

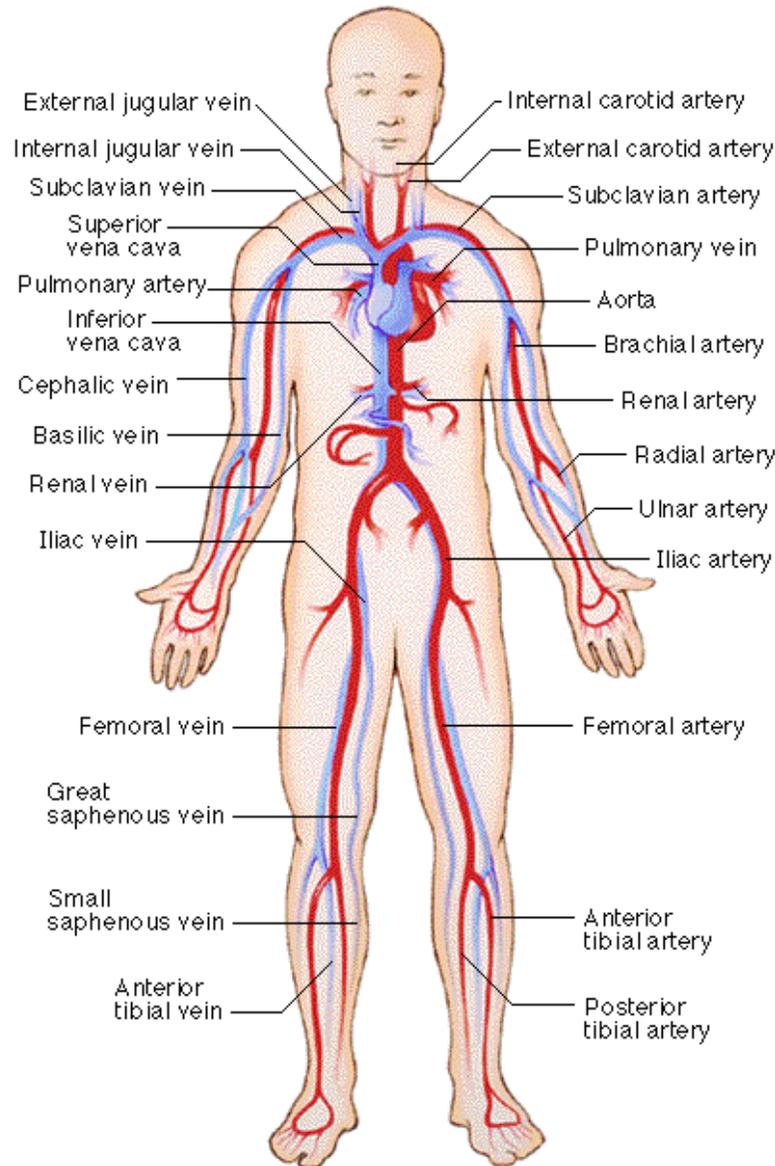
Escola Politécnica da  
Universidade de São Paulo



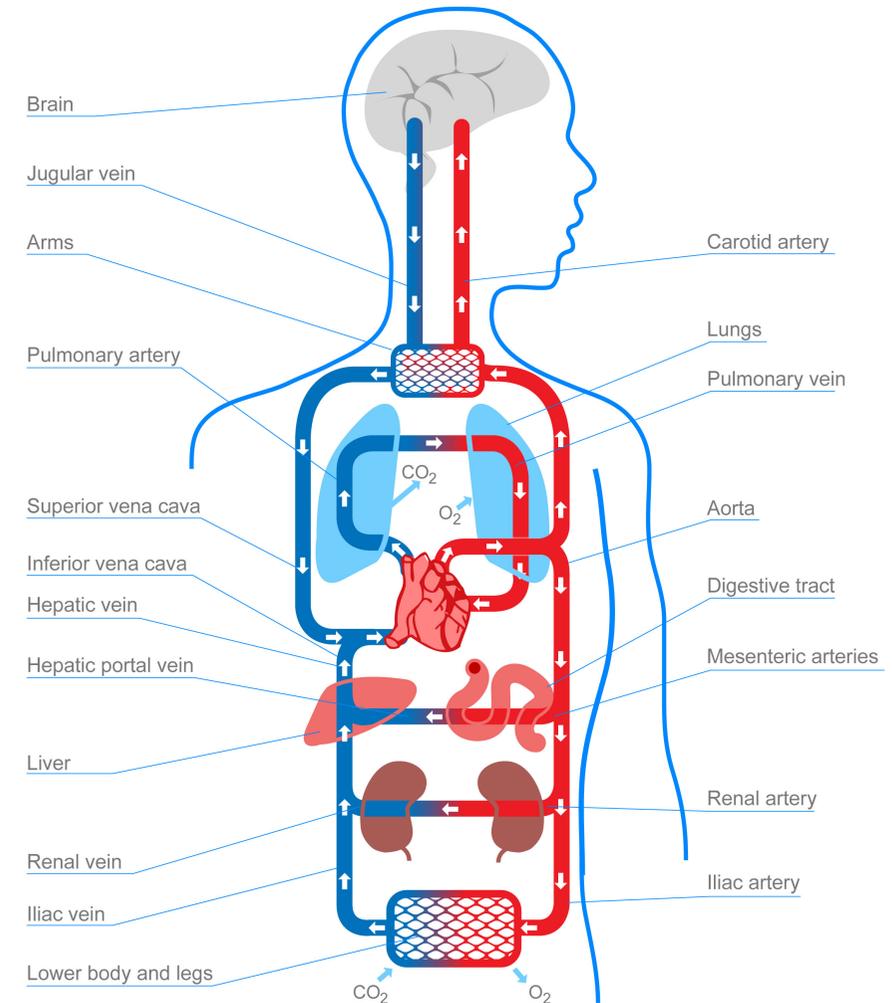
---

# Transferência de calor entre sangue e tecido

# Um pouco de anatomia



## THE CIRCULATORY SYSTEM



# Um pouco de anatomia



Aorta ( $D = 5.000\mu\text{m}$ )



Vasos que suprem os músculos (300 a  $1.000\mu\text{m}$ )  
Principais artérias e veias de suprimento (SAV)



Dividem-se

Artérias primárias (P, 100 a  $300\mu\text{m}$ )



