ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Kinematics and dynamics of manipulators

Laboratório de Biomecatrônica
Departamento de Engenharia Mecatrônica e Sistemas Mecânicos

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PMR5005
Biomechatronics and Biorobotics

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Goals of the lecture

- Provide the most relevant concepts for the kinematic and dynamic modeling of manipulator robots

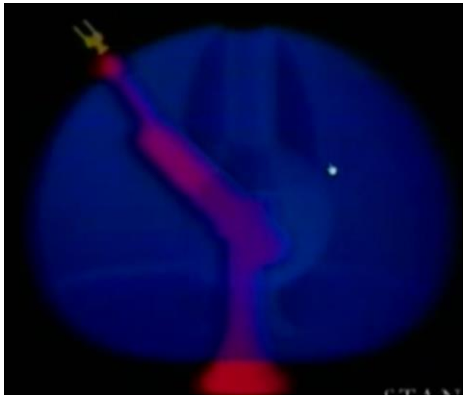
- Overview of the kinematic and dynamic modeling process oriented to robot design

- Study a bio-robotic application of the acquired knowledge

Contents

Introduction

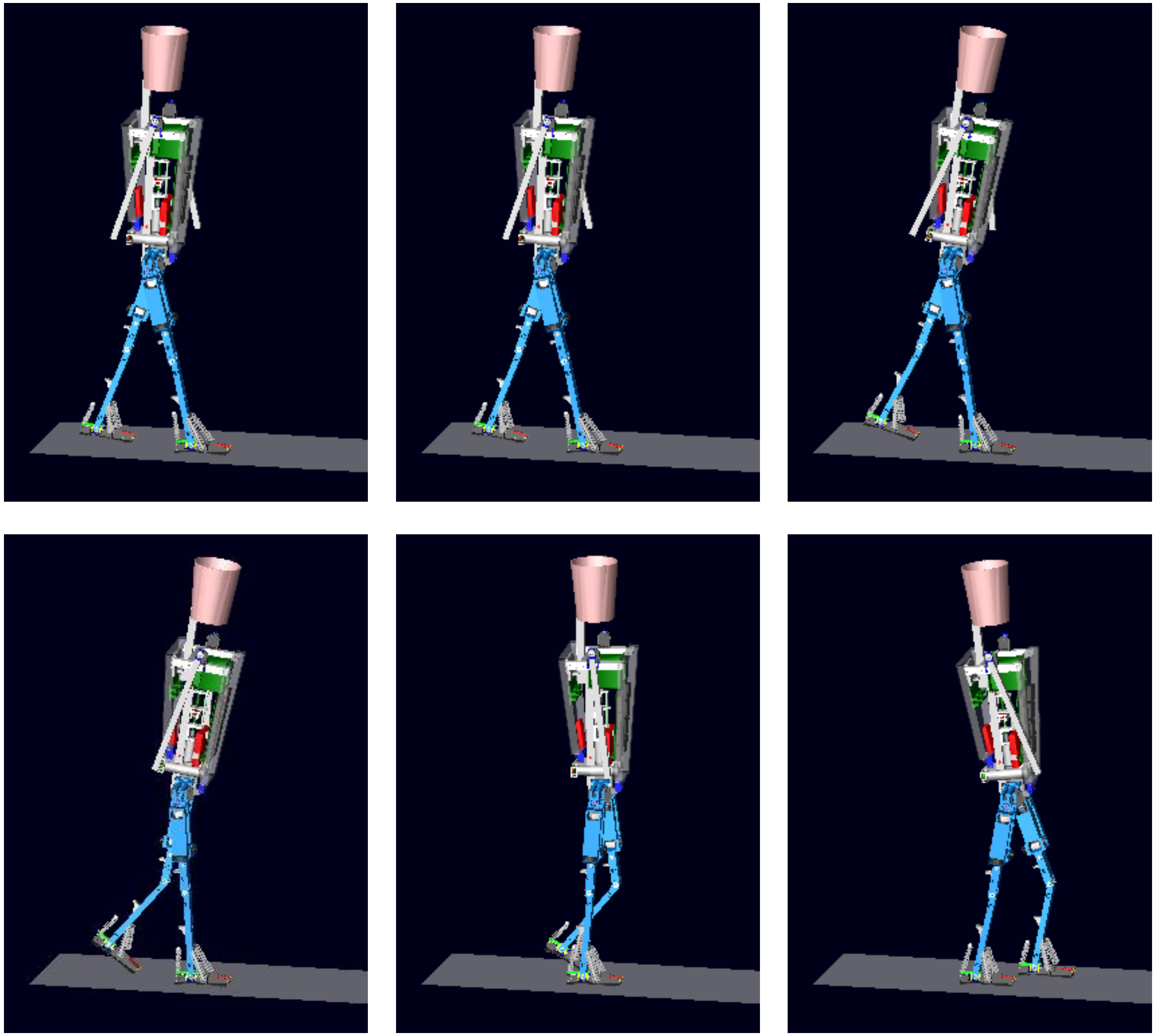
- Definitions
  - Robot: programmable machine, capable of performing automatically complex series of actions
  - **Manipulator with several degrees of freedom**, mobile or with fixed base
    - Kinematics: Movement description
    - Dynamics: Movement causes



- Why and what for?
  - Planning joint trajectories
  - Robot workspace

Examples: Manipulators. Widely used in industry.

Analysis of a biped robot

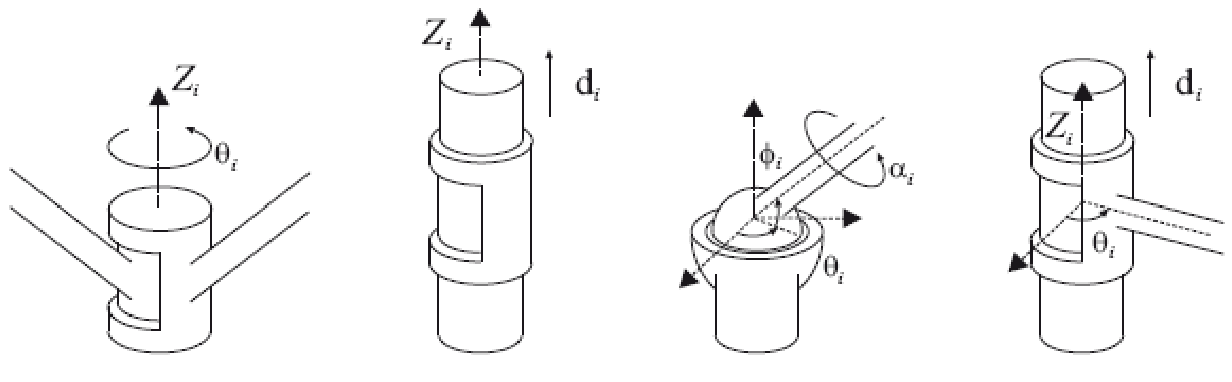


Determine the position of the foot in the air (effector) from the position of the foot on the ground (base)

Contents

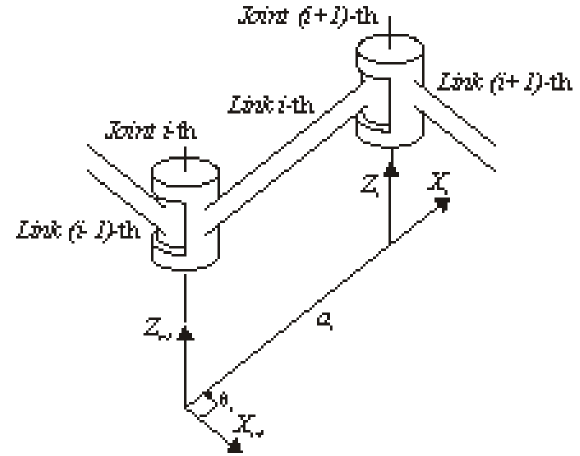
- Introduction:
  - Definitions and formulation of the problem
- **Kinematic modeling**
  - **Direct kinematic problem**
  - **Kinematic description of a robot**
  - **Inverse kinematic problem**
  - **Jacobian Matrix**
- Dynamic modeling
  - Direct and inverse dynamic problem
  - Newton-Euler formulation
  - Lagrange formulation
- Case study: bio-robotics and bipeds

Joints and segments



•Spherical joint = 3 revolute
•Cylindrical =1 prismatic (sliding) + 1 revolute (hinge)

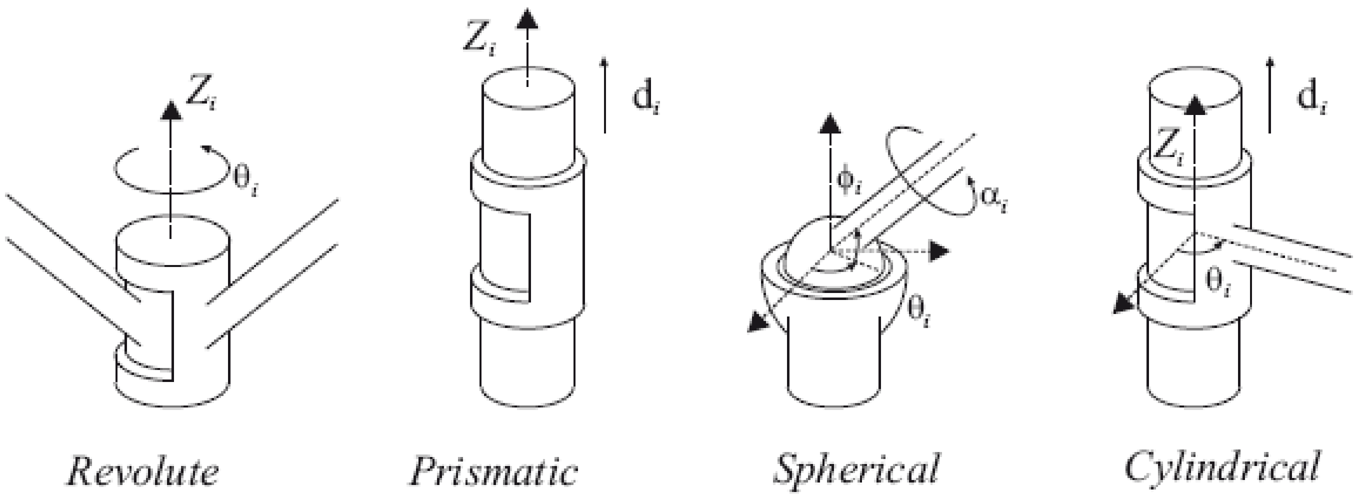
•Name according to the joint type (R or P).



Degrees of freedom

- Minimum number of independent position variables that need to be specified to define the location of all parts of the mechanism (joints).
- Single axis articulation have one degree of freedom
- Movement along more than one axis: the joint has more degrees of freedom.
- Most robots have between 4 and 6 degrees of freedom.
- Human arm (shoulder to wrist), 7 d.o.f..

