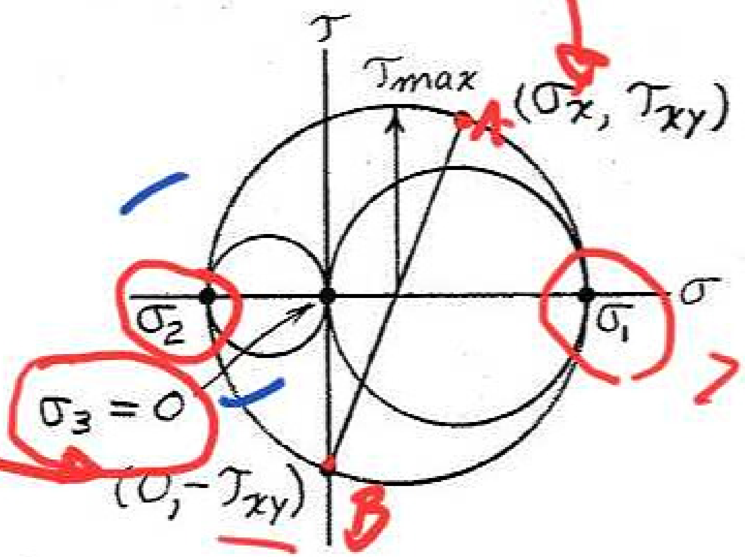
$d = f\,|M, T, \sigma_0, X|$



$\sigma_1 > 0,\ \sigma_2 < 0$

$\sigma_3 = \sigma_z = 0$

σ_1, σ_2 give τ_{max}



> 0

$$\left(\frac{\sigma_x}{2}\right)^2 < \left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2$$

$$\sigma_{1,2} = \frac{\sigma_x + \cancel{\sigma_y}}{2} \pm \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_2 < 0$

$\sigma_1 > 0$

TROSCA

$$\sigma_{ef} = \text{máx}\left(|\sigma_1 - \sigma_2| ; |\sigma_2 - \sigma_3| ; |\sigma_3 - \sigma_1| \right)$$

$$\sigma_3 = 0 \qquad \sigma_1 > 0 \qquad \sigma_2 < 0$$

$$\downarrow$$

$$\sigma_{ef} = \sigma_1 - \sigma_2 = 2\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \rightarrow |\sigma_1 - \sigma_2| =$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\bar{\sigma}_{ef} = 2\left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right] \leq \frac{\sigma_0}{X}$$

FLEXÃO

$$\sigma_x = \frac{M}{I} y = \frac{M}{\frac{\pi d^4}{64}} \cdot \frac{d}{2}$$

$$\sigma_x = \frac{32 M}{\pi d^3}$$

TORÇÃO

$$\tau_{xy} = \frac{16 T}{\pi d^3}$$

$$\frac{\sigma_0}{X} = 2\left[\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2\right]^{1/2}$$

$$\downarrow$$

$$\frac{\sigma_0}{X} = \frac{32}{\pi \underline{\underline{d}}^3}\left[M^2 + T^2\right]^{1/2}$$

$$\downarrow$$

GEOMETRIA

CARICAMENTO

$$d_{TRESCA} = \left[\frac{32X}{\pi \sigma_0}\sqrt{M^2 + T^2}\right]^{1/3}$$

↳ PROPRIETÀ MATERIALE

von Mises

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right]^{1/2}$$

$\sigma_x\,,\ \tau_{xy} \neq 0 \qquad \sigma_y=\sigma_z=\tau_{yz}=\tau_{xz}=0$

$$\bar{\sigma}_{ef} = \frac{1}{\sqrt{2}}\left[2\sigma_x^2+6\tau_{xy}^2\right]^{1/2}$$

$$\bar{\sigma}_{ef} = \left[\sigma_x^2+3\tau_{xy}^2\right]^{1/2}$$

$$\sigma_{ef} \leq \frac{\sigma_0}{X}$$

$$\sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \frac{\sigma_0}{X}$$

$$\left[\left(\frac{32M}{\pi d^3}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2\right]^{1/2} = \frac{\sigma_0}{X}$$

$$\frac{16}{\pi \underline{d}^3}\left[4M^2 + 3T^2\right]^{1/2} = \frac{\sigma_0}{X}$$

$$d = \left[\frac{16X}{\pi \sigma_0} \sqrt{4M^2 + 3T^2} \right]^{1/3}$$

RESTRIÇÕES

- CUSTO
- DISPONIBIL
- PESO
- VOLUME

CARREGAMENTO

MATERIAL

GEOMETRIA

- AL
- AÇO
- POLÍMERO