Domínio da Frequência

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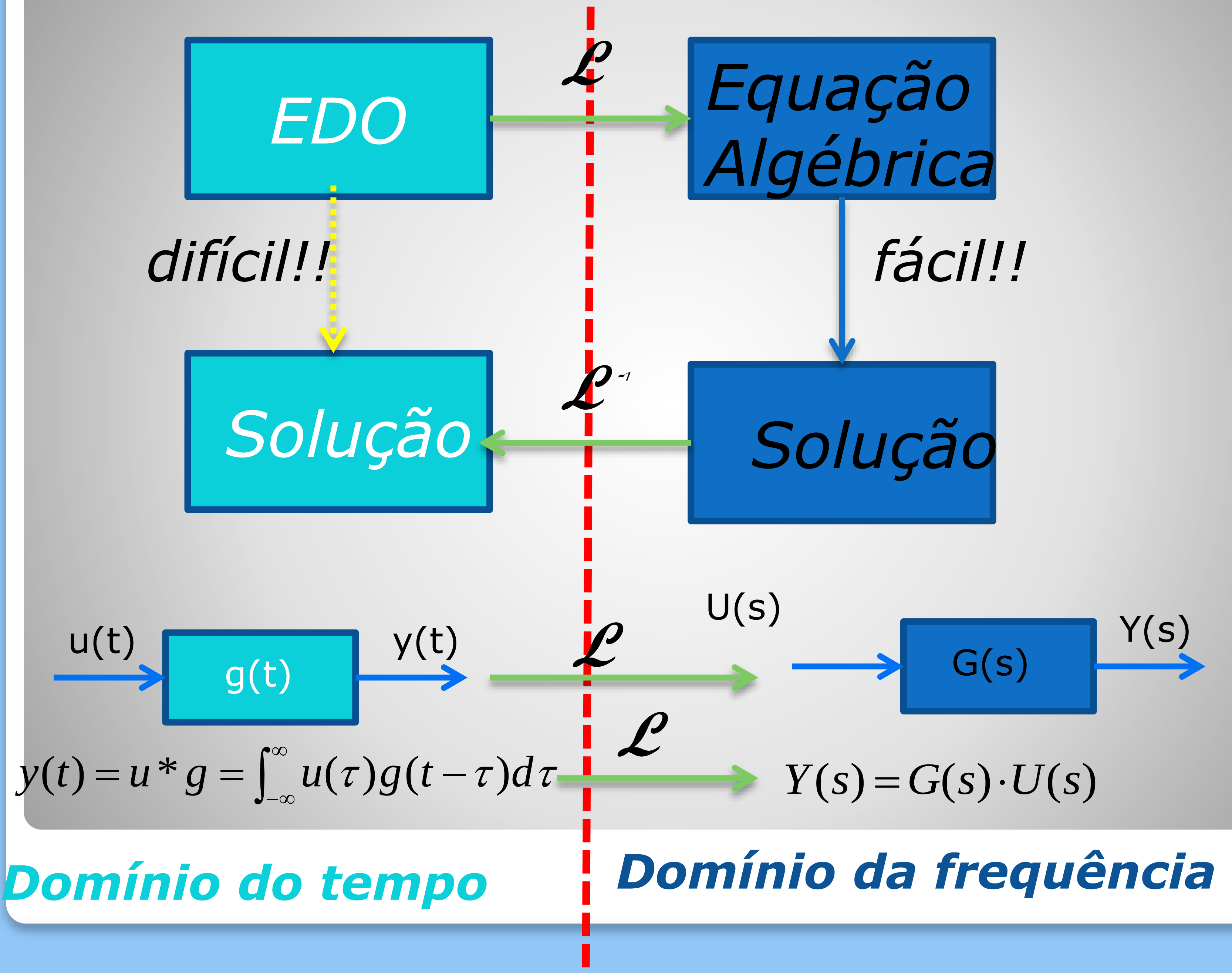
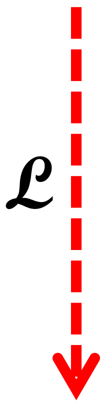
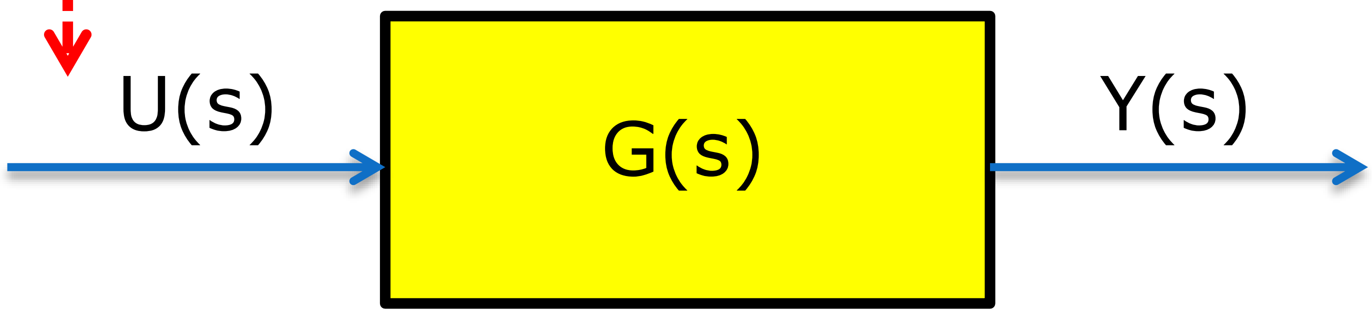
Motivação


EDO
$\mathcal{L}$
Equação Algébrica
difícil!!
fácil!!
Solução
$\mathcal{L}^{-1}$
Solução
u(t)
g(t)
y(t)
$\mathcal{L}$
U(s)
G(s)
Y(s)
$y(t) = u * g = \int_{-\infty}^{\infty} u(\tau) g(t-\tau) d\tau$
$\mathcal{L}$
$Y(s) = G(s) \cdot U(s)$
Domínio do tempo
Domínio da frequência

$$y(t) = \int_0^\infty g(t-\tau)u(\tau)d\tau] = u * g$$





$$Y(s) = G(s) \cdot U(s) \Rightarrow \mathcal{L}^{1}(Y(s)) = y(t)$$

Definição

$$s = \sigma + \omega j$$

$$F(s) = \mathcal{L}[f(t)] = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$$

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st}ds$$

$$F(s) = \int_{-\infty}^{+\infty} f(t)e^{-(\sigma+\omega j)t}dt = \int_{-\infty}^{+\infty} e^{-(\sigma)t} f(t)e^{-(\omega t)j}dt$$

