Lista 8 - Sobre derivadas parciais

(I) Calcule $\dfrac{\partial z}{\partial x}$ e $\dfrac{\partial z}{\partial y}$, se existir:

1. $z = 4x^2y^3 - \dfrac{1}{xy} + 4$

2. $z = sen(x^3 + y^2)$

3. $z = e^{xseny}$

4. $z = xy^2 ln(x^4 + y^4 + 3)$

5. $z = arctg\dfrac{y}{x}$

6. $z = \dfrac{x^3 + 2y^2}{2x^2 + 4y}$

7. $z = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{se } (x, y) \neq (0, 0) \\ 0 & \text{se } (x, y) = (0, 0) \end{cases}$

8. $z = sec(x^4 y)$

9. $f(x, y) = \begin{cases} \dfrac{(x-1)^3}{(x-1)^2 + (y-2)^2} + 5x & \text{se } (x, y) \neq (1, 2) \\ 5 & \text{se } (x, y) = (1, 2) \end{cases}$

10. $z = x^y$

11. $f(x, y) = \begin{cases} \dfrac{3(x+3)^2 y}{(x+3)^2 + y^2} + 2x + y & \text{se } (x, y) \neq (-3, 0) \\ -6 & \text{se } (x, y) = (-3, 0) \end{cases}$

12. $z = ln(xy^7)$

(II) Seja $z = \dfrac{x^2 y}{x^2 + y^2}$. Verifique que $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$.

(III) Seja $g : \mathbb{R} \longrightarrow \mathbb{R}$ uma função derivável tal que $g(0) = 2$ e $g'(0) = -1$.

Considere a função $f(x, y) = 3x.g(x^2 - y^2)$. Calcule $\dfrac{\partial f}{\partial x}(1, 1)$ e $\dfrac{\partial f}{\partial y}(1, 1)$.

(IV) Seja $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ uma função derivável tal que $\phi(1) = 3$ e $\phi'(1) = 2$.

Considere a função $g(x,y) = (2x^3 + 3y)\phi(\frac{y}{x})$. Calcule $\frac{\partial g}{\partial x}(3,3)$ e $\frac{\partial g}{\partial y}(3,3)$.

(V) Seja $f(x,y) = \int_0^{x^2-y^4} e^{-t^2} dt$. Calcule $\frac{\partial f}{\partial x}(4,2)$ e $\frac{\partial f}{\partial y}(4,2)$.

(VI) Calcule $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$, sendo

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^3}{x^2 + y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$