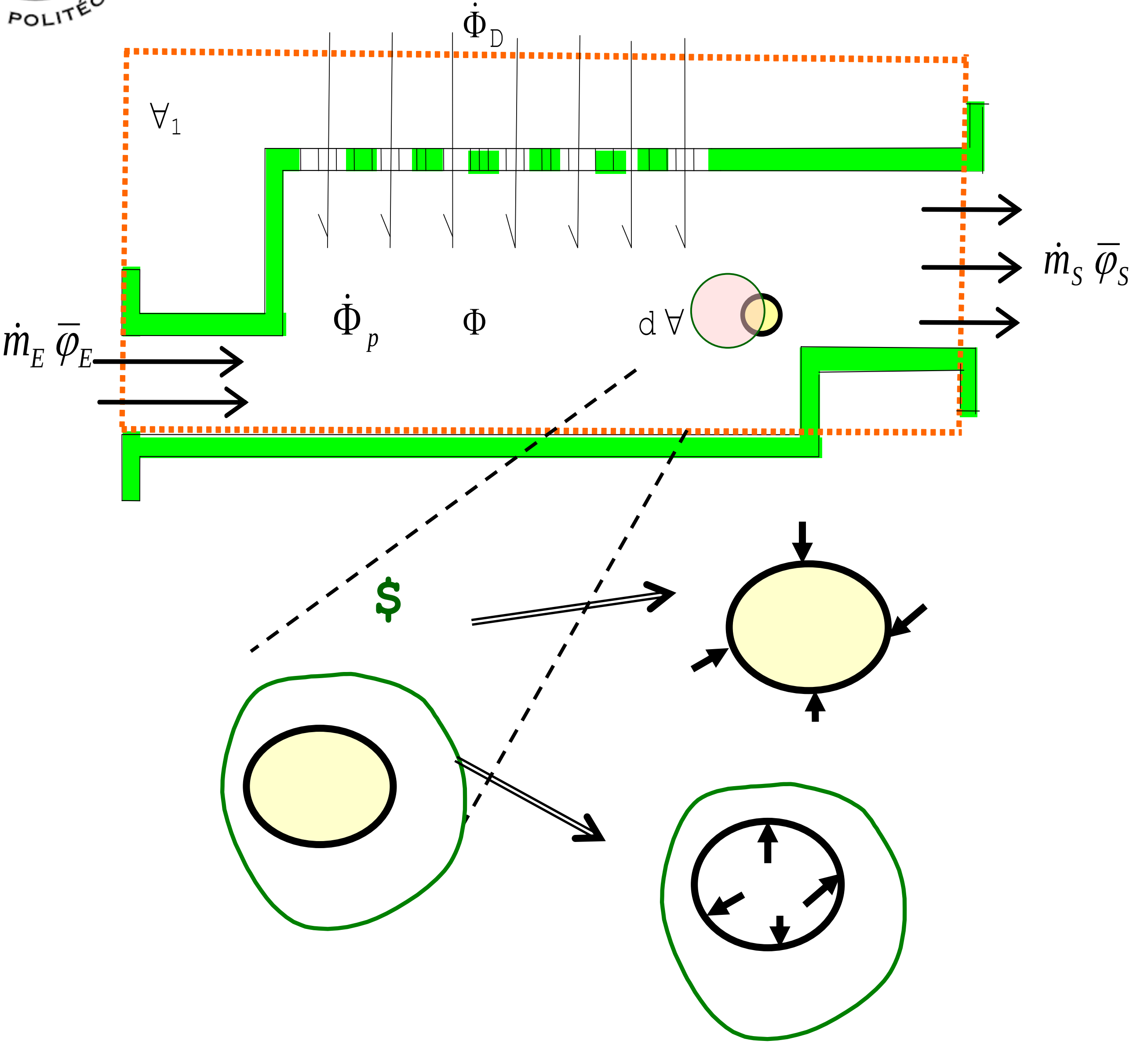
PQI 3202: FENÔMENOS DE TRANSPORTE I

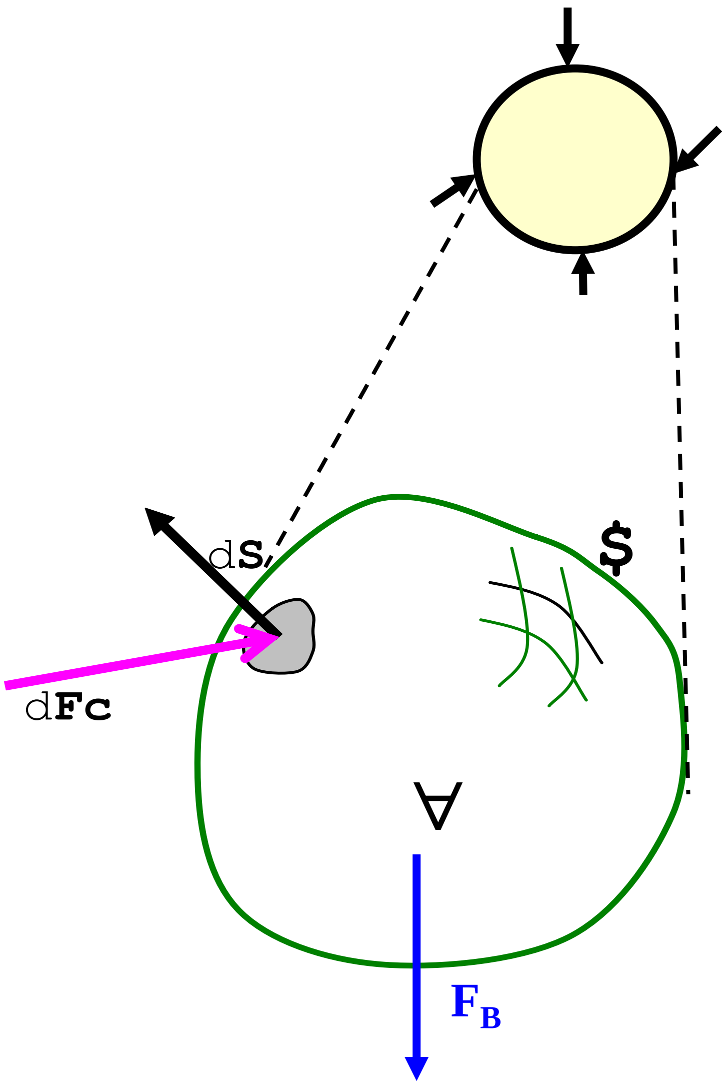
Tensão em fluidos

Equações Constitutivas – Escoamento - **CAUCHY**



Equações Constitutivas

FORÇA DE CONTATO E DE CAMPO



F_B - força de campo – produção

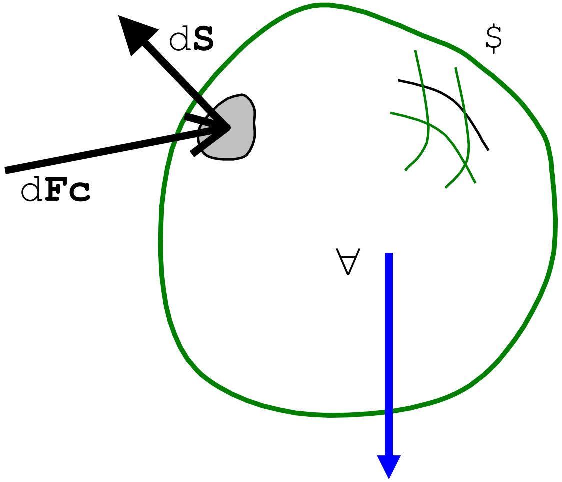
F_C - força de contato - difusão

Equações Constitutivas

FORÇA DE CONTATO E DE CAMPO

