AULA MECÂNICA

07/10

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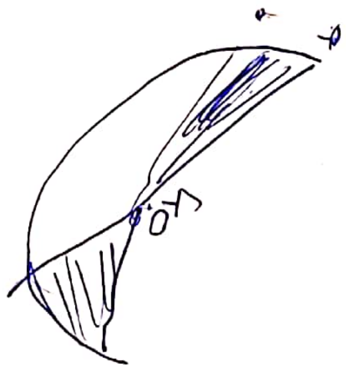
- FORÇAS CENTRAIS
- CONSERVAÇÃO DO MOMENTO ANGULAR.
- MOVIMENTO PLANAR EM SISTEMAS C/ FORÇAS CENTRAIS.
- LEI DE KEPLER SOBRE A VELOCIDADE DE PERCORRER ÁREA.

$r^2 \cdot \dot{\theta}$ É CONSTANTE.



SISTEMAS BIDIMENSIONAIS COM FORÇAS CENTRAIS

→ O SISTEMA É CONSERVATIVO.

$$\Rightarrow \exists\, U: \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$$

$$\ddot{X} = -\nabla U(X)$$

$$U(X) = \Phi(\|X\|)$$

$$\nabla U(x) = \Phi'(\|x\|) \cdot \frac{x}{\|x\|}$$

SEI QUE O MOMENTO ANGULAR É CONSTANTE

$\exists M$ TAL QUE $\boxed{r^2 \dot{\theta} = M}$.

$$\frac{\dot{x}^2}{2} + U(x) \equiv E$$ NUMA SOLUÇÃO.

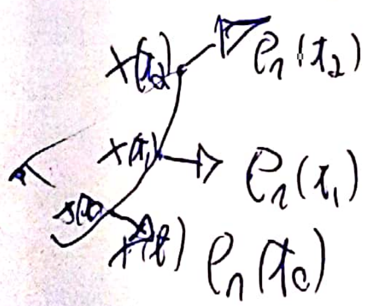
$$r^2 \dot{\theta} \equiv M$$

- ESTRATÉGIA! REDUZIR O PROBLEMA A DESCOBRIR A SOLUÇÃO DE UM PROBLEMA C/ 1 GRAU DE LIBERDADE P/ $r(t)$.

$$\ddot{x} = -\Phi'(\|x\|) \cdot e_r.$$

$e_r(t)$

$$x(t) = r(t) \cdot e_r(t).$$



$$e_r(t) = \frac{x(t)}{\|x(t)\|}$$

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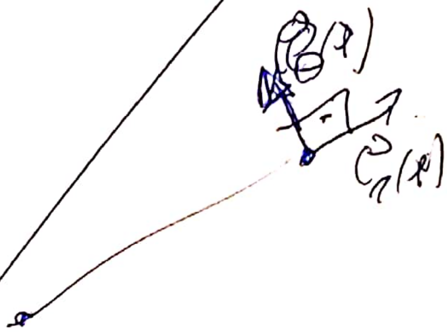
$$\dot{X}(t) = \dot{r}(t)\, e_r + r \cdot \dot{e}_r(t).$$

$$\dot{e}_r(t) = \boxed{\cancel{r}\,\dot{\theta} \cdot e_\theta}$$



$$e_\theta(t) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot e_r(t).$$

$$\dot{X} = \dot{r} e_r + r\dot{\theta} e_\theta$$

$$\cancel{\ddot{X} = \ddot{r} e_r + \dot{r}\dot{e}_r + \dot{r}\dot{\theta} e_\theta + r\ddot{\theta} e_\theta + r\dot{\theta}\dot{e}_\theta =}$$

