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Design Procedure for the Buckling Analysis of Reinforced Concrete Shells

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ABSTRACT

The paper presents a simple procedure to establish the buckling load of shell structures. It is essentially based on the assumption that the various factors influencing buckling can be assessed as multipliers of the 'classical' critical load.

These factors are: imperfections, creep, plasticity, cracks and the steel reinforcement. The paper gives the values of these factors, thus establishing a method which can be used in practical design.

1 INTRODUCTION

Shell buckling is a rather complicated phenomenon even if the material of the shell is homogeneous, isotropic and linearly elastic. The specific properties of reinforced concrete: its nonlinear elasticity, creep, plastic behaviour under higher loads, cracking and the effect of the reinforcement, make the problem even more complicated. Although large computer programs exist that take all these aspects into account, it seems desirable to develop a simple, approximate method, by which a designer can assess the safety against buckling with adequate reliability. Such a method will be presented in this paper.

2 BASIC CONCEPT OF THE METHOD

The basic idea of the method is that, first, the 'classical' critical load of the shell (computed by linear theory, disregarding any imperfections and assuming a linearly elastic material) should be established. This value has to be modified due to imperfections, creep, plasticity, cracking and the role of the reinforcement; all these effects will be taken into account with the aid of appropriate factors by which the classical critical load should be multiplied.[1] Hence the actual critical load of the shell is obtained in the following form:

$$p_{cr} = p_{cr}^{lin} \cdot \rho_{imp} \cdot \rho_{creep} \cdot \rho_{crack} \cdot \rho_{pl}$$

where

p_{cr}^{lin} = classical critical load
ρ_{imp} = factor taking effect of geometric imperfections into account
ρ_{creep} = factor considering creep of concrete
ρ_{crack} = factor that takes cracking of concrete and role of reinforcement into account
ρ_{pl} = factor of plasticity

3 CLASSICAL CRITICAL LOAD AND THE INFLUENCE OF IMPERFECTIONS

The value of the classical critical load has been derived for many shell forms and can be found in books dealing with shell buckling. In the following we will assume that its value for the shell in question, or for a shell with a geometry sufficiently close to it, can be found.

The presence of imperfections may considerably reduce the classical critical load, depending on the post-buckling behaviour of the shell. Many shells exhibit a decreasing post-buckling load-bearing capacity, and consequently imperfections (of amplitude w_0) sharply reduce the maximum 'upper' load, p_{cr}^{u}, the shell can carry, as compared with the classical critical value (Fig. 1). Other shells exhibit an increasing post-buckling load-bearing capacity. In these cases geometric imperfections only cause the bifurcation point to disappear, but the increasing load-bearing capacity of the shell remains (Fig. 2). Such shells are not sensitive to initial imperfections, and we will not deal with them in the following discussion.

For shells with decreasing post-buckling load-bearing behaviour, with curves like those in Fig. 1, we can establish p_{cr}^{u} as a function of the imperfection amplitude w_0. Such curves are shown in Fig. 3, where L denotes the length of the cylinder and t the wall thickness.[2]

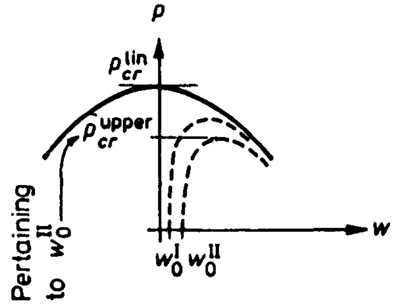


Fig. 1. Shell with decreasing post-buckling load-bearing capacity.

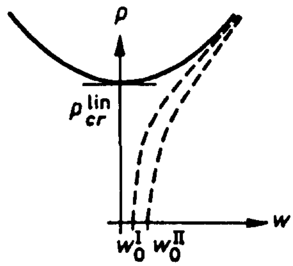


Fig. 2. Shell with increasing post-buckling load-bearing capacity.

4 ESTABLISHING THE VALUES OF THE VARIOUS FACTORS

4.1 Effect of creep

The effect of creep may be estimated by reducing the value of the modulus of elasticity, E_{co}, of concrete according to the formula

$$E_c = \frac{E_{co}}{1 + \phi_c}$$

Here E_c is the reduced modulus of elasticity of the concrete (due to creep), E_{co} is the initial modulus of elasticity (without creep), and ϕ_c is the final value of the creep factor.