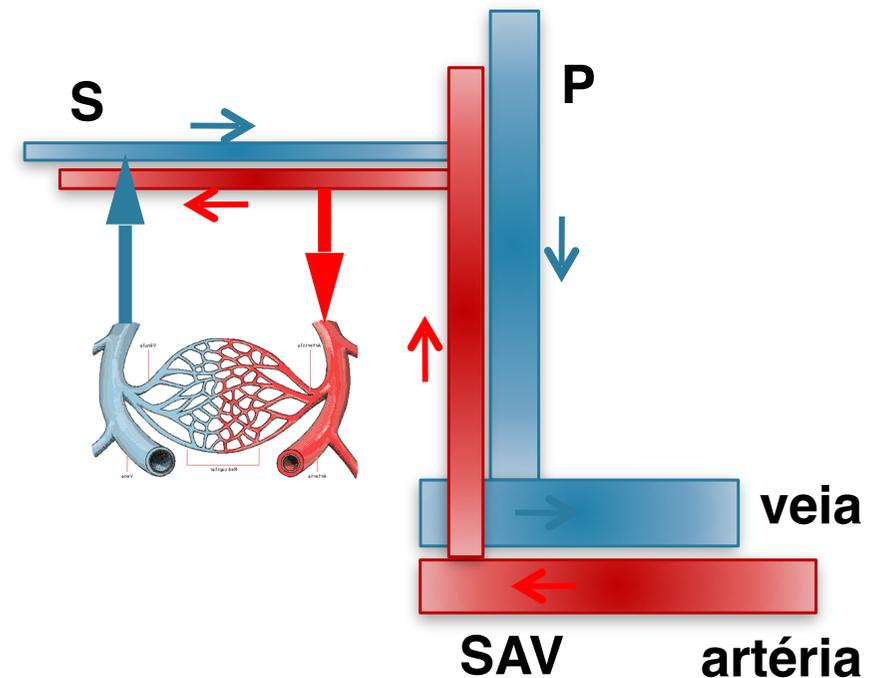
Artérias secundárias (S, 50 a  $100\mu\text{m}$ )



Arteriolas (20 a  $40\mu\text{m}$ )



Capilares (5 a  $15\mu\text{m}$ )





$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T + w_b \cdot \rho_b \cdot c_b \cdot [T_{ar} - T] + q''$$

## Hipóteses:

- *Região de equilíbrio* – a transferência de calor entre o sangue e o tecido adjacente ocorre principalmente nos leitos capilares;
- *Perfusão sanguínea* – o escoamento de sangue nos capilares é assumido isotrópico;
- *Arquitetura vascular* – os grandes vasos na vizinhança dos leitos capilares não são importantes na transferência de calor;
- *Temperatura do sangue* – o sangue chega no leito capilar na temperatura do núcleo do corpo e instantaneamente equilibra com a temperatura local do tecido.

## Aplicabilidade:

- *Região com vasos com  $D > 500\mu\text{m}$  e  $L_e/L > 0,3$ .*



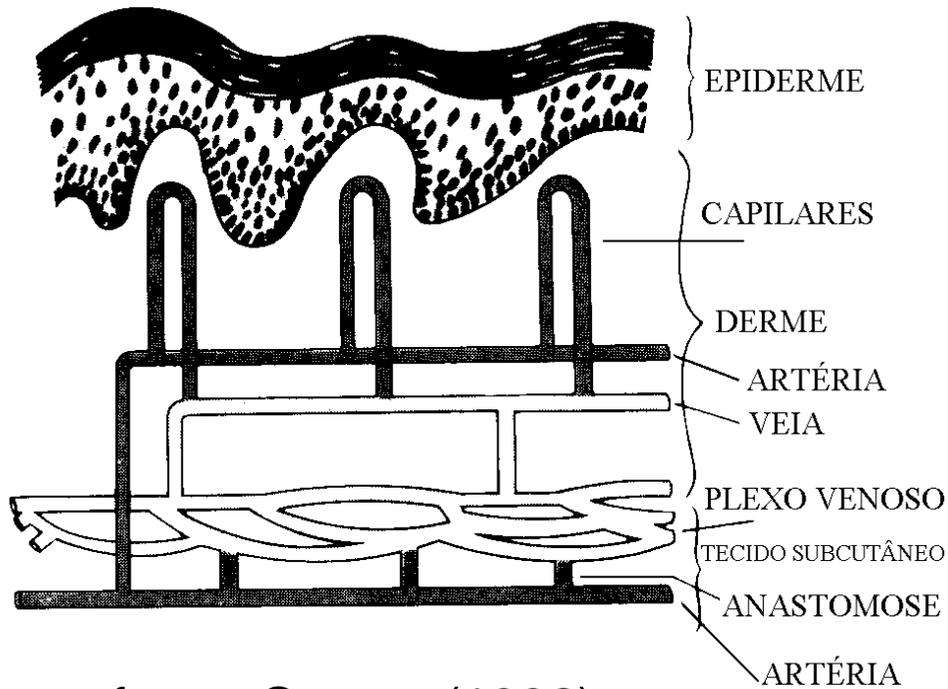
## Críticas:

- *Região de equilíbrio* – o equilíbrio térmico ocorre nas pré-arteríolas e nas pós-vênulas (70 a 500 $\mu$ m) e não nos leitos capilares;
- *Perfusão sanguínea* – a direção da perfusão sanguínea é importante para a transferência de calor. A perfusão nos capilares não é isotrópica, pois procede das arteríolas para as veias;
- *Arquitetura vascular* – a geometria vascular não é considerada. Fatores como a transferência de calor com os grandes vasos e transferência de calor contracorrente não são considerados;
- *Temperatura do sangue* – a temperatura do sangue arterial varia continuamente de seu valor no interior do corpo (aorta) a medida que percorre os vasos. A temperatura do sangue arterial não é a interna e a do que deixa o tecido (venosa) não é igual a do tecido. Essa hipótese superestima a transferência de calor.

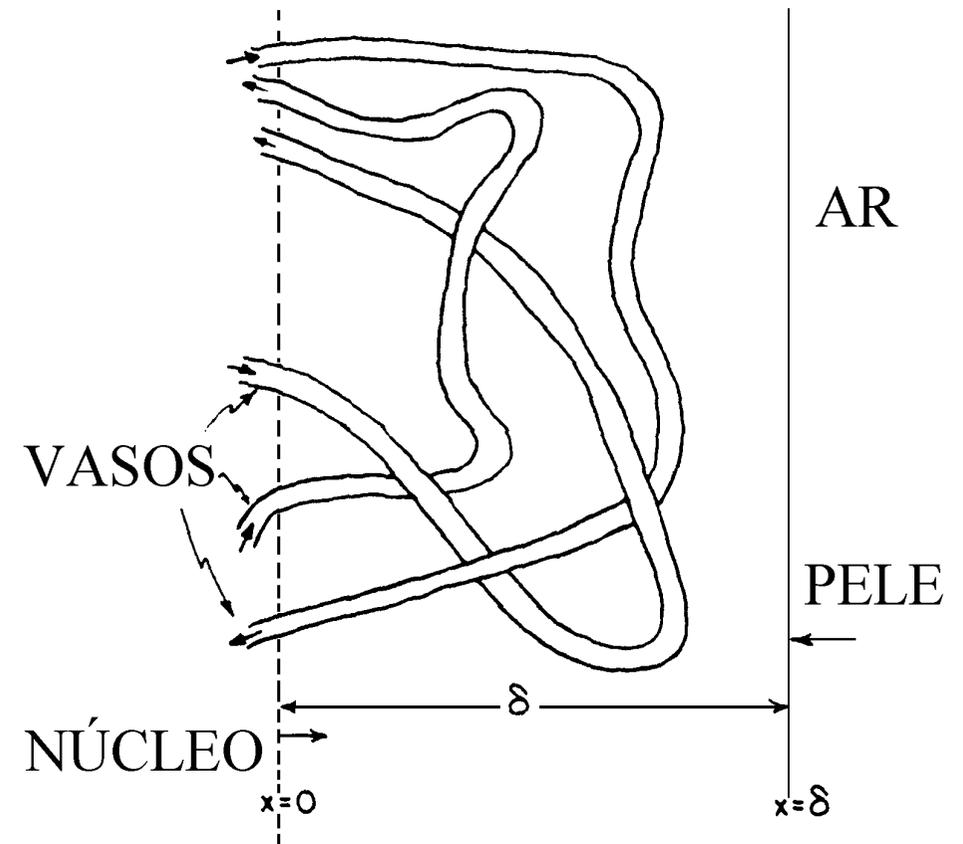
# Modelo de Keller; Seiler (1971):