$$\dot{e}_r = \cancel{r\dot{\theta}} \; \cancel{r}\,\dot{\theta}\, e_\theta$$

$$\dot{e}_\theta.$$

$$\ddot{r} = -\frac{d}{dr}\left(\Phi(r) + \frac{M^2}{2r^2}\right)$$

$$\boxed{\ddot{r} = -\frac{d}{dr}\left(\Phi(r) + \frac{M^2}{2r^2}\right)}$$

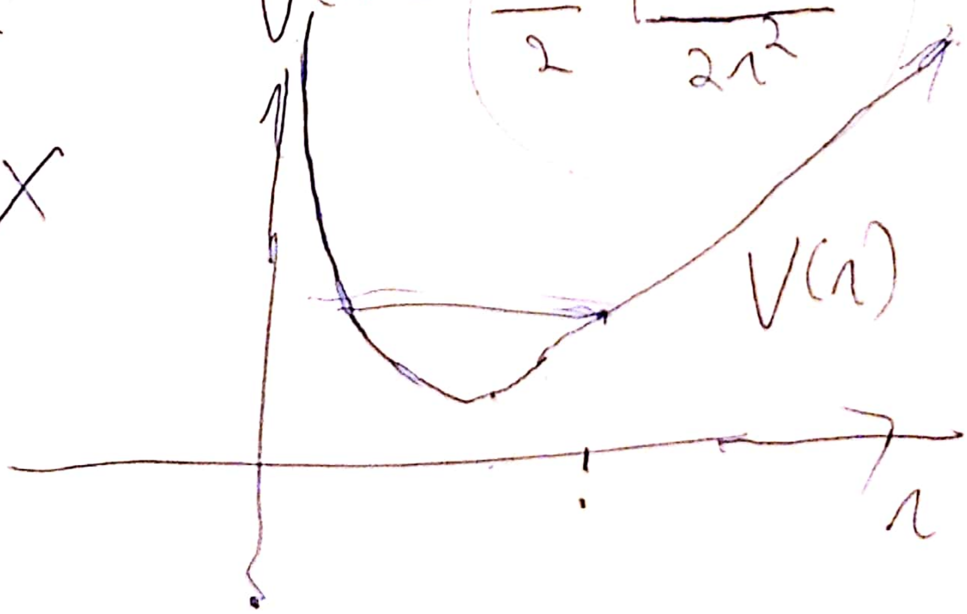
$$\Phi(r) = U(\|x\|)$$

$V(r)$ ENERGIA POTENCIAL EFETIVA.

$$V(r) = \Phi(r) + \frac{M^2}{2r^2}$$

ex: $\phi(r) = \frac{r^2}{2}$ $\qquad V(r) = \frac{r^2}{2} + \frac{2M^2}{2r^2}$

$$\ddot{X} = -X$$



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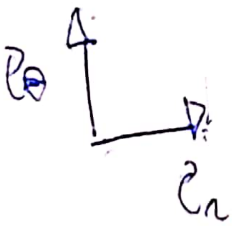
$$\dot X = \dot r e_r + r \cdot \dot e_r = \dot r e_r + r\dot\theta e_\theta$$

$$\ddot X = \ddot r e_r + \dot r \dot e_r + \dot r\dot\theta e_\theta + r\ddot\theta e_\theta + r\dot\theta\dot e_\theta$$

$$r\dot\theta\dot e_\theta = -r\dot\theta\dot\theta e_r$$

$$= \ddot r e_r + \dot r\dot\theta e_\theta + \dot r\dot\theta e_\theta + r\ddot\theta e_\theta - r\dot\theta\dot\theta e_r =$$