If only a part of the load p_0 is acting from the beginning, while another part, p_t, begins to act only at a later date t, we may reduce the creep factor ϕ_c accordingly.

In summary, the factor taking the effect of creep into account is

$$\rho_{creep} = \frac{E_c}{E_{co}}$$

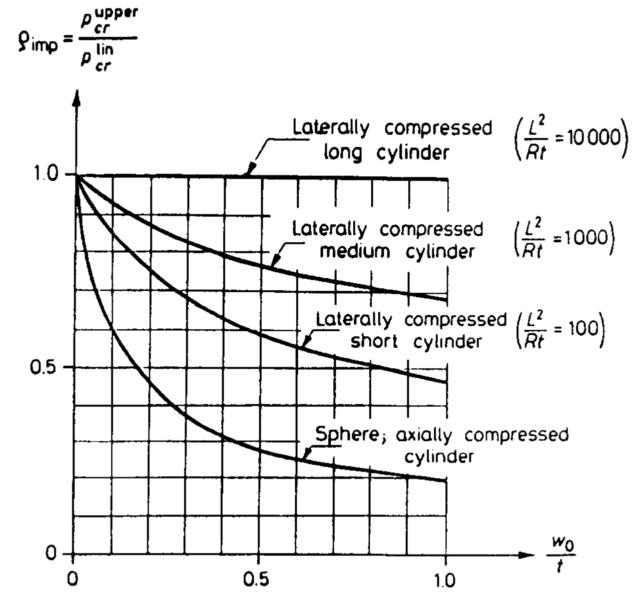


Fig. 3. The 'upper' load as a function of the imperfection amplitude.

4.2 Relationship between imperfection and eccentricity

In order to establish the values of the factors taking cracks, reinforcement and plasticity into account, we have to consider that the imperfection amplitude w_0 is not equal to the eccentricity e_0 of the compressive force. The imperfection amplitude is relevant to the reduction of the classical critical load, while to assess the influence of cracking (and of the reinforcement) and of plasticity we need the actual eccentricity of the compressive force.

If we impose a small deformation w onto a shell with a given geometry and state of stress, we can determine the corresponding bending moment and the change in the membrane forces at any point and in any direction with the aid of the classical bending theory.

Dividing the bending moment by the modified value of the membrane force we arrive at the magnitude of the eccentricity e_0, which is not necessarily equal to the imperfection amplitude w_0.

Performing this investigation for several shell surfaces, and for various states of stress, we obtained the following results for the ratio e_0/w_0 (see Ref. 3):

— cylindrical shells: $e_0/w_0 = 1{\cdot}0$,
— shells with positive Gaussian curvature (domes): $e_0/w_0 = 0{\cdot}67$,

— shells with negative Gaussian curvature (hyperbolic shells): $e_0/w_0 = 0{\cdot}50$.

We still need the expected imperfection amplitude w.

The initial imperfection consists of two parts. One is the calculable imperfection computed by the bending theory of shells. Its amplitude will be denoted by $w_{0,\text{calc}}$.

The other part is the accidental imperfection due to inaccuracies of erection, the amplitude of which will be denoted by $w_{0,\text{accid}}$. This can be assumed generally to be 1/4 to 3/4 times the wall thickness, or the value guaranteed by the contractor.

4.3 Effect of cracks and reinforcement

In order to assess this effect, we need the stiffness of the shell in the cracked state and compare this to the stiffness of the uncracked shell. This ratio will be called ψ, and can be calculated with the usual methods of strength of materials. We first calculate the quotient

$$n\mu = \frac{E_s}{E_c}\frac{A_s}{A_c}$$

with E_s and E_c as the modulus of elasticity of steel and concrete, respectively, A_s and A_c as the area of steel (in one direction) and of concrete, respectively. Knowing $n\mu$ and the ratio η (which is the concrete covering of the reinforcement relative to the wall thickness), we calculated ψ and compiled its values in Table 1 (see Ref. 2).