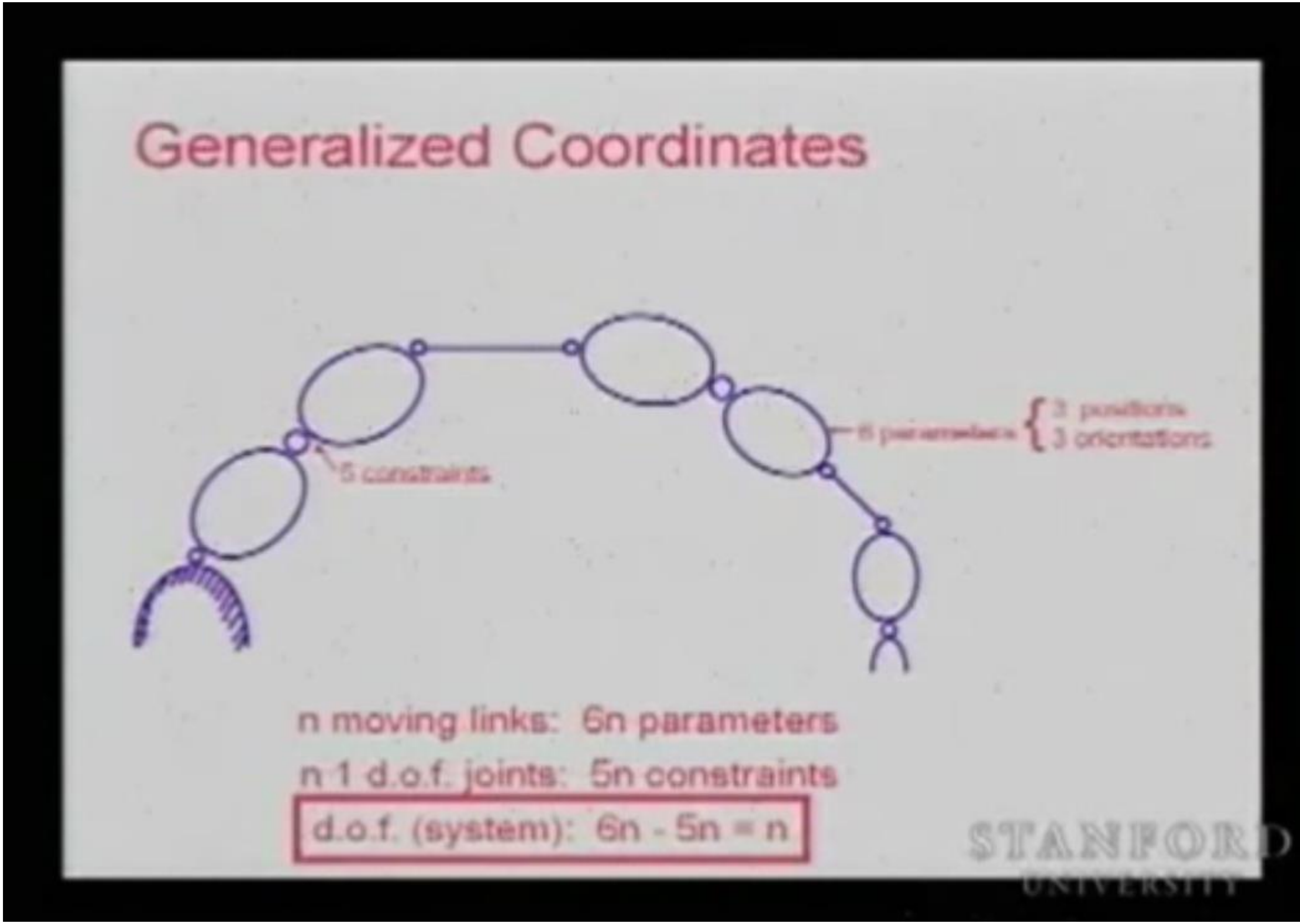
Degrees of freedom



1 1 ~~2~~ 3 2

Joints and segments

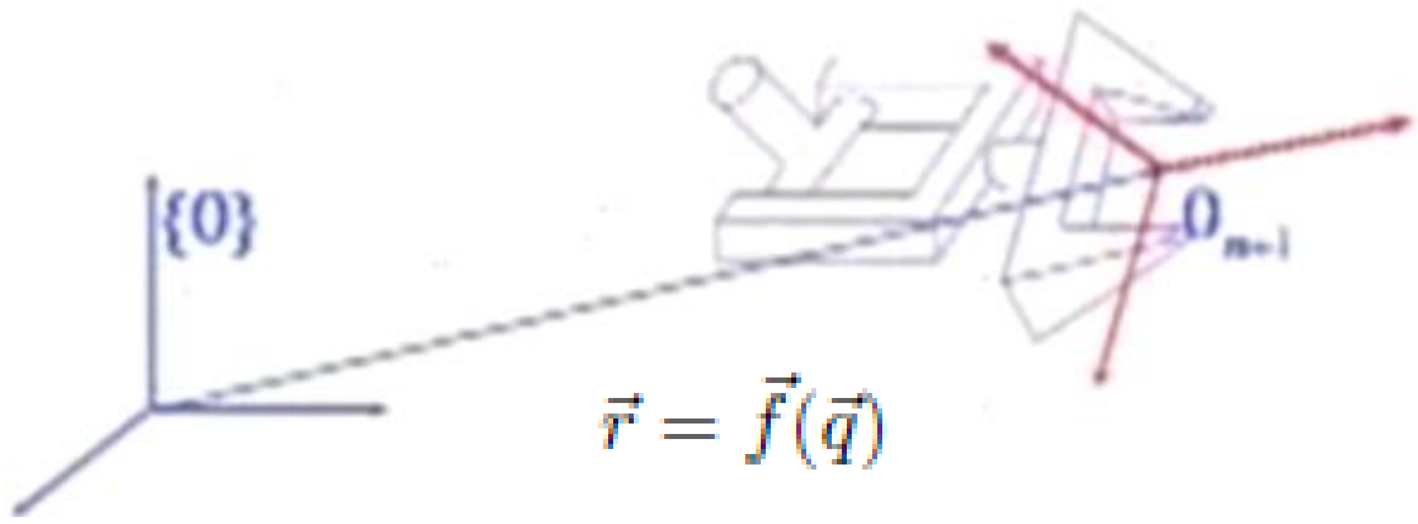
- Reduce d.o.f.
    - 1 d.o.f.: Revolute (hinge), prismatic (deslizante)
    - More d.o.f.: Cylindrical, spherical
- N segments: 6N d.o.f. (each joint: 6 d.o.f.)
- N Joints with 1 d.o.f.: 5N constraints
- 6N-5N= N d.o.f. (fixed base)
- E.g. humanoid biped robot: N+6
    – (6 unactuated d.o.f.)



Introduction to Robotics. Oussama Khatib. Stanford University

Direct kinematics

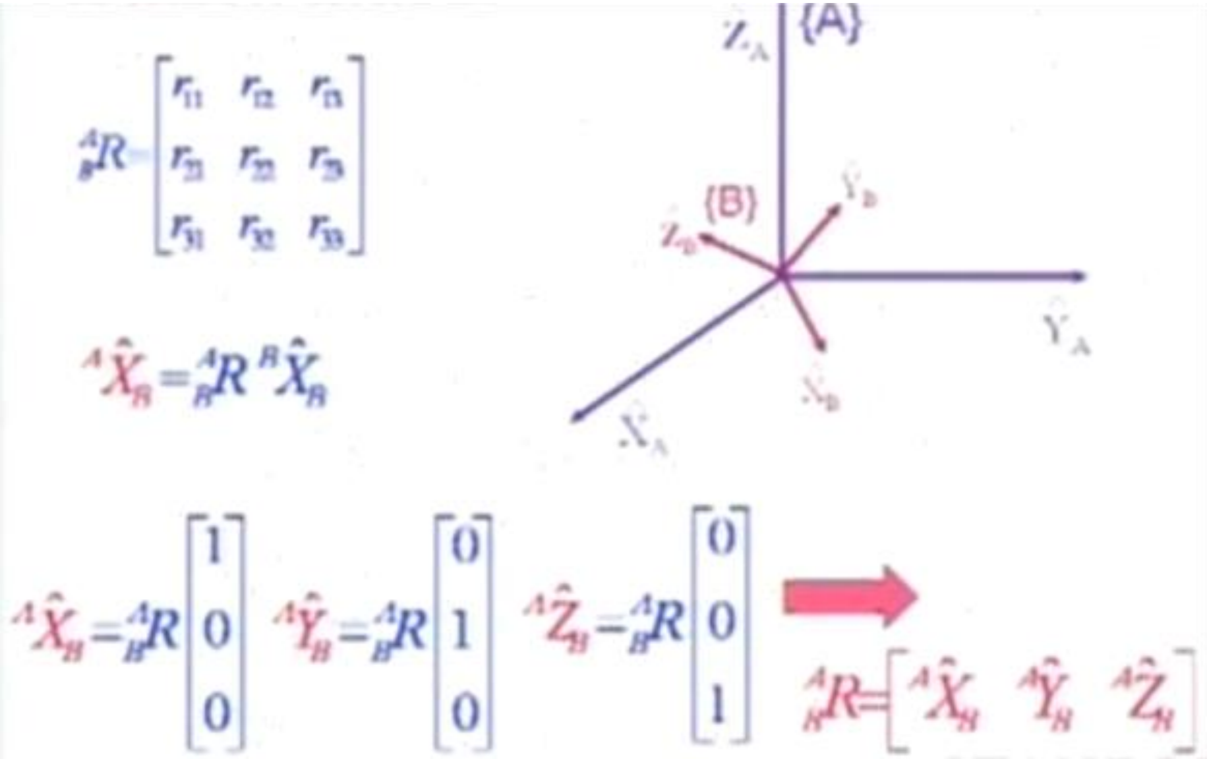
- Determine the position and orientation of the end-effector with respect to a coordinate system



$$\vec{r} = \vec{f}(\vec{q})$$

Rotation matrix

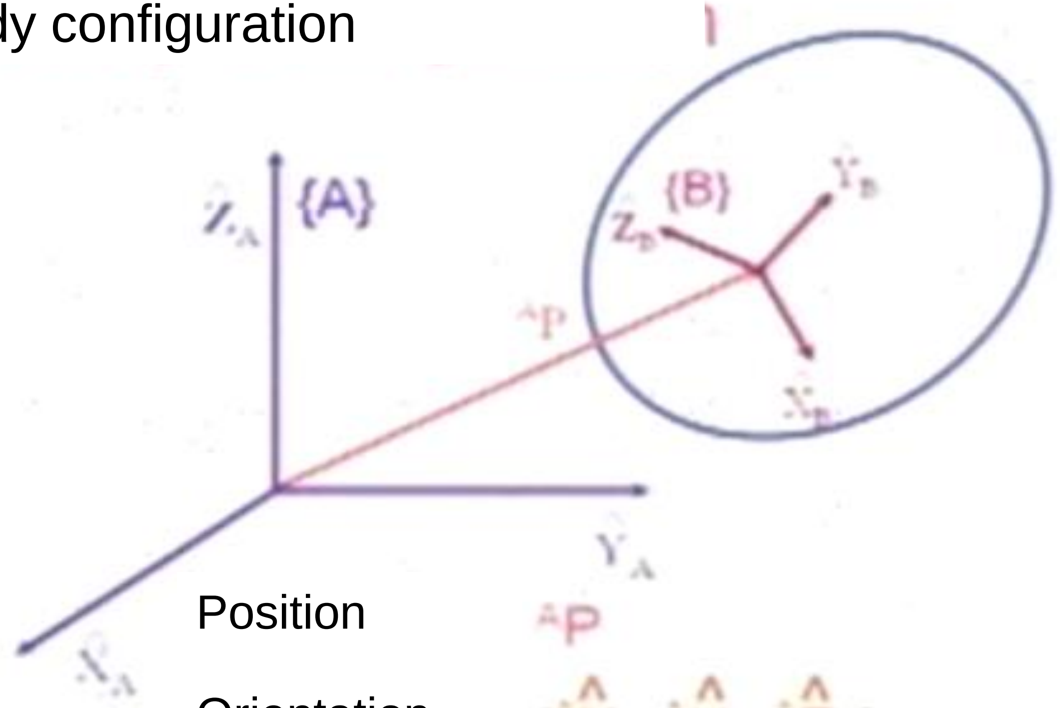
- Unity-vector componentes of the new axis with respect to the reference.