$$\boxed{\dot e_\theta = -\dot\theta e_r}$$

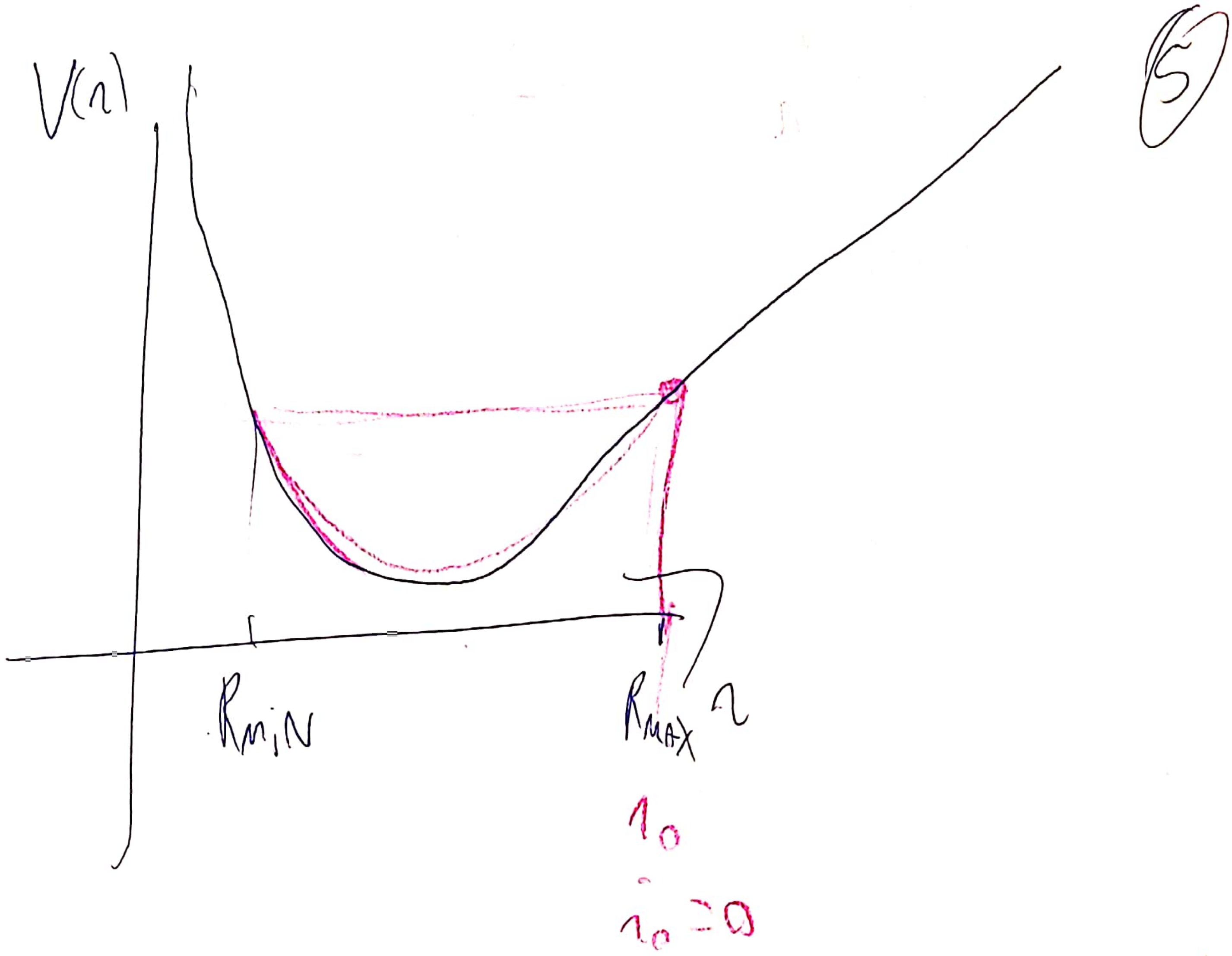


$$= \left(\ddot r - r\dot\theta^2\right) e_r + \boxed{\left(2\dot r\dot\theta + r\ddot\theta\right) e_\theta} = \ddot X$$

|| 0 pois $\ddot X = F(x)$ CENTRAL.

Logo $-\Phi'(r) = \ddot r - r\dot\theta^2$ ou

$$\ddot r = -\Phi'(r) + r \cdot \frac{M^2}{r^4} = -\Phi'(r) + \frac{M^2}{r^3}$$



LOGO r(t) NESTE EXEMPLO OSCILA PERIODICA/E ENTRE O VALOR R_{MAX} E O VALOR R_{min}.

