Shs 5896

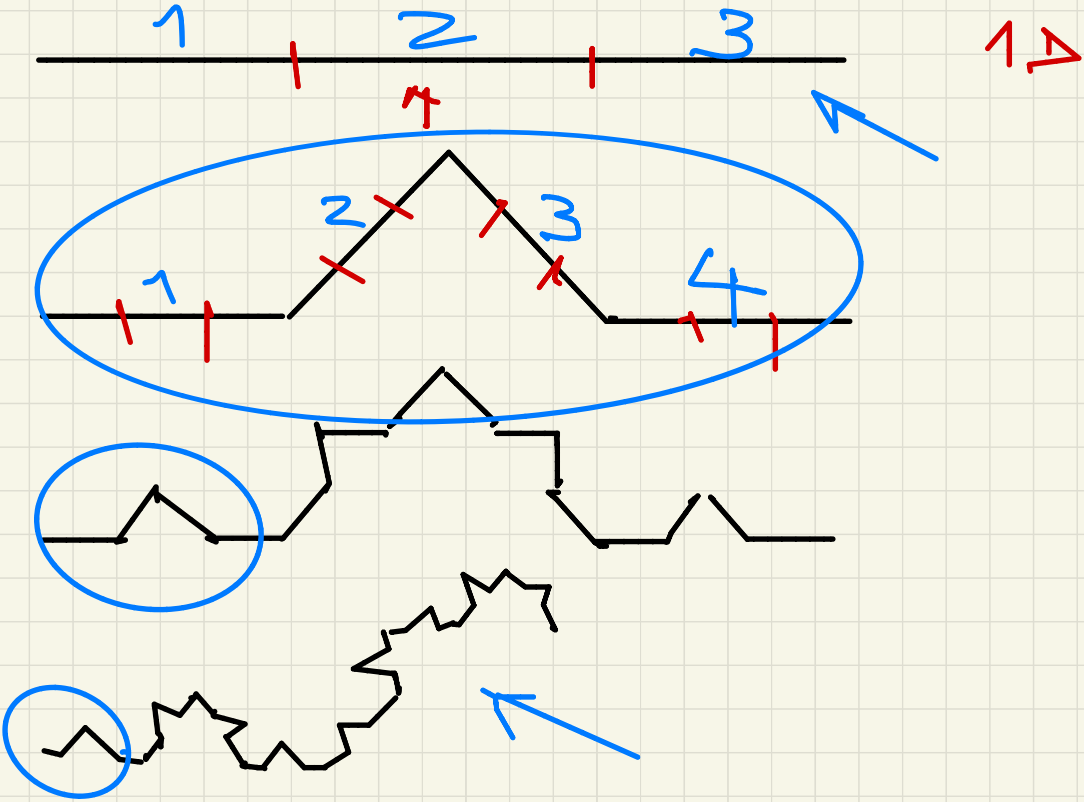
2020

Edson Wendland

EESC / USP

número fractal

auto-similaridade

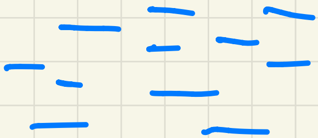


$$N = \frac{4}{3} = 1{,}33\ D$$

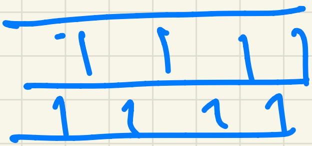
Simbología

arenosos

lutita = arcilla.



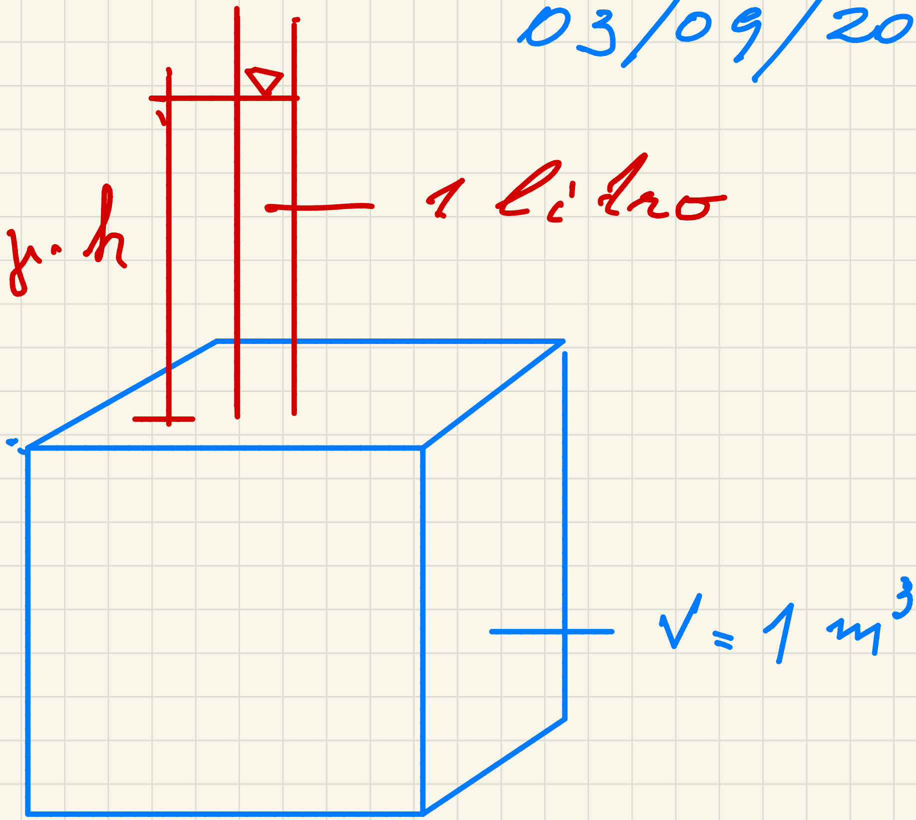
calcáreos
carbonatos



magmáticas

03/09/2020

$p = \gamma \cdot h$



$h = 20\ m \rightarrow 100\ m,\ 1000\ m \rightarrow$

$250\ m,\ 100\ m,\ 1000\ m$

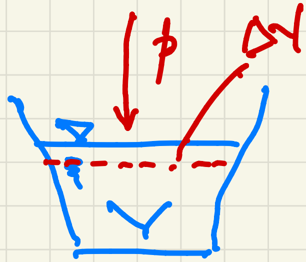
$500\ m$

$p = 1\ MPa = 10^6\ Pa$

Compressibilidade

$$\beta = 4{,}8 \cdot 10^{-10}\ m^2/N$$

$$\beta = -\frac{\Delta V}{V}\,\frac{1}{p}$$



$\rightarrow$ $\gamma = 9789\ N/m^3$

$V = 1\ m^3$

$\Delta V = -1\ \ell = -10^{-3}\ m^3$

$$p = -\frac{\Delta V}{V}\cdot\frac{1}{\beta}$$

$$p = \frac{10^{-3}}{1}\cdot\frac{1}{4{,}8\cdot10^{-10}}\left[\frac{m^3}{m^3}\cdot\frac{N}{m^2}\right]$$

$$p = \frac{10\cdot10^{-4}\cdot10^{10}}{4{,}8} \cong 2\cdot10^{6}\ \frac{N}{m^2}$$

$$p \cong 2\ MPa$$

$$p = \gamma\cdot h \ \therefore\ h = \frac{p}{\gamma} = \frac{2\cdot10^{6}}{9789} \cong 200\ \text{mca}$$

