

Lista 10: Problemas de valores de contorno (extraídos do livro de Boyce e DiPrima)

Problems

In each of Problems 1 through 13, either solve the given boundary value problem or else show that it has no solution. (Problems 11 through 13 involve Euler equations; see Section 5.4.)

1. $y'' + y = 0$, $y(0) = 0$, $y'(\pi) = 1$
2. $y'' + 2y = 0$, $y'(0) = 1$, $y'(\pi) = 0$
3. $y'' + y = 0$, $y(0) = 0$, $y(L) = 0$
4. $y'' + y = 0$, $y'(0) = 1$, $y(L) = 0$
5. $y'' + y = x$, $y(0) = 0$, $y(\pi) = 0$
6. $y'' + 2y = x$, $y(0) = 0$, $y(\pi) = 0$
7. $y'' + 4y = \cos x$, $y(0) = 0$, $y(\pi) = 0$
8. $y'' + 4y = \sin x$, $y(0) = 0$, $y(\pi) = 0$
9. $y'' + 4y = \cos x$, $y'(0) = 0$, $y'(\pi) = 0$
10. $y'' + 3y = \cos x$, $y'(0) = 0$, $y'(\pi) = 0$
11. $x^2y'' - 2xy' + 2y = 0$, $y(1) = -1$, $y(2) = 1$
12. $x^2y'' + 3xy' + y = x^2$, $y(1) = 0$, $y(e) = 0$
13. $x^2y'' + 5xy' + (4 + \pi^2)y = \ln x$, $y(1) = 0$, $y(e) = 0$

In each of Problems 14 through 20, find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real.

14. $y'' + \lambda y = 0$, $y(0) = 0$, $y'(\pi) = 0$
15. $y'' + \lambda y = 0$, $y'(0) = 0$, $y(\pi) = 0$
16. $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\pi) = 0$
17. $y'' + \lambda y = 0$, $y'(0) = 0$, $y(L) = 0$
18. $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(L) = 0$
19. $y'' - \lambda y = 0$, $y(0) = 0$, $y'(L) = 0$
20. $x^2y'' - xy' + \lambda y = 0$, $y(1) = 0$, $y(L) = 0$, $L > 1$