In the case of a single-layer reinforcement (placed in the middle of the thickness) $\eta = 0{\cdot}5$, while for a double layer (placed on the two sides of the cross section) we may take $\eta \approx 0{\cdot}2$.

Knowing ψ, i.e. the stiffness of the cracked shell, we can calculate the upper critical load of the cracked shell, i.e. the factor ρ_{crack} representing the ratio of this value to that of the uncracked shell.

The factor ρ_{crack} depends upon the parameter ψ, on the ratio e_0/t (eccentricity to wall thickness) and on the ratio e_0/w_0. For example, for shells characterized by the lowest curve in Fig. 3, values of ρ_{crack} are compiled in Table 2.

TABLE 1
Values of Factor ψ

$n\mu$:	*0*	*0·05*	*0·10*	*0·15*	*0·20*	*0·3*	*0·4*	*0·5*
$\eta = 0{\cdot}2$	0	0·178	0·285	0·373	0·453	0·579	0·730	0·850
$\eta = 0{\cdot}5$	0	0·139	0·212	0·269	0·316	0·393	0·457	0·510

TABLE 2
Values of the Factor ρ_{crack}

ψ	e_0/w_0	$e_0/t = 0$	0·1	0·2	0·3	0·4	≥0·5
	0·5						
1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0
	0·5		0·96	0·87	0·81	0·80	0·8
0·8	1·0	1·0	0·96	0·90	0·84	0·81	0·8
	0·5		0·92	0·74	0·63	0·60	0·6
0·6	1·0	1·0	0·93	0·79	0·67	0·61	0·6
	0·5		0·88	0·60	0·44	0·40	0·4
0·4	1·0	1·0	0·89	0·69	0·51	0·42	0·4
	0·5		0·84	0·47	0·26	0·20	0·2
0·2	1·0	1·0	0·86	0·58	0·34	0·22	0·2
	0·5		0·81	0·34	0·07	0	0
0·0	1·0	1·0	0·82	0·48	0·18	0·03	0

4.4 Effect of plasticity

So far, the material of the reinforced concrete (rc) shell has been assumed to be elastic (el). Plastic (pl) properties of materials can be taken into account with the aid of the 'semi-quadratic Dunkerley interaction formula':[2]

$$\left(\frac{p_{cr}}{p_{pl}}\right)^2 + \left(\frac{p_{cr}}{p_{cr,el,rc}}\right) = 1$$

from which the critical load p_{cr} of the plastic shell can be obtained. In this equation p_{pl} denotes the load that the shell is able to carry with the initial eccentricity e_0 of the compressive in-plane forces (buckling disregarded), when both concrete and reinforcement yield, and

$$p_{cr,el,rc} = p_{cr}^{lin} \cdot \rho_{imp} \cdot \rho_{creep} \cdot \rho_{crack}$$

Knowing p_{cr} the factor ρ_{pl} becomes

$$\rho_{pl} = \frac{p_{cr}}{p_{cr,el,rc}}$$

5 FACTOR OF SAFETY

The factor of safety γ depends mainly upon the 'slenderness' of the shell, i.e. upon the ratio p_p/p_{cr}^{lin}, where p_p denotes the plastic failure load under central compression (without buckling).

If $p_p/p_{cr}^{lin} = 0$, then plastic failure prevails and the safety factor should be 1·5, i.e. equal to that used for reinforced concrete structures without risk of buckling. On the other hand, if $p_p/p_{cr}^{lin} = \infty$, then buckling prevails, and the factor of safety should be increased at least to 3·0. Thus in Table 3 we arrive at the (minimum) values:[4]

TABLE 3
Safety Factors

p_p/p_{cr}^{lin}	γ
0	1·50
0·5	1·90
1·0	2·35
2·0	2·75
∞	3·00

6 CHECKING THE METHOD BY COMPARISON WITH MODEL TEST RESULTS

To check the reliability of the method proposed we computed the critical loads of experimental reinforced concrete shells and compared them with the measured values. The results are plotted in Fig. 4. (Numbers in brackets correspond to references in Ref. 2.) The mean value of the ratio