Introduction to Robotics. Oussama Khatib. Stanford University

Position and orientation

Rigid body configuration



Position ^AP

Orientation $\{^A\hat{X}_B, ^A\hat{Y}_B, ^A\hat{Z}_B\}$

Describes the rotation and translation of {B} with respect to {A}

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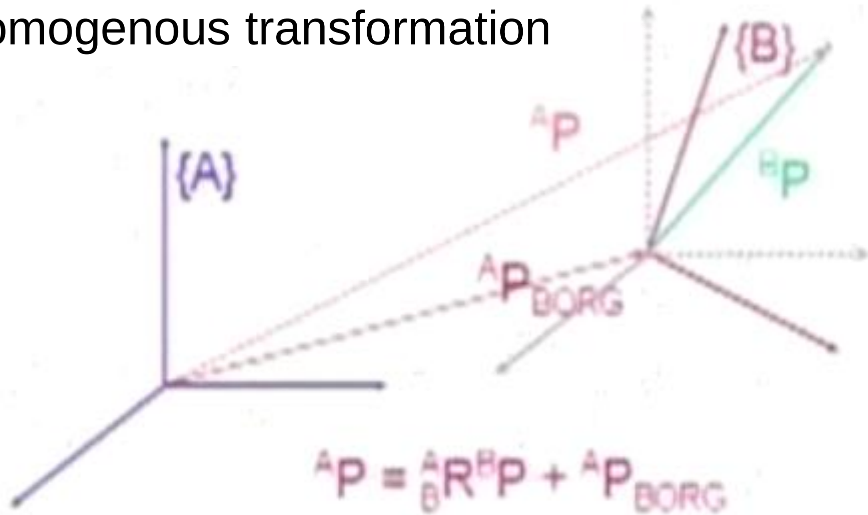
Homogenous Transformation

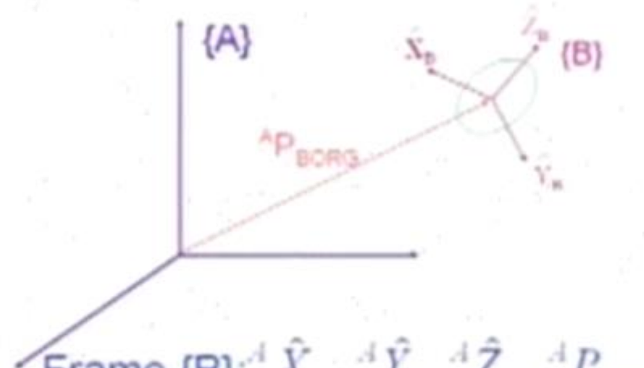
Homogenous transformation



$$^AP = {}^A_BR\,{}^BP + {}^AP_{BORG}$$

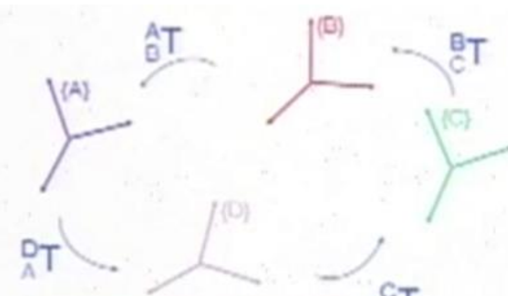
$$\begin{bmatrix} ^AP \\ 1 \end{bmatrix} = \begin{bmatrix} ^A_BR & ^AP_{Borg} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^BP \\ 1 \end{bmatrix}$$

$$\underset{(4x1)}{^AP} = \underset{(4x4)}{^A_BT}\ \underset{(4x1)}{^BP}$$



Frame {B}: $^A\hat{X}_B, {}^A\hat{Y}_B, {}^A\hat{Z}_B, {}^AP_{Borg}$

$$\{B\} = \{ {}^A_BR \quad {}^AP_{Borg} \}$$



$$^A_BT\,{}^B_CT\,{}^C_DT\,{}^D_AT = I$$

$$\Rightarrow \quad {}^B_AT = {}^B_CT\,{}^C_DT\,{}^D_AT$$

Introduction to Robotics. Oussama Khatib. Stanford University

Rotation matrices

- Euler angles
  - Series of successive rotations
  - Axes orientation change
- Roll-Pitch-Yaw (RPY)
  - Fixed axes
- Quaternions
  - Euler parameters

Rotation matrices

- Individual rotations about x axis

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

- Analogously, rotation about y axis

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation matrices

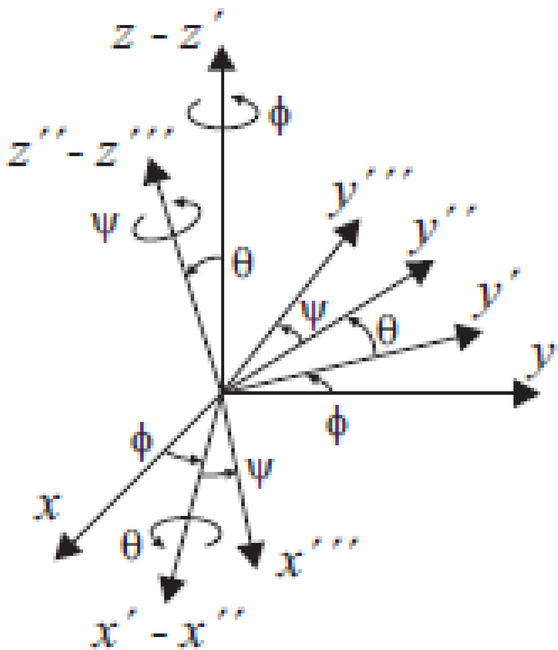
- A general rotation matrices can be expressed as three rotations around individual axes:

$$
\begin{aligned}
R &= R_z(\phi)R_y(\theta)R_x(\psi) \\
&= \begin{bmatrix} \cos\theta\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \cos\theta\sin\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi \\ -\sin\theta & \sin\psi\cos\theta & \cos\psi\cos\theta \end{bmatrix}
\end{aligned}
$$

Now we can calculate the individual rotations from R

Euler angles

- Convention x

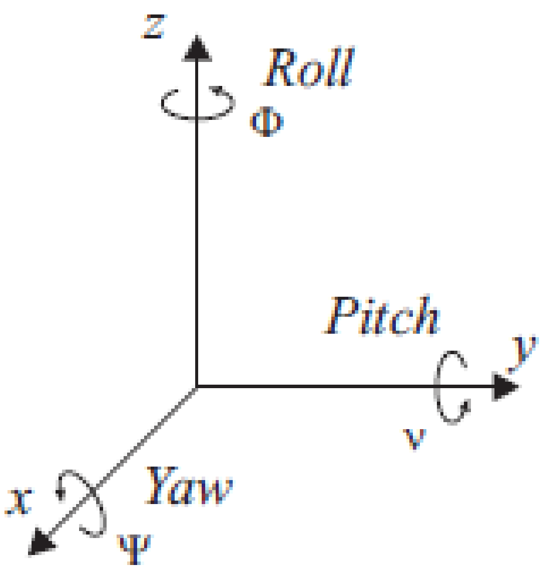


$$\mathbf{T}_{Euler}(\phi,\theta,\psi) = \mathbf{T}(z,\phi)\mathbf{T}(x,\theta)\mathbf{T}(z,\psi) =$$

$$\begin{bmatrix} c\phi & -s\phi & 0 & 0 \\ s\phi & c\phi & 0 & \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & -s\theta & \\ 0 & s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\psi & -s\psi & 0 & 0 \\ s\psi & c\psi & 0 & \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

s seno; c coseno

RPY

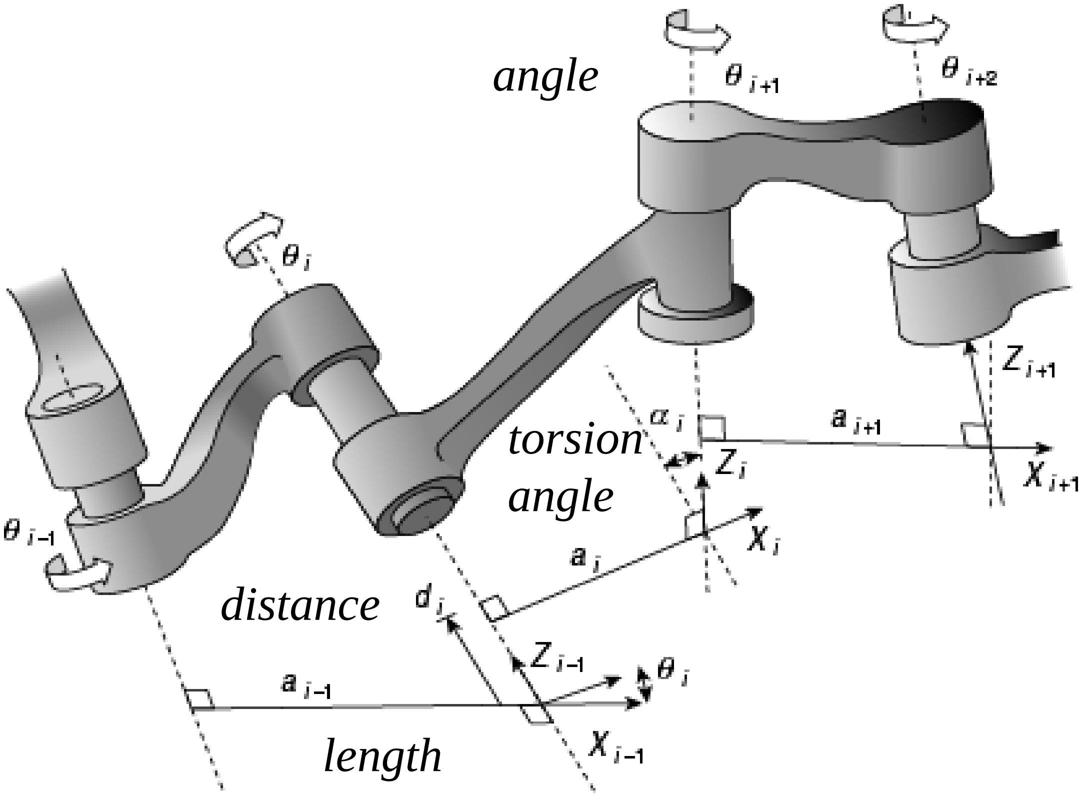


$$\mathbf{T}_{RPY}(\Psi, \nu, \Phi) = \mathbf{T}(z, \Psi)\mathbf{T}(y, \nu)\mathbf{T}(x, \Phi) =$$

$$\begin{bmatrix} c\Phi & -s\Phi & 0 & 0 \\ s\Phi & c\Phi & 0 & \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\nu & 0 & s\nu & 0 \\ 0 & 1 & 0 & \\ -s\nu & 0 & c\nu & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\Psi & -s\Psi & \\ 0 & s\Psi & c\Psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

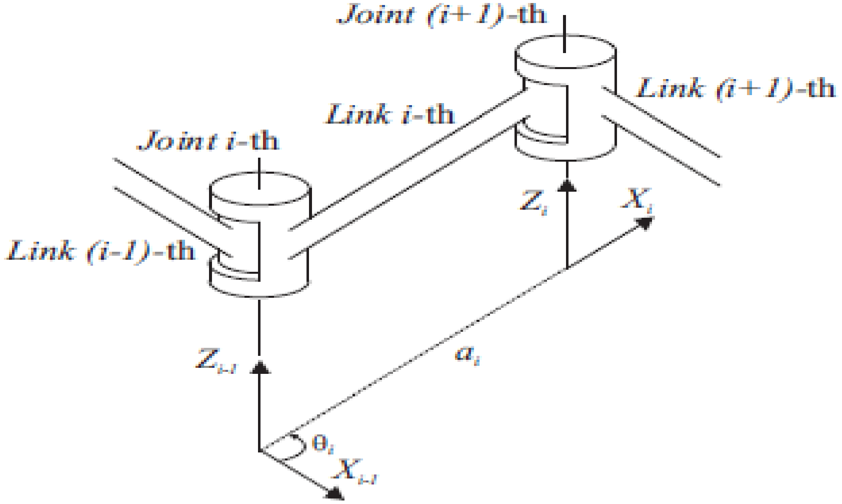
Kinematic description of a robot

- System joints and segments:
  - Joints with 1 d.o.f.
  - Joints with zero length.
- Description:
  - a_i, α_i structure
  - d_i, θ_i relative position



Denavit-Hartenberg (D-H)

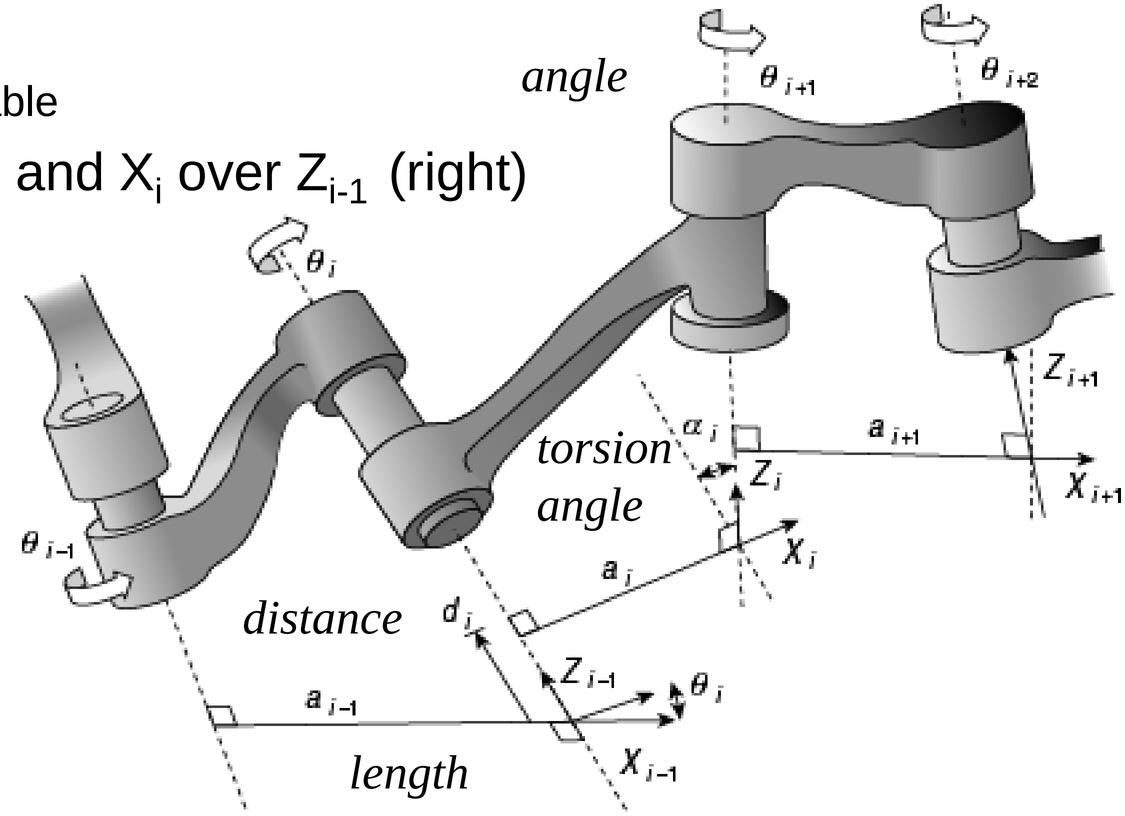
- Definition of the axes:
  - Base: system 0 (X_0 Y_0 Z_0)
  - Z_{i-1}, is the axis of the movement of joint i
  - X_i normal to Z_{i-1}
  - Y_i right-handed reference system