Força de campo/volume: GRAVIDADE $\vec{F}_B = \rho \vec{g}$

O tensor das **Tensões**: $$\vec{\vec{\sigma}} = \frac{d\vec{F}_C}{d\vec{S}}$$



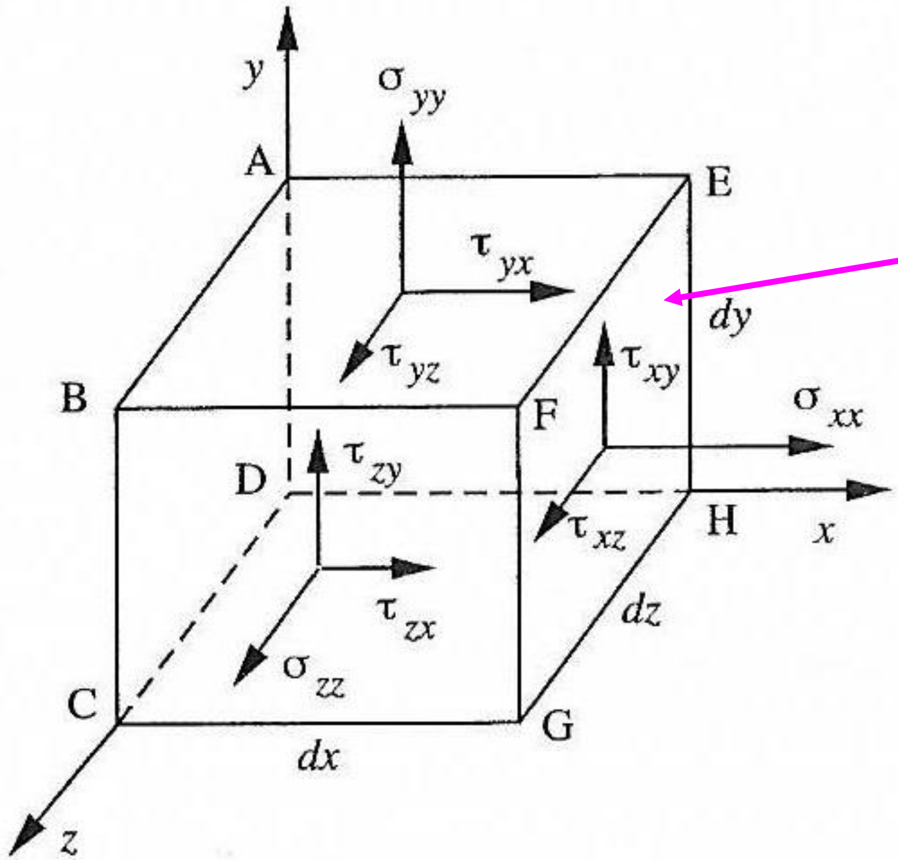
F_B - força de campo – produção

F_C - força de contato - difusão

Força de contato

$$\vec{\vec{\sigma}} = \frac{d\vec{F}_C}{d\vec{A}} \quad \begin{cases} d\vec{F_c} = dF_{cx}\vec{\imath} + dF_{cy}\vec{\jmath} + dF_{cz}\vec{k} \\ d\vec{A} = dA_x\vec{\imath} + dA_y\vec{\jmath} + dA_z\vec{k} \end{cases}$$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}.$$



$$\sigma_{xx} = \frac{dF_{cx}}{dA_x}$$

$$\tau_{xy} = \frac{dF_{cy}}{dA_x}$$

$$\tau_{xz} = \frac{dF_{cz}}{dA_x}$$

Força de contato

Decomposição do Tensor das **TENSÕES** σ

$$\vec{\vec{\sigma}} = -p\,\vec{\vec{\delta}} + \vec{\vec{\tau}}$$

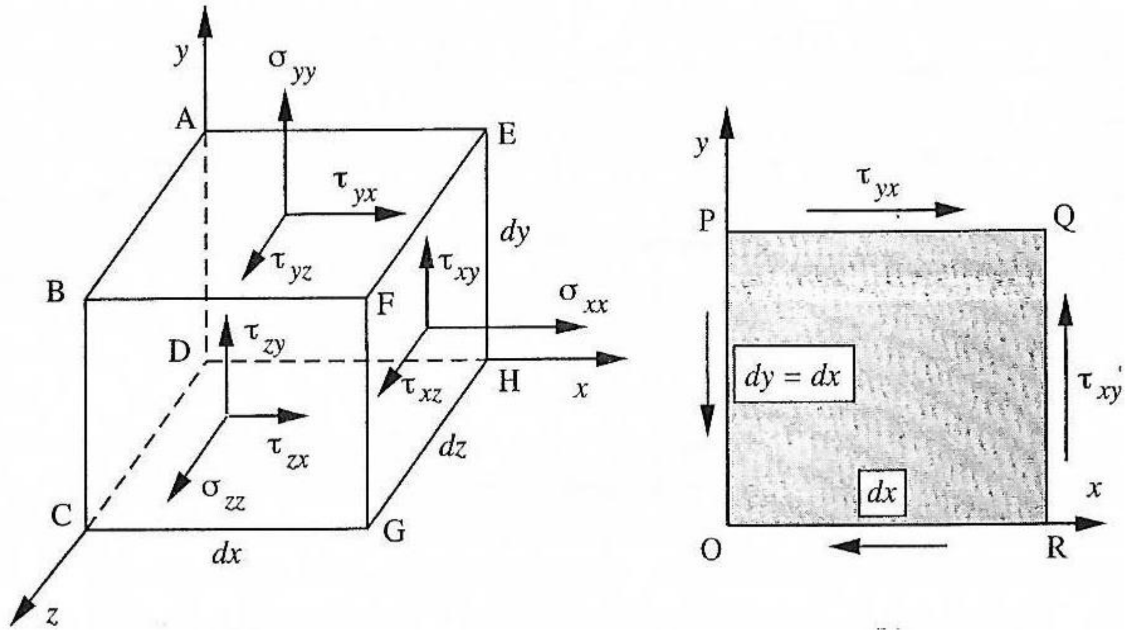
tensões normais: $\sigma_{ii} = -p + \tau_{ii}$