POR OUTRO LADO.

$$\dot{\Theta} = \frac{M}{r^2}. \qquad \Theta(t) = \Theta(t_0) + \int_{t_0}^{t} \frac{M}{R^2(t)}\, dt$$

LOGO $\dot{\Theta}$ É UMA FUNÇÃO PERIÓDICA. E

$\Theta(t)$ É UMA FUNÇÃO LINEAR + UMA FUNÇÃO PERIÓDICA.

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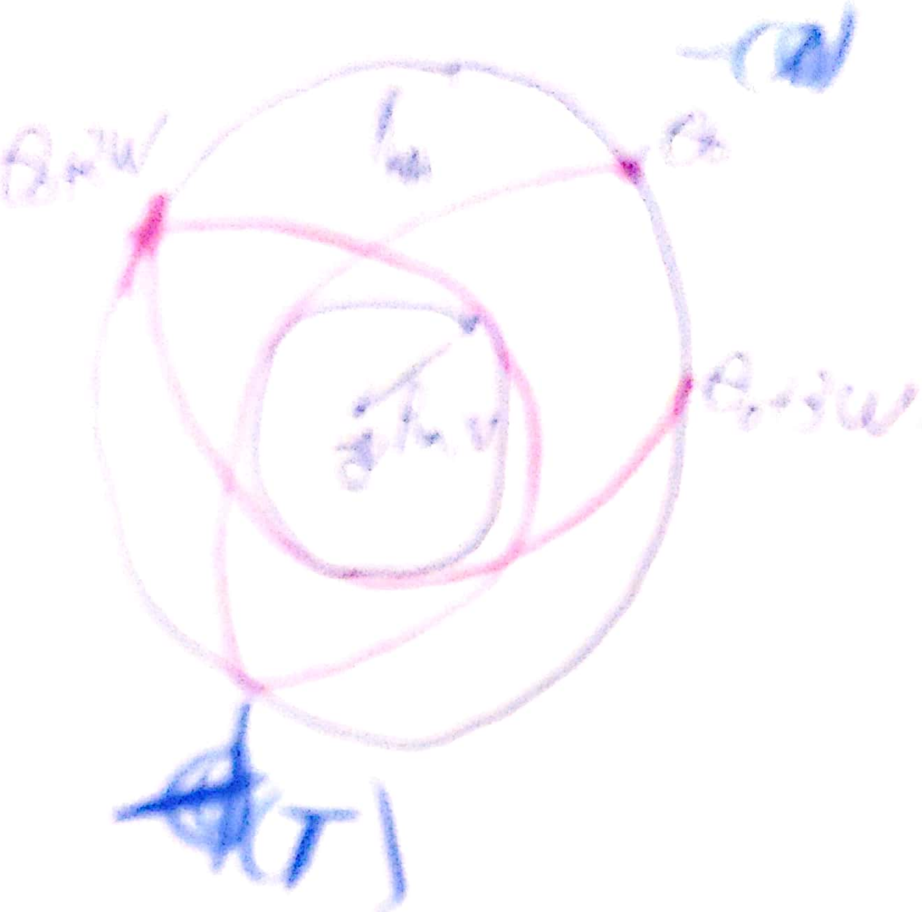