$$\dot{m}_x = \rho \cdot u_x \cdot A_x$$

$$= \frac{kg}{\cancel{m^3}} \cdot \frac{\cancel{m}}{s} \cdot \cancel{m^2}$$

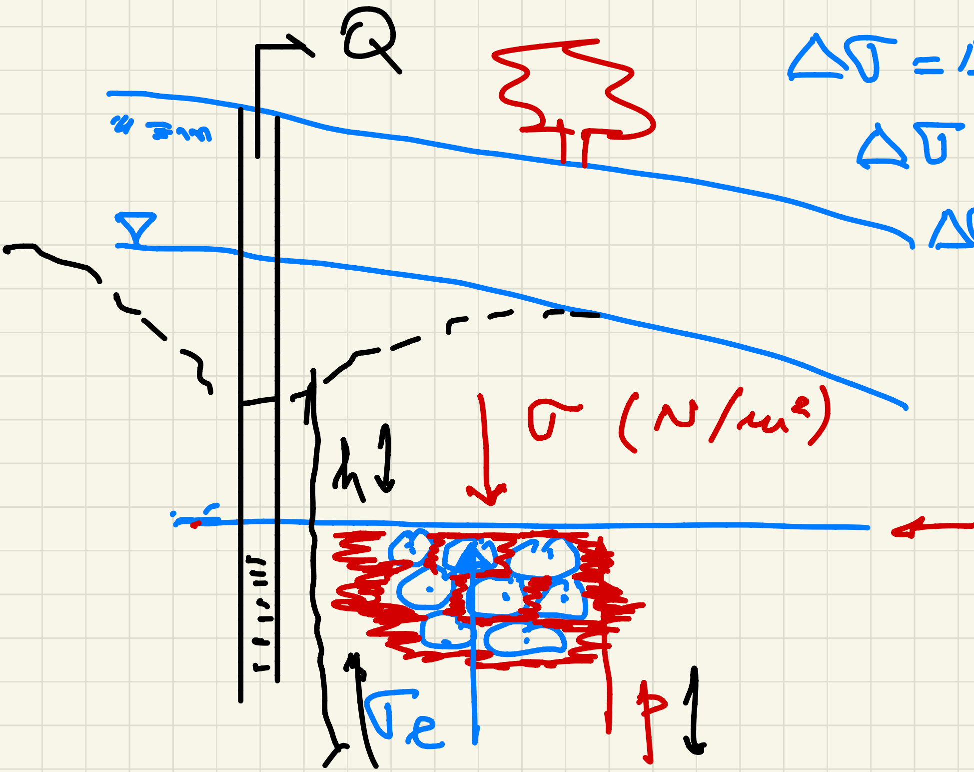
$$\dot{m}_x = kg/s \qquad \frac{\partial m}{\partial t} = \frac{kg}{s}$$

$$\sigma = \sigma_e + p$$

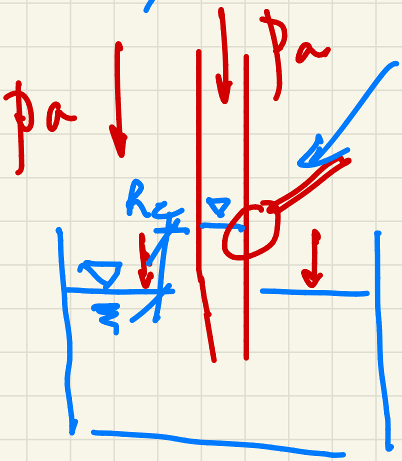
$$\Delta\sigma = \Delta\sigma_e + \Delta p$$

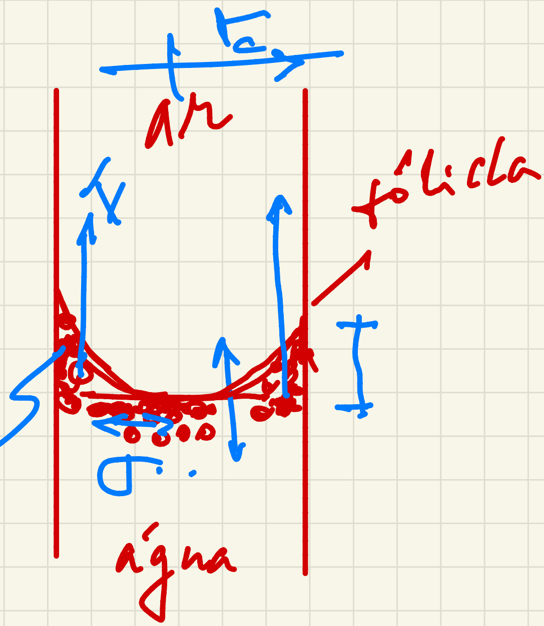
$$\Delta\sigma = 0$$

$$\Delta\sigma_e = -\Delta p$$

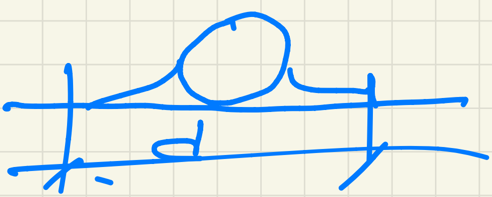


Capilaridade



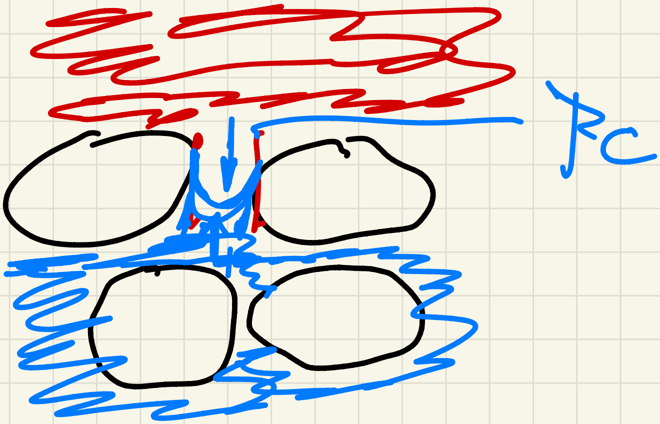


molhabilidade

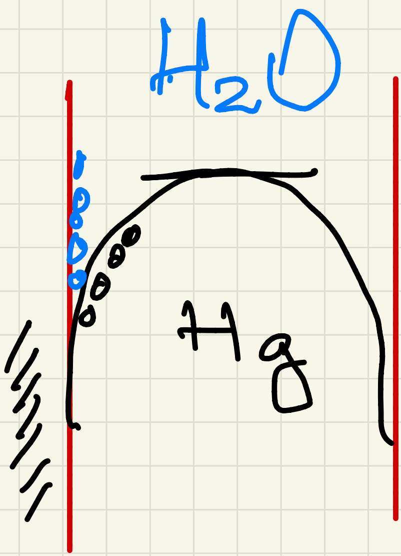


$$h_c = f(\sigma_e, d_t)$$

$$\gamma_c = \gamma_{H_2O} - \gamma_{ar}$$



$$p_c = p_a - p_o$$



10/09/2020

EDP flusso

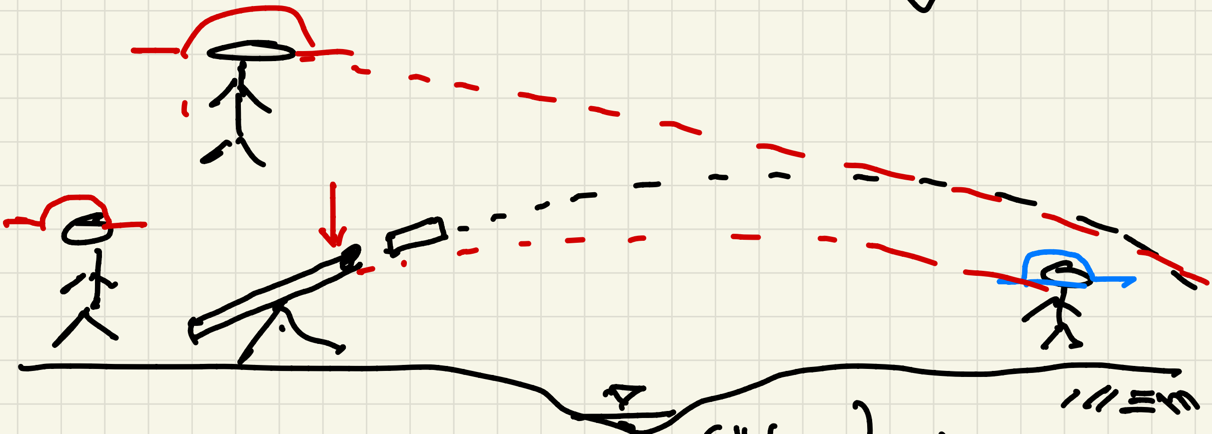
$$S_0 \frac{\partial h}{\partial t} = -\nabla (K \nabla h) + Q$$