$\sigma \rightarrow$ *fator* de amortecimento

obs : Fórmula de *Euler*

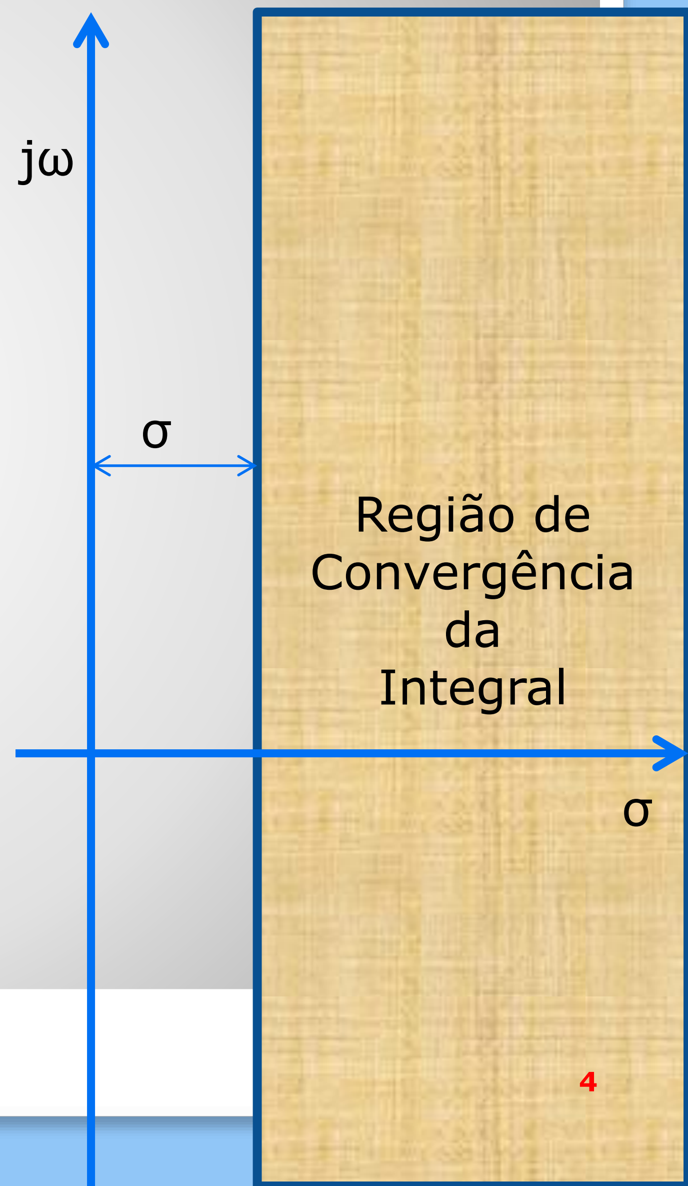
$$e^{j\theta} = \cos\theta + jsen\theta$$

$$e^{-j\theta} = \cos\theta - jsen\theta$$

$$\sigma > 0 \Rightarrow \begin{cases} t > 0 \Rightarrow convergência \\ t < 0 \Rightarrow pode \text{ haver } divergência \end{cases}$$

Transformada Unilateral :

$$F(s) = \mathcal{L}_1[f(t)] = \int_{0}^{+\infty} f(t)e^{-st}dt$$



Exemplos de transformadas e Propriedades

$$\mathcal{L}(1) = \frac{1}{s}$$

$$\mathcal{L}\left[\delta(t)\right] = 1$$

$$\mathcal{L}[\dot{f}(t)] = sF(s) - f(0)$$

$$\mathcal{L}[\ddot{f}(t)] = s^2F(s) - sf(0) - \dot{f}(0)$$

$$\mathcal{L}[\int_0^\infty f(t-\tau)g(\tau)d\tau] = \mathcal{L}[f * g] = F(s)\cdot G(s)$$

$$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)G(\tau - s)ds = \frac{1}{2\pi j}F * G$$

Transformadas de sinais elementares

$$F(s) = \mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}dt$$

1) Degrau unitário: $f(t) = u(t-\tau) = 1(t-\tau)$

$$F(s) = \int_{\tau}^{\infty} 1 \cdot e^{-st}dt = \frac{-1}{s}e^{-st}\Big|_{\tau}^{\infty} = 0 + \frac{1}{s}e^{-s\tau}$$

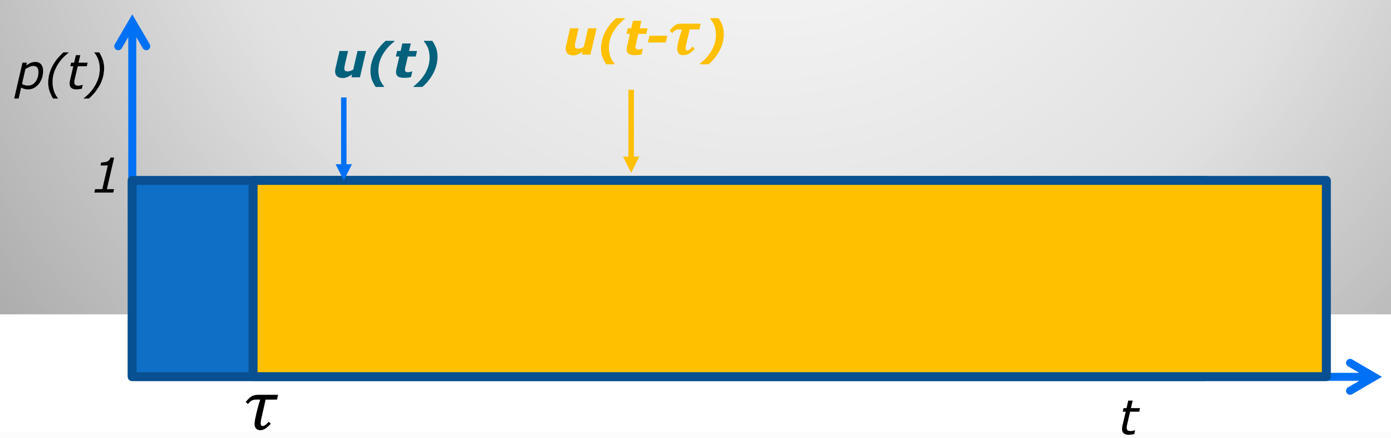
Se τ=0 ➔ F(s)=$\frac{1}{s}$

- Pulso unitário na origem: *p(t)*

$$F(s)=\int_0^{\infty}\underbrace{\frac{u(t)-u(t-\tau)}{\tau}}_{p(t)}e^{-st}dt=$$

$$\therefore F(s)=\frac{1}{\tau}\left[\int_0^{\infty}e^{-st}dt-\frac{1}{s}e^{-st}\Big|_{\tau}^{\infty}\right]=\frac{1}{\tau}\left[\frac{1}{s}-\frac{1}{s}e^{-s\tau}\right]$$