Denavit-Hartenberg (D-H)

- Definition of the parameters
  - a_i minimal distance (perpendicular) between Z_{i-1} and Z_i
  - α_i angle between Z_{i-1} e Z_i over X_i (right-handed)
  - d_i distance between the origin of system i-1 until intersection of Z_{i-1} with X_i (over Z_{i-1})
    - Prismatic joint = variable
  - θ_i angle between X_{i-1} and X_i over Z_{i-1} (right)
    - Rotative joint
- Table

	a_i	α_i	d_i	θ_i
i-1				
i				



Homogeneous Transformation Matrix

The product of 4 matrices:

Traslation (d_i) Rotation (θ_i) Traslation (a_i) Rotation (α_i)

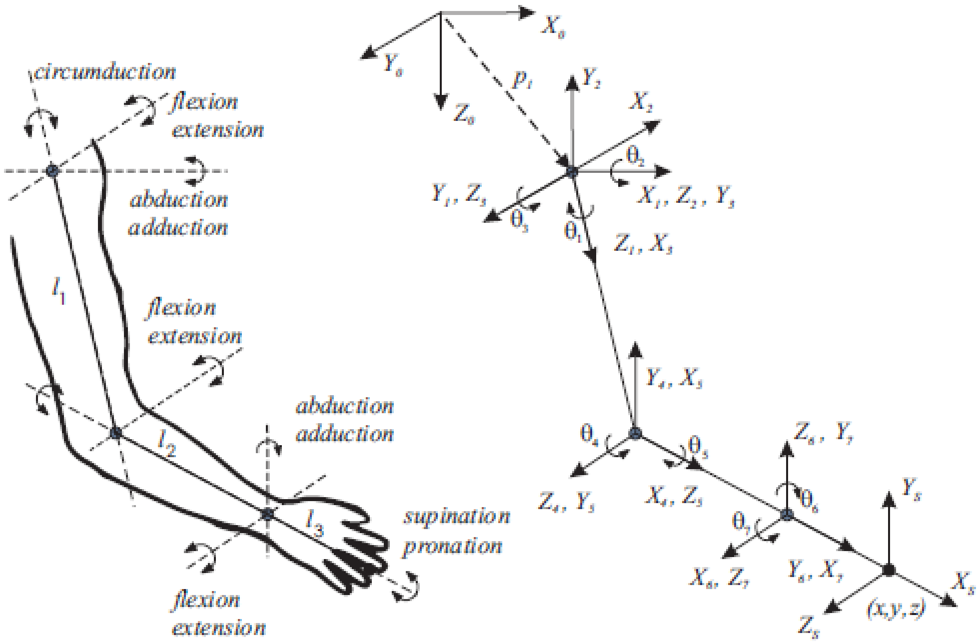
$$\mathbf{T}_{\mathbf{i-1}}^{\mathbf{i}} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composition of rotations from base to end-effector

$$\mathbf{T}_{\mathbf{0}}^{\mathbf{n}} = \mathbf{T}_{\mathbf{0}}^{\mathbf{1}}\mathbf{T}_{\mathbf{1}}^{\mathbf{2}}\cdots\mathbf{T}_{\mathbf{n-1}}^{\mathbf{n}}$$

Application example

- Obtain the D-H parameters of the human arm:



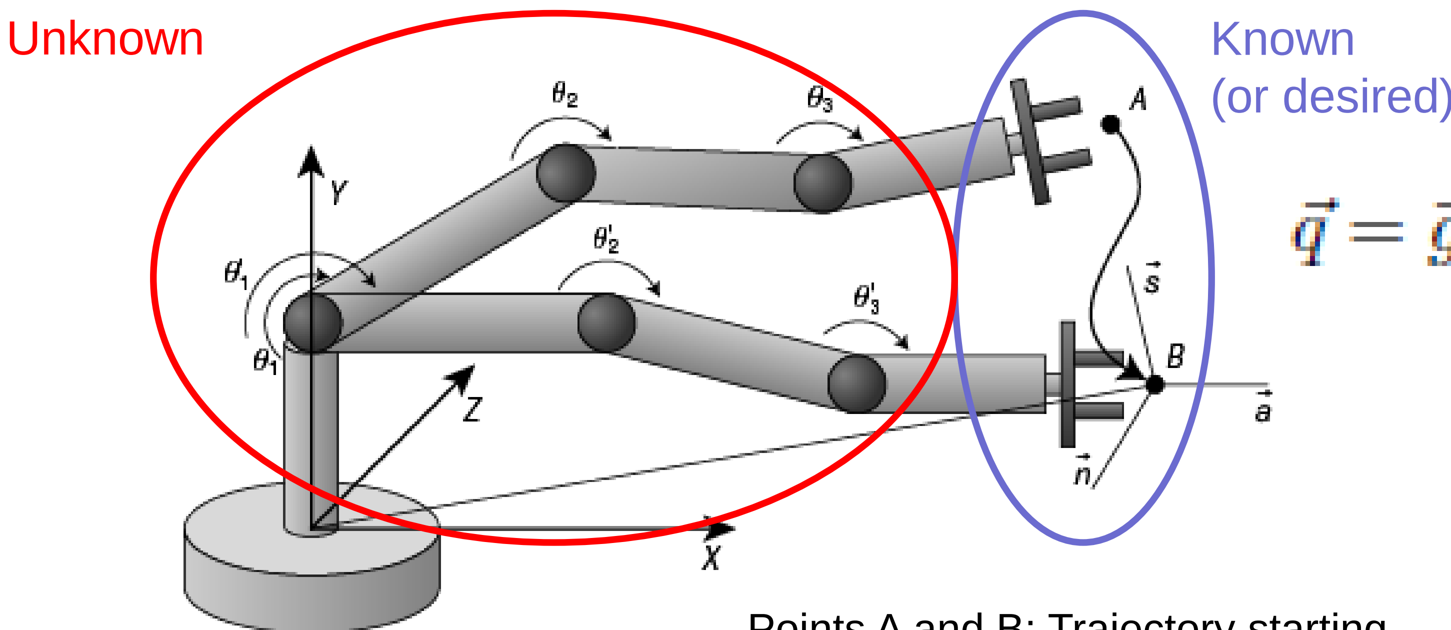
Chapter 3. Kinematics and dynamics of wearable robots. A. Forner–Cordero et al. In: Wearable Robots: Biomechatronic Exoskeletons. Editor: J. L. Pons (2008) John Wiley & Sons, Ltd.

Solução

Joint	β_i	No.	α_i	a_i	d_i	θ_i
base	0	$1_{(0 \longrightarrow 1)}$	0	a_0	d_0	0
shoulder	(-90) medial rot. / lateral rot. (+90)	$2_{(1 \longrightarrow 2)}$	-90^o	0	0	$\beta_1 + 90^o$
shoulder	(-180) abduction / adduction (+50)	$3_{(2 \longrightarrow 3)}$				
shoulder	(-180) flexion / extension(+80)	$4_{(3 \longrightarrow 4)}$				
elbow	(-10) extension / flexion (+145)	$5_{(4 \longrightarrow 5)}$				
elbow	(-90) pronation / supination (+90)	$6_{(5 \longrightarrow 6)}$				
wrist	(-90) flexion / extension (+70)	$7_{(6 \longrightarrow 7)}$				
wrist	(-15) abduction / adduction (+40)	$8_{(7 \longrightarrow 8)}$				

Inverse kinematics

- Obtain the robot joint angles from the orientation and position of the end-effector



$$\vec{q} = \vec{g}(\vec{r})$$

Points A and B: Trajectory starting and ending point, respectively

Inverse kinematics

- Solution procedures:
  - Geometric: for robots with low d.o.f. number
  - Analytic:
    - Given: $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$, position/orientation of end-effector solve the system of trigonometric equations:

      $T = T_0^n(q_1, q_2, \ldots, q_n)$ com $\mathbf{T_0^n} = \mathbf{T_0^1 T_1^2} \cdots \mathbf{T_{n-1}^n}$
    - Method of the inverse transformation (Paul, 1981)
  - Numerical (iterative):
    - Convergence to a possible solution
    - Real-time viability

Inverse kinematics

- Problem
  - Known the orientation of the end of the manipulator:

$$T = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}$$

  - Determine the Euler angles that represent this orientation

Jacobian matrix

- Relationship between effector speed and joint speeds is a multidimensional form of derivative

  $\dot{x} = J(q)\dot{q}$ can be expressed as: $\Delta \underline{x} = J \, \Delta \underline{\theta}$
- Singularities:
  - q configurations that reduce the rank of J
    - Unreachable points
    - Unreachable directions from setup
    - There is no solution to the inverse kinematic problem
    - Small speeds of the effector => high articular speeds

http://ijr.sagepub.com/content/3/4/66.abstract

Jacobian matrix

Given the joint angles θ_i and the velocity (x, y, z) and orientation (γ, β, α) of a certain point of the manipulator

$$J = \begin{bmatrix} \frac{\partial\theta_1}{\partial x} & \frac{\partial\theta_2}{\partial x} & \cdots & \frac{\partial\theta_n}{\partial x} \\ \frac{\partial\theta_1}{\partial y} & \frac{\partial\theta_2}{\partial y} & \cdots & \frac{\partial\theta_n}{\partial y} \\ \frac{\partial\theta_1}{\partial z} & \frac{\partial\theta_2}{\partial z} & \cdots & \frac{\partial\theta_n}{\partial z} \\ \frac{\partial\theta_1}{\partial \gamma} & \frac{\partial\theta_2}{\partial \gamma} & \cdots & \frac{\partial\theta_n}{\partial \gamma} \\ \frac{\partial\theta_1}{\partial \beta} & \frac{\partial\theta_2}{\partial \beta} & \cdots & \frac{\partial\theta_n}{\partial \beta} \\ \frac{\partial\theta_1}{\partial \alpha} & \frac{\partial\theta_2}{\partial \alpha} & \cdots & \frac{\partial\theta_n}{\partial \alpha} \end{bmatrix}$$

Matriz Jacobiana

- If there exists the inverse of J

$$\dot{q} = J(q)^{-1}\dot{x}$$

$$\ddot{q} = J(q)^{-1}(\ddot{x} - \left(\frac{d}{dt}J(q)\right)\dot{q})$$

- Inverse jacobian:

$$\ddot{x} = J(q)\ddot{q} + (\frac{d}{dt}J(q))\dot{q}$$

Jacobian Matrix

- If the inverse of J does not exist: Pseudoinverse

$$\dot{q} = J^{+}\dot{x} + (I - J^{+}J)\dot{z}$$

  - Term affecting the end-effector
  - Term of joint motions that do not affect the position/orientation of the effector
    - Redundancies (more d.o.f.)
    - Uncontrolled manifold

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  - **Direct and inverse dynamic problem**
  - **Newton-Euler formulation**
  - **Lagrange formulation**
- Case study: bio-robotics and bipeds

Dynamic modeling

– Direct dynamics:

- Evolution of joint angles as a function of forces and torques:

$$\ddot{\vec{r}} = f(F, T)$$

$$\dot{\vec{r}} = \int \ddot{\vec{r}} dt$$

$$\vec{r} = \int \dot{\vec{r}} dt$$

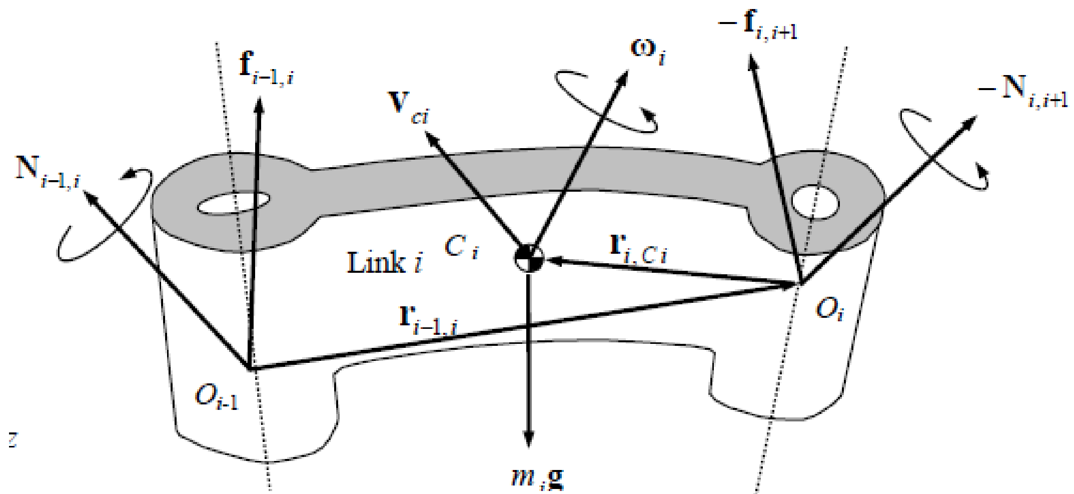
– Inverse dynamics:

- Determine forces and torques as a function of joint angles:

$$F = g\left(\vec{r}, \dot{\vec{r}}, \ddot{\vec{r}}\right)$$

Newton-Euler formulation

- Free-body
- Recursive



$$\mathbf{f}_{i-1,i} - \mathbf{f}_{i,i+1} + m_i\mathbf{g} - m_i\dot{\mathbf{v}}_{ci} = \mathbf{0}, \qquad i = 1, \cdots, n$$

$$\mathbf{N}_{i-1,i} - \mathbf{N}_{i,i+1} - (\mathbf{r}_{i-1,i} + \mathbf{r}_{i,Ci}) \times \mathbf{f}_{i-1,i} + (-\mathbf{r}_{i,Ci}) \times (-\mathbf{f}_{i,i+1}) - \mathbf{I}_i\dot{\boldsymbol{\omega}}_i - \boldsymbol{\omega}_i \times (\mathbf{I}_i\boldsymbol{\omega}_i) = \mathbf{0}, \quad i = 1, \cdots, n$$

$$\mathbf{I} = \begin{pmatrix} \int_{body}\{(y-y_c)^2+(z-z_c)^2\}\rho\, dV & -\int_{body}(x-x_c)(y-y_c)\rho\, dV & -\int_{body}(z-z_c)(x-x_c)\rho\, dV \\ -\int_{body}(x-x_c)(y-y_c)\rho\, dV & \int_{body}\{(z-z_c)^2+(x-x_c)^2\}\rho\, dV & -\int_{body}(y-y_c)(z-z_c)\rho\, dV \\ -\int_{body}(z-z_c)(x-x_c)\rho\, dV & -\int_{body}(y-y_c)(z-z_c)\rho\, dV & \int_{body}\{(x-x_c)^2+(y-y_c)^2\}\rho\, dV \end{pmatrix}$$

$$\tau_i = \sum_{j+1}^{n} H_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}\dot{q}_j\dot{q}_k + G_i, \qquad i = 1, \cdots, n$$

Asada H. Introduction to robotics. MIT Lecture Notes

Lagrange formulattion

- Multibody holonomous systems
  - Joint constraints $f(r_1, r_2, \cdots, r_n, t) = 0$
- Coordenates $\vec{q}$ and generalized forces τ_i
- Defines the Lagrangian
  - L= T-V, with T and V kinetic and potential energies
- It can be shown $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad \forall\ i = 1, \cdots, n$

Lagrange formulation

- For n segments in matrix form:

$$\mathbf{M}\left(\vec{q}\right)\ddot{\vec{q}} + \mathbf{C}\left(\vec{q}, \dot{\vec{q}}\right) + \mathbf{K}\left(\vec{q}\right) = \vec{\tau}$$

  - M mass matrix (square and invertible)
  - C Centrifugal and Coriolis forces vector
  - K gravity force vectors
- Improve the model:
  - Friction: $\mathbf{F}(\dot{\vec{q}}) = \mathbf{F_u}\dot{\vec{q}} + F_d$
  - Generalized perturbation forces: $\vec{\tau}_d$

Lagrange formulation

- We have:

$$\mathbf{M}(\vec{q})\ddot{\vec{q}} + \mathbf{C}(\vec{q},\dot{\vec{q}}) + \mathbf{F}(\dot{\vec{q}}) + \mathbf{K}(\vec{q}) + \vec{\tau}_d = \vec{\tau}$$

- That can be expressed as:

$$\mathbf{M}(\vec{q})\ddot{\vec{q}} + \mathbf{N}(\vec{q},\dot{\vec{q}}) + \vec{\tau}_d = \vec{\tau}$$

- Matrix N contains the non-linearities:

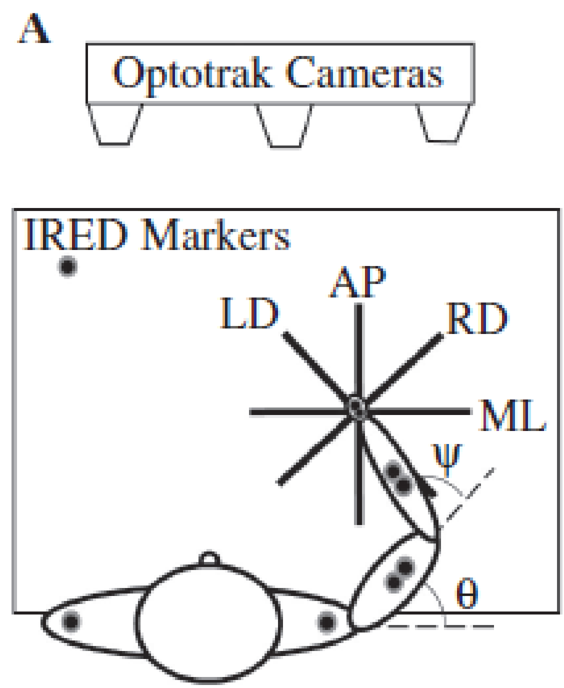
$$\mathbf{N}(\vec{q},\dot{\vec{q}}) = \mathbf{C}(\vec{q},\dot{\vec{q}}) + \mathbf{F}(\dot{\vec{q}}) + \mathbf{K}(\vec{q})$$

Example

Evidence for Adaptive Shoulder-Elbow Control in Cyclical Movements With Different Amplitudes, Frequencies, and Orientations

Oron Levin[1], Arturo Forner-Cordero[1,2], Yong Li[1], Mourad Ouamer[1], Stephan P. Swinnen[1]
[1]Department of Biomedical Kinesiology, Katholieke Universiteit Leuven, Leuven, Belgium. [2]Instituto de Automática Industrial, Madrid, Spain.

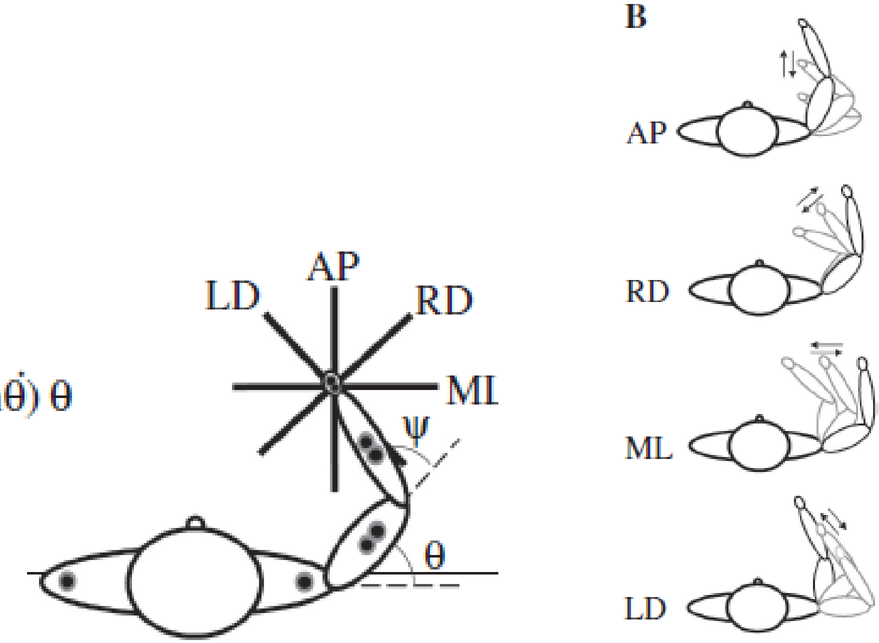
- Motor control study
- Star-drawing task



Example: star-drawing

Elbow (distal joint)

$$NT_{elbow} = (I_d + m_d r_d^2)\ddot{\psi}$$
$$IT_{elbow} = -(I_d + m_d r_d l_p \cos\theta)\ddot{\theta} - (m_d r_d l_p \sin\theta)\dot{\theta}$$
$$MT_{elbow} = NT_{elbow} - IT_{elbow}$$



Shoulder (proximal joint):

$$NT_{shoulder} = (I_p + m_p r_p^2)\ddot{\theta}$$
$$IT_{shoulder} = -(I_d + m_d[l_p^2 + r_d^2 + 2 l_p r_d \cos\theta])\ddot{\psi} - (I_d + m_d r_d l_p \cos\theta)\ddot{\theta} + (m_d r_d l_p \sin\theta)\dot{\theta} + (2 m_d r_d l_p \sin\theta)\dot{\theta}\dot{\psi}$$
$$MT_{shoulder} = NT_{shoulder} - IT_{shoulder}$$

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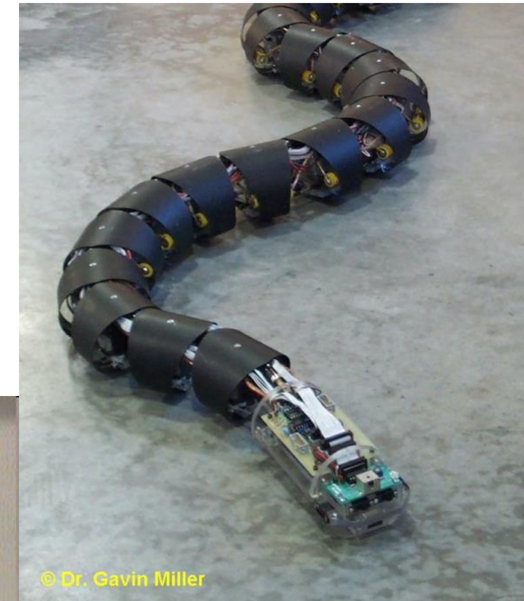
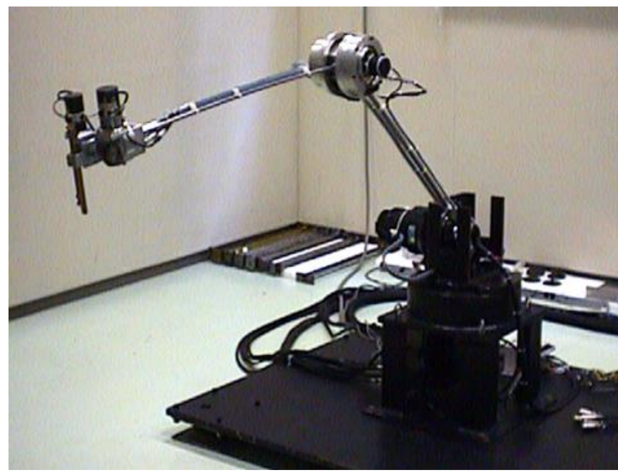
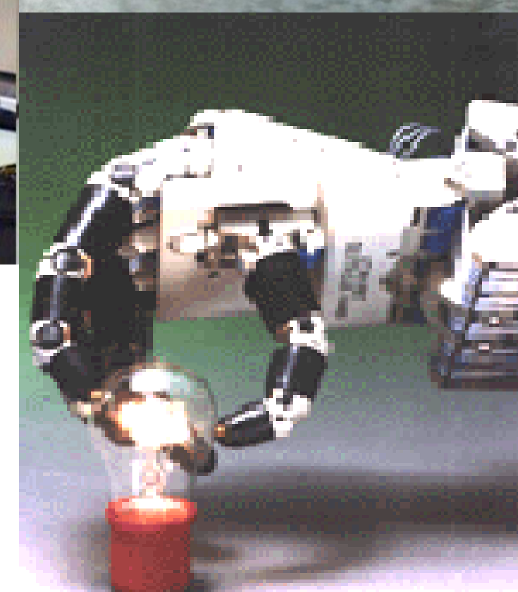
Manipulators biorobot structures

