Fenômenos de Transporte

ADIMENSIONALIZAÇÃO

Expressão das equações de Fenômenos de Transporte na forma adimensionalizada

- **Números ADIMENSIONAIS**
  (Re, Pr, Sc, Pe, Gr, Sh, Nu etc..)
  - Correlações semi-empíricas generalizadas
  - Analogia dos FT's

- **"SCALING"**
  Identificação das ordens de grandeza
  - Simplificação das equações (equações aproximadas)
  - Resultados semi-quantitativos

- **CFD _Fluidodinâmica Computacional**
  - Simplificação das equações (equações aproximadas)
  - Elaboração de critérios e modelos

ADIMENSIONALIZAÇÃO

PARÂMETROS CARACTERÍSTICOS: t_0, ρ_0, v_0, L

ADIMENSIONALIZAÇÃO DAS VARIÁVEIS: $\hat{\rho} = \dfrac{\rho}{\rho_0}$; $\hat{\vec{v}} = \dfrac{\vec{v}}{v_0}$; $\hat{\vec{r}} = \dfrac{\vec{r}}{L}$; $\hat{t} = \dfrac{t}{t_0}$

$$\rho = \rho_0\,\hat{\rho} \quad ; \quad \vec{v} = v_0\,\hat{\vec{v}} \quad ; \quad \vec{r} = L\,\hat{\vec{r}} \quad ; \quad t = t_0\,\hat{t}$$

ADIMENSIONALIZAÇÃO DOS OPERADORES:

$$\hat{\text{gr}\vec{\text{a}}\text{d}}(\ ) = L\,\text{gr}\vec{\text{a}}\text{d}(\ ) \quad ; \quad \hat{\text{div}}(\ ) = L\,\text{div}(\ ) \quad ; \quad \hat{\text{lap}}(\ ) = \hat{\text{div}}\,\hat{\text{gr}\vec{\text{a}}\text{d}}(\ ) = L^2\,\text{lap}(\ )$$

$$\frac{\partial}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \hat{t}}$$

$$\frac{\partial f}{\partial x} = \frac{\partial(\hat{f}\,f_0)}{\partial(\hat{x}\,x_0)} = \frac{f_0}{x_0}\frac{\partial \hat{f}}{\partial \hat{x}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial(\hat{x}\,x_0)}\left(\frac{\partial(\hat{f}\,f_0)}{\partial(\hat{x}\,x_0)}\right) = \frac{f_0}{{x_0}^2}\frac{\partial}{\partial \hat{x}}\left(\frac{\partial \hat{f}}{\partial \hat{x}}\right) = \frac{f_0}{{x_0}^2}\frac{\partial^2 \hat{f}}{\partial \hat{x}^2}$$

Equação da Continuidade

$$\frac{\partial \rho}{\partial t} = -\operatorname{div} \rho \vec{v}$$

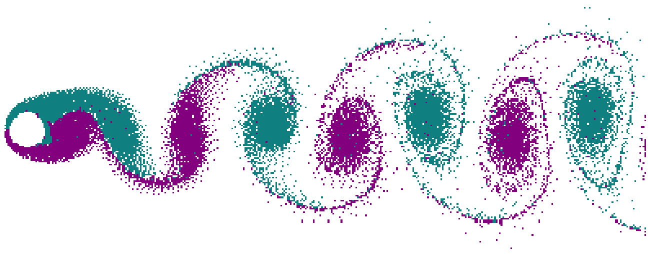
$$\frac{\rho_0}{t_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\frac{\rho_0 v_0}{L}\hat{\operatorname{div}}\,\hat{\rho}\,\hat{\vec{v}} \quad \Rightarrow \quad \frac{L}{t_0 v_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\hat{\operatorname{div}}\,\hat{\rho}\,\hat{\vec{v}}$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\hat{\operatorname{div}}\,\hat{\rho}\,\hat{\vec{v}}$$

STROUHAL

$$Sr = \frac{t_0 v_0}{L}$$

$$Sr = \frac{\Omega D}{v_0}$$



t_0 **: tempo para o escoamento "atingir o regime permanente"**
ou associado à frequência em processo periódico

Ω **: frequência em processo periódico**

Equação de Conservação Generalizada

Balanço microscópico de φ:

$$\rho \frac{D\varphi}{Dt} = \frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\left(\rho\varphi\vec{v}\right) = -\mathrm{div}\,\vec{j}_{\phi} + \dot{\sigma}_{\forall\Phi}$$

$$\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\left(\rho\vec{v}\varphi + \vec{j}_{\Phi}\right) = \dot{\sigma}_{\forall_{\Phi}}$$