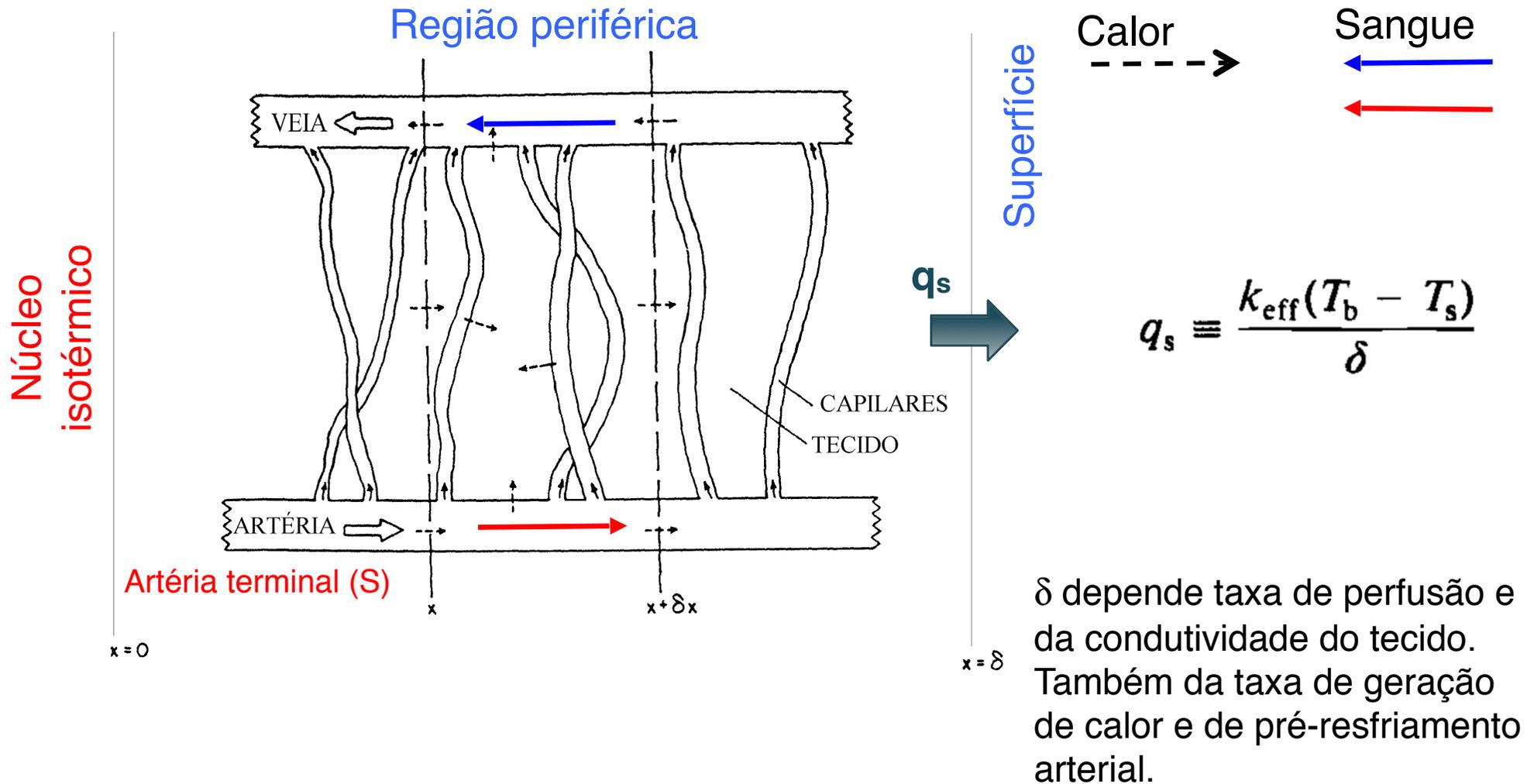
## Circulação cutânea



fonte: Guyton (1992)



# Modelo de Keller; Seiler (1971):



# Modelo de Keller; Seiler (1971):

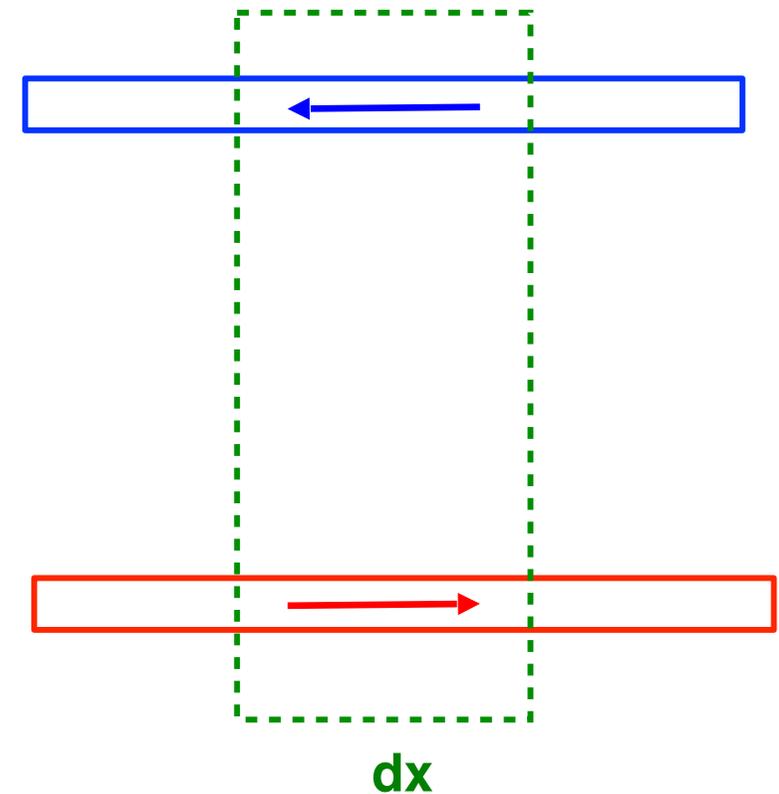
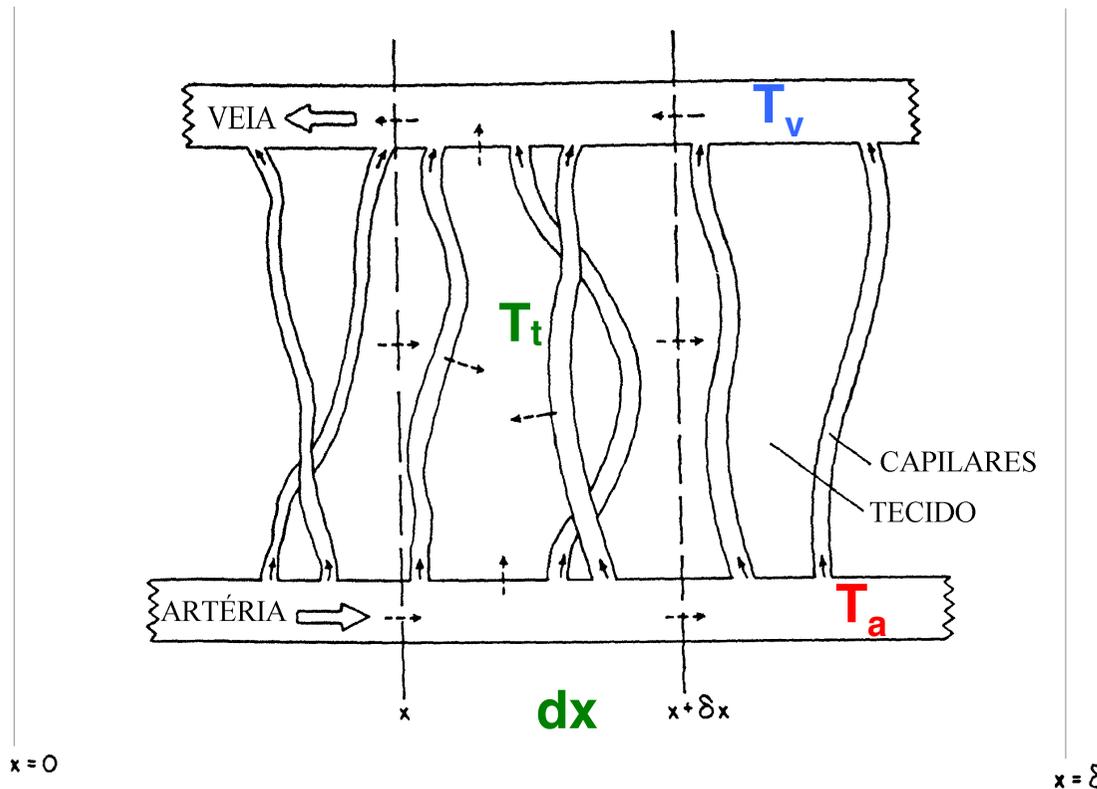