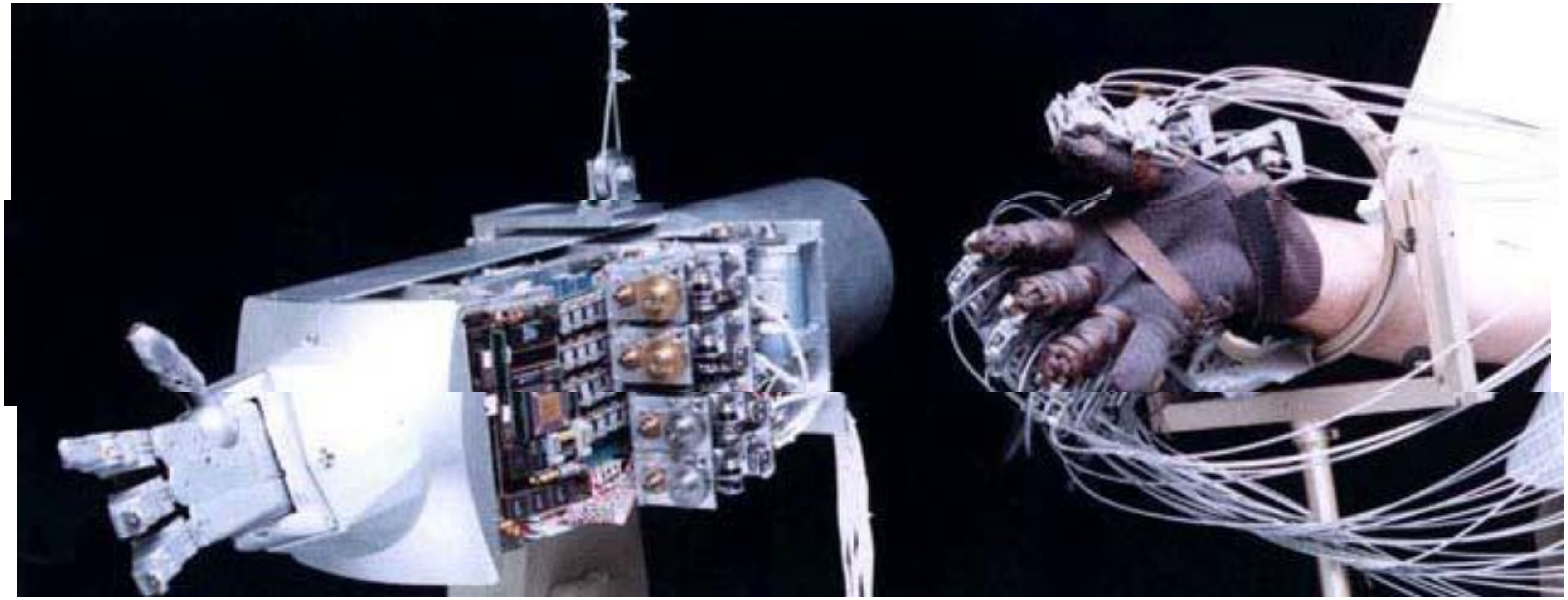
- Redundant robots:
  - Snake robot (Miller)
- Flexible robots:
  - Univ. Kyoto:
- Hands
  - Biomimetism
  - UTAH/MIT hand





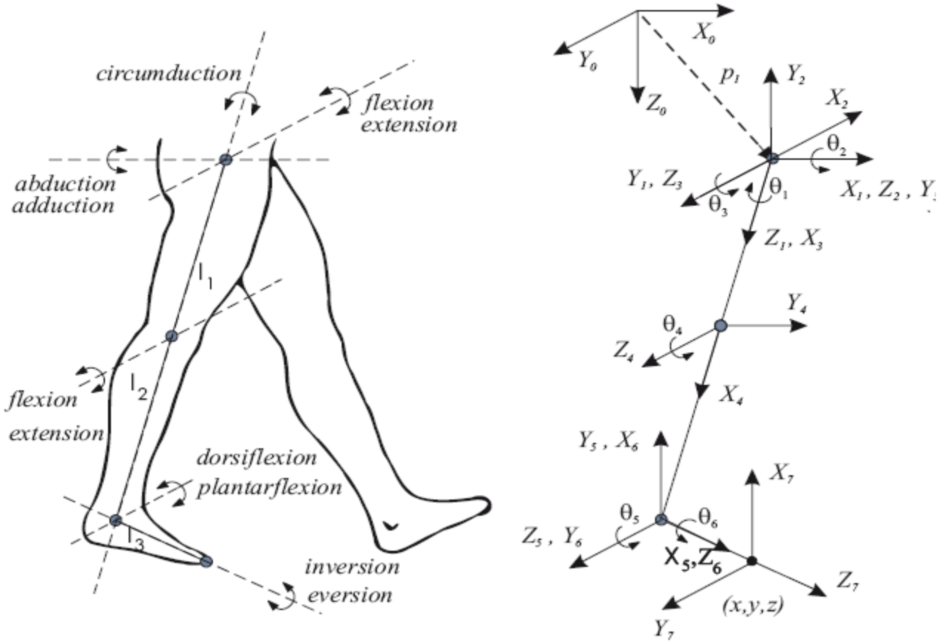


Human vs robotic hand



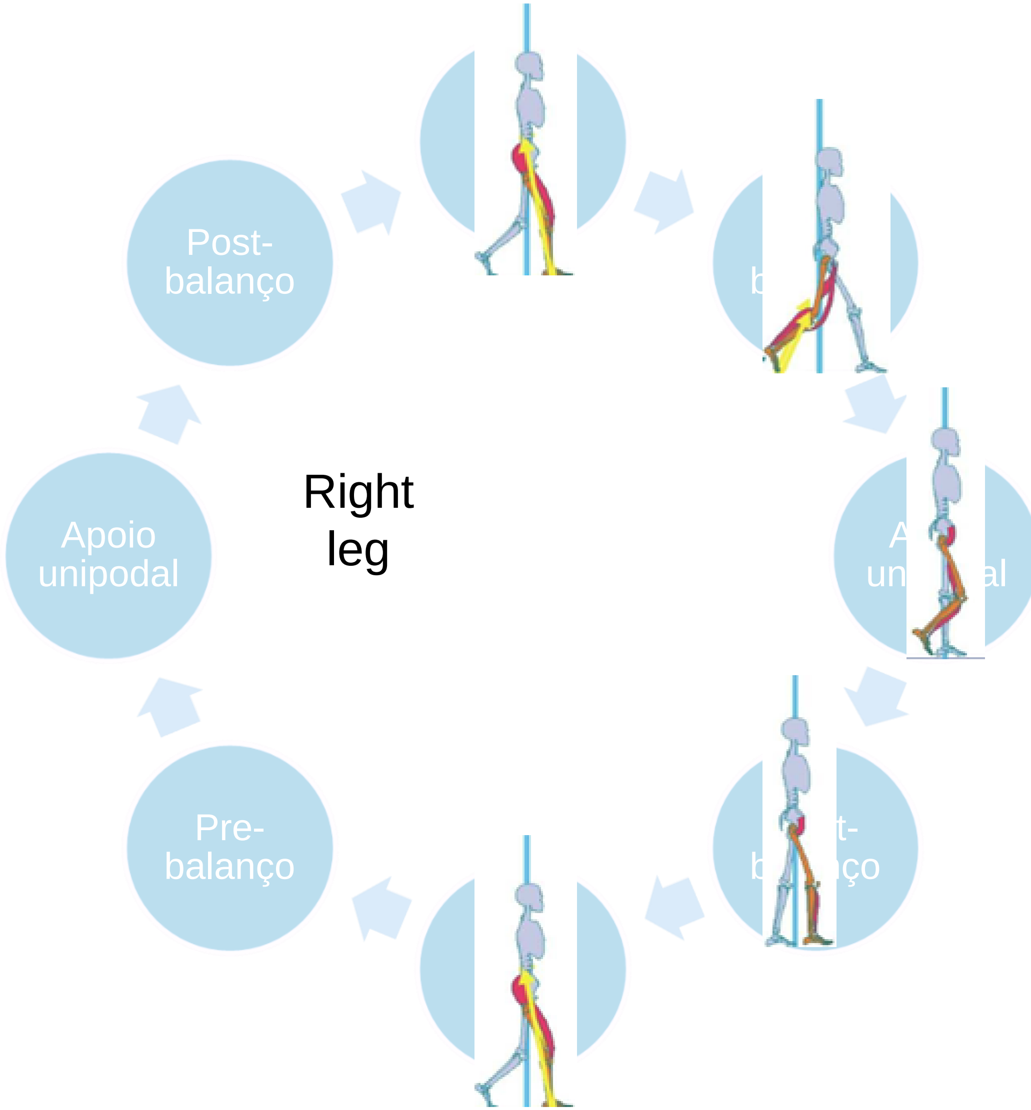
Biomechatronic mechanisms: Homework

- Calculate D-H parameters of a human leg

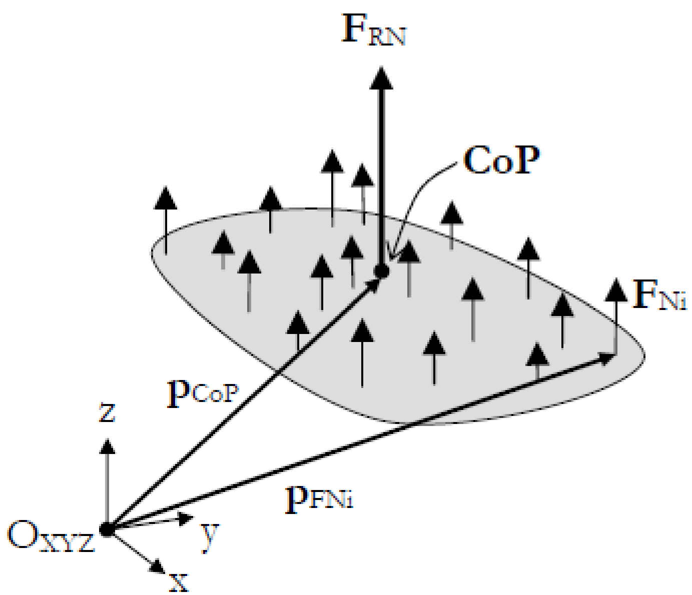


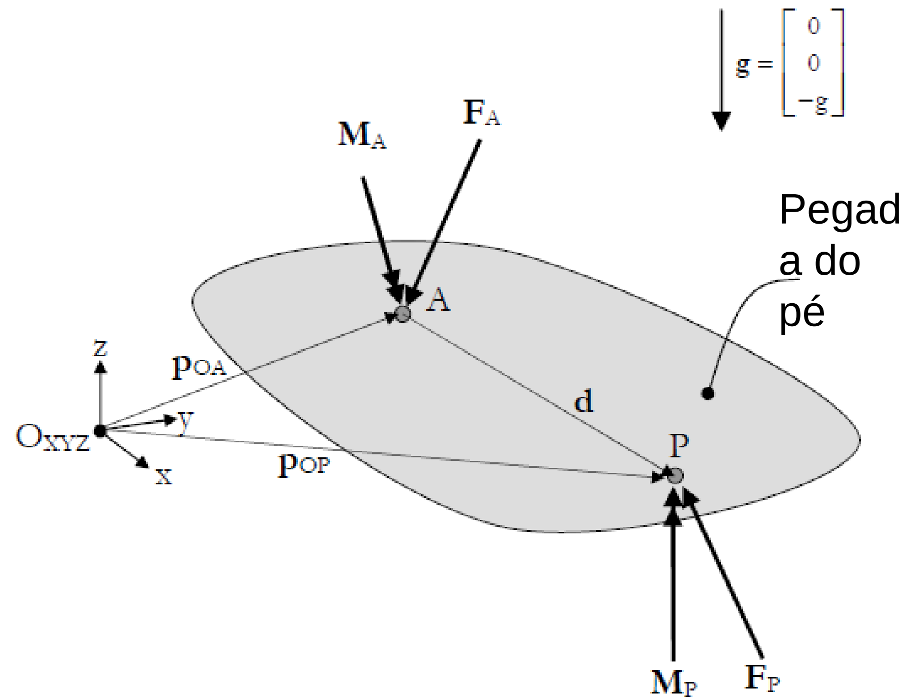
Chapter 3. Kinematics and dynamics of wearable robots. A. Forner–Cordero et al. In: Wearable Robots: Biomechatronic Exoskeletons. Editor: J. L. Pons (2008) John Wiley & Sons, Ltd.