$$\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

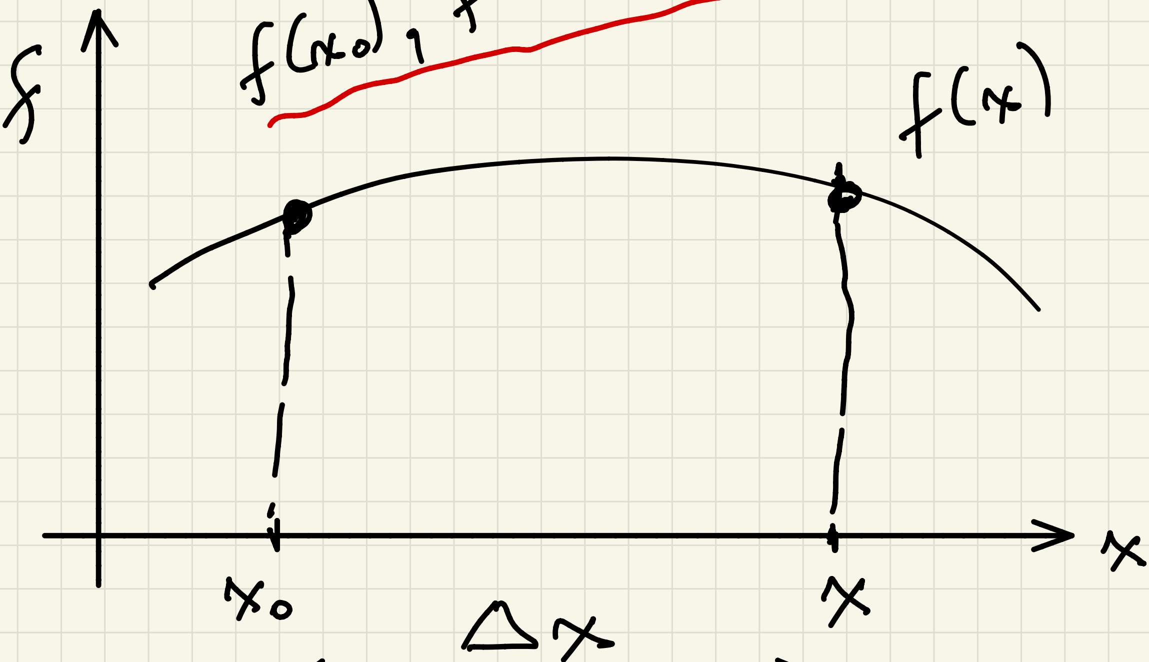
$$\bar{h} \cong \sum_{i=1}^{n} a_i \cdot \varphi^i$$

$$= a_1 \varphi^1 + a_2 \varphi^2 + a_3 \varphi^3 + \dots$$

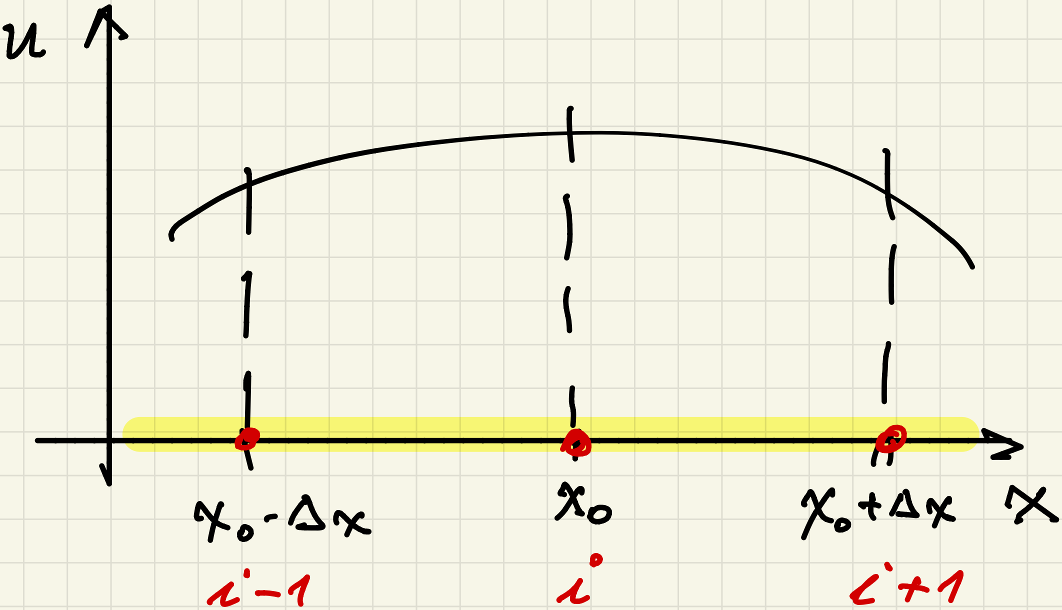
Séries de Taylor



$$f(x_0),\ f'(x_0),\ f''(x_0)\ \ldots$$



$$f(x) = f_{x_0} + f'_{x_0}\cdot\Delta x + f''_{x_0}\cdot\frac{\Delta x^2}{2!} + f'''_{x_0}\frac{\Delta x^3}{3!} + \ldots$$



$i = 1, 2, 3, 4 \ldots$

modelo mat.

$$\boxed{\frac{\partial^2 u}{\partial x^2}} = \frac{\partial u}{\partial t} \Rightarrow u''_x = u'_t$$

$$\frac{\partial u}{\partial x} \cong \frac{u(x_0 + \Delta x) - u(x_0)}{\Delta x} + O(\Delta x)$$

$$\frac{\partial u}{\partial x} \cong \frac{u_{i+1} - u_i}{\Delta x} + O(\Delta x)$$

$$u'_t = \frac{u^{n+1} - u^n}{\Delta t} + O(\Delta t)$$

$$f(x_0+\Delta x) = f(x_0) + \cancel{\Delta x f'} + \frac{\Delta x^2}{2!} f'' + \dots$$

$$+\ f(x_0-\Delta x) = f_{x_0} - \cancel{\Delta x f'} + \frac{\Delta x^2}{2!} f'' - \dots$$

$$\underbrace{f_{x_0+\Delta x}}_{i+1} + \underbrace{f_{x_0-\Delta x}}_{i-1} = 2 \underbrace{f_{x_0}}_{i} + \cancel{2} \frac{\Delta x^2}{\cancel{2}!} f'' + \dots$$

$$f'' = \frac{f_{i+1} - 2 f_i + f_{i-1}}{\Delta x^2} + \frac{1}{\Delta \cancel{x^2}} \left[\frac{2 \Delta x^{\cancel{4}\,2}}{4!} f^{IV} \dots \right.$$

$$f'' = \frac{f_{i+1} - 2 f_i + f_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

↑ central difference

$O(\Delta x^2)$

$$u_x^{4} = \frac{U_{i+1} - 2U_i + U_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

modelo mat.

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \Rightarrow \quad u''_{x} = u'_{t}$$