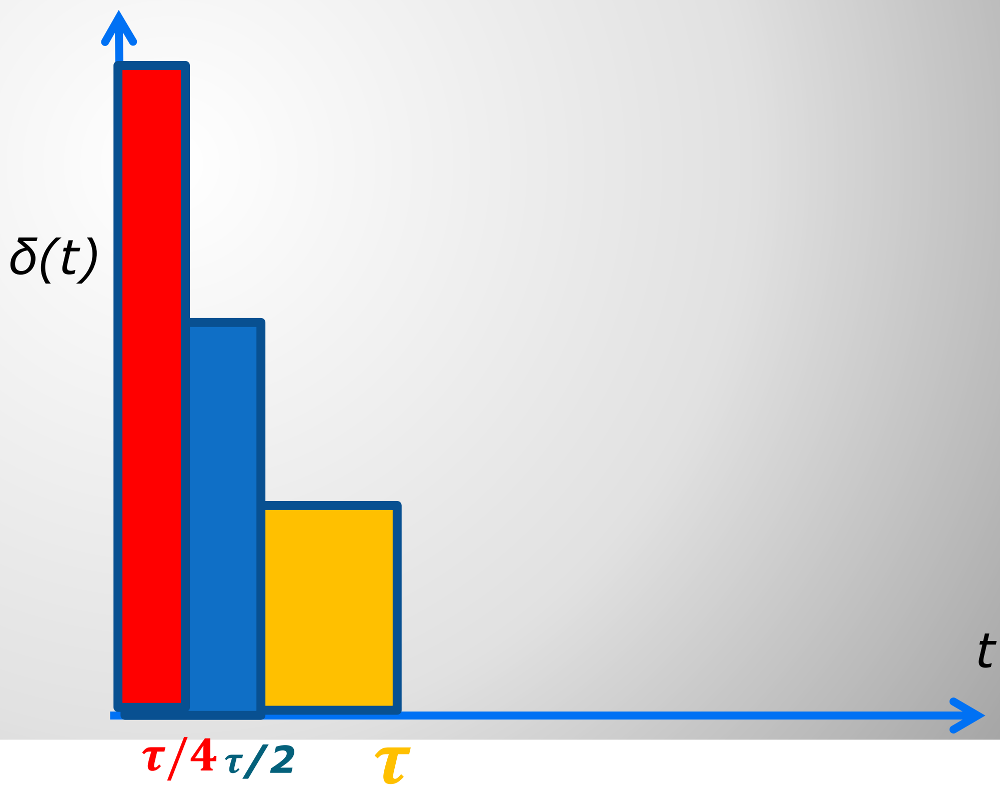
$$F(s)=\frac{1-e^{-s\tau}}{\tau s}$$



Impulso unitário na origem: $f(t) = \delta(t)$

$$F(s) = \lim_{\tau \to 0} \frac{1 - e^{-s\tau}}{\tau s} = \frac{1-1}{0} \xrightarrow{L'Hospital}$$

$$\lim_{\tau \to 0} \frac{+se^{-s\tau}}{s} = 1$$



Transformadas de funções elementares

$$F(s) = \mathcal{L}[\dot{f}(t)] = \int_0^\infty \dot{f}(t) e^{-st} dt$$

Integral por partes: $\int u dv = uv - \int v du$

$$u = e^{-st} \rightarrow \frac{du}{dt} = -se^{-st}$$

$$dv = \dot{f}(t) dt$$

$$\int_0^\infty \dot{f}(t) e^{-st} dt = e^{-st} f(\mathrm{t}) \Big|_0^\infty - \int_0^\infty f(t)(-se^{-st}) dt$$

$$F(s) = \mathcal{L}\left[\dot{f}(t)\right] = -f(0) + s\int_0^\infty f(t)(e^{-st}) dt$$

$$F(s) = \mathcal{L}\left[\dot{f}(t)\right] = -f(0) + sF(s)$$

Transformadas de funções elementares

$$F(s) = \mathcal{L}[\ddot{f}\ (t)] = \int_0^\infty \ddot{f}\ (t) e^{-st} dt$$

Seja : $\dot{f}(t) = g(t) \Rightarrow F(s) = \mathcal{L}[\ddot{f}\ (t)] = \mathcal{L}[\dot{g}\ (t)] = -g(0) + sG(s)$

$$F(s) = \mathcal{L}[\ddot{f}\ (t)] = -\dot{f}(0) + s\int_0^\infty g(t)(e^{-st})dt$$

$$F(s) = \mathcal{L}[\ddot{f}\ (t)] = -\dot{f}(0) + s\int_0^\infty \dot{f}(t)(e^{-st})dt$$

$$F(s) = \mathcal{L}[\ddot{f}\ (t)] = -\dot{f}(0) + s[-f(0) + sF(s)]$$

$$F(s) = \mathcal{L}[\ddot{f}\ (t)] = -\dot{f}(0) - sf(0) + s^2F(s)$$

$$\Rightarrow \boldsymbol{F(s) = \mathcal{L}[\overset{(n)}{f}(t)] = \int_0^\infty \overset{(n)}{f}(t) e^{-st} dt = s^n F(s) - s^{n-1} f(0) - \cdots}$$

$$\boldsymbol{\cdots - s\overset{(n-2)}{f}(0) - \overset{(n-1)}{f}(0)}$$

Propriedades das Transformadas de Laplace

1	$\mathscr{L}[Af(t)] = AF(s)$	Ex. 1 casa
2	$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$	Ex. 2 casa
3	$\mathscr{L}_{\pm}\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0\pm)$	
4	$\mathscr{L}_{\pm}\left[\frac{d^2}{dt^2} f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$	
5	$\mathscr{L}_{\pm}\left[\frac{d^n}{dt^n} f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm)$ onde $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}} f(t)$	
6	$\mathscr{L}_{\pm}\left[\int f(t)\, dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\, dt\right]_{t=0\pm}$	Ex. 3 casa
7	$\mathscr{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$	
8	$\mathscr{L}\left[\int_0^t f(t)\, dt\right] = \frac{F(s)}{s}$	
9	$\int_0^\infty f(t)\, dt = \lim_{s\to 0} F(s)$ se $\int_0^\infty f(t)\, dt$ existir	
10	$\mathscr{L}[e^{-at} f(t)] = F(s + a)$	Ex. 4 casa
11	$\mathscr{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$	
12	$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$	

7	$\mathcal{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}} \left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$
8	$\mathcal{L}\left[\int_0^t f(t)\, dt\right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t)\, dt = \lim_{s\to 0} F(s)$ se $\int_0^\infty f(t)\, dt$ existir
10	$\mathcal{L}[e^{-at} f(t)] = F(s + a)$
11	$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$
12	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathcal{L}[t^2 f(t)] = \frac{d^2}{ds^2} F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \qquad n = 1, 2, 3, \ldots$
15	$\mathcal{L}\left[\frac{1}{t} f(t)\right] = \int_s^\infty F(s)\, ds$ se $\lim_{t\to 0} \frac{1}{t} f(t)$ existir
16	$\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = aF(as)$ Ex. 5 casa
17	$\mathcal{L}\left[\int_0^t f_1(t - \tau) f_2(\tau)\, d\tau\right] = F_1(s)F_2(s)$ Ex. 6 casa
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s - p)\, dp$