Biped gait



Zero Moment Point





Resulting normal force under the foot (FRN) and ceter of pressure (CoP) under static conditions

CoP position:

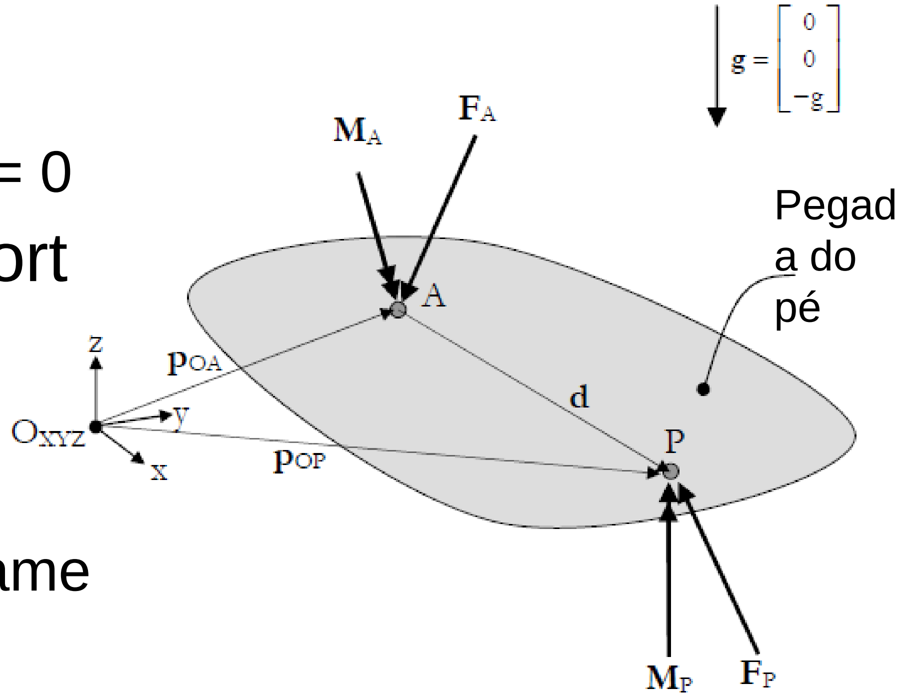
$$\overrightarrow{p_{CoP}} = \frac{\sum_{i=1}^{p} \overrightarrow{p_{FNi}} F_{Ni}}{F_{RN}}$$

Zero Moment Point

- $\mathbf{F}_A$, $\mathbf{M}_A$ biped forces are compensated by ground reaction forces: $\mathbf{F}_P$, $\mathbf{M}_P$

  $\mathbf{F}_P + \mathbf{F}_A = 0$

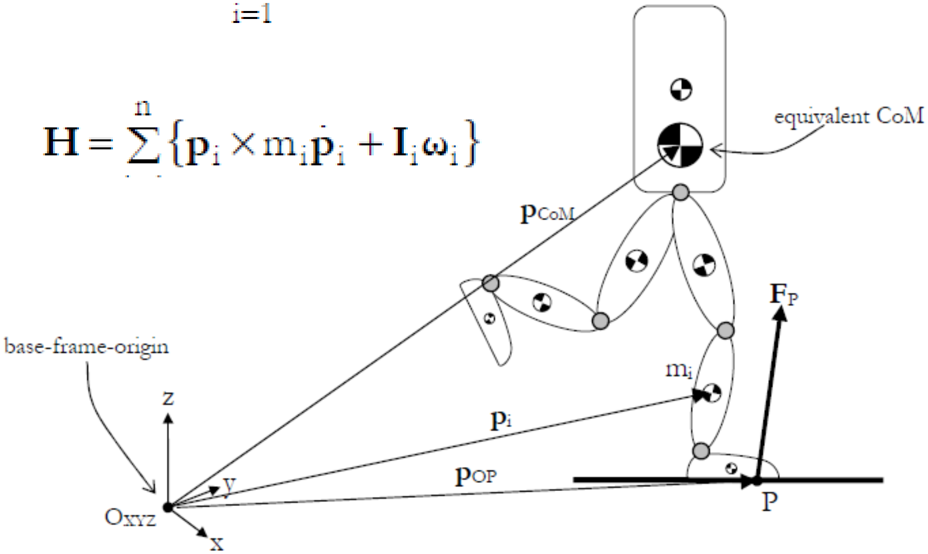
  $\mathbf{p}_{OP} \times \mathbf{F}_P + \mathbf{M}_A + \mathbf{p}_{OA} \times \mathbf{F}_A + \mathbf{M}_P = 0$
- Point P inside of the support polygon of the feet.
- Relation with the CoP:
  - Force balance:
    CoP and ZMP are on the same point
  - Necessary and sufficient stability criterion.



ZMP calculation

$$\mathbf{P} = \sum_{i=1}^{n} m_i \dot{\mathbf{p}}_i$$

$$\mathbf{H} = \sum_{i=1}^{n} \{\mathbf{p}_i \times m_i \dot{\mathbf{p}}_i + \mathbf{I}_i \boldsymbol{\omega}_i\}$$



$$\mathbf{F}_P = -\mathbf{F}_A = \dot{\mathbf{P}} - m_{tot}\mathbf{g}$$

$$\mathbf{M}_O = \dot{\mathbf{H}} - \mathbf{p} \times m_{tot}\mathbf{g}$$

$$\mathbf{M}_O = \mathbf{p}_{OP} \times \mathbf{F}_P + \mathbf{M}_P$$

$$\mathbf{M}_P = \dot{\mathbf{H}} - \mathbf{p}_{CoM} \times m_{tot}\mathbf{g} + (\dot{\mathbf{P}} - m_{tot}\mathbf{g}) \times \mathbf{p}_{OP}$$

$$x_{ZMP} = \frac{m_{tot} g_z p_{CoMx} + z_{ZMP} \dot{P}_x - \dot{H}_y}{m_{tot} g_z + \dot{P}_z}$$

$$y_{ZMP} = \frac{m_{tot} g_z p_{CoMy} + z_{ZMP} \dot{P}_y + \dot{H}_x}{m_{tot} g_z + \dot{P}_z}$$

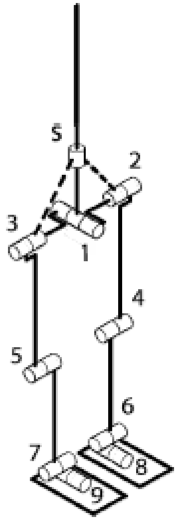
$z_{MP}=0$

$$x_{ZMP} = \frac{\sum_{i=1}^{n} \left(m_i (p_{ix}(\ddot{p}_{iz} + g_z) - p_{iz}(\ddot{p}_{ix} + g_x)) - I_{iy}\omega_{iy} \right)}{\sum_{i=1}^{n} m_i (\ddot{p}_{iz} + g_z)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n} \left(m_i (p_{iy}(\ddot{p}_{iz} + g_z) - p_{iz}(\ddot{p}_{iy} + g_y)) - I_{ix}\omega_{ix} \right)}{\sum_{i=1}^{n} m_i (\ddot{p}_{iz} + g_z)}$$

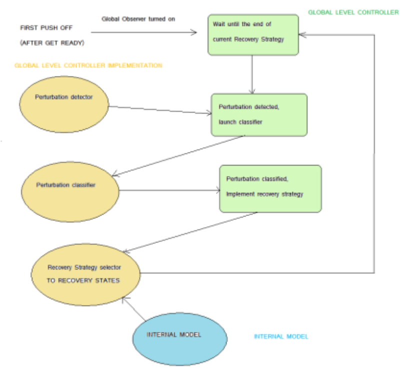
Aplications

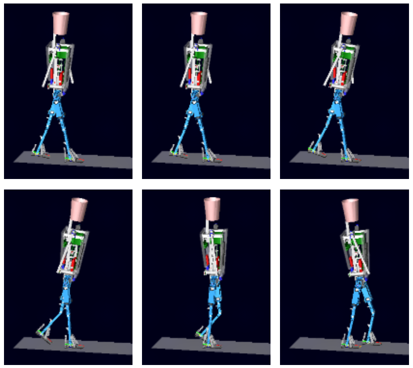
- Stability analysis of a biped robot:
  - Similar to a manipulator:
    - Supporting foot as base and swing foot as end-effector
  - Determine the points on the ground such that the ZMP will be inside of the support polygon.

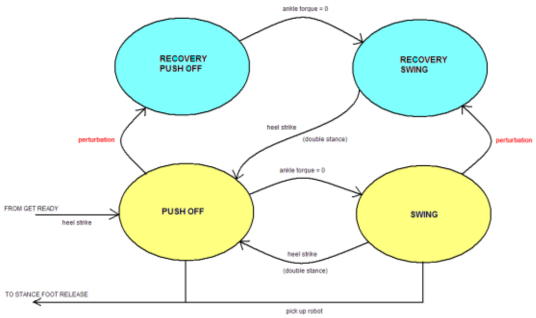


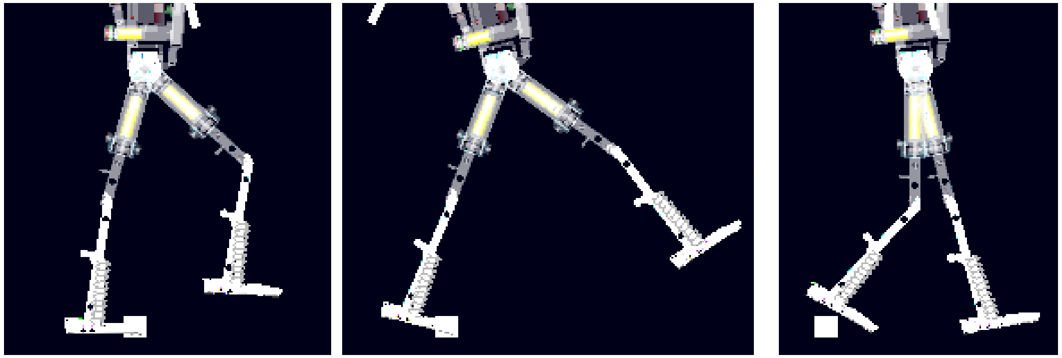
Simulation

ESBiRRoBot model with ADAMS and perturbation tests

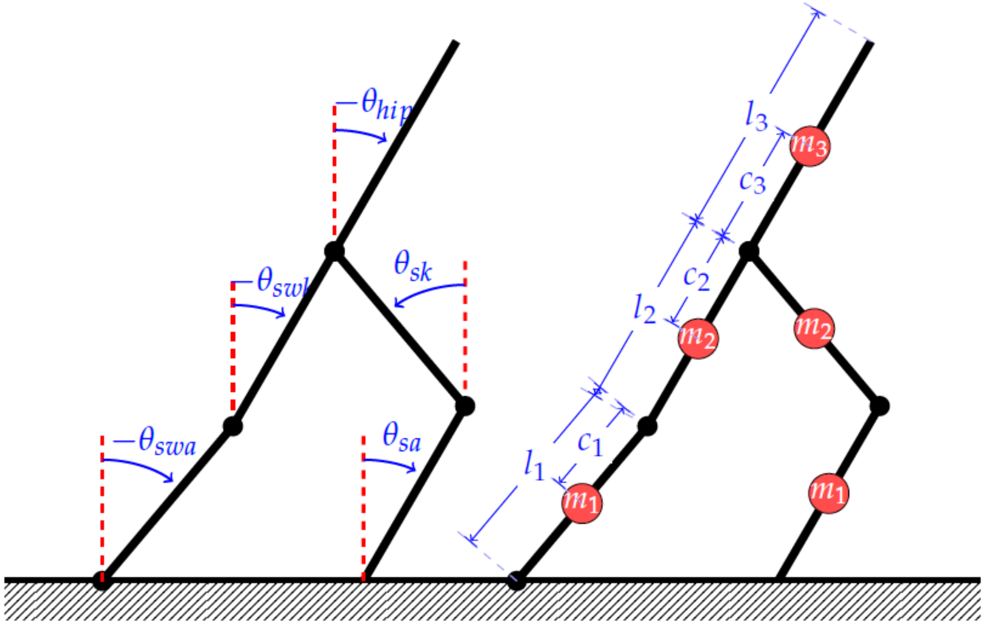








Five-segments model



$$\mathbf{M}(\theta)\ddot{\theta} + \mathbf{N}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta) = \mathbf{B}\mathbf{u}$$

$$\mathbf{M}(\theta) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{bmatrix}$$