tensões de cisalhamento: $\sigma_{ij} = \tau_{ij}$

σ_{ij} : componente da força (de contato) na direção j por unidade de área perpendicular à direção i.

$$\tau = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}, \qquad \sigma = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}. \qquad (5$$

$$\vec{\vec{\sigma}} = \frac{d\vec{F}_C}{d\vec{S}}$$



Força de contato na direção x

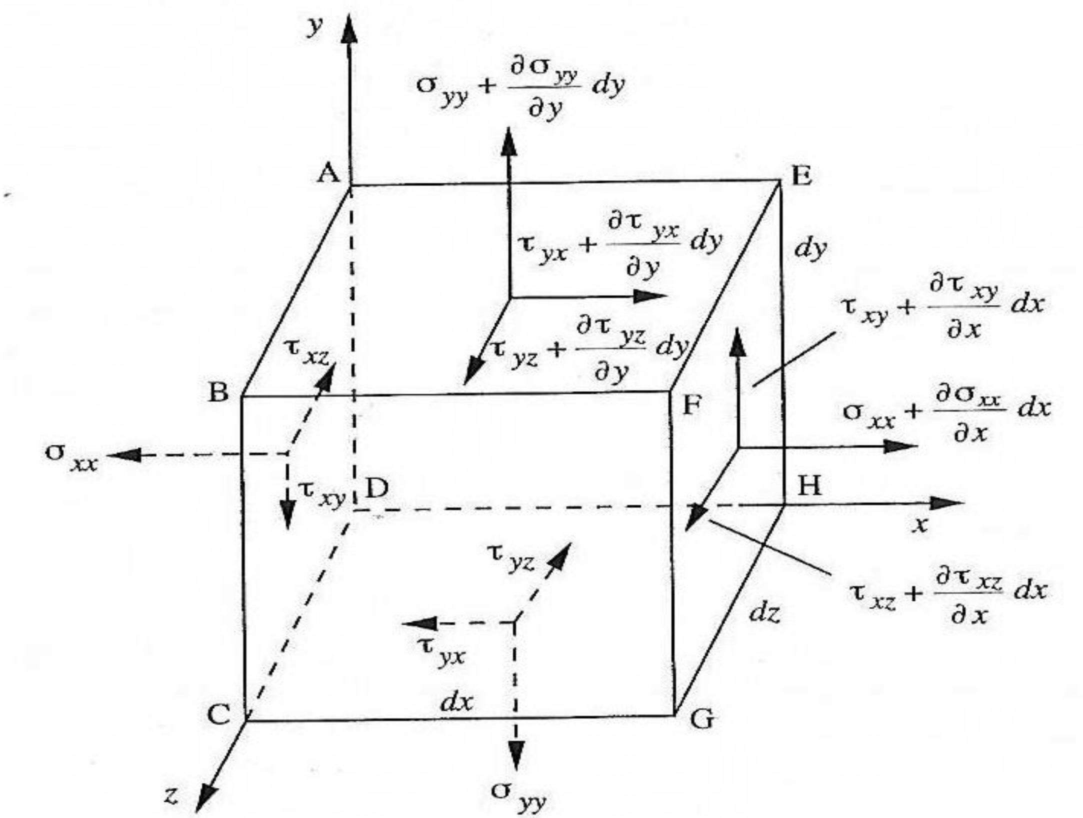
$$d\vec{F_c} = dF_{cx}\vec{i} + dF_{cy}\vec{j} + dF_{cz}\vec{k}$$