Teorema do Valor Final e Teorema do Valor Inicial

Multiplicação por t:

Seja $g(t) = tf(t) \Rightarrow \mathcal{L}(g(t)) = \mathcal{L}(t \cdot f(t)) = G(s)$

$$\boxed{\mathcal{L}(t \cdot f(t)) = -\frac{dF(s)}{ds}}$$

$$\therefore \quad \frac{dF(s)}{ds} = \frac{d}{ds}\int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty f(t)\frac{d}{ds}\left(e^{-st}\right)dt$$

$$\therefore \quad \frac{dF(s)}{ds} = -\int_0^\infty \underbrace{f(t)\,t}_{g(t)} \cdot e^{-st}\,dt = \underbrace{-\int_0^\infty g(t)e^{-st}\,dt}_{G(s)}$$

$$G(s) = -\frac{dF(s)}{ds}$$

Teorema do Valor Final

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

sempre que: [redacted] i) $\exists\ \mathcal{L}(f(t))$ e $\mathcal{L}(\dot{f}(t))$

ii) $\exists \lim_{t \to \infty} f(t)$

iii) $sF(s)$ NÃO tenha POLOS no plano direito

(semi-plano DIREITO e eixo imaginário)

DEMONSTRAÇÃO

Sabemos que: $\mathcal{L}(\dot{f}) \overset{iff}{=} sF(s) - f(0) \Rightarrow \lim_{s \to 0} \mathcal{L}(\dot{f}) = \lim_{s \to 0} [sF(s) - f(0)]$

$\therefore \lim_{s \to 0} \left[\int_0^\infty \dot{f} e^{-st} dt \right] = -f(0) + \lim_{s \to 0} sF(s)$

$\therefore \int_0^\infty \dot{f}(t) \lim_{s \to 0} e^{-st} dt \overset{iff}{=} -f(0) + \lim_{s \to 0} sF(s)$ (com $\lim_{s \to 0} e^{-st} = 1$)

$\therefore \lim_{s \to 0} sF(s) - f(0) \overset{iff}{=} f(t) \Big|_0^\infty = f(\infty) - f(0)$

$$\lim_{s \to 0} sF(s) = f(\infty) \overset{iff}{=} \lim_{t \to \infty} f(t)$$

c.q.d.

DEGRAU → 1) $f(t) = 1 \Rightarrow F(s) = \frac{1}{s}$

RAMPA → 2) $g(t) = t \cdot f(t) = t \Rightarrow G(s) = -\frac{dF(s)}{ds} = -\left(-\frac{1}{s^2}\right) = \frac{1}{s^2}$

PARÁBOLA → 3) $h(t) = t \cdot g(t) = t^2 \cdot f(t)$

$$H(s) = -\frac{dG(s)}{ds} = -\left(-\frac{2s}{s^4}\right) = \frac{2}{s^3}$$

4) $x(t) = t^k f(t)$

$$\Rightarrow \boxed{X(s) = \frac{k!}{s^{(k+1)}}}$$

PARES DE TRANSFORMADAS

	$f(t)$	$F(s)$
1	Impulso unitário $\delta(t)$	1
2	Degrau unitário $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\operatorname{sen} \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\operatorname{senh} \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$

Ex. 7 casa

Ex. 8 casa

Ex. 9 casa

PARES DE TRANSFORMADAS

6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\operatorname{sen} \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\operatorname{senh} \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

$$F(s) = \mathcal{L}[f(t)] = \int^{\infty} f(t)e^{-st}\,dt$$

PARES DE TRANSFORMADAS

20	$e^{-at} \operatorname{sen} \omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at} \cos \omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
22	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen} \omega_n \sqrt{1-\zeta^2}\, t$	$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen}(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1} \dfrac{\sqrt{1-\zeta^2}}{\zeta}$	$\dfrac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1-\dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \operatorname{sen}(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1} \dfrac{\sqrt{1-\zeta^2}}{\zeta}$	$\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos \omega t$	$\dfrac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t - \operatorname{sen} \omega t$	$\dfrac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\operatorname{sen} \omega t - \omega t \cos \omega t$	$\dfrac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\dfrac{1}{2\omega} t \operatorname{sen} \omega t$	$\dfrac{s}{(s^2+\omega^2)^2}$
29	$t \cos \omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$