$\omega = \Theta(T) = \Theta(0)$

$\theta_0+\omega$

N_0, L naturales

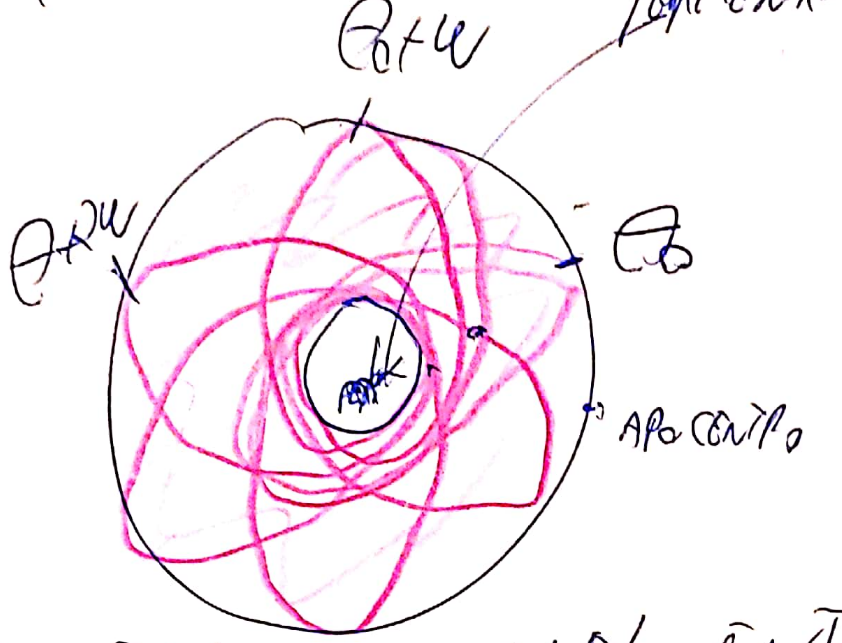
si $\frac{\omega}{\pi} \in \mathbb{Q} \Rightarrow \exists N_0 \in \mathbb{N}$,

$N_0\,\omega = L\cdot 2\pi$

$L \in \mathbb{N}$

SE $\frac{\omega}{2\pi} \notin \mathbb{Q}$

ENTÃO



ÓRBITA DENSA NO ANEL ENTRE R_{min} E R_{MAX}.

OBS:

SISTEMAS CONSERVATIVOS:

2 SISTEMAS

$1^o \quad \frac{\|\dot{X}\|^2}{2} + U(X) \qquad E = \frac{\|\dot{X}(t)\|^2}{2} + U(X).$

$2^o = \frac{\dot{r}^2}{2} + V(r)$

$\dot{X} = \dot{r}e_r + r\dot{\theta}e_\theta$

$\|\dot{X}\|^2 = \dot{r}^2 + r^2\dot{\theta}^2$

$\frac{\|\dot{x}\|^2}{2} = \frac{\dot{r}^2}{2} + \frac{r^2\dot{\theta}^2}{2} = \frac{\dot{r}^2}{2} + \frac{M^2}{2r^2}$

LOGO SE EM t_0

ENERGIA DO SISTEMA INICIAL.

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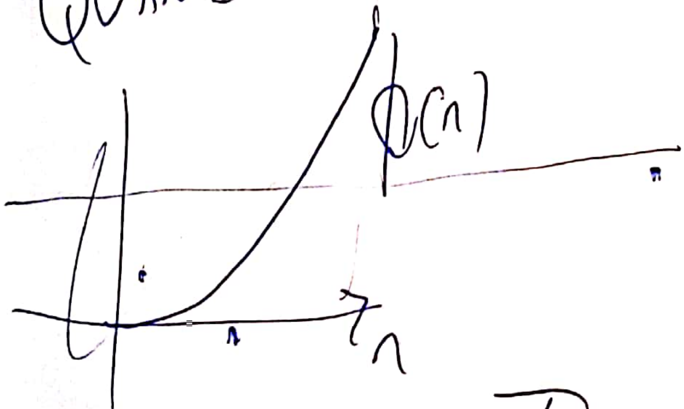
$$E_0 = \frac{\|\dot{X}(t_0)\|^2}{2} + U(X(t_0)) =$$

$$= \frac{\dot{r}^2(t_0)}{2} + \frac{\mu^2}{2r^2(t_0)} + \Phi(r(t_0)) =$$

ENERGIA DO SISTEMA DERIVADO

QUANDO UMA ÓRBITA PODE SER ILIMITADA?



SE $\lim_{r \to +\infty} \Phi(r) = +\infty$ ENTÃO TODAS AS ÓRBITAS SÃO LIMITADAS.