$$dF_{cx} = \{[(\tau_{xx})_{x+\Delta x} - (\tau_{xx})_x]\Delta y\Delta z + \left[(\tau_{yx})_{y+\Delta y} - (\tau_{yx})_y\right]\Delta x\Delta z$$

$$+[(\tau_{zx})_{z+\Delta z} - (\tau_{zx})_z]\Delta x\Delta y\} - \{[(p)_{x+\Delta x} - (p)_x]\Delta y\Delta z\}$$

$$\sigma_{ii} = -p + \tau_{ii}$$

Volume de controle finito $\Delta V = \Delta x\Delta y\Delta z.$



Força de contato na direção x

$$dF_{cx} = \{[(\tau_{xx})_{x+\Delta x} - (\tau_{xx})_x]\Delta y\Delta z + \left[(\tau_{yx})_{y+\Delta y} - (\tau_{yx})_y\right]\Delta x\Delta z$$

$$+[(\tau_{zx})_{z+\Delta z} - (\tau_{zx})_z]\Delta x\Delta y\} - \{[(p)_{x+\Delta x} - (p)_x]\Delta y\Delta z\}$$

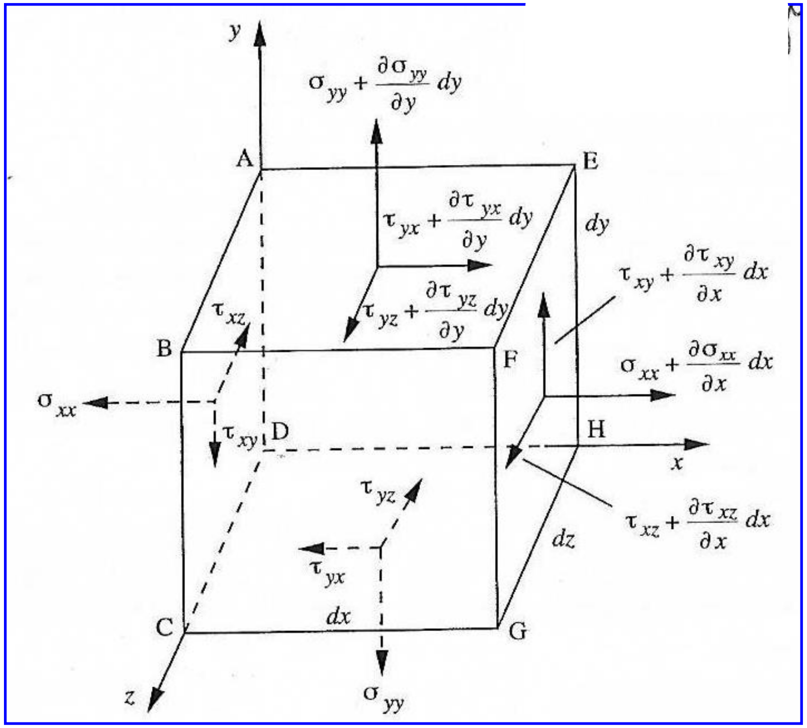
$$dF_{cx}$$

$$= -\left[\frac{(\tau_{xx})_{x+\Delta x} - (\tau_{xx})_x}{\Delta x}\right]\Delta x\Delta y\Delta z$$

$$+\left[\frac{(\tau_{yx})_{y+\Delta y} - (\tau_{yx})_y}{\Delta y}\right]\Delta y\Delta x\Delta z$$

$$+\left[\frac{(\tau_{zx})_{z+\Delta z} - (\tau_{zx})_z}{\Delta z}\right]\Delta z\Delta x\Delta y$$

$$-\left\{[\frac{(p)_{x+\Delta x} - (p)_x}{\Delta x}]\Delta x\Delta y\Delta z\right\}$$



Força de contato e campo na direção x

$$dF_{cx} = \left[\frac{(\tau_{xx})_{x+\Delta x} - (\tau_{xx})_x}{\Delta x}\right]\Delta x\Delta y\Delta z + \left[\frac{(\tau_{yx})_{y+\Delta y} - (\tau_{yx})_y}{\Delta y}\right]\Delta y\Delta x\Delta z$$

$$+ \left[\frac{(\tau_{zx})_{z+\Delta z} - (\tau_{zx})_z}{\Delta z}\right]\Delta z\Delta x\Delta y - \left\{ \left[\frac{(p)_{x+\Delta x} - (p)_x}{\Delta x}\right]\Delta x\Delta y\Delta z \right\}$$