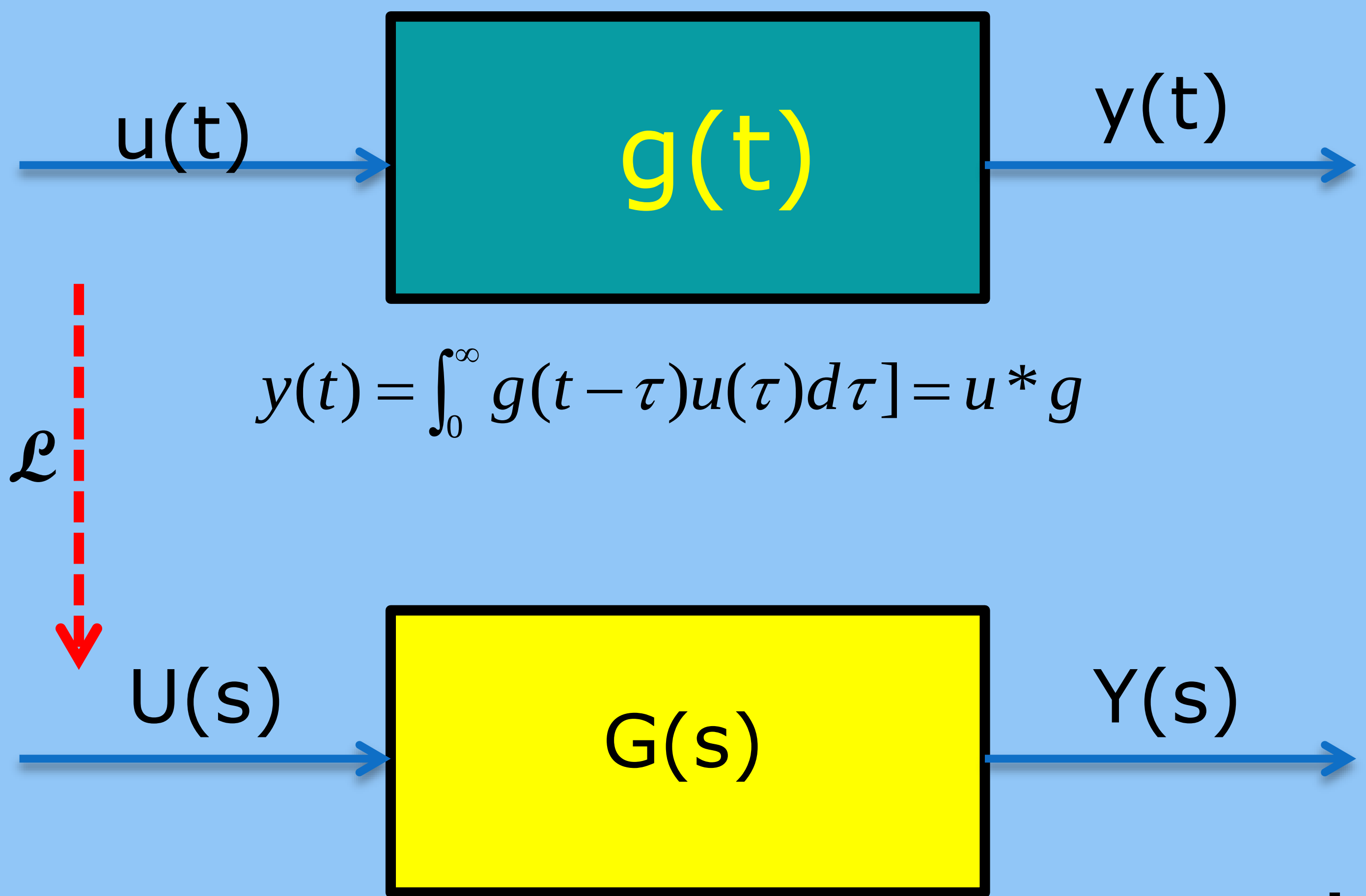
Transformada da Convolução

Ex. 10 para casa:

Mostre que:

$$F(s) = \mathcal{L}[\int_0^\infty f(t-\tau g(\tau)\,d\tau] = \mathcal{L}[f * g] = F(s)G(s)$$

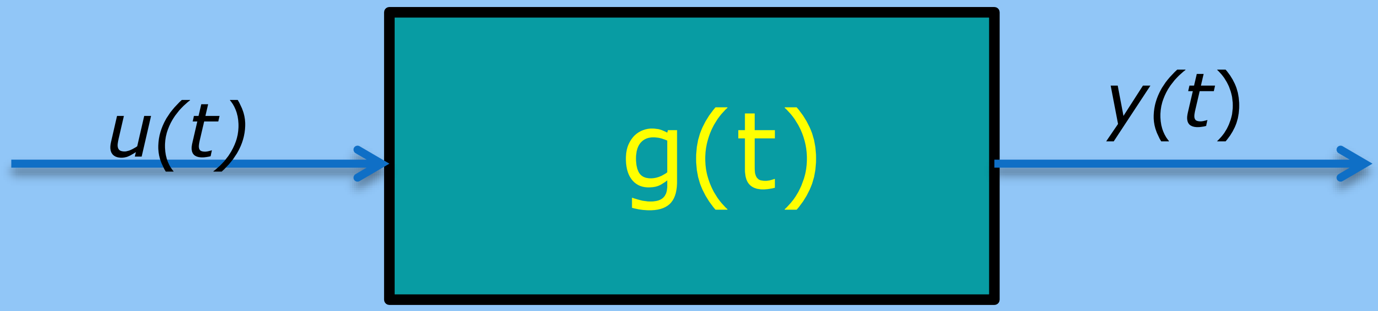
Definição de Função de Transferência



$$y(t) = \int_0^{\infty} g(t-\tau)u(\tau)d\tau] = u * g$$

$$Y(s) = G(s) \cdot U(s) \Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}\Bigg|_{CI \equiv 0}$$

Definição de Função de Transferência



Considere o SLIT descrito pela EDO:

$$\overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y(t) = b_1 \overset{(n-1)}{u} + \cdots + b_{n-1}\dot{u} + b_n u(t)$$

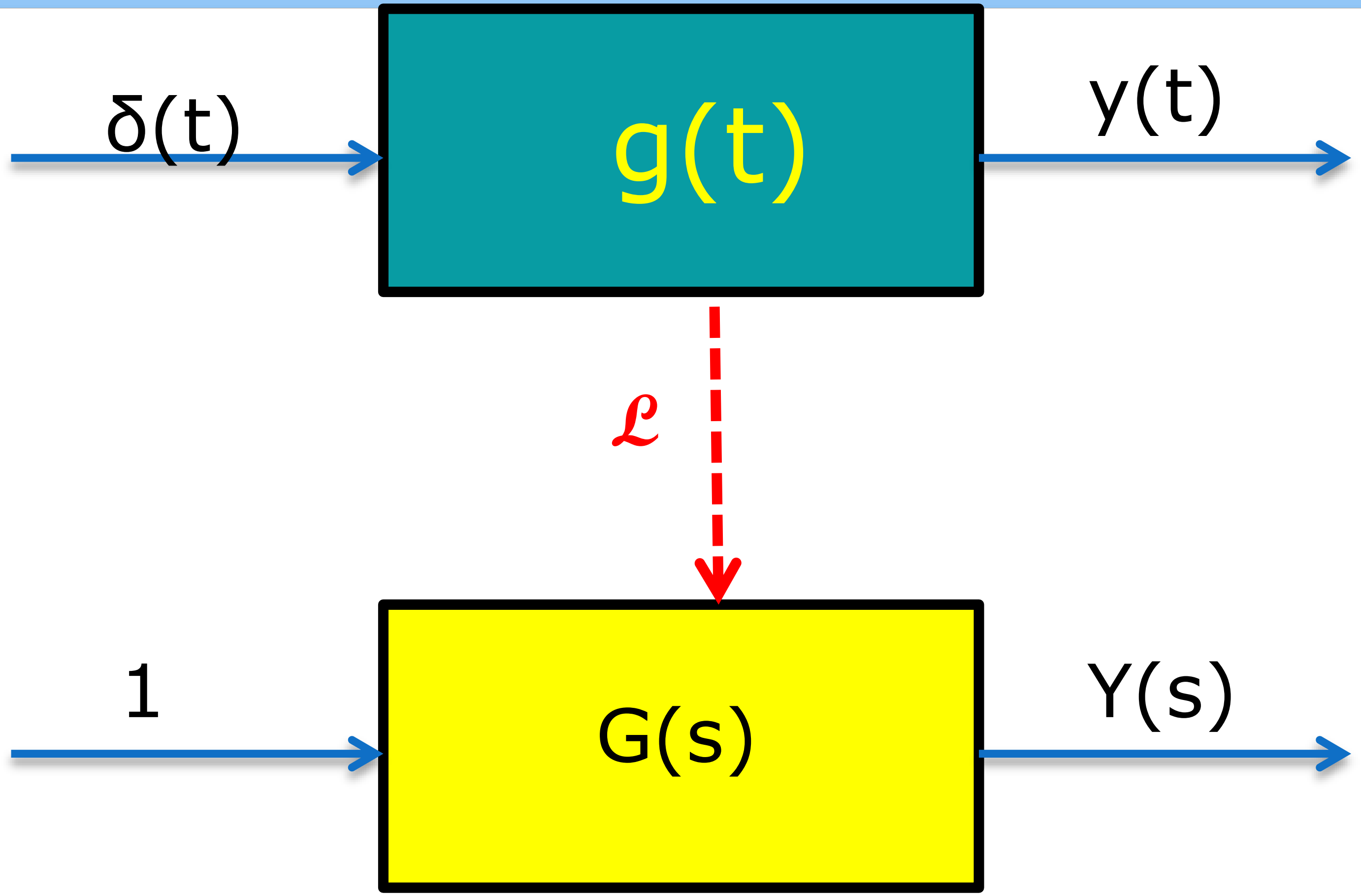
Aplicando a Transformada de Laplace com CI≡0:

$$\mathcal{L}[\ \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots] = \mathcal{L}[b_1 \overset{(n-1)}{u} + \cdots + b_{n-1}\dot{u} + b_n u(t)]$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} = \frac{b_n + b_{n-1}s + \cdots b_1 s^{n-1}}{a_n + a_{n-1}s + \cdots a_1 s^{n-1} + s^n}$$

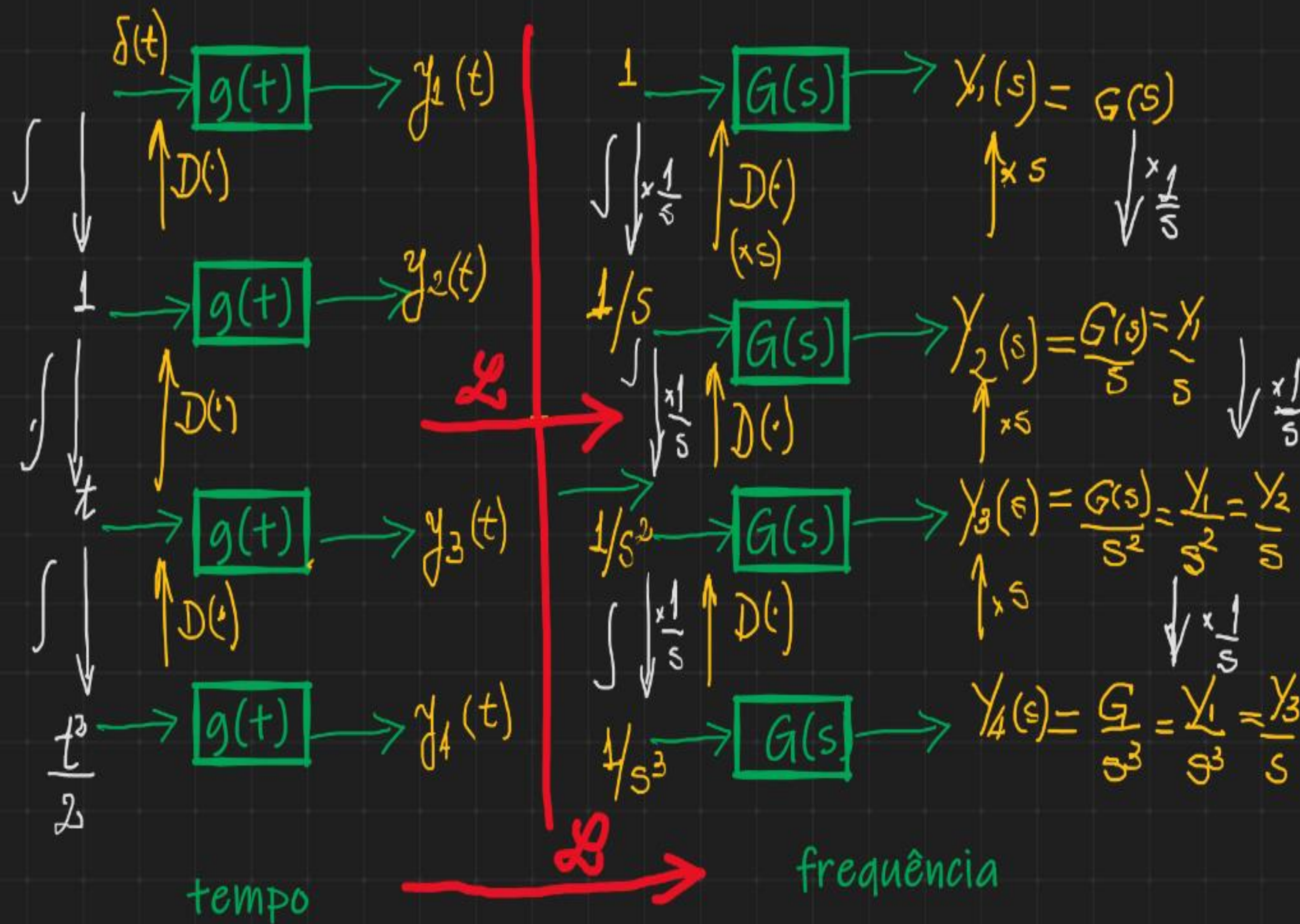
FUNÇÃO DE TRANSFERÊNCIA

- Usando a FT é possível representar um SLIT por uma equação algébrica em s, cuja ordem é dada pela maior potência do denominador (n);
- A FT é uma característica dos sistema e independe da magnitude e da natureza do sinal de entrada;
- Sistemas diferentes podem ter a mesma FT (não há informação física numa FT);
- A FT pode ser determinada experimentalmente.