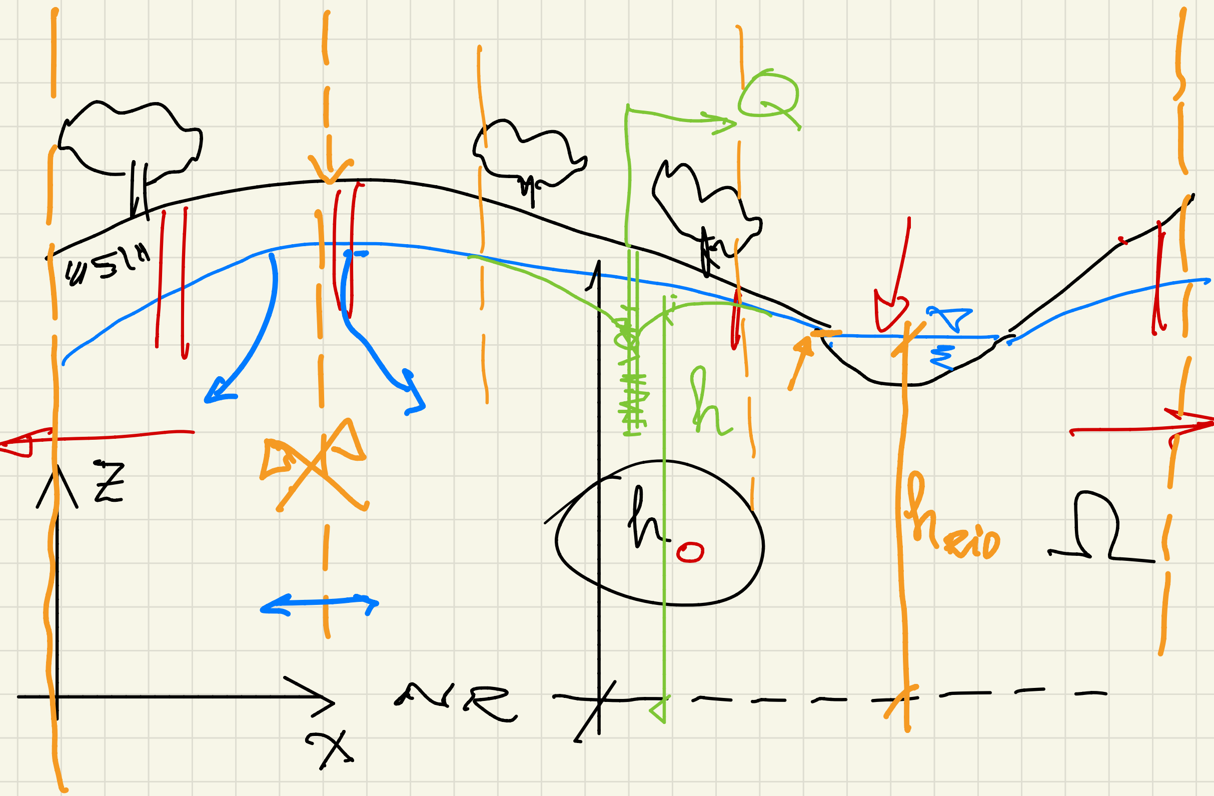
$$u''_{x} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

$$u'_{t} = \frac{u^{n+1} - u^n}{\Delta t} + O(\Delta t)$$

$$\boxed{\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \simeq \frac{u^{n+1} - u^n}{\Delta t} + O(\Delta x^2, \Delta t)}$$

17/09/2020

Condições de contorno



EDP

$$S_o \frac{\partial h}{\partial t} = -\nabla(K \nabla h) + Q \quad , \Omega, t$$

$$h = f(h_o, x) \quad , \forall \Omega, \quad t = t_o$$

↳ condição inicial

Condições de contorno

1 - carga hid. conhecida

$h = h_{RIO}$ → condição do 1º tipo (Dirichlet)

→ variável de interesse é conhecida

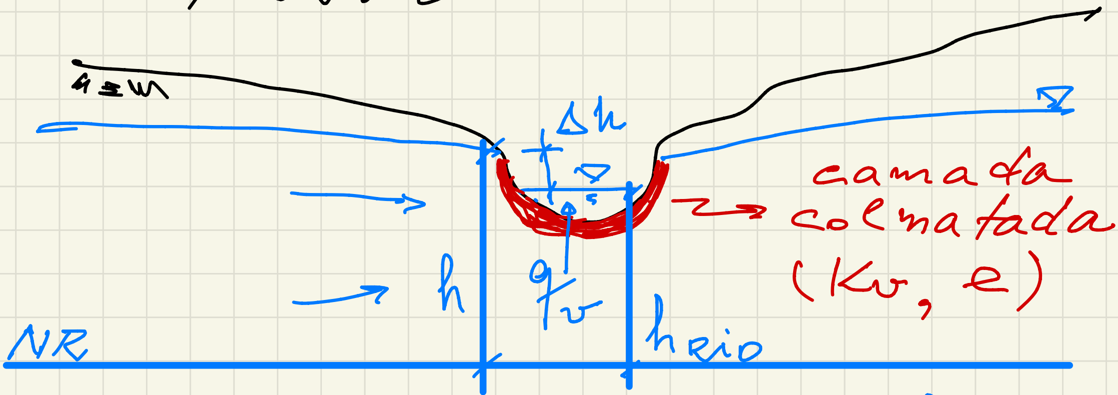
2 → derivada normal é conhecida

$\frac{\partial h}{\partial n}$ → cond. de Neuman

$q_0 = -K\frac{\partial h}{\partial n}$ ⇒ fluxo através do contorno (fronteira) é conhecido

$q_0 = 0$ (fluxo nulo)

3 - Condição de Cauchy ou Robin



$$q_v = K_v \frac{\Delta h}{e} \quad ; \quad \Delta h = h - h_{rio}$$

$$q_v = \frac{K_v}{e}(h - h_{rio})$$

coef. de drenança (leakage coef.) = α

$$q_v = \alpha (h - h_{rio})$$

$$\boxed{h = \frac{q_v}{\alpha} + h_{rio}}$$

↳ Cond. 3ª tipo

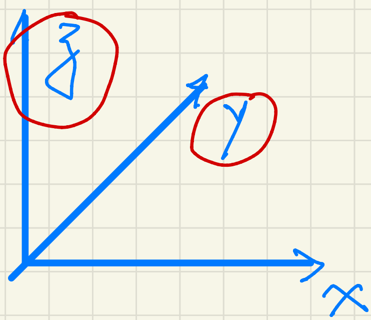
01/10

$$S_0 \cancel{\frac{\partial h}{\partial t}} = -\nabla (K \nabla h) + \cancel{Q}$$

$$0 = K \nabla^2 h$$

$$\nabla^2 h = 0 \quad \text{(Eq. Laplace)}$$

$$\frac{\partial^2 h}{\partial x^2} + \cancel{\frac{\partial^2 h}{\partial y^2}} + \cancel{\frac{\partial^2 h}{\partial z^2}} = 0$$



$q_z = 0$

$q_y = 0$

$$\boxed{\frac{\partial^2 h}{\partial x^2} = 0}$$

EDO

$$\frac{\partial^2 h}{\partial x^2} = 0 \quad \overset{\sim}{\Rightarrow} \quad \frac{\Delta^2 h}{\Delta x^2} = 0$$

$$h_{i-1} - 2h_i + h_{i+1} = 0 \Leftarrow$$

$i=2$ $$h_1 - 2h_2 + h_3 = 0$$

$$+2h_2 = +h_1 + h_3$$

$$h_2 = \frac{h_1 + h_3}{2}$$

$$h_1 = 50\,m$$

$$h_2 = \frac{50 + h_3}{2}$$

$$\boxed{h_i^{\nu} = \frac{h_{i-1}^{\nu-1} + h_{i+1}^{\nu-1}}{2}}$$ Jacobi

Iteração $\nu = 1$

$$h_3^{\nu} = \frac{h_2^{\nu-1} + h_4^{\nu-1}}{2}$$

$$\boxed{h_i^{\nu} = \frac{h_{i-1}^{\nu-1} + h_{i+1}^{\nu-1}}{2}}$$ Jacobi

$$h_i^{\nu} = \frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2} \Leftarrow$$

Gauss - Seidel

SOR (Successive Over-relaxation)

p/ nó i $\quad \Delta_i = h_i^{\nu} - h_i^{\nu-1}$

$$h_i^{\nu} = h_i^{\nu-1} + \omega\,\Delta$$

$\omega \rightarrow$ coef. de relaxação

$$h_i^{\nu} = h_i^{\nu-1} + \omega\left(h_i^{\nu} - h_i^{\nu-1}\right)$$

$$h_i^{\nu} = \underline{h_i^{\nu-1}} + \underline{\omega} \left(h_i^{\nu} - \underline{h_i^{\nu-1}} \right)$$

$$h_i^{\nu} = h_i^{\nu-1} (1 - \omega) + \omega \cdot \underline{h_i^{\nu}}$$

Gauss - Seidel

$$h_i^{\nu} = \frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2}$$

$$\boxed{h_i^{\nu} = (1-\omega) h_i^{\nu-1} + \omega \left(\frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2} \right)}$$

SOR

$\omega > 1 \Rightarrow$ over relaxat.

