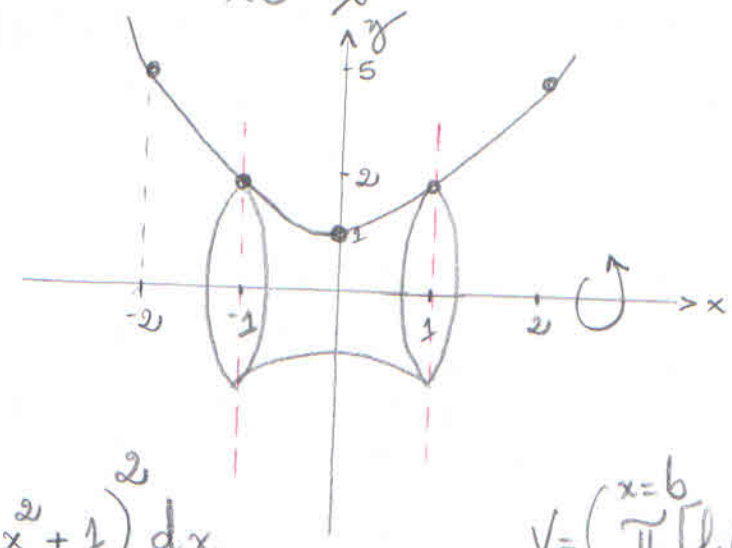
(2) Encontrar o volume do sólido obtido pela rotação da região limitada por $y = x^2 + 1$, $x = -1$, $x = 1$, $y = 0$ ao redor do eixo x

x	y
-2	5
-1	2
0	1
1	2
2	5



$$V = \int_{-1}^{1} \pi \left(x^2 + 1\right)^2 dx \qquad V = \int_{x=a}^{x=b} \pi \left[f(x)\right]^2 dx$$

$$V = \int_{-1}^{1} \pi \left(x^4 + 2x^2 + 1\right) dx$$

$$V = \pi \left(\frac{x^5}{5} + \frac{2x^3}{3} + x\right)\Bigg|_{-1}^{1}$$

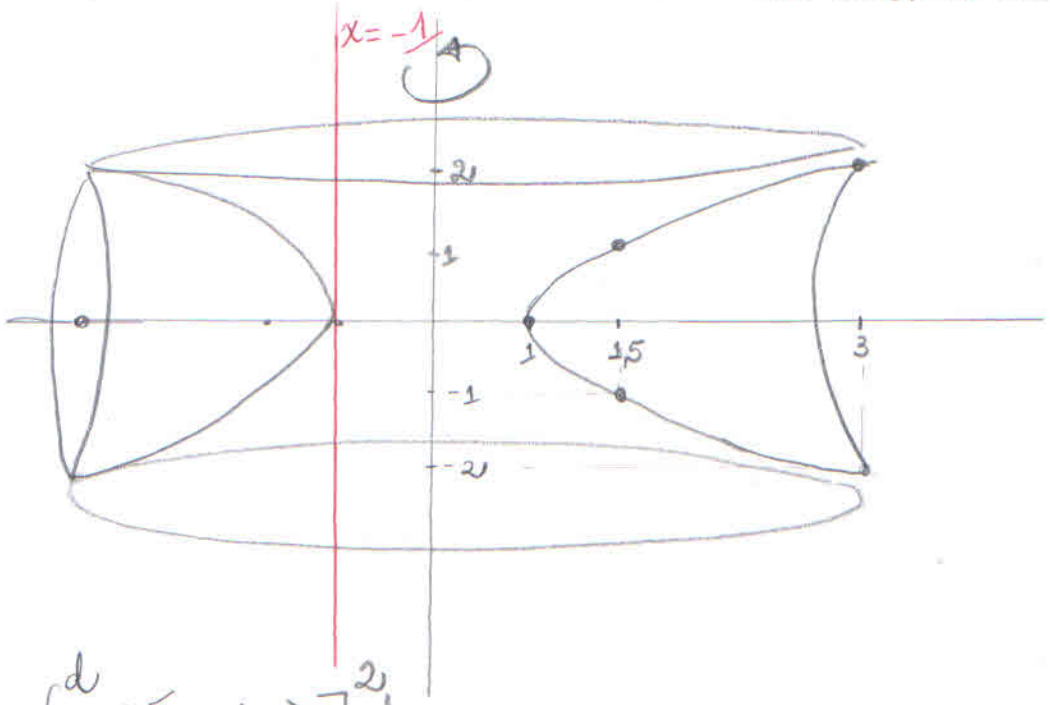
$$V = \pi \left[\left(\frac{1}{5} + \frac{2}{3} + 1\right) - \left(\frac{-1}{5} - \frac{2}{3} - 1\right)\right]$$

$$V = \pi \left[\left(\frac{3 + 10 + 15}{15}\right) - \left(\frac{-3 - 10 - 15}{15}\right)\right]$$

$$V = \pi \left[\frac{28}{15} + \frac{28}{15}\right] \Rightarrow V = \frac{56\pi}{15} \text{ u.v.}$$