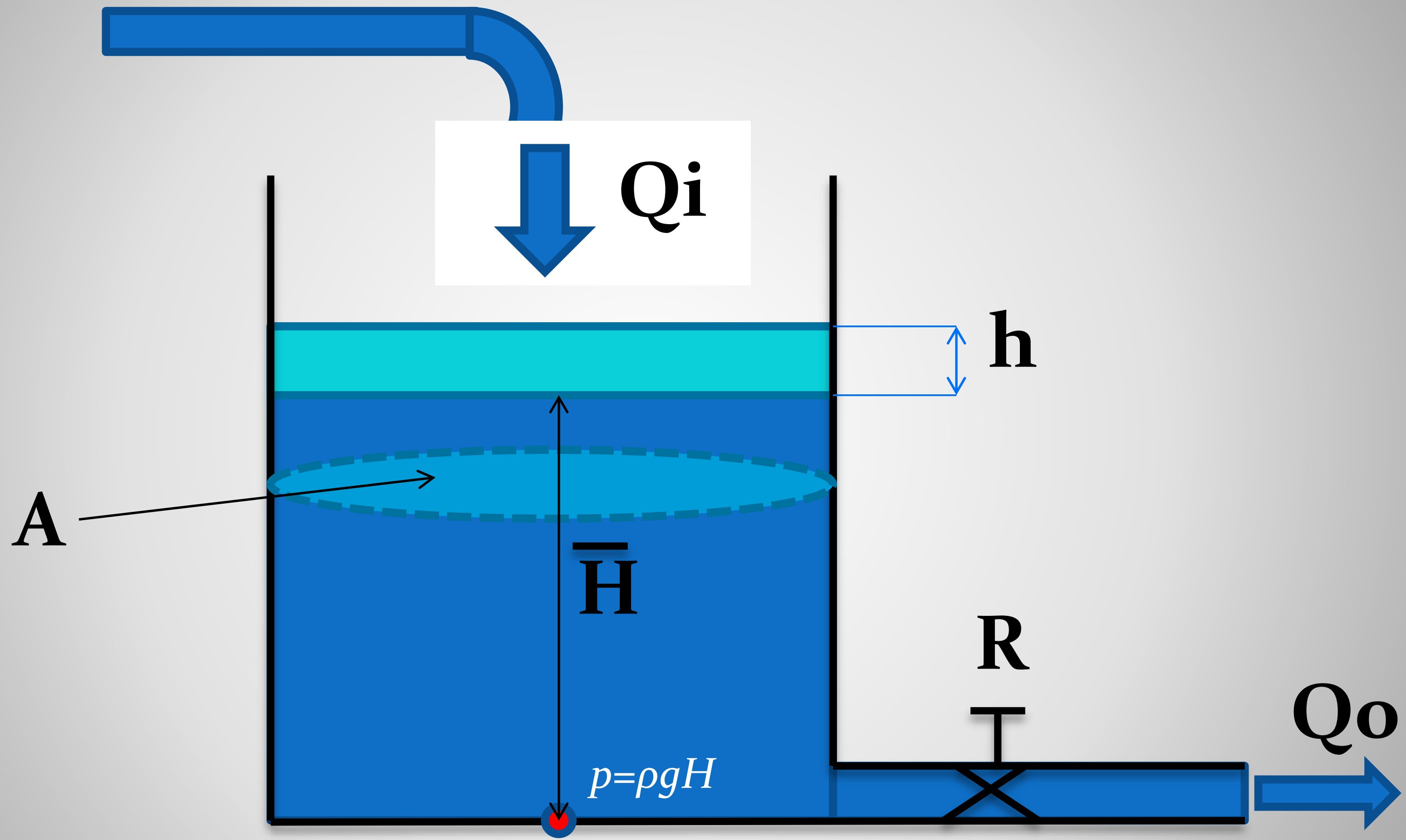
$Y(s) = G(s) * 1 = G(s) \rightarrow Y(s) = G(s)$ ➔ A saída é a própria FT.

O IMPULSO IDENTIFICA O SISTEMA!!!



Integrando o sinal de entrada ➔ integral do sinal de saída

Tanque com fluido incompressível



Escoamento turbulento na válvula

$$Qi = \overline{Q} + q(t)$$

$$H = \overline{H} + h(t)$$

$$Qo = K\sqrt{H}$$

Lei da conservação da massa, fluído incompressível.

$$Qi - Qo = A\frac{dH}{dt} \Rightarrow A\frac{dh}{dt} = Qi - K\sqrt{H}$$

$$\therefore \dot{h} = \frac{1}{A}\left(Qi - K\sqrt{H}\right)$$

$$\dot{h} = \frac{1}{A}\left(Qi - K\sqrt{H}\right)$$

$$\therefore \dot{h} = f(Qi, H)$$

$$\dot{h} \cong f(\overline{Q}, \overline{H}) + \left.\frac{\partial f}{\partial Qi}\right|_{\frac{\overline{Q}}{\overline{H}}}\left(Qi - \overline{Q}\right) + \left.\frac{\partial f}{\partial H}\right|_{\frac{\overline{Q}}{\overline{H}}}\left(H - \overline{H}\right)$$

$$\text{Obs.: } f(\overline{Q}, \overline{H}) \rightarrow R.P. \rightarrow \dot{h} = 0 \Rightarrow \overline{Q} = K\sqrt{\overline{H}}$$

$$\therefore f(\overline{Q}, \overline{H}) = \frac{1}{A}\left(\overline{Q} - K\sqrt{\overline{H}}\right) = 0$$

$$\therefore \dot{h} \cong \frac{1}{A}\left(q(t) - \left.\frac{K}{2\sqrt{H}}\right|_{\frac{\overline{Q}}{\overline{H}}} h(t)\right) = \frac{1}{A}\left(q(t) - \frac{K}{2\sqrt{\overline{H}}} h(t)\right)$$

$$K = ?$$

$$R \triangleq \frac{dH}{dQ}$$

$$Qo = K\sqrt{H} \Rightarrow \frac{dQo}{dH} = \frac{K}{2\sqrt{H}} \Rightarrow R = \frac{2\sqrt{H}}{K}$$

$$\therefore K = \frac{2\sqrt{H}}{R} \text{ em R.P.} \Rightarrow K = \frac{2\sqrt{\overline{H}}}{R}$$

$$\dot{h} = \frac{1}{A}\left(q(t) - \frac{K}{2\sqrt{\overline{H}}}h(t)\right) = \frac{1}{A}\left(q(t) - \frac{2\sqrt{\overline{H}}}{R \cdot 2\sqrt{\overline{H}}}h(t)\right)$$

$$\dot{h} + \frac{h}{RA} = \frac{q}{A} \Rightarrow \text{EDO Linear!}$$

$$\dot{h} = \frac{1}{A}\left(Qi - K\sqrt{H}\right) \Rightarrow \text{EDO Não-linear!}$$

Função de transferência usando o operador D(.)

$$\dot{h} + \frac{h}{RA} = \frac{q}{A}$$

usando o operador $D[\bullet]$:

$$\left(D + \frac{1}{RA}\right)h = \frac{q}{A}$$

$$\frac{saída}{entrada} = \frac{h}{q} = \frac{1}{\left(AD + {}^{1}\!/\!_{R}\right)} = \frac{R}{ARD + 1}$$

$[AR] = cte$ de $tempo \Rightarrow AR = \tau$

$$\frac{h}{q} = \frac{R}{\tau D + 1}$$

$\Rightarrow$ se τ é grande $\Rightarrow$ sistema lento!