$\omega < 1 \Rightarrow$ under relaxation

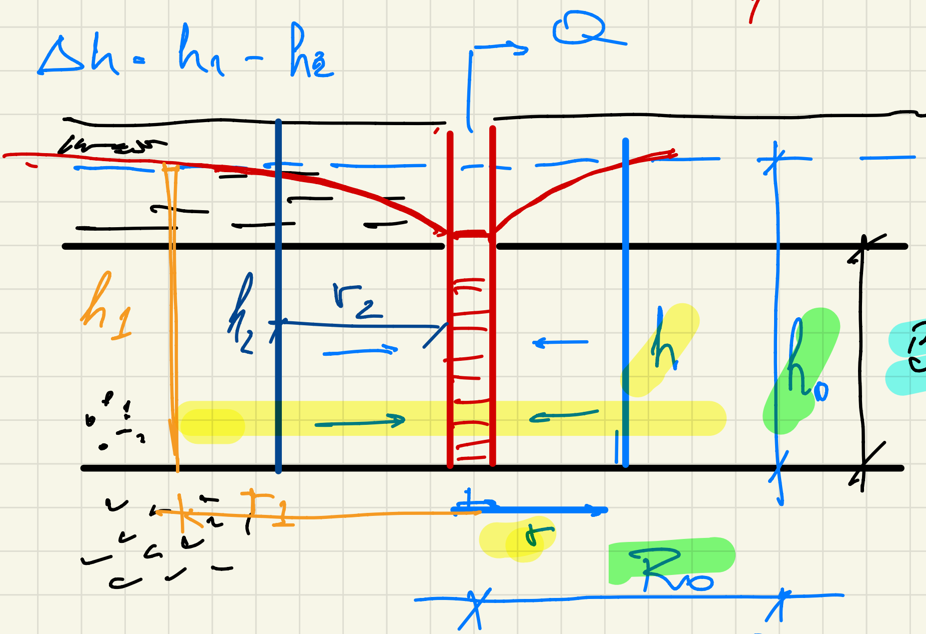
$\omega = 1$ = Gauss-Seidel

08/10/2020

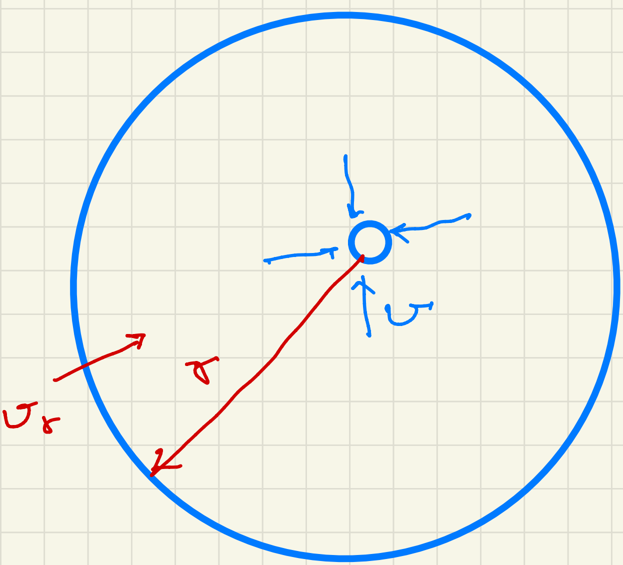
Equação de Thiem

Aq. Conf.

$\Delta h = h_1 - h_2$



$B = 4m$



$Q = v \cdot A$

$v_r = K \cdot \frac{\partial h}{\partial r}$

$A = 2\pi r \cdot B$

$Q = K \cdot \frac{\partial h}{\partial r} 2\pi r B$

$Q \frac{\partial r}{r} = K\, 2\pi B\, \partial h$

$$Q\int_{r}^{R_0}\frac{\partial r}{r} = K\,2\pi\,3\int_{h}^{h_0}\partial h$$

$$Q\,\ln r\Big|_{r}^{R_0} = 2\pi\,\underbrace{K\cdot 3}_{T}\cdot h\Big|_{h}^{h_0}$$

$$Q\left[\ln R_0 - \ln r\right] = 2\pi\,T\left(h_0 - h\right)$$

$$Q\left[\ln r - \ln R_0\right] = 2\pi T\left(h - h_0\right)$$

$$\boxed{Q\cdot\ln\frac{r}{R_0} = 2\pi T\left(h - h_0\right)}$$

Eq. de Thiem

$$h - h_0 = \frac{Q}{2\pi T}\ln\frac{r}{R_0}$$

$$h = \frac{Q}{2\pi T}\ln\frac{r}{R_0} + h_0$$

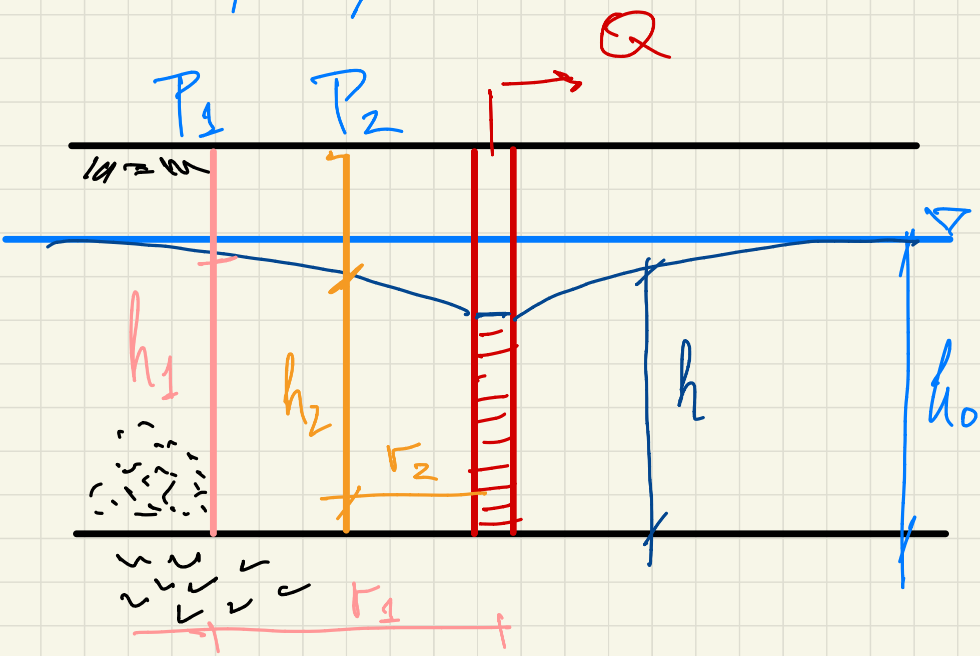
$$Q \ln \frac{r_1}{r_2} = 2\pi T (h_1 - h_2)$$

$$T = \frac{Q}{2\pi \; \Delta h} \ln \frac{r_1}{r_2} = K \cdot B$$

$$\Delta h = h - h_2$$

$$\boxed{K = \frac{Q}{2\pi \Delta h \cdot B} \ln \frac{r_1}{r_2}}$$

Aquífero Livre



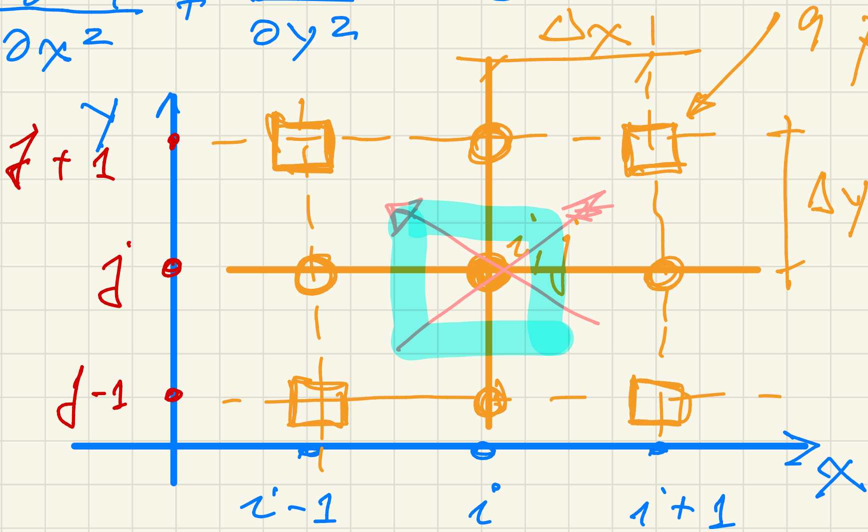
Desenvolver uma solução analítica para determinação da cond. hidráulica em regime permanente, usando os poços de observação P1 e P2.