$$\frac{dF_{cx}}{dV} = \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x}$$

$$\mathrm{dV} = \Delta z\Delta x\Delta y$$

$$dF_{bx} = \rho(\Delta x\Delta y\Delta z)g_x$$

$$\frac{dF_{bx}}{dV} = \rho g_x$$

$$(\rho dV)\frac{\mathrm{D}v_x}{\mathrm{Dt}} = dF_{bx} + dF_{cx}$$

2ª lei de Newton – Forças de campo e de contato

$$\vec{a} = \frac{\mathrm{D}\vec{v}}{\mathrm{Dt}} = \frac{\partial \vec{v}}{\partial \mathrm{t}} + \vec{v}.\,\mathrm{gr\vec{a}d}\vec{v}$$

$$(\rho dV)\vec{a} = (\rho dV)\frac{\mathrm{D}\vec{v}}{\mathrm{Dt}} = (\rho dV)\frac{\partial \vec{v}}{\partial \mathrm{t}} + (\rho dV)\vec{v}.\,\mathrm{gr\vec{a}d}\vec{v}$$

$$(\rho dV)\frac{\mathrm{D}v_x}{\mathrm{Dt}} = (\rho dV)\left[\frac{\partial v_x}{\partial \mathrm{t}} + \frac{\partial v_x}{\partial x}v_x + \frac{\partial v_x}{\partial y}v_y + \frac{\partial v_x}{\partial z}v_z\right]$$

$$(\rho dV)\frac{\mathrm{D}v_x}{\mathrm{Dt}} = dF_{bx} + dF_{cx}$$

2ª lei de Newton – Forças de campo e de contato

$$(\rho dV)\frac{\mathrm{D}v_x}{\mathrm{Dt}} = dF_{bx} + dF_{cx}$$

$$\rho\frac{\mathrm{D}v_x}{\mathrm{Dt}} = \frac{dF_{bx}}{dV} + \frac{dF_{cx}}{dV}$$

$$\frac{dF_{cx}}{dV} = \left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x}$$

$$\frac{dF_{bx}}{dV} = \rho g_x$$

$$\rho\frac{\mathrm{D}v_x}{\mathrm{Dt}} = \rho\left[\frac{\partial v_x}{\partial \mathrm{t}} + \frac{\partial v_x}{\partial x}v_x + \frac{\partial v_x}{\partial y}v_y + \frac{\partial v_x}{\partial z}v_z\right] =$$
$$= \left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \overrightarrow{grad}\, p + div\,\overline{\overline{\tau}}$$

Fluido Newtoniano: Tensão x deformação angular

$$\vec{\vec{\sigma}} = -p\vec{\vec{\delta}} + \vec{\vec{\tau}}$$

tensões normais: $\sigma_{ii} = -p + \tau_{ii}$

tensões de cisalhamento: $\sigma_{ij} = \tau_{ij}$

$\dot{\gamma}$ **:** deformação angular

$$\dot{\gamma} = \begin{pmatrix} 2\frac{\partial v_x}{\partial x} & \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) & \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) \\ \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) & 2\frac{\partial v_y}{\partial y} & \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) \\ \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) & \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right) & 2\frac{\partial v_z}{\partial z} \end{pmatrix} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T,$$

Fluido Newtoniano: Tensão x deformação angular

τ_{ii}

$$\tau_{xx} = 2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu \nabla \cdot \mathbf{v},$$
$$\tau_{yy} = 2\mu \frac{\partial v_y}{\partial y} - \frac{2}{3}\mu \nabla \cdot \mathbf{v},$$
$$\tau_{zz} = 2\mu \frac{\partial v_z}{\partial z} - \frac{2}{3}\mu \nabla \cdot \mathbf{v}.$$

τ_{ij}

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right),$$
$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right),$$
$$\tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right).$$

σ_{ii}

$$\sigma_{xx} = -p + 2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu \nabla \cdot \mathbf{v},$$
$$\sigma_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y} - \frac{2}{3}\mu \nabla \cdot \mathbf{v},$$
$$\sigma_{zz} = -p + 2\mu \frac{\partial v_z}{\partial z} - \frac{2}{3}\mu \nabla \cdot \mathbf{v}.$$