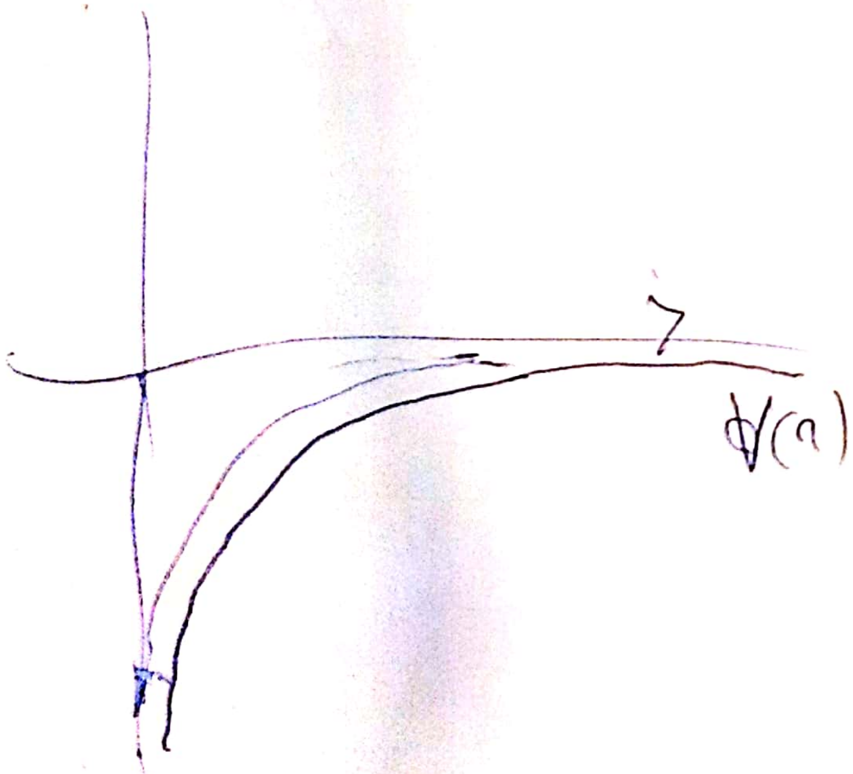
$$\lim_{r \to +\infty} \phi(r) = \lim_{r \to +\infty} V(r)$$

$$V(r) = \phi(r) + \frac{\ell^2}{2r^2}$$

SE $\lim_{r \to 0} \left(\phi(r)\, r^2\right) \to -\infty$ FOR NEGATIVO.

ENTÃO PODEMOS TER SOLUÇÕES QUE CHEGAM ATÉ A ORIGEM.

EX: $\phi(r) = -\frac{1}{r^3}$



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~~Ex:~~ CASO MAIS RELEVANTE É QUANDO

$$\boxed{\Phi(r) = -\frac{K}{r}} \qquad K>0$$

$$\ddot{x} = -\frac{K}{\|x\|^2} \cdot \frac{x}{\|x\|}$$

$$V(r) = -\frac{K}{r} + \frac{M^2}{2r^2}.$$

$$\lim_{r \to 0} V(r) = +\infty.$$

RAYN

$$\dot{\theta} = \frac{M}{r^2}$$

$$r^2 \dot{\theta} = M$$

$$\dot{\theta} = \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dr}\frac{dr}{dt}$$