$$Q = \frac{2\pi K}{2}\left(h^2 - h_0^2\right) \cdot \ln\frac{R_0}{r}$$

12/10/2020

$$\rightarrow \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

esquema 9 pontos



$$\ast \quad \frac{\partial^2 h}{\partial x^2} \simeq \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{\Delta x^2} + O(\Delta x^2)$$

$$\ast \quad \frac{\partial^2 h}{\partial y^2} = \frac{h_{j-1,i} - 2h_{i,j} + h_{j+1,i}}{\Delta y^2} + O(\Delta y^2)$$

$$\frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{\Delta x^2} + \frac{h_{j-1,i} - 2h_{i,j} + h_{j+1,i}}{\Delta y^2} = 0$$

$$\Delta x = \Delta y$$

$$h_{i-1,j} - 2h_{ij} + h_{i+1,j} + h_{i,j-1} - 2h_{i,j} + h_{i,j+1} = 0$$

$$4\,h_{i,j} = h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}$$

$$\boxed{h_{ij} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}}$$

esquema de 5 puntos

Exercício 1

$$S_o \cancel{\frac{\partial h}{\partial t}}^{\,0} = +\nabla(K\nabla h) + Q$$

$$0 = K\nabla^2 h + Q$$

$$\int_0^B \left[0 = K\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \cancel{\frac{\partial^2 h}{\partial z^2}}^{\,0}\right) + Q \right] dz$$

bidimensional $h = f(x, y)$

$$\frac{\partial h}{\partial z} = 0$$

aq. confinado $\longrightarrow$ esp. constante (B)

$$0 = \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) \cdot K z \Big|_0^B + Q$$

$$0 = \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) \cdot \underbrace{\underline{K} \cdot \underline{B}}_{T} + Q$$

$$\underbrace{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}}_{\text{esquema 5 pontos}} = -\frac{Q}{T} \cdot \delta_{ij}$$

$\delta_{ij} \downarrow$ poço

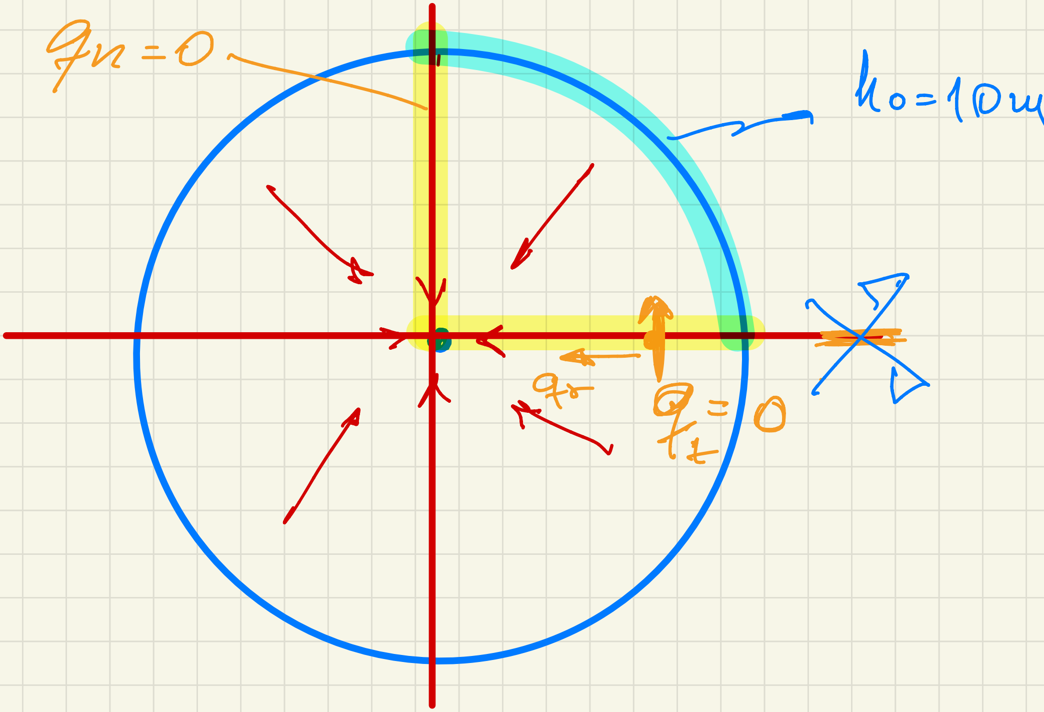
$$h_{ij} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}$$

$$h_{ij} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4} - \frac{Q}{T}$$

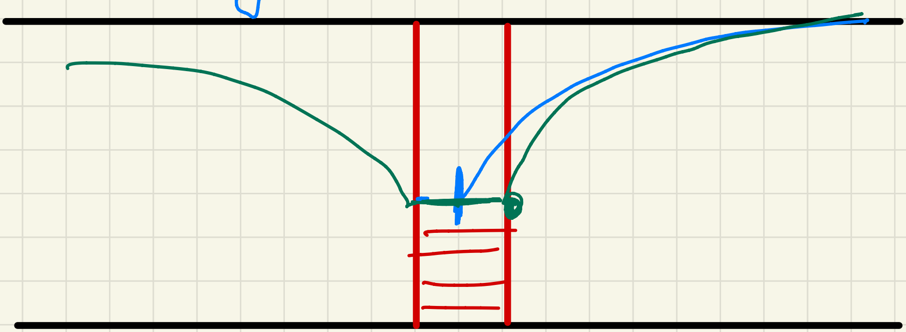
$h_{ij} \downarrow$ m

$\uparrow$ poço

$$\frac{\cancel{m^3/s}}{\cancel{m/s} \cdot m} = m$$



Singularidade $(r \to 0)$



$r \to 0$

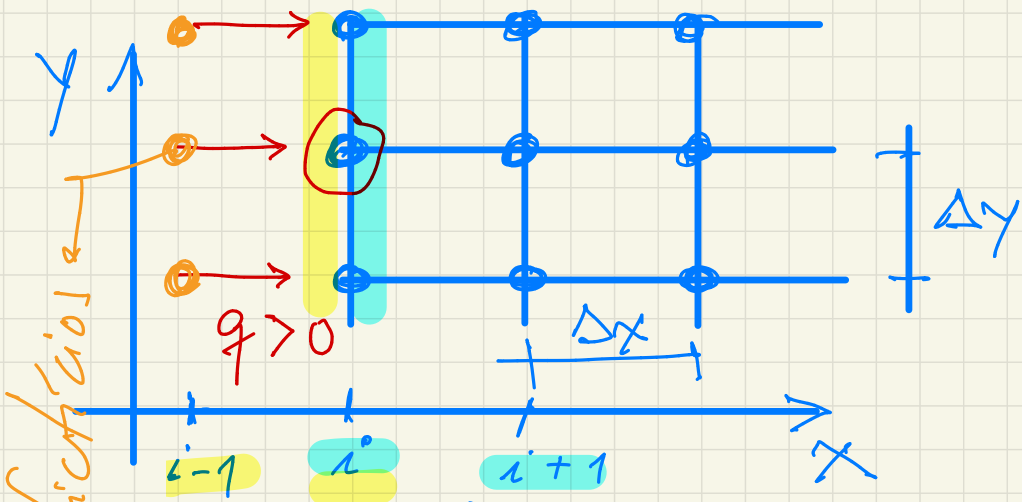
$\ln \frac{r}{r_0} \to \ln \frac{r_w}{R_0}$

D

$r_w \Rightarrow$ raio do poço

15/10/2020

Implementação da condição do 2º tipo (fluxo conhecido)



$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \text{(Laplace)}$$

esquema 5 pontos (interior)

Lei de Darcy

$$q_x = -K_x \frac{\partial h}{\partial x} \Leftarrow$$

$$\frac{\partial h}{\partial x} \cong \frac{\Delta h}{\Delta x}$$

forward difference

$$\frac{\partial h}{\partial x} \cong \frac{h_{i+1} - h_i}{\Delta x}$$

$$q_x = -\frac{K_x}{\Delta x}\left(h_{i+1} - h_i\right)$$

$$\boxed{h_i = \frac{q_x \cdot \Delta x}{K_x} + h_{i+1}}$$

backward difference

$$\frac{\partial h}{\partial x} \cong \frac{h_i - h_{i-1}}{\Delta x}$$

$$q_x = -\frac{K_x}{\Delta x}\left(h_i - h_{i-1}\right)$$

$$\boxed{h_{i-1} = \frac{q_x \cdot \Delta x}{K_x} + h_i}$$

$$h_i = \frac{h_{i+1} + h_{i-1}}{2}$$

$$2h_i = h_{i+1} + h_{i-1}$$

$$\cancel{2}h_i = h_{i+1} + \frac{q_x \cdot \Delta x}{K_x} + \cancel{h_i}$$

$$\boxed{h_i = h_{i+1} + \frac{q_x \cdot \Delta x}{K_x}}$$

cond. fluxo nulo $(q_x = 0)$

$$\boxed{h_i = h_{i+1}}$$

central difference

$$\frac{\partial h}{\partial x} = \frac{h_{i+1} - h_{i-1}}{2\Delta x} + O(\Delta x^2)$$

$$q_x = \frac{-K_x}{2\Delta x}(h_{i+1} - h_{i-1})$$

$$h_{i-1} = \frac{2 q_x \Delta x}{K_x} + h_{i+1}$$

$q_x = 0$ $\quad h_{i-1} = h_{i+1}$

Aproximação temporal

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} = \frac{u_i^{n+1} - u_i^{n}}{\Delta t} + O(\Delta t)$$

explícito

$$\frac{u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}}{\Delta x^2} = \frac{u_i^{n+1} - u_i^{n}}{\Delta t} + O(\Delta t)$$

implícito

$$\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{\Delta x^2} = \frac{u_i^{n+1} - u_i^{n}}{\Delta t} + O(\Delta t)$$

combinação

$$\frac{u_i^{n+1} - u_i^{n}}{\Delta t} = \theta \frac{u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}}{\Delta x^2} + (1-\theta)\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{\Delta x^2}$$

$$\frac{u_i^{n+1} - u_i^{n}}{\Delta t} = \theta \, \frac{u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}}{\Delta x^2} + (1-\theta) \, \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{\Delta x^2}$$

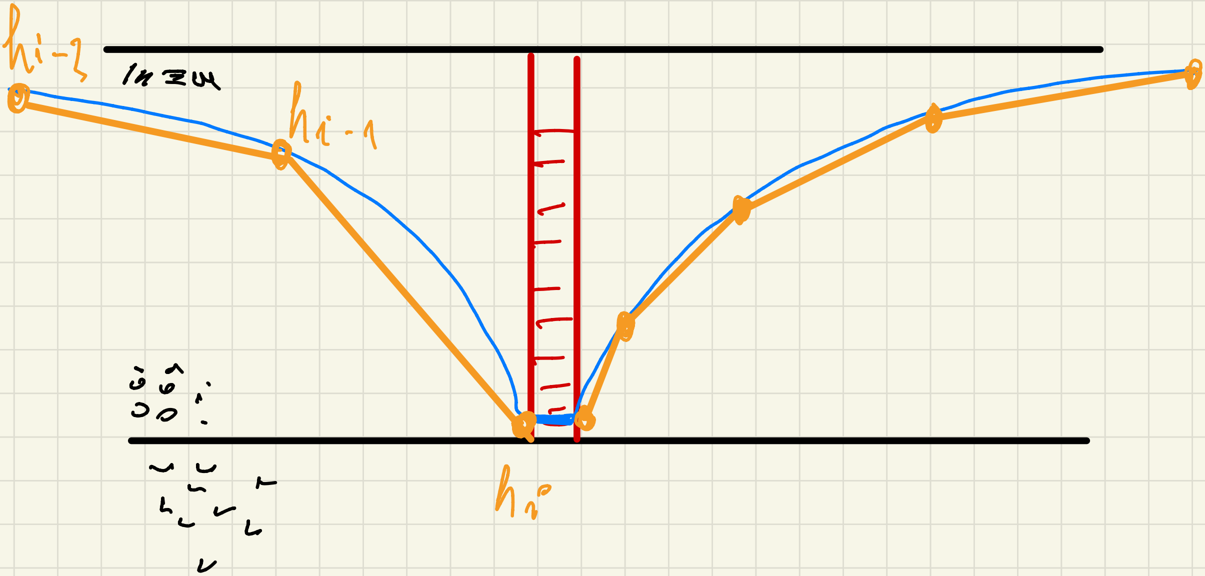
$\theta = 1 \longrightarrow$ explícito $O(\Delta t)$

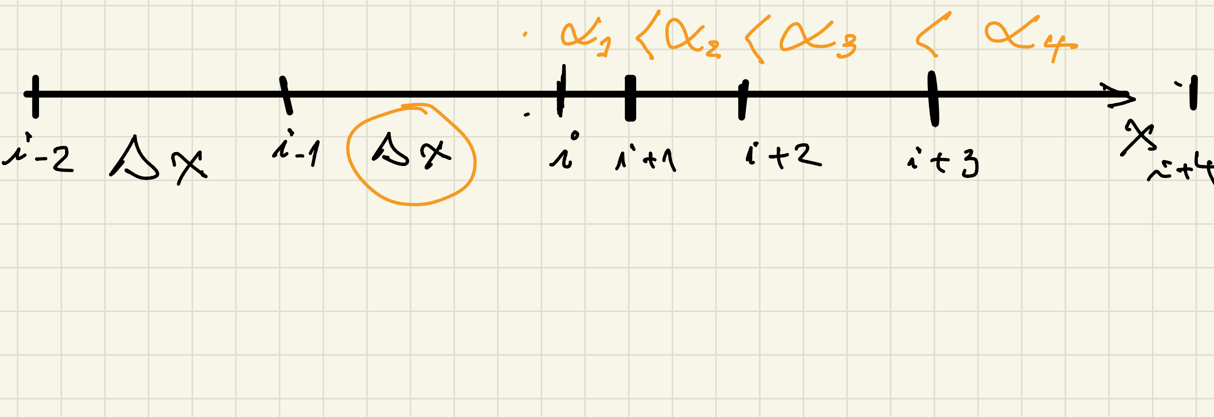
$\theta = 0 \longrightarrow$ implícito $O(\Delta t)$