Fluido Newtoniano: Tensão x deformação angular

τ_{ii}

$$\tau_{xx} = 2\mu \frac{\partial \mathrm{v}_x}{\partial x} - \frac{2\mu}{3}\left[\frac{\partial \mathrm{v}_x}{\partial x} + \frac{\partial \mathrm{v}_y}{\partial y} + \frac{\partial \mathrm{v}_z}{\partial z}\right]$$

$$\tau_{yy} = 2\mu \frac{\partial \mathrm{v}_y}{\partial y} - \frac{2\mu}{3}\left[\frac{\partial \mathrm{v}_x}{\partial x} + \frac{\partial \mathrm{v}_y}{\partial y} + \frac{\partial \mathrm{v}_z}{\partial z}\right]$$

$$\tau_{zz} = 2\mu \frac{\partial \mathrm{v}_z}{\partial z} - \frac{2\mu}{3}\left[\frac{\partial \mathrm{v}_x}{\partial x} + \frac{\partial \mathrm{v}_y}{\partial y} + \frac{\partial \mathrm{v}_z}{\partial z}\right]$$

Caso de escoamento incompressível:

$$\mathrm{div}\,\vec{\mathrm{v}} = 0$$

τ_{ij}

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial \mathrm{v}_x}{\partial y} + \frac{\partial \mathrm{v}_y}{\partial x}\right]$$

$$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial \mathrm{v}_y}{\partial z} + \frac{\partial \mathrm{v}_z}{\partial y}\right]$$

$$\tau_{xz} = \tau_{zx} = \mu\left[\frac{\partial \mathrm{v}_z}{\partial x} + \frac{\partial \mathrm{v}_x}{\partial z}\right]$$

$$\rho \frac{Dv_x}{Dt} = \rho\left[\frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x} v_x + \frac{\partial v_x}{\partial y} v_y + \frac{\partial v_x}{\partial z} v_z\right] =$$

$$= \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\tau_{xx} = 2\mu \frac{\partial v_x}{\partial x} - \frac{2\mu}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$

$$\tau_{xz} = \tau_{zx} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

$$\rho \frac{Dv_x}{Dt} = \rho\left[\frac{\partial v_x}{\partial t} + \frac{\partial v_x}{\partial x} v_x + \frac{\partial v_x}{\partial y} v_y + \frac{\partial v_x}{\partial z} v_z\right] =$$

$$\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \frac{\mu}{3}\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

incompressível: $\mathrm{div}\, \vec{v} = 0$

NAVIER - STOKES

$$\rho\frac{Dv_x}{Dt}=\rho\left[\frac{\partial v_x}{\partial t}+\frac{\partial v_x}{\partial x}v_x+\frac{\partial v_x}{\partial y}v_y+\frac{\partial v_x}{\partial z}v_z\right]=$$

$$\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\frac{\mu}{3}\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

$$\mathrm{div}\,\vec{v}=0$$

$$\rho\frac{Dv_x}{Dt}=\rho\left[\frac{\partial v_x}{\partial t}+\frac{\partial v_x}{\partial x}v_x+\frac{\partial v_x}{\partial y}v_y+\frac{\partial v_x}{\partial z}v_z\right]=$$

$$\mu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2}+\frac{\partial^2 \mathrm{v}_x}{\partial y^2}+\frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

Escoamento incompressível, fluido newtoniano e viscosidade constante

$$\rho\frac{D\vec{v}}{Dt}=\rho\vec{g}-\vec{\mathrm{grad}}\,p+\mathrm{div}\,\vec{\vec{\tau}}$$

NAVIER - STOKES

$$\rho\frac{D\vec{v}}{Dt}=\rho\frac{\partial\vec{v}}{\partial t}+\rho\,\vec{v}.\vec{\mathrm{grad}}\vec{v}=\rho\vec{g}-\vec{\mathrm{grad}}\,p+\mu\,\mathrm{lap}\,\vec{v}$$

QUANTIDADE DE MOVIMENTO E NAVIER - STOKES

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}+\rho g_x,$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}+\rho g_y,$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}+\rho g_z.$$

NAVIER - STOKES

Escoamento incompressível, newtoniano e viscosidade constante

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x,$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y,$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z.$$

QUANTIDADE DE MOVIMENTO – Coord. Cilíndricas

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$
$$= -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial (r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z} + \rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{r^2}\frac{\partial (r^2\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$
$$= -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \rho g_z.$$

NAVIER – STOKES : Coord. Cilíndricas

Escoamento incompressível, newtoniano
e viscosidade constante

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z.$$