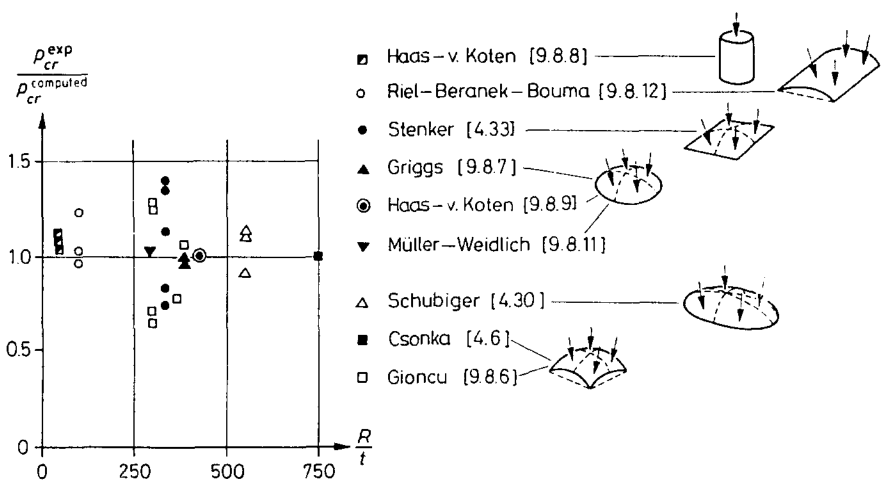
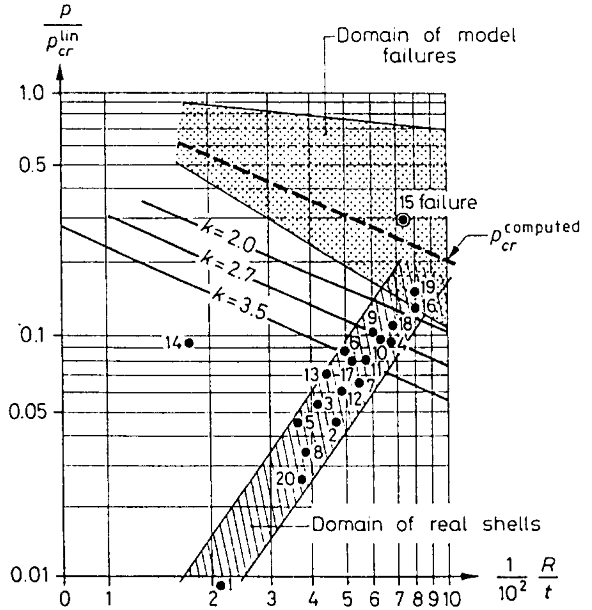


Fig. 4. Critical loads of experimental shells.



1. Jena, GDR
2. Jena, GDR
3. Matsuyama, Japan
4. Ingoviskoza Werke, FRG
5. Hilling, USA
6. Hamburg, FRG
7. Windward, USA
8. Wales, GB
9. Albuquerque, USA
10. Belgrade, Yugoslavia
11. Belgrade, Yugoslavia
12. Algeciras, Spain
13. Novosibirsk, SU
14. Rome, Italy
15. Gödöllő, Hungary
16. Saloniki, Greece
17. Puerto Rico, USA
18. Cleadon, GB
19. Lyons, France
20. Massachusetts, USA

Fig. 5. Critical loads of erected shells.

of experimental to computed critical loads is 1·02; the standard deviation is 20%, which is amply covered by the safety factor.

We also determined the critical loads of erected large reinforced concrete domes and plotted their ratio k to the actual loads of the shells in Fig. 5. These data show that most structures have a safety factor greater than 2. Two domes exhibited a safety factor somewhat less than 2, and one showed a safety factor less than unity. This structure, in fact, collapsed. Hence the safety factors proposed in Table 3 seem to be realistic.

7 CONCLUDING REMARKS

We propose that the method presented is simple and at the same time accurate enough to be used in engineering practice. It has the further advantage that the influence of the various phenomena is clearly shown, so that the engineer has a visual overview and can easily decide how he can improve the buckling safety of his structure. We may also mention that the International Association for Shell and Spatial Structures based its recommendations for the stability analysis of reinforced concrete shells on this method.[5,6]

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