(3) A região R, delimitada pela parábola $x = \frac{1}{2}y^2 + 1$ e pelas retas $x = -1$, $y = -2$ e $y = 2$ gira em torno da reta $x = -1$. Determinar o vol. do sólido obtido.

y	x
-2	3
-1	3/2
0	1
1	3/2
2	3



$$V = \int_c^d \pi [w(y)]^2 dy$$

$$V = \int_{-2}^{2} \pi \left[\frac{1}{2}y^2 + 1 - (-1)\right]^2 dy \qquad \frac{1}{2}y^2 + 1 + \underbrace{1}_{2}$$

$$V = \int_{-2}^{2} \pi \left[\frac{1}{4}y^4 + 2y^2 + 4\right] dy$$

$$V = \pi \left[\frac{1}{4}\frac{y^5}{5} + 2\frac{y^3}{3} + 4y\right]_{-2}^{2} \qquad \frac{64}{20} + \frac{32}{3} + 16$$

$$V = \pi\left[\left(\frac{32}{20} + \frac{16}{3} + 8\right) - \left(\frac{-32}{20} - \frac{16}{3} - 8\right)\right]$$

$$V = \frac{192 + 640 + 960}{60} = \frac{1892}{60} = \frac{964}{30}$$

$$V = \frac{482}{15}\pi \text{ u.v.}$$

Exercícios Extras

(1) Encontre o volume do sólido limitado pelas curvas $y = x$, $y = x^2$, girando em torno da reta $y = 2$.

$y = x^2$

$x = 0 \rightarrow y = 0$

$x = \pm 1 \rightarrow y = 1$

$x = \pm 2 \rightarrow y = 4$

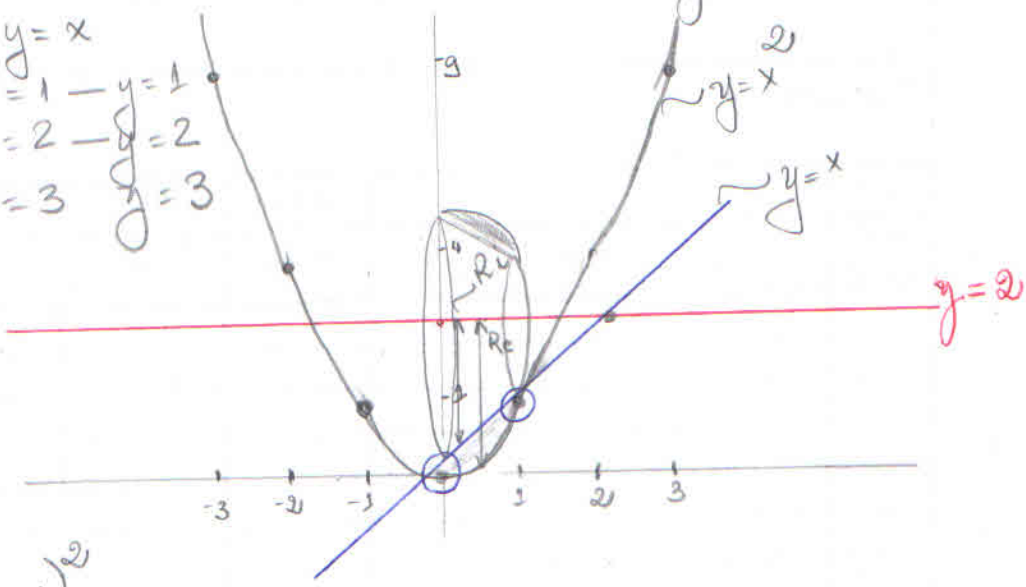
$x = \pm 3 \rightarrow y = 9$

$y = x$

$x = 1 \rightarrow y = 1$

$x = 2 \rightarrow y = 2$

$x = 3 \rightarrow y = 3$



$A(y) = \pi (re)^2 - \pi (ri)^2$

$re = 2 - x^2$ $\quad ri = 2 - x$

$$V = \pi \int_0^1 (2 - x^2)^2 - (2 - x)^2 \, dx$$

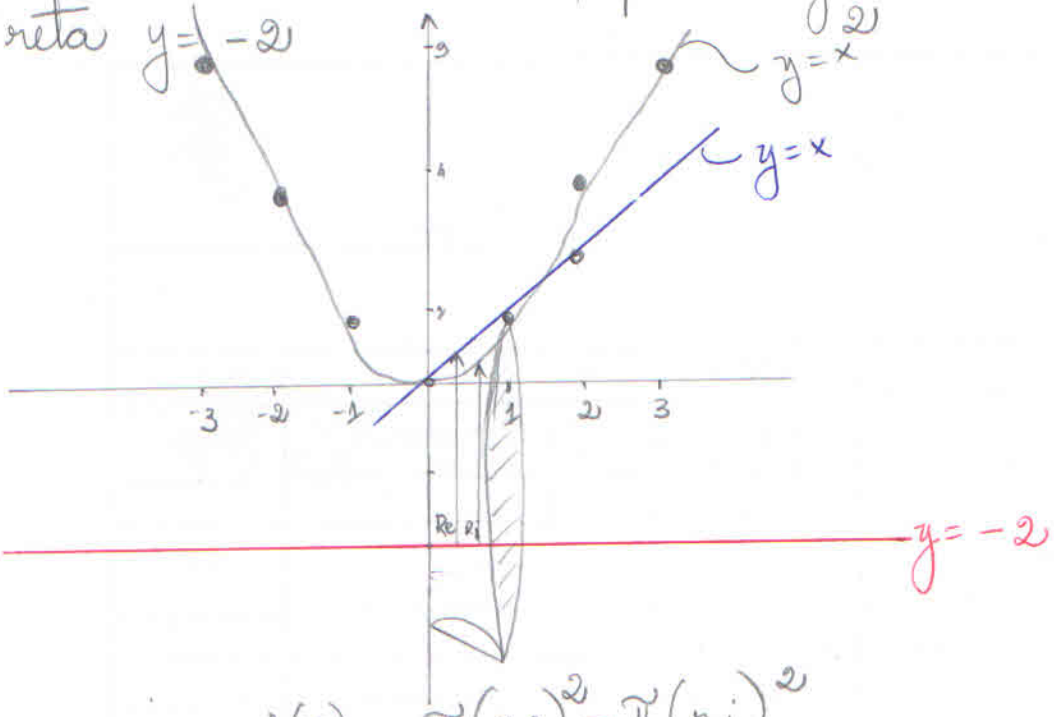
$$V = \pi \int_0^1 \left[(4 - 4x^2 + x^4) - (4 - 4x + x^2) \right] dx \rightarrow -5x^2 + x^4 + 4x$$

$$V = \int_0^1 \pi \left(-5x^2 + x^4 + 4x \right) dx$$

$$V = \pi \left(-\frac{5x^3}{3} + \frac{x^5}{5} + \frac{4x^2}{2} \Big|_0^1 \right)$$

$$V = -\frac{5}{3} + \frac{1}{5} + 2 = \frac{-25 + 3 + 30}{15} = \frac{8\pi}{15} \text{ u.v.}$$

(2) Idem ao exercício 1, porém girando em torno da reta $y = -2$ (2)



$$A(x) = \pi (re)^2 - \pi (ri)^2$$

$$re = x - (-2) = x + 2 \qquad ri = x^2 - (-2) = x^2 + 2$$

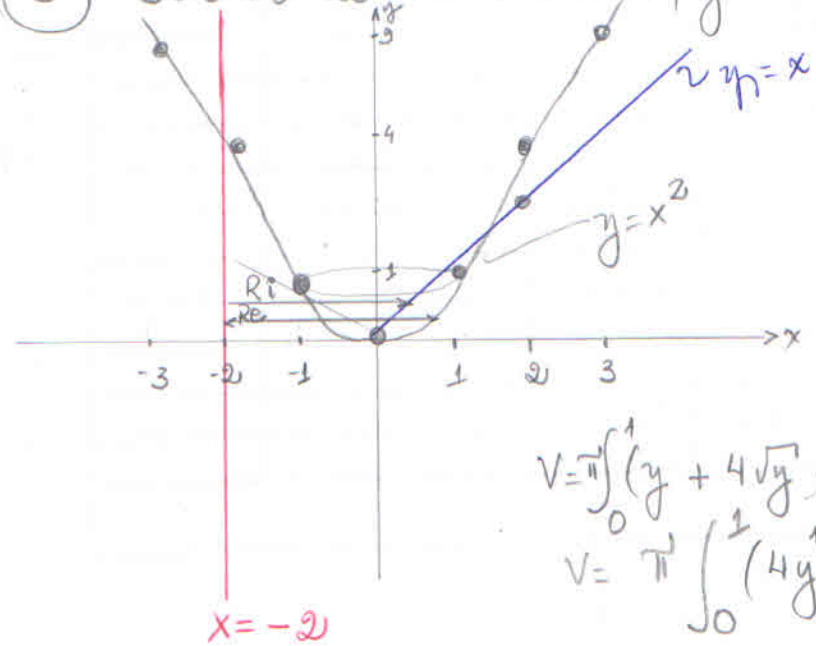
$$V = \int_0^1 \pi \left[(x+2)^2 - (x^2+2)^2 \right] dx$$

$$V = \pi \int_0^1 (x^2 + 4x + 4) - (x^4 + 4x^2 + 4)\, dx$$

$$\pi \int_0^1 (-3x^2 + 4x - x^4)\, dx \qquad -\frac{3x^3}{3} + \frac{4x^2}{2} - \frac{x^5}{5} \Big|_0^1$$

$$V = -1 + 2 - \frac{1}{5} = \frac{-5+10-1}{5} = \boxed{\frac{4}{5}\pi \ u.v.}$$

(3) Idem ao exercício 1, girando ao redor da reta $x = -2$



$$A(y) = \pi (re)^2 - \pi (ri)^2$$

$y = x^2 \rightarrow x = y^{1/2}$

$$re = y^{1/2} - (-2)$$

$$ri = y - (-2)$$

$$V = \int_0^1 \pi \left[(y^{1/2} + 2)^2 - (y+2)^2 \right] dy$$

$$V = \pi \int_0^1 (y + 4\sqrt{y} + 4 - y^2 - 4y - 4)\, dy$$

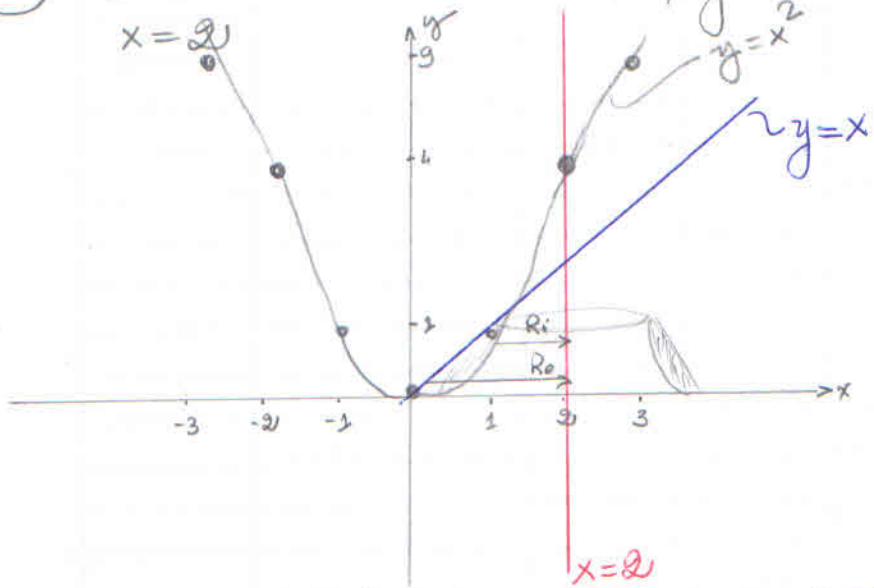
$$V = \pi \int_0^1 (4y^{1/2} - y^2 - 3y)\, dy$$

$$V = \pi \left[4\frac{y^{3/2}}{3/2} - \frac{y^3}{3} - 3\frac{y^2}{2} \right]_0^1$$

$$V = \pi \left[\frac{8}{3} y^{3/2} - \frac{y^3}{3} - \frac{3y^2}{2} \right]_0^1$$

$$\frac{8}{3} - \frac{1}{3} - \frac{3}{2} = \frac{16 - 2 - 9}{6} = \frac{5}{6}\pi \text{ u.v.}$$

(4) Idem ao exercício 1, girando ao redor da reta



$$A(y) = \pi(re)^2 - \pi(ri)^2$$

$$re = 2 - y$$

$$ri = 2 - y^{1/2}$$

$$V = \int_0^1 \left[\pi(2-y)^2 - \pi(2-y^{1/2})^2 \right] dy$$

$$V = \pi \int_0^1 \left[(4 - 4y + y^2) - (4 - 4\sqrt{y} + y) \right] dy$$

$$(4 - 4y + y^2 - 4 + 4\sqrt{y} - y)\, dy$$

$$V = \pi \int_0^1 \left(-5y + y^2 + 4y^{1/2} \right) dy$$

$$V = \pi \left[-\frac{5y^2}{2} + \frac{y^3}{3} + \frac{4y^{3/2}}{3/2} \right]_0^1$$

$$V = -\frac{5}{2} + \frac{1}{3} + \frac{8}{3} = \frac{-15 + 2 + 16}{6} = \frac{3}{6}$$

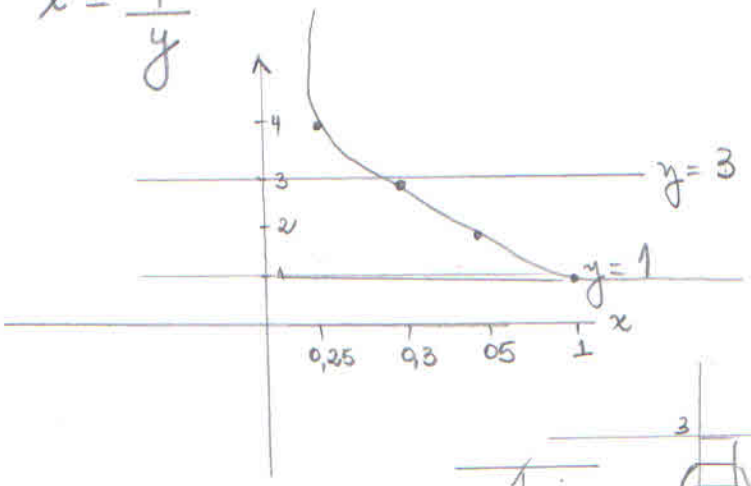
$$V = \frac{\pi}{2} \text{ u.v.}$$

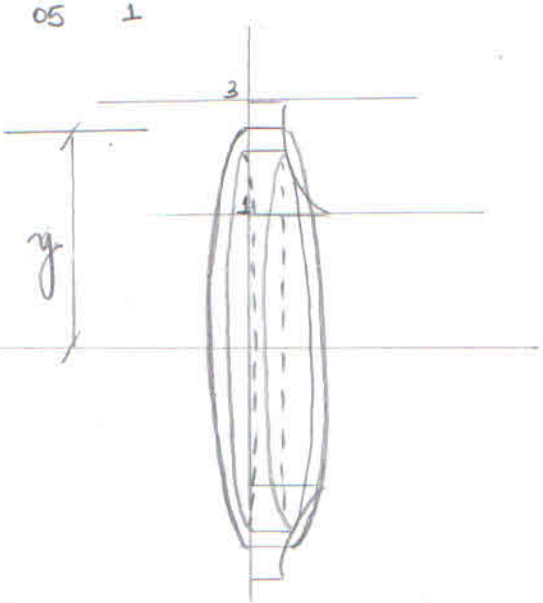
9-14 Use o método das cascas cilíndricas para encontrar o vol do sólido pela rotação da região delimitada pelas curvas dadas em torno do eixo x.

9- $xy = 1$, $x = 0$, $y = 1$, $y = 3$

$xy = 1 \Rightarrow x = \frac{1}{y}$

x	y
#	0
1	1
1/2	2
1/3	3
1/4	4



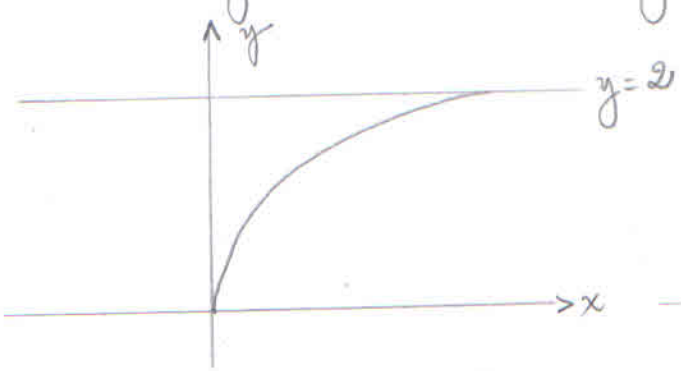


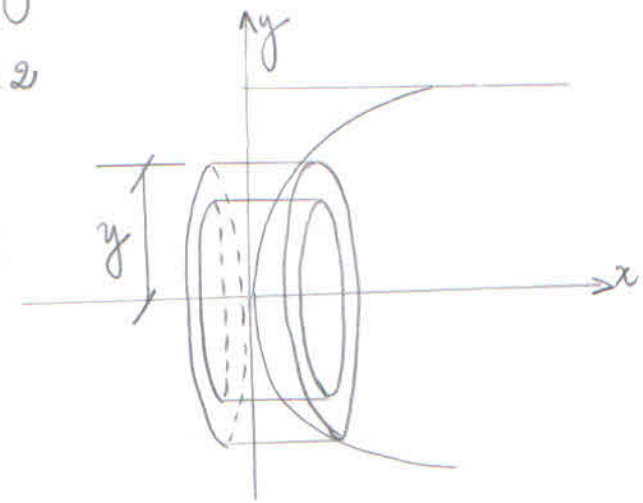
$$V = 2\pi \int_1^3 y\left(\frac{1}{y}\right) dy$$

$$= 2\pi \int_1^3 dy = 2\pi y \Big|_1^3$$

$$V = 4\pi$$

10- $y = \sqrt{x}$, $x = 0$, $y = 2$





$$V = \int_0^2 2\pi y\,(y^2)\, dy$$

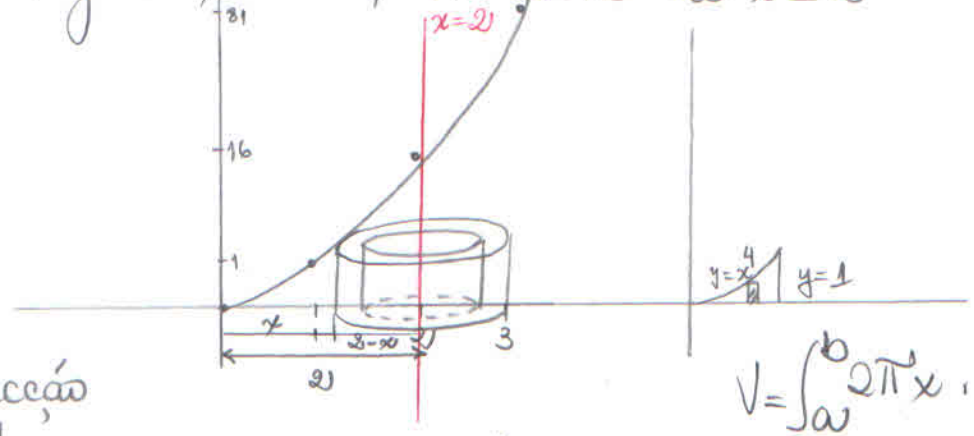
$$= \int_0^2 2\pi y^3 dy$$

$$= 2\pi \frac{y^4}{4}\Big|_0^2 \Rightarrow V = 2\pi \frac{16}{4} = 8\pi$$

15-20 Use o método das cascas cilíndricas p/ achar o volume gerado pela rotação da região delimitada pelas curvas dadas em torno do eixo especificado.

(15) $y = x^4$, $y = 0$, $x = 1$; em torno de $x = 2$

x	y
0	0
1	1
2	16
3	81



Interseção

$y = x^4$, $x = 1$

$y = 1$

$$V = \int_a^b 2\pi x \cdot h(x) \cdot dx$$

$$V = \int_0^1 2\pi (2 - x) \cdot x^4 \, dx$$

$$= 2\pi \int_0^1 (2x^4 - x^5) \, dx$$

$$= 2\pi \left[\frac{2x^5}{5} - \frac{x^6}{6} \right]_0^1$$

$$= 2\pi \left[\frac{2}{5} - \frac{1}{6} \right] = \frac{12 - 5}{30}$$

$$V = \frac{14\pi}{30} = \frac{7\pi}{15}$$