In each of Problems 1 through 7, find the inverse Laplace transform of the given function.-

1.
$$F(s) = \frac{3}{s^2 + 4}$$

2.
$$F(s) = \frac{4}{(s-1)^3}$$

3.
$$F(s) = \frac{2}{s^2 + 3s - 4}$$

4. $F(s) = \frac{2s + 2}{s^2 + 2s + 5}$

5.
$$F(s) = \frac{2s-3}{s^2-4}$$

6. $F(s) = \frac{8s^2 - 4s + 12}{s(s^2 + 4)}$

7.
$$F(s) = \frac{1-2s}{s^2+4s+5}$$

In each of Problems 8 through 16, use the Laplace transform to solve the given initial value problem.

8.
$$y'' - y' - 6y = 0;$$
 $y(0) = 1, y'(0) = -1$

9. y'' + 3y' + 2y = 0; y(0) = 1, y'(0) = 0

In each of Problems 1 through 8:

a. Find the solution of the given initial value problem.

G b. Plot a graph of the solution.

- **1.** $y'' + 2y' + 2y = \delta(t \pi); \quad y(0) = 1, \quad y'(0) = 0$
- **2.** $y'' + 4y = \delta(t \pi) \delta(t 2\pi); \quad y(0) = 0, \quad y'(0) = 0$
- **3.** $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t);$ y(0) = 0, y'(0) = 1/2

10. y'' - 2y' + 2y = 0; y(0) = 0, y'(0) = 1 **11.** y'' - 2y' + 4y = 0; y(0) = 2, y'(0) = 0 **12.** y'' + 2y' + 5y = 0; y(0) = 2, y'(0) = -1 **13.** $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0;$ y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1 **14.** $y^{(4)} - y = 0;$ y(0) = 1, y'(0) = 0, y''(0) = 1, y'''(0) = 0 **15.** $y'' + \omega^2 y = \cos(2t),$ $\omega^2 \neq 4;$ y(0) = 1, y'(0) = 0**16.** $y'' - 2y' + 2y = e^{-t};$ y(0) = 0, y'(0) = 1

In each of Problems 17 through 19, find the Laplace transform $Y(s) = \mathcal{L}{y}$ of the solution of the given initial value problem. A method of determining the inverse transform is developed in Section 6.3. You may wish to refer to Problems 16 through 18 in Section 6.1.

17.
$$y'' + 4y = \begin{cases} 1, & 0 \le t < \pi, \\ 0, & \pi \le t < \infty; \end{cases}$$
 $y(0) = 1, & y'(0) = 0$
18. $y'' + 4y = \begin{cases} t, & 0 \le t < 1, \\ 1, & 1 \le t < \infty; \end{cases}$ $y(0) = 0, & y'(0) = 0$
19. $y'' + y = \begin{cases} t, & 0 \le t < 1, \\ 2 - t, & 1 \le t < 2, \\ 0, & 2 \le t < \infty; \end{cases}$ $y(0) = 0, & y'(0) = 0$

4.
$$y'' + 2y' + 3y = \sin t + \delta(t - 3\pi);$$
 $y(0) = 0, y'(0) = 0$
5. $y'' + y = \delta(t - 2\pi) \cos t;$ $y(0) = 0, y'(0) = 1$
6. $y'' + 4y = 2\delta(t - \pi/4);$ $y(0) = 0, y'(0) = 0$
7. $y'' + 2y' + 2y = \cos t + \delta(t - \pi/2);$ $y(0) = 0, y'(0) = 0$
8. $y^{(4)} - y = \delta(t - 1);$ $y(0) = 0, y'(0) = 0,$
 $y''(0) = 0, y'''(0) = 0$