Energy balance:

Tissue

$$\left(-k_t A_x \frac{dT_t}{dx}\right)_x + (UA') dx [(T_a - T_t) + (T_v - T_t)] + \omega \rho_b c_b (T_a - T_t) A_x dx + Q_m A_x dx = \left(-k_t A_x \frac{dT_t}{dx}\right)_{x+dx}$$

area per unit length



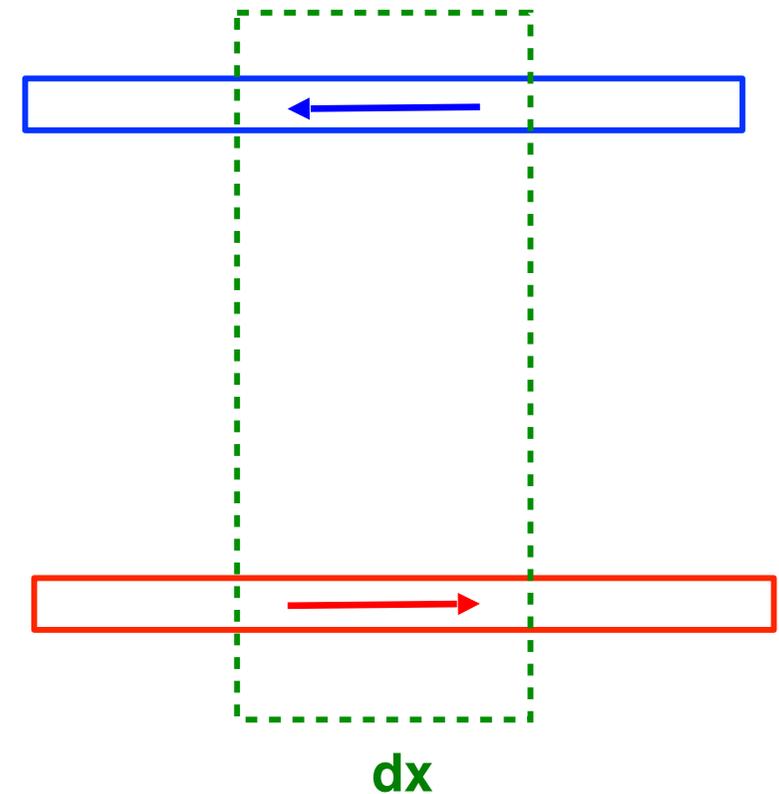
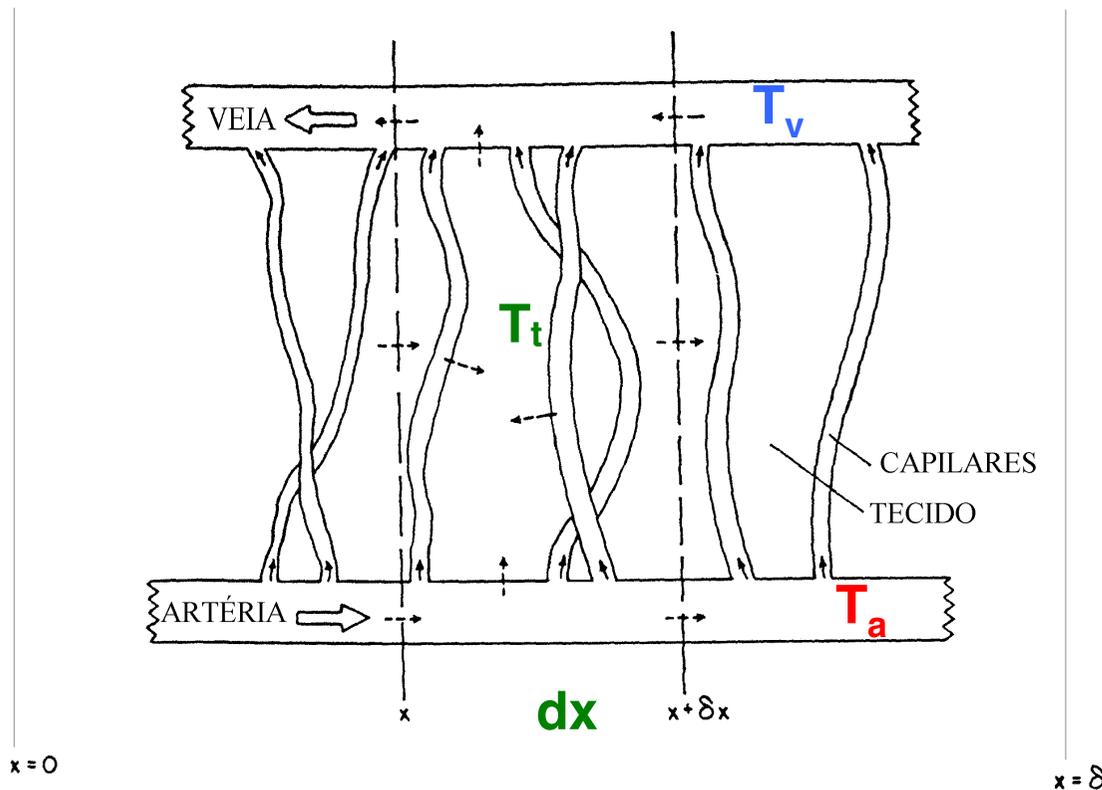
# Modelo de Keller; Seiler (1971):