Equação constitutiva de difusão:

$$\vec{j}_{\Phi} = -\rho\,\Gamma_{\varphi}\,\mathrm{gr\vec{a}d}\;\varphi$$

φ	Γ_{Φ}	ν/Γ_{Φ}
V	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

$$\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\,\rho\left(\vec{v}\varphi - \Gamma_{\varphi}\mathrm{grad}\;\varphi\right) = \dot{\sigma}_{\forall_{\Phi}}$$

Equação de Conservação Generalizada

$$\frac{\partial \rho\varphi}{\partial t} + \operatorname{div}\rho\left(\vec{v}\varphi - \Gamma_{\varphi}\operatorname{grad}\varphi\right) = \dot{\sigma}_{\forall_{\Phi}}$$

$$\rho = \rho_0\,\hat{\rho} \quad ; \quad \vec{v} = v_0\,\hat{\vec{v}} \quad ; \quad \vec{r} = L\,\hat{\vec{r}} \quad ; \quad t = t_0\,\hat{t} \quad ; \quad \varphi = \hat{\varphi}\,\Delta\varphi + \varphi_0$$

$$\frac{1}{t_0}\frac{\partial \rho_0\,\hat{\rho}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)}{\partial \hat{t}} + \frac{1}{L}\operatorname{d\hat{i}v}\hat{\rho}\left(\rho_0 v_0\,\hat{\vec{v}}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right) - \frac{\rho_0}{L}\Gamma_{\varphi}\operatorname{gr\hat{a}d}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)\right) = \dot{\sigma}_{\forall_{\Phi}}$$

Reagrupando-se os termos:

$$\varphi_0\frac{\rho_0 v_0}{L}\underbrace{\left[\frac{L}{t_0 v_0}\frac{\partial\hat{\rho}}{\partial\hat{t}} + \operatorname{d\hat{i}v}\left(\hat{\rho}\hat{\vec{v}}\right)\right]}_{=0\text{ , continuidade}} + \frac{\rho_0\Delta\varphi}{t_0}\frac{\partial\left(\hat{\rho}\hat{\varphi}\right)}{\partial\hat{t}} + \frac{\rho_0 v_0\Delta\varphi}{L}\left[\operatorname{d\hat{i}v}\left(\hat{\rho}\hat{\vec{v}}\hat{\varphi}\right) - \operatorname{d\hat{i}v}\hat{\rho}\left(\frac{\Gamma_{\varphi}}{v_0 L}\operatorname{gr\hat{a}d}\hat{\varphi}\right)\right] = \dot{\sigma}_{\forall_{\Phi}}$$

Equação de Conservação Generalizada

$$\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\,\rho\left(\vec{v}\varphi - \Gamma_{\varphi}\,\mathrm{grad}\,\varphi\right) = \dot{\sigma}_{\forall_\Phi}$$

$$\frac{L}{v_0 t_0}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi} - \frac{\Gamma_{\varphi}}{v_0 L}\,\hat{\mathrm{grad}}\,\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\hat{\mathrm{grad}}\,\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 \Delta\varphi\, v_0}$$

PECLET

$$Pe = \frac{v_0 L}{\Gamma_{\varphi}} = \frac{\nu}{\Gamma_{\varphi}}\frac{v_0 L}{\nu} = Re\frac{\nu}{\Gamma_{\varphi}}$$

$$Pe = Re\frac{\nu}{\Gamma_{\varphi}}$$

Equação de Conservação Generalizada

ADIMENSIONAIS

PECLET

$$Pe = Re \frac{\nu}{\Gamma_\varphi}$$

CONVECÇÃO / DIFUSÃO

PECLET MÁSSICO

$$Pe = Re\,Sc$$

PECLET TÉRMICO

$$Pe = Re\ Pr$$

PRANDTL

$$Pr = \frac{\nu}{\alpha}$$

$$\alpha = \frac{k}{\rho Cp}$$

SCHMIDT

$$Sc = \frac{\nu}{D_{AB}}$$

φ	Γ_Φ	ν/Γ_Φ
$\mathbf{V}$	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

Equação de Energia

$$\frac{1}{Sr}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial\hat{t}}+\hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi}-\frac{1}{Pe}\hat{grad}\;\hat{\varphi}\right)=\frac{\dot{\sigma}_{\forall_\Phi}L}{\rho_0 v_0\Delta\varphi}\Rightarrow$$

φ= CpT, