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Microeconomia II
Programa de Pós-Graduação em Economia
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Seção 13.B - Adverse Selection

Exercise 1 (MWG 13.B.2). Suppose that $r(\cdot)$ is a continuous and strictly increasing function and that there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) > \theta$ for $\theta > \hat{\theta}$ and $r(\theta) < \theta$ for $\theta < \theta$. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Show that a competitive equilibrium with unobservable worker types necessarily involves a Pareto inefficient outcome.

Exercise 2 (MWG 13.B.3). Consider a *positive selection* version of the model discussed in Section 13.B in which $r(\cdot)$ is a continuous, strictly *decreasing* function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

(a) Show that the *more capable* workers are the ones choosing to work at any given wage.

(b) Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

(c) Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves *too much* employment relative to the Pareto Optimal allocation of workers.

Exercise 3. Let $\Theta = [a, b]$ and assume that the productivity θ is distributed according to a density f with $f(\theta) > 0, \forall \theta \in \Theta$. Let $r : [a, b] \mapsto [0, \infty)$ be a strictly increasing and continuous function, with $r(a) < a$. In this case, the function $w \mapsto E[\theta | r(\theta) w]$ is continuous in $[r(a), \infty)$. Use the Intermediate Value Theorem to demonstrate that a competitive equilibrium exists.

Exercise 4. Consider $\Theta = [0, 1]$. Suppose that the population of workers is uniformly distributed in this interval. Let $r(\theta) = \alpha\theta$ with $0 < \alpha < 1$ be the opportunity cost of a worker with productivity θ. Show that:

(a) If $\alpha > 1/2$, there is no competitive equilibrium.

(b) If $\alpha = 1/2$, there are an infinite number of competitive equilibria. Which one employs a bigger set of workers?

(c) If $\alpha < 1/2$, there is only one competitive equilibrium.

(d) Solve for $\alpha < 1$ and $\theta \sim U[0, 2]$.

Exercise 5 (**MWG 13.B.7**). Suppose that it is impossible to observe worker types and consider a competitive equilibrium with wage rate w^*. Show that there is a Pareto-improving market intervention $(\tilde{w}_e, \tilde{w}_u)$ that reduces employment if and only if there is one of the form $(w_e, w_u) = (w^*, \hat{w}_u)$ with $\hat{w}_u > 0$. Similarly, argue that there is a Pareto-improving market intervention $(\tilde{w}_e, \tilde{w}_u)$ that increases employment if and only if there is one of the form $(w_e, w_u) = (\hat{w}_e, 0)$ with $\hat{w}_e > w^*$. Can you use there facts to give a simple proof of Proposition 13.B.2?

Exercise 6 (**MGW 13.B.8**). Consider the following alteration to the adverse selection model in Section 13.B. Imagine that when workers engage in home production, they use product x. Suppose that the amount consumed is related to a worker's type, with the relation given by the increasing function $x(\theta)$. Show that if a central authority can observe purchases of good x but not worker types, then there is a market intervention that results in a Pareto improvement even if the market is at the highest-wage competitive equilibrium.

Exercise 7. Prove que $E[\theta | r(\theta) \leqslant w]$ é crescente em w.

Exercise 8. Prove a Proposição 13.B.1. Em particular mostre que $h(w') \equiv E[\theta | r(\theta) \leqslant w'] - w' < 0$ para todo $w' > w^*$.