Rossi, L, 2017, PhD Thesis

Five-segments model

$$
\begin{aligned}
M_{11} = & -I_1 - m_1(c_1\cos(\theta_1) - l_1\cos(\theta_1))^2 - m_1(c_1\sin(\theta_1) - l_1\sin(\theta_1))^2 \\
& - l_1 m_3 \sin(\theta_1)(-c_3\sin(\theta_3) + l_1\sin(\theta_1) + l_2\sin(\theta_2) + l_3\sin(\theta_3)) \\
& + l_1 m_5 \sin(\theta_1)(c_5\sin(\theta_5) - l_1\sin(\theta_1) - l_2\sin(\theta_2) + l_4\sin(\theta_4)) \\
& - l_1 m_2 \cos(\theta_1)(-c_2\cos(\theta_2) + l_1\cos(\theta_1) + l_2\cos(\theta_2)) \\
& - l_1 m_4 \cos(\theta_1)(-c_4\cos(\theta_4) + l_1\cos(\theta_1) + l_2\cos(\theta_2)) \\
& - l_1 m_2 \sin(\theta_1)(-c_2\sin(\theta_2) + l_1\sin(\theta_1) + l_2\sin(\theta_2)) \\
& - l_1 m_4 \sin(\theta_1)(-c_4\sin(\theta_4) + l_1\sin(\theta_1) + l_2\sin(\theta_2)) \\
& - l_1 m_3 \cos(\theta_1)(-c_3\cos(\theta_3) + l_1\cos(\theta_1) + l_2\cos(\theta_2) + l_3\cos(\theta_3)) \\
& + l_1 m_5 \cos(\theta_1)(c_5\cos(\theta_5) - l_1\cos(\theta_1) - l_2\cos(\theta_2) + l_4\cos(\theta_4))
\end{aligned}
$$

Rossi, L, 2017, PhD Thesis

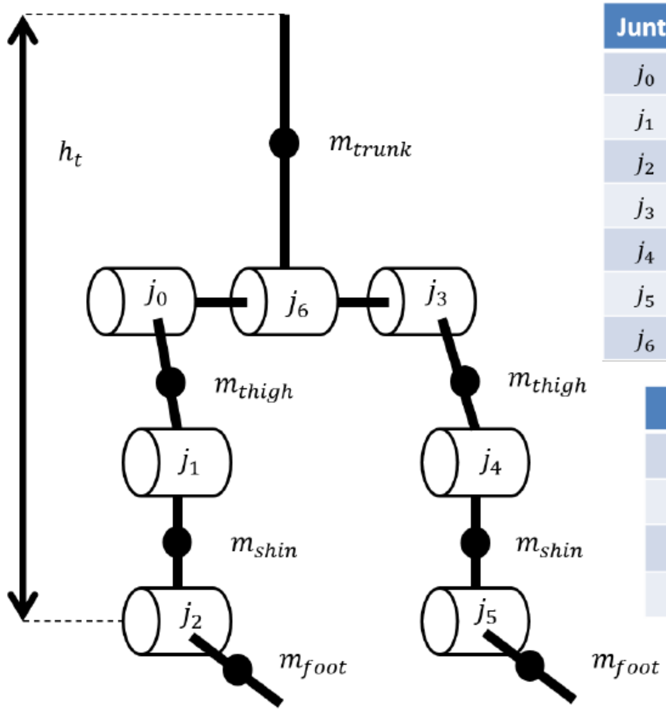
Five-segments model

$$\mathbf{f}(\theta, \dot{\theta}) = \mathbf{B}\mathbf{u} - (\mathbf{N}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta))$$

$$\mathbf{f}(\theta, \dot{\theta}) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}$$

Rossi, L, 2017, PhD Thesis

Model to analyze gait



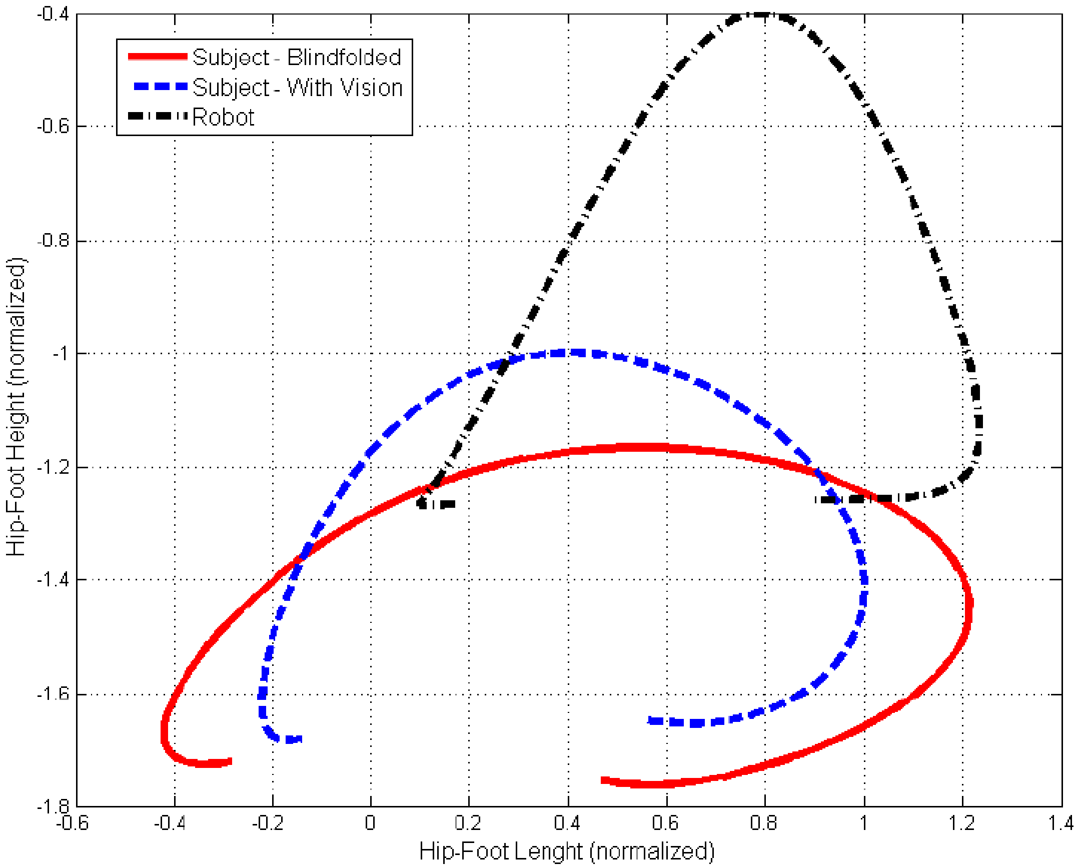
Junta	a_i	d_i	α_i	θ_i
j_0	0	0	$0.245h_t$	θ_0
j_1	0	0	$0.246h_t$	θ_1
j_2	0	0	$0.122h_t$	θ_2
j_3	0	0	$0.245h_t$	θ_3
j_4	0	0	$0.246h_t$	θ_4
j_5	0	0	$0.122h_t$	θ_5
j_6	0	0	$0.509h_t$	θ_6

m_i	*Value*
m_{thigh}	$0.1b_m$
m_{shin}	$0.0465b_m$
m_{foot}	$0.0145b_m$
m_{trunk}	$0.678b_m$

$b_m = Massa\ do\ Corpo$

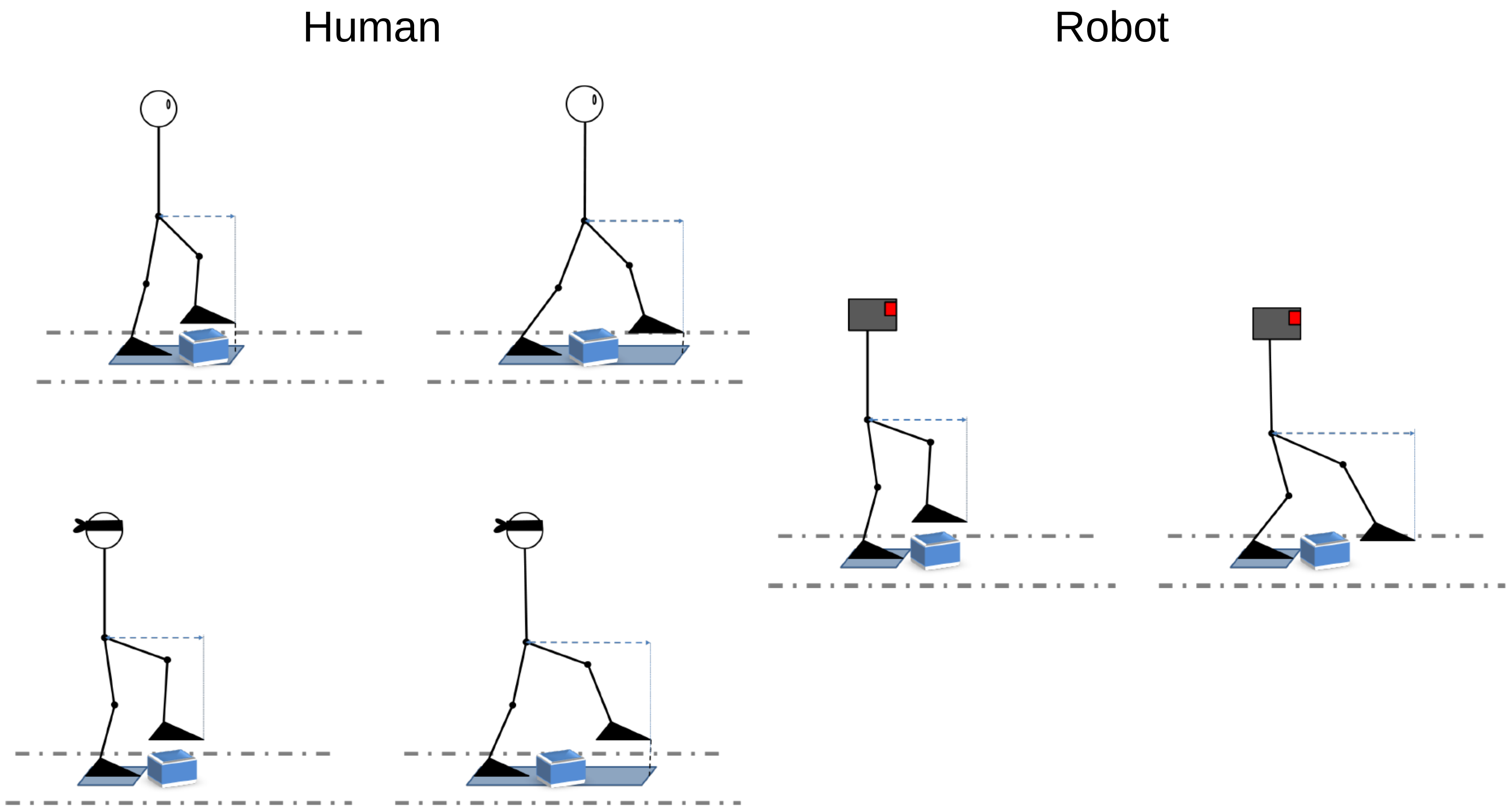
Rossi et al. BioRob 2014

Model to analyze gait



Rossi et al. BioRob 2014

Model to analyze gait



Rossi et al. BioRob 2014

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Take home message!

- Description of the kinematic model:
  - Analysis methodology
- Dynamic model
- Importance and usefulness:
  - Diagrama de Blocos
    - Explicar
- Inspiração biológica em robôtica

- Thanks
- See you next week

Solução D-H perna

Table 3.3 D-H parameters for leg segments.

Joint	β_i	No.	α_i	a_i	d_i	θ_i
base	0	$1_{(0 \longrightarrow 1)}$	0	a_0	d_0	0
hip	(-50) medial rot. /lateral rot.(+40)					
hip	(-20) abduction / adduction(+45)					
hip	(-30) extension / flexion(+120)					
knee	0 extension / flexion (+150)					
ankle	(-40) plantarflex. / dorsiflex. (+20)					
ankle	(-35) inversion / eversion (+20)					