$$\frac{d\theta}{dr} \cdot \dot{r} = \dot{\theta} = \frac{M}{r^2}^{\star}$$

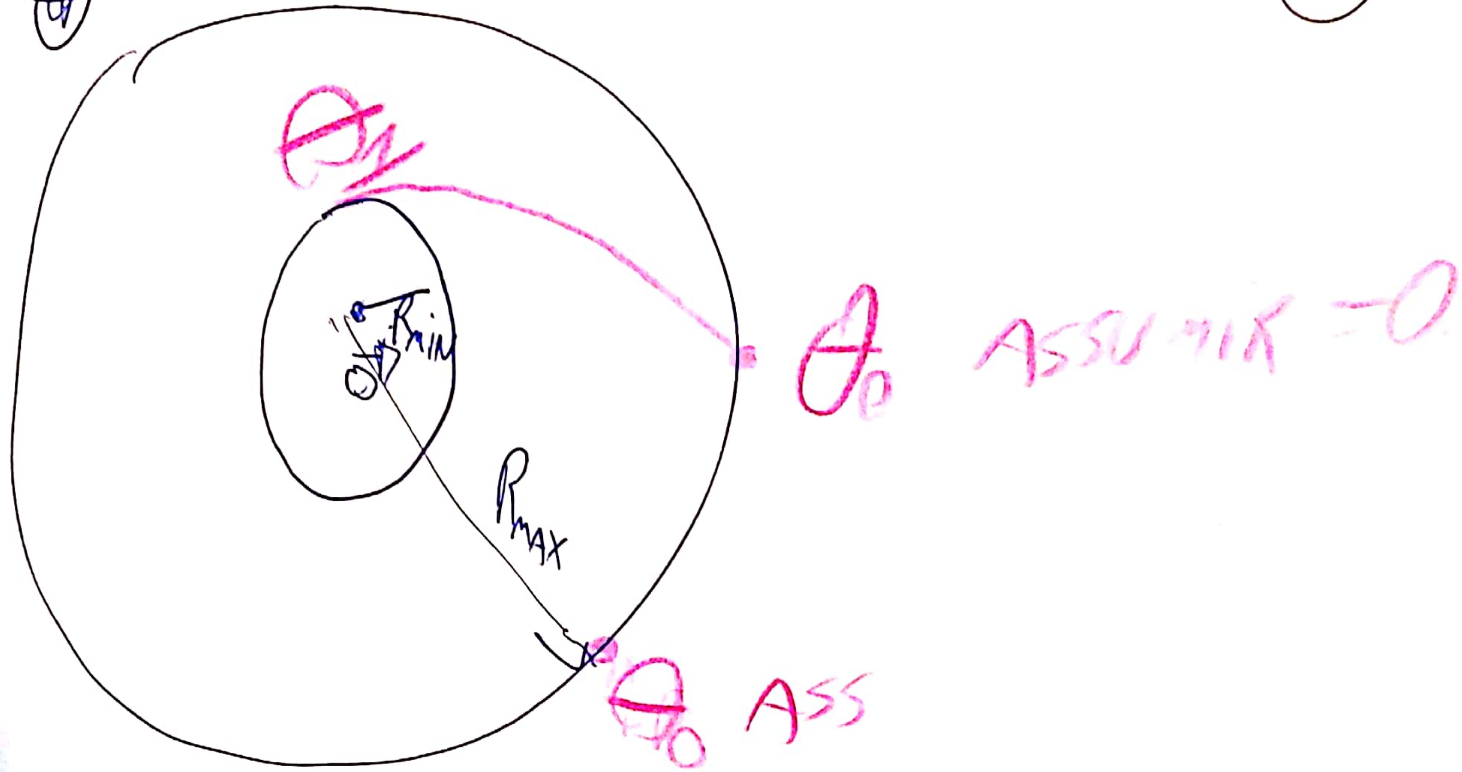
$$\frac{\dot{r}^2}{2} = E - V(r)$$

$$\dot{r} = \sqrt{2(E-V(r))} \quad \text{SE } \dot{r} > 0$$

$$\dot{r} = -\sqrt{2(E-V(r))} \quad \text{SE } \dot{r} < 0$$

$$\frac{d\theta}{dr} = \frac{M}{r^2} \cdot \frac{1}{\sqrt{2(E-V(r))}}$$

SE r ESTÁ VARIANDO DE r_{min} P/ r_{max}



$$\theta(r) = \cancel{\theta_0 + \int^{R_{MAX}} \frac{M}{r^2} \frac{1}{r\sqrt{2(E-V(r))}}}$$

$$= \theta_0 + \int_r^{R_{MAX}} \frac{M}{r^2}\left(\frac{1}{\sqrt{2(E-V(R))}}\right) dr =$$

$$\theta(r) = \operatorname{ARC\,COS} \frac{\frac{M}{R} - \frac{K}{M}}{\sqrt{2E - \frac{K^2}{M^2}}}$$