Função de transferência usando o operador D(.)

$$\frac{saída}{entrada} = \frac{h}{q_i} = \frac{1}{\left(AD + \frac{1}{R}\right)} = \frac{R}{ARD + 1}$$

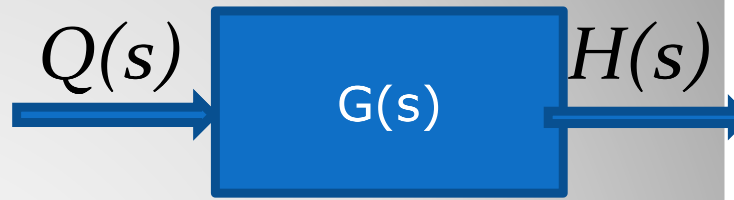
$$q_o = Kh = \frac{h}{R_f} \Rightarrow h = Rq_o$$

$$\frac{Rq_o}{q_i} = \frac{R}{ARD + 1}$$

$$\frac{q_o}{q_i} = \frac{1}{ARD + 1} = \frac{1}{\tau D + 1}$$

Função de transferência usando a Transf. de Laplace

$$\dot{h} + \frac{h}{RA} = \frac{q}{A}$$



Transf. de Laplace com condições iniciais nulas:

$$\left(s + \frac{1}{RA}\right) H(s) = \frac{Q(s)}{A}$$

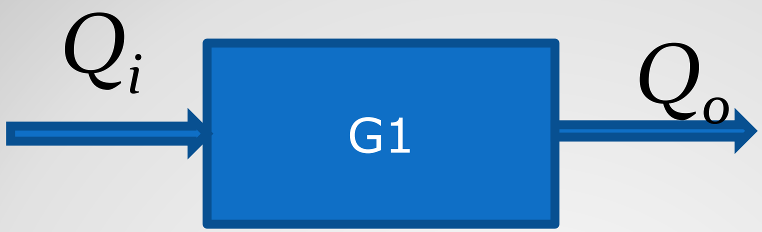
$$\frac{saída}{entrada} = \frac{H(s)}{Q(s)} = \frac{1}{\left(As + {}^{1}\!/\!_{R}\right)} = \frac{R}{ARs + 1}$$

$$[AR] = cte \text{ de } tempo \Rightarrow AR = \tau$$

$$\frac{H}{Q} = G(s) = \frac{R}{\tau s + 1}$$

$\Rightarrow$ se τ é grande $\Rightarrow$ sistema lento!

Função de transferência usando a Transf. de Laplace



$$\frac{saída}{entrada} = \frac{H}{Q_i} = \frac{1}{\left(As + \frac{1}{R}\right)} = \frac{R}{ARs+1}$$

$$Q_o = KH = \frac{H}{R_f} \Rightarrow H = RQ_o$$

$$\frac{RQ_o}{Q_i} = \frac{R}{ARs+1}$$

$$\frac{Q_o}{Q_i} = G1 = \frac{1}{ARs+1} = \frac{1}{\tau s+1}$$

1) A resposta de um sistema ao impulso é:

$$y(t) = \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$$

Qual sua Função de Transferência (FT)?

2) A resposta de um sistema ao degrau é:

$$y(t) = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$$

Qual sua Função de Transferência (FT)?

3) Dado o sistema no Espaço de Estado (E):

$$A = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \qquad C = [1 \quad 0] \qquad D = 0$$

$$\mathbf{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \qquad \Delta t = 0{,}1 \qquad t_f = 1{,}0$$

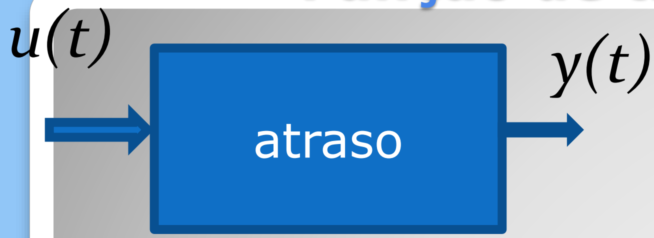
Simule e determine a resposta. Qual é FT?

4) Dado o sistema no EE:

$$A = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad D = 0$$

Determine a Equação Característica e verifique a estabilidade do sistema. Há polos dominantes?

Função de transferência transcendentais



$y(t) = u(t-\tau) \rightarrow \tau$ tempo de atraso (delay)

$\mathscr{L} \downarrow$

$$Y(s) = \int_0^{\infty} u(t-\tau) e^{-st} dt$$

$$seja: t - \tau = \xi$$

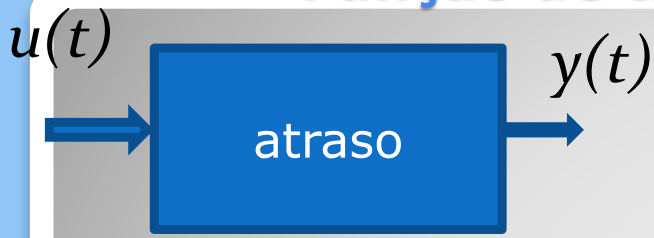
$$Y(s) = \int_{-\tau}^{\infty} u(\xi) e^{-s(\xi+\tau)} d\xi = \int_0^{\infty} u(\xi) e^{-s(\xi)} e^{-s(\tau)} d\xi$$

$$Y(s) = e^{-s(\tau)} \int_0^{\infty} u(\xi) e^{-s(\xi)} d\xi$$

$$Y(s) = e^{-s(\tau)} U(s)$$

$$\therefore G(s) = e^{-s\tau} = 1 - s\tau + \frac{(s\tau)^2}{2!} - \frac{(s\tau)^3}{3!} \cdots$$

Função de transferência transcendentais



$$G(s) = e^{-s\tau} = 1 - s\tau + \frac{(s\tau)^2}{2!} - \frac{(s\tau)^3}{3!} \cdots$$

$$G(s) = \frac{1}{e^{s\tau}} = \frac{1}{1 + s\tau + \frac{(s\tau)^2}{2!} + \frac{(s\tau)^3}{3!} \cdots}$$

$\Rightarrow FT$ com infinitos polos!

Se τ for pequeno:

Aproximação de $1^{\underline{a}}$ *ordem*:

$$G(s) = e^{-s\tau} = \frac{1}{e^{s\tau}} \cong \frac{1}{1 + s\tau} = 1 - s\tau$$

Função de transferência transcendentais

$$Aproximação\ Padé\ de\ 1^{\underline{a}}\ ordem:$$

$$G(s) = e^{-s\tau} = \frac{1}{e^{s\tau}} \cong \frac{1 - s\tau/2}{1 + s\tau/2}$$

$$Aproximação\ Padé\ de\ 2^{\underline{a}}\ ordem:$$

$$G(s) = e^{-s\tau} = \frac{1}{e^{s\tau}} \cong \frac{1 - s\tau/2 + (s\tau)^2/12}{1 + s\tau/2 + (s\tau)^2/12}$$