$\theta = 1/2 \longrightarrow$ Crank-Nicholson $O(\Delta t^2)$

$\theta = 1/3$

22/10/2020



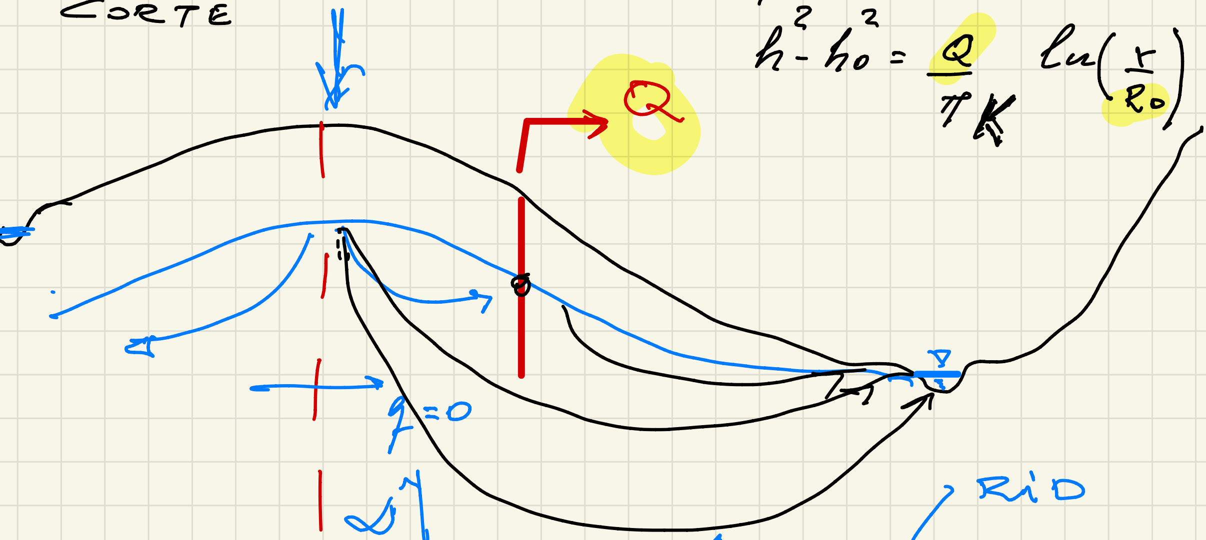


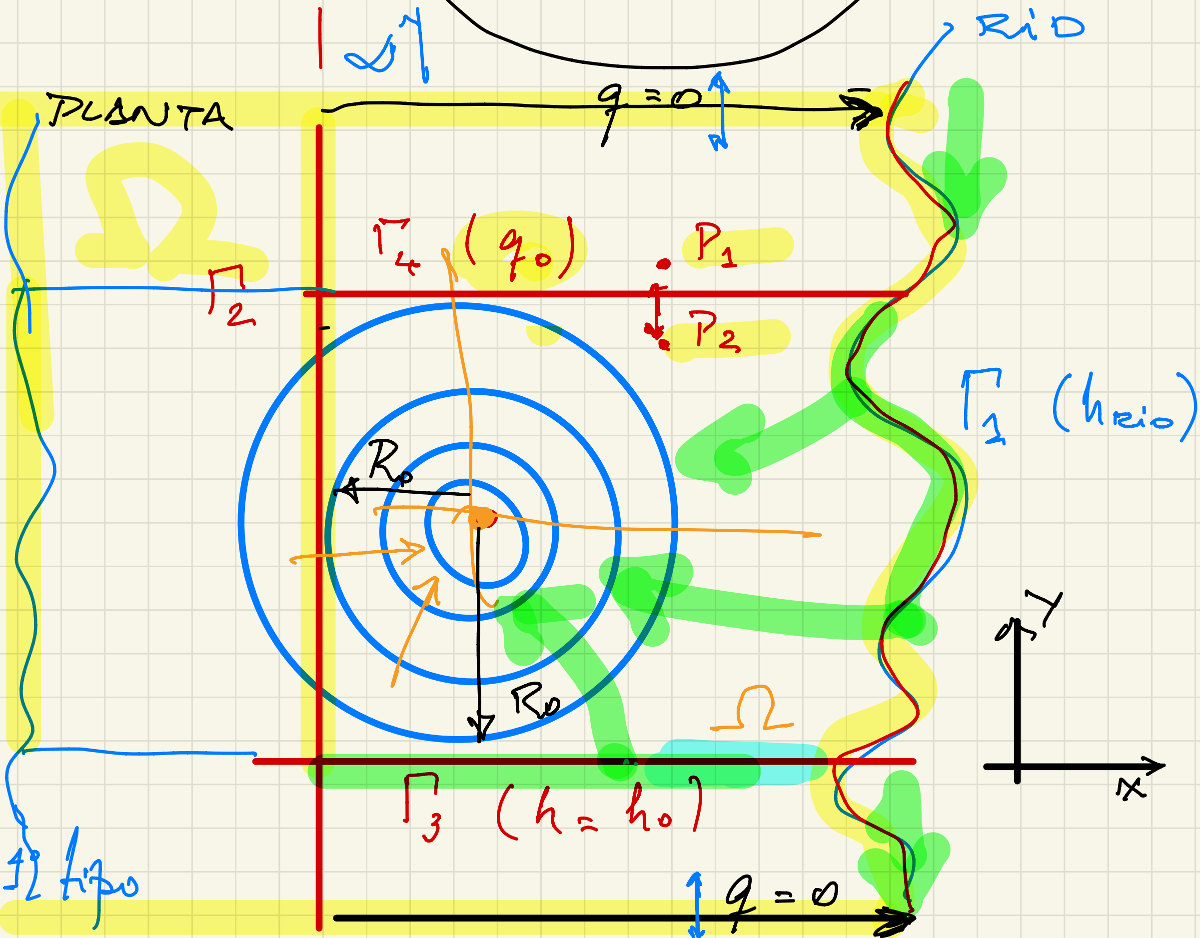
MODELO CONCEITUAL

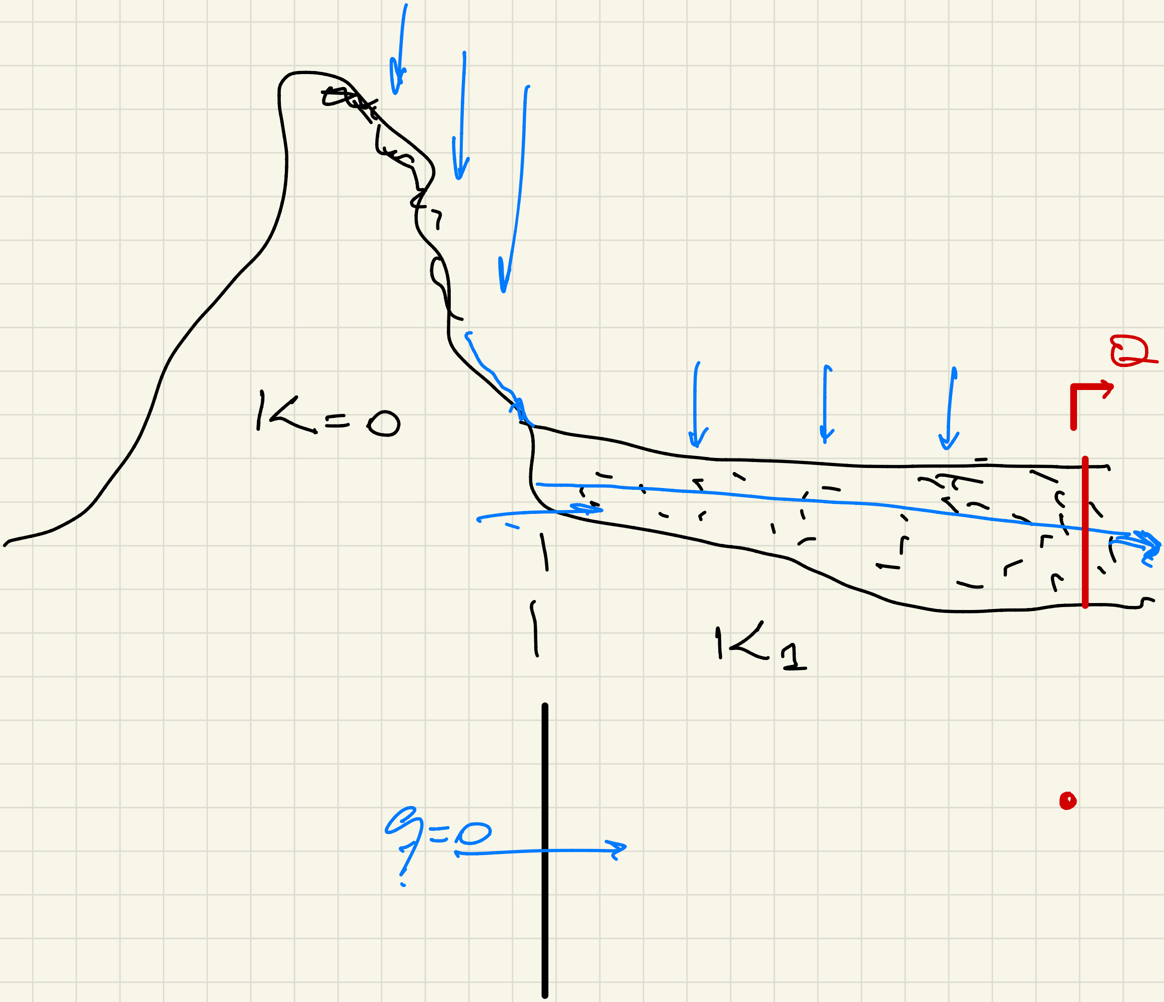
Aq. Libre

$$h^2 - h_0^2 = \frac{Q}{\pi K} \ln\left(\frac{r}{R_0}\right)$$

CORTE






K=0
K1
Q
q=0