Energy balance:

$$\text{Artery } (m_a c_b T_a)_x = (m_a c_b T_a)_{x+dx} + c_b \omega \rho_b T_a A_x dx + (UA')_a dx (T_a - T_t)$$

$$\frac{d}{dx} \left( \frac{m_a c_b T_a}{A_x} \right) = \frac{(UA')_a}{A_x} (T_t - T_a) - c_b \omega \rho_b T_a$$



# Modelo de Keller; Seiler (1971):

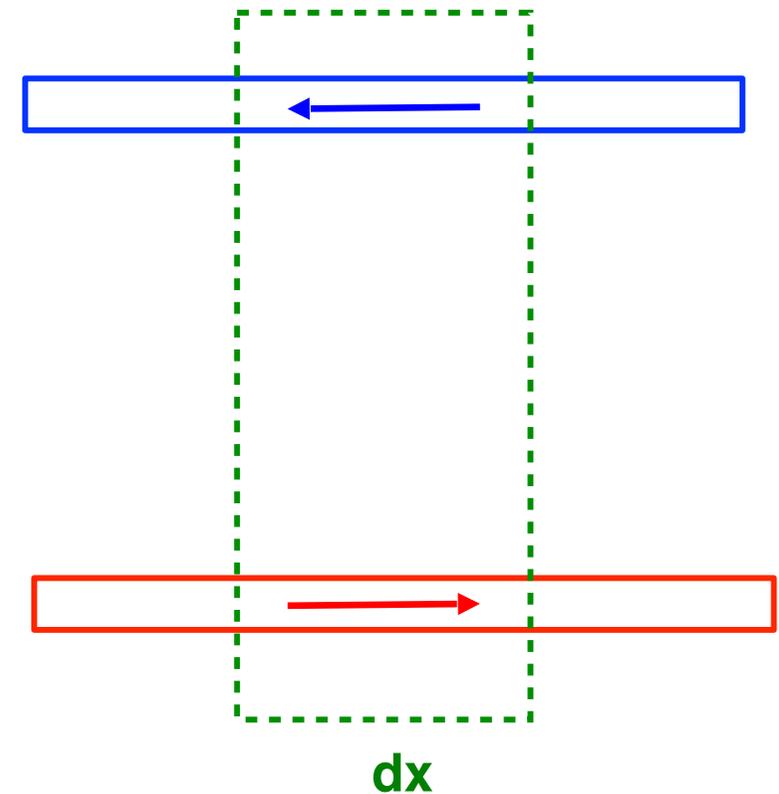
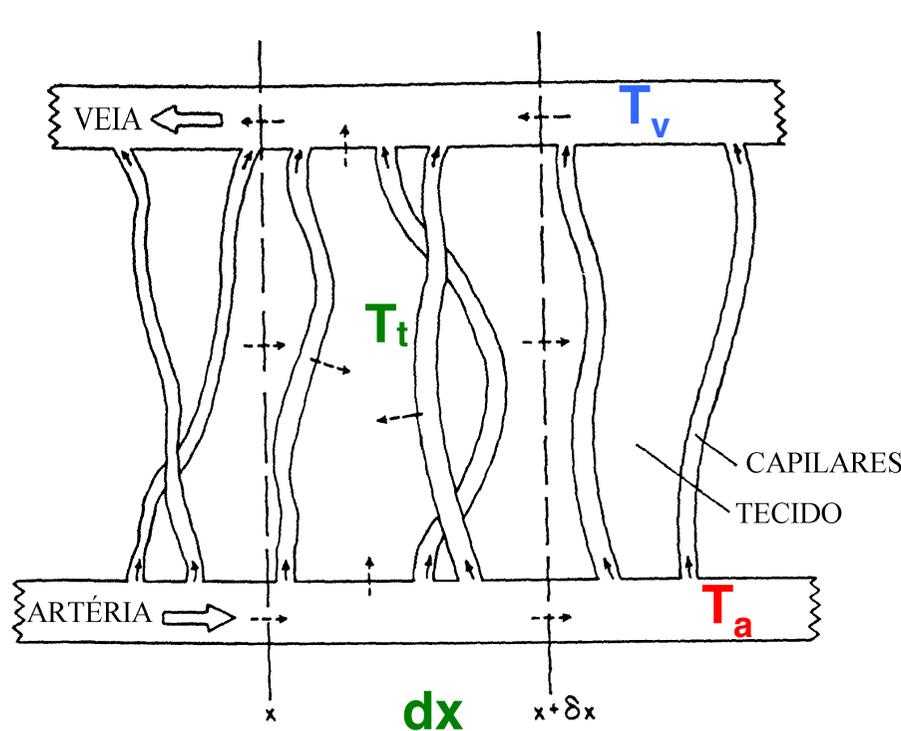


Energy balance:

temperature of the venous blood as it drains from the capillary

Vein  $(m_v c_b T_v)_{x+dx} + c_b \omega \rho_b T_{v0} A_x dx = (m_v c_b T_v)_x + (UA')_v dx (T_t - T_v)$

$$-\frac{d}{dx} \left( \frac{m_v c_b T_v}{A_x} \right) = \frac{(UA')_v}{A_x} (T_t - T_v) + c_b \omega \rho_b T_{v0}$$



x=0

x=delta

# Modelo de Keller; Seiler (1971):

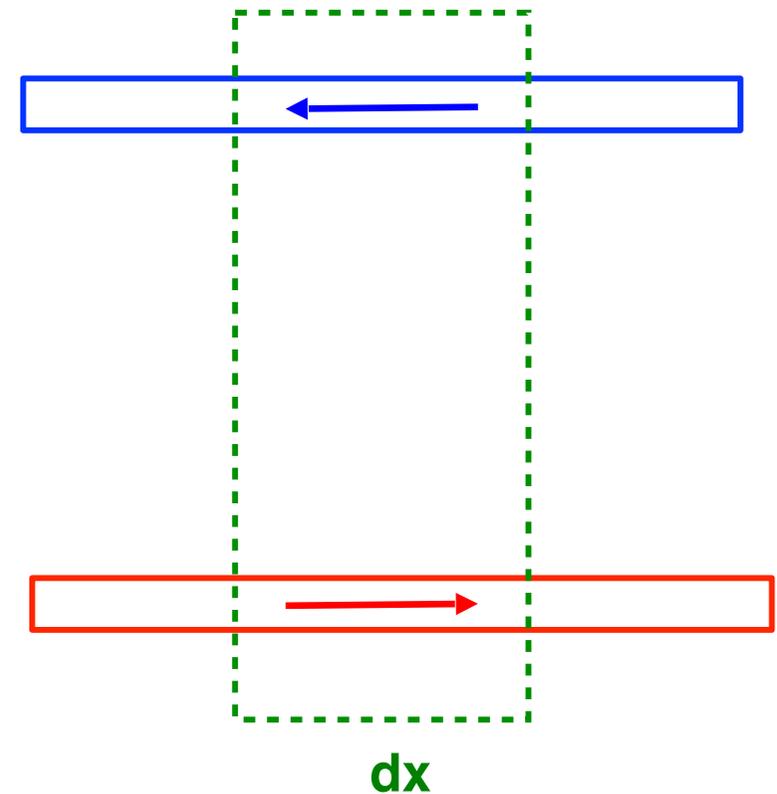
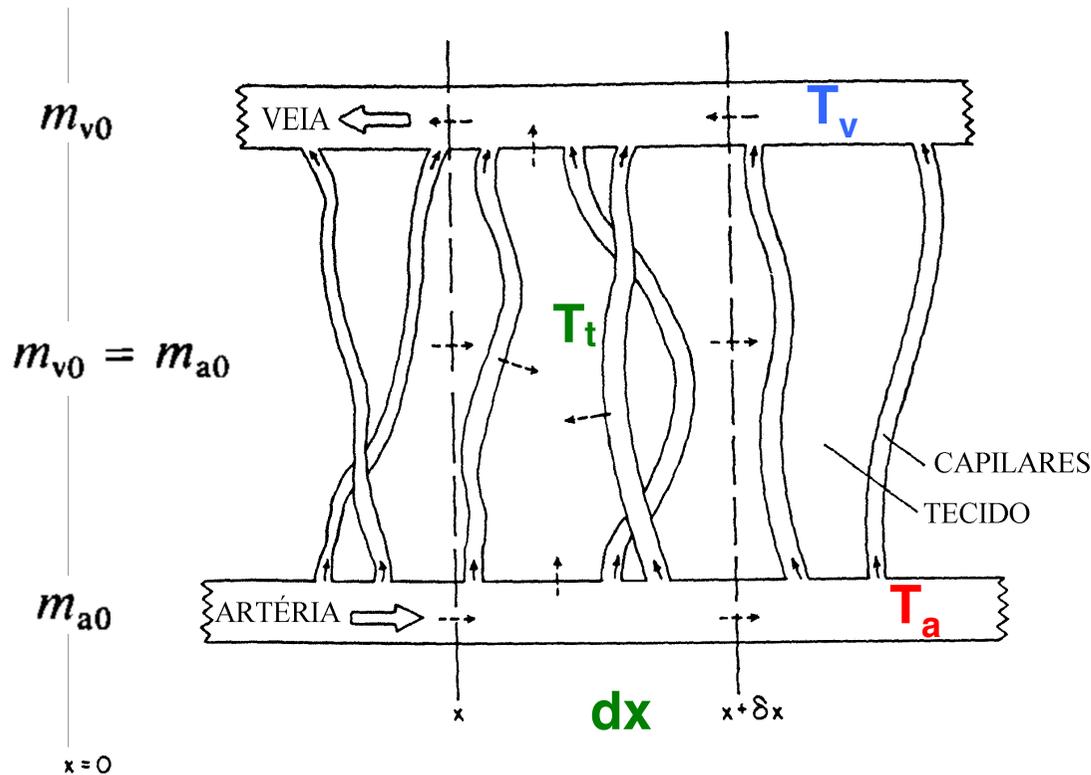


Mass balances:

artery  $m_a = m_{a0} - \int_0^x \omega \rho_b A_x dx$

vein  $m_v = m_{v0} - \int_0^x \omega \rho_b A_x dx$

$$\frac{dm_a}{dx} = -\omega \rho_b A_x = \frac{dm_v}{dx}$$





*simplifications*

$$(UA')_a = (UA')_v = (UA')$$

$$mc_b \frac{dT_a}{dx} = (UA')(T_i - T_a)$$

$$-mc_b \frac{dT_v}{dx} = (UA')(T_i - T_v) + c_b \omega \rho_b A_x (T_{v0} - T_v)$$

$T_{v0} = T_t$

# Modelo de Keller; Seiler (1971):



ODE:

Tissue  $k_t \frac{d^2 T_t}{dx^2} + \frac{(UA')}{A_r} [(T_a - T_t) + (T_v - T_t)] + \omega \rho_b c_b (T_a - T_t) + Q_m = 0$

Artery  $mc_b \frac{dT_a}{dx} = (UA')(T_t - T_a)$

Vein  $-mc_b \frac{dT_v}{dx} = (UA')(T_t - T_v) + c_b \omega \rho_b A_r (T_t - T_v)$

Boundary conditions:

$$T_t(0) = T_a(0) = T_b$$

$$T_t(\delta) = T_v(\delta) = T_s$$

$$m_{v0} = m_{a0}$$



Analytical solution:

Tissue  $k_t \frac{d^2 T_t}{dx^2} + \frac{(UA')}{A_r} [(T_a - T_t) + (T_v - T_t)] + \omega \rho_b c_b (T_a - T_t) + Q_m = 0$

Artery  $mc_b \frac{dT_a}{dx} = (UA')(T_t - T_a) \Rightarrow T_a(x) = T_b$

Vein  $-mc_b \frac{dT_v}{dx} = (UA')(T_t - T_v) + c_b \omega \rho_b A_x (T_t - T_v)$

Boundary conditions:

$$T_t(0) = T_a(0) = T_b$$

$$T_t(\delta) = T_v(\delta) = T_s$$

$$m_{v0} = m_{a0}$$



Analytical solution:

Tissue

$$k_t \frac{d^2 T_t}{dx^2} + \frac{(UA')}{A_r} [(T_a - T_t) + (T_v - T_t)] + \omega \rho_b c_b (T_a - T_t) + Q_m = 0$$

*(Note: A red arrow points from the fraction  $\frac{(UA')}{A_r}$  to a red '0' below it, indicating that this term is zero.)*

$$T_t = C_1 \cosh \lambda x + C_2 \sinh \lambda x + \frac{Q_m}{\omega \rho_b c_b} + T_b \quad \lambda = \sqrt{\frac{\omega \rho_b c_b}{k_t}}$$

$$\theta = \frac{T_t(x) - T_b}{T_s - T_b} = \Phi [\cosh \lambda x - 1] + [1 + \Phi(1 - \cosh \lambda \delta)] \frac{\sinh \lambda x}{\sinh \lambda \delta}$$

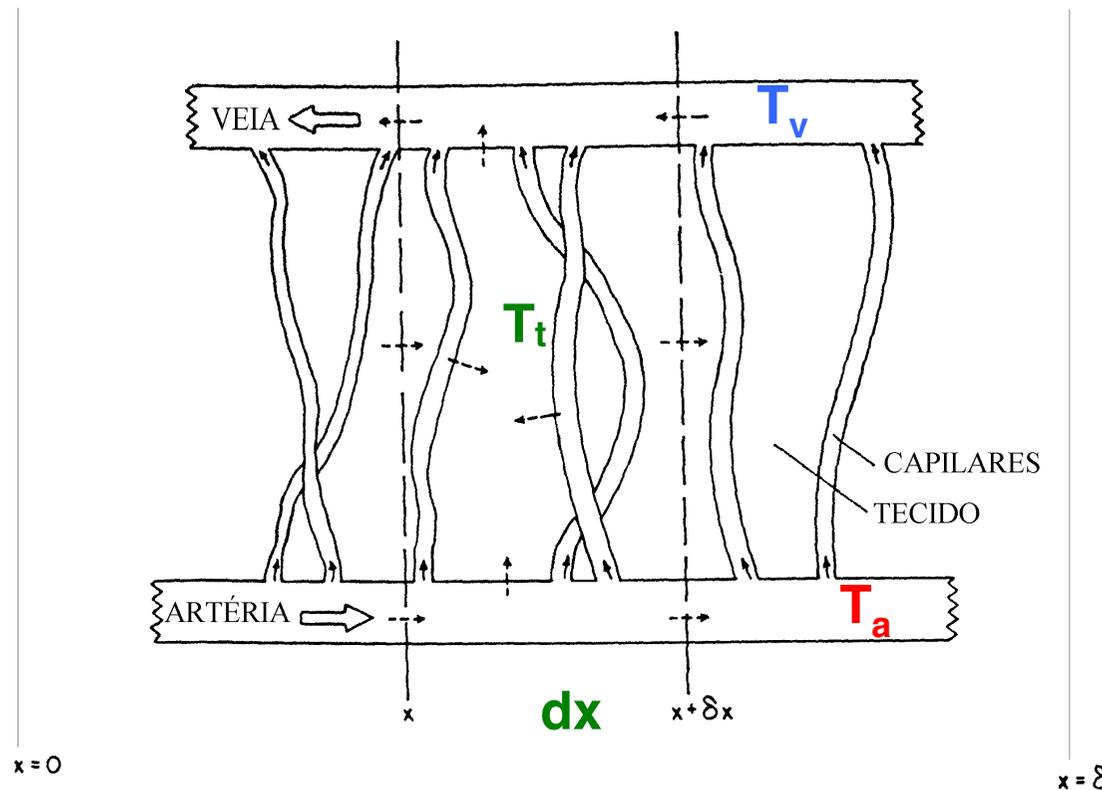
$$\Phi = \frac{Q_m}{\omega \rho_b c_b (T_b - T_s)}$$

# Effective thermal conductivity

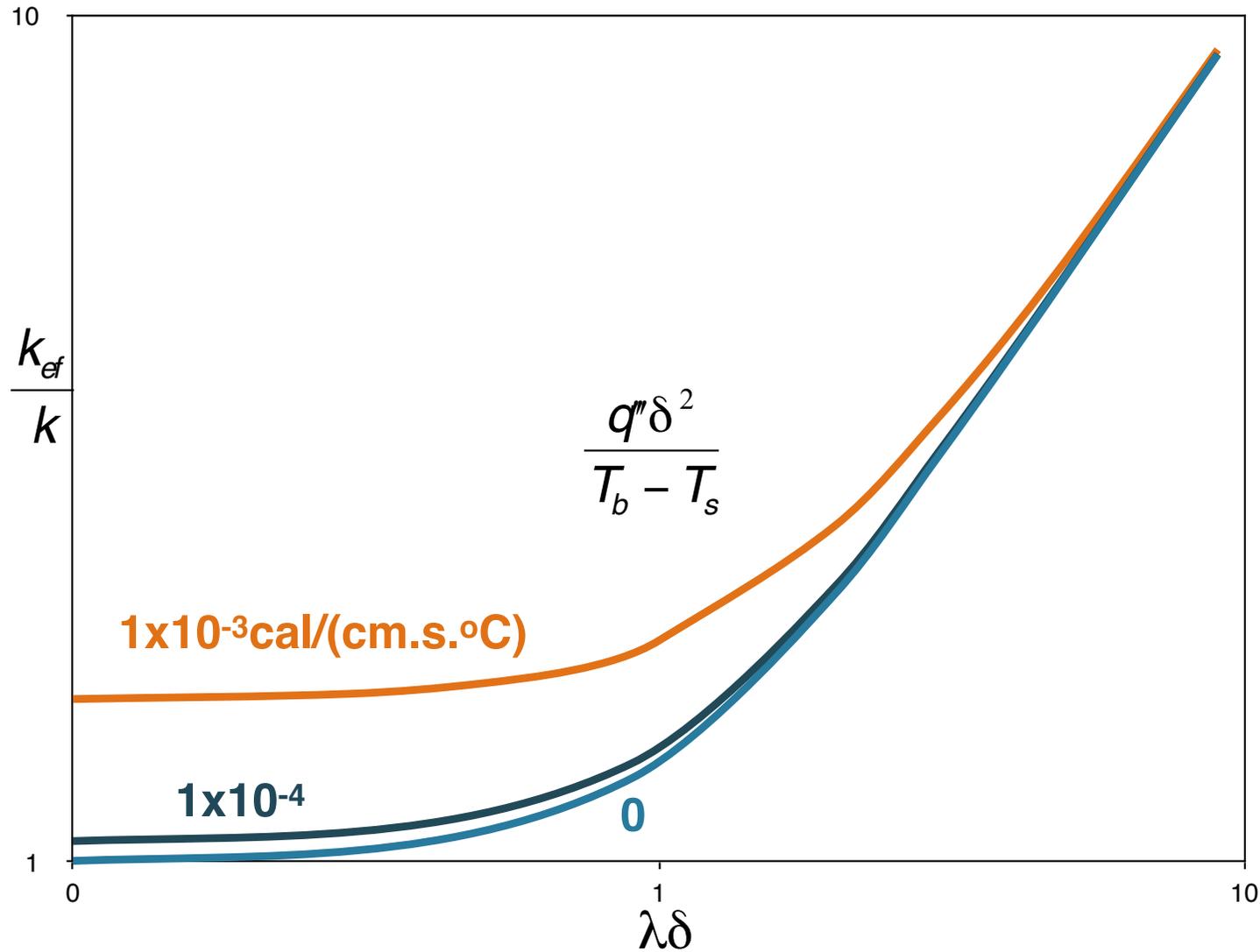


$$q_s \equiv \frac{k_{\text{eff}}(T_b - T_s)}{\delta} \Rightarrow \frac{k_{\text{eff}}}{k_t} = \frac{-\delta}{(T_b - T_s)} \frac{dT_t}{dx} \Big|_{x=\delta} \Rightarrow \frac{k_{\text{eff}}}{k_t} = \lambda \delta \left\{ \Phi \sinh \lambda \delta + \frac{1 + \Phi(1 - \cosh \lambda \delta)}{\tanh \lambda \delta} \right\}$$

$$\frac{k_{\text{eff}}}{k_t} = \frac{\lambda \delta}{\tanh \lambda \delta} + \frac{Q_m \delta^2}{(T_b - T_s)} \frac{\tanh\left(\frac{\lambda \delta}{2}\right)}{k_t \lambda \delta} \quad \lambda = \sqrt{\frac{\omega \rho_b c_b}{k_t}}$$

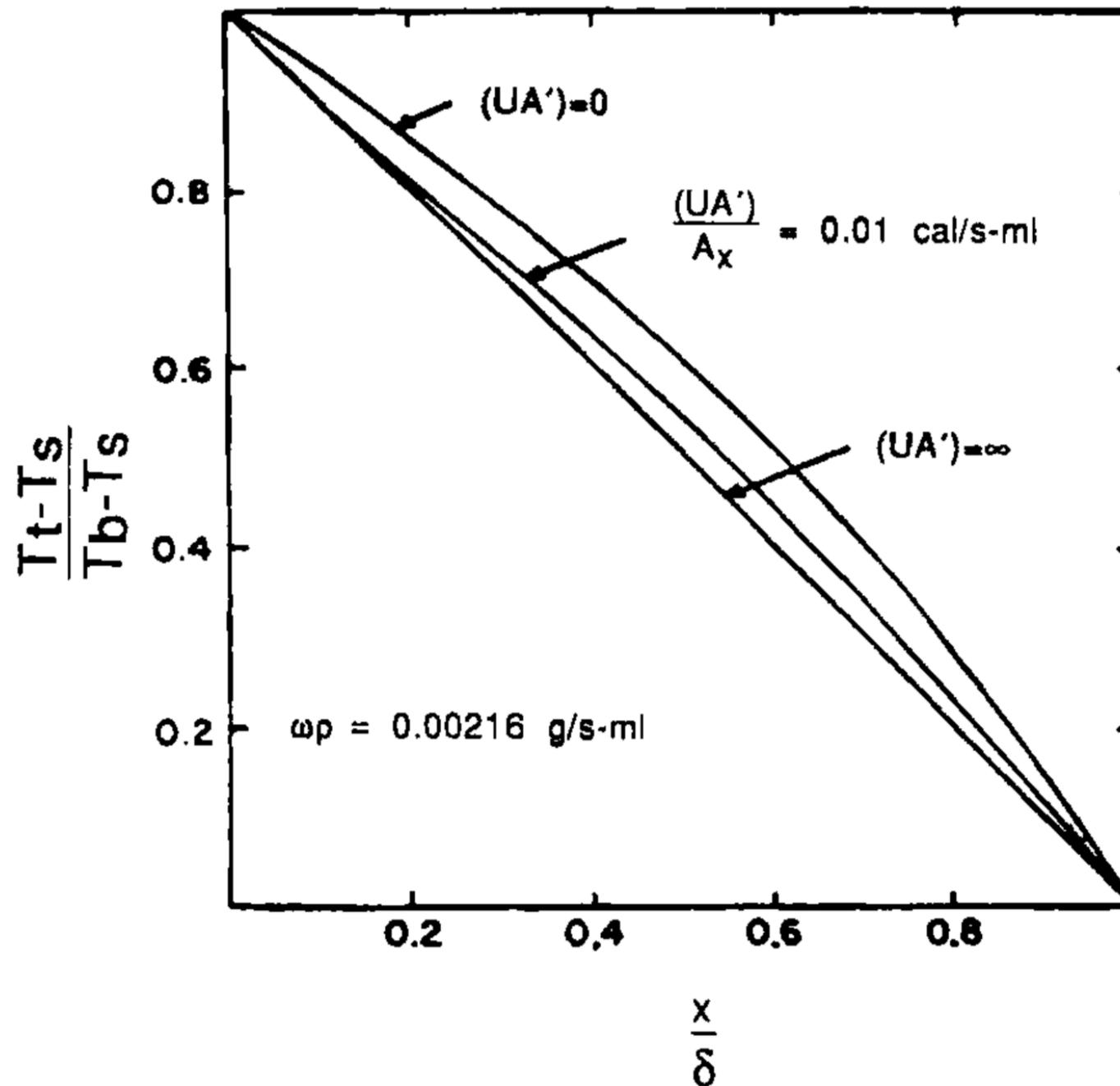


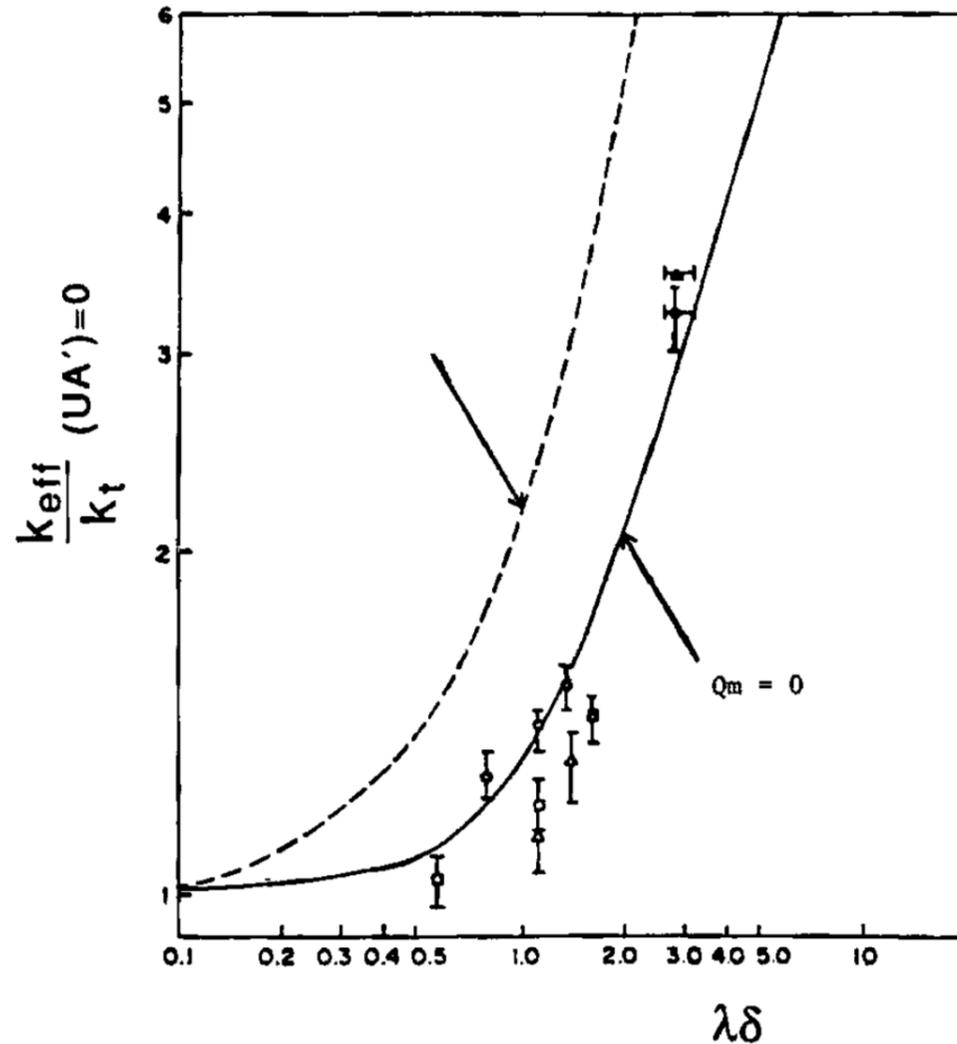
# Effective thermal conductivity



$$\lambda = \sqrt{\frac{\omega\rho_b c_b}{k_t}}$$

# General analysis





Predictions of the Keller and Seiler model, (with  $Q_m = 0$ ), compared with experimental data from the literature as compiled by Keller and Seiler. The dashed line indicates the predictions of a simple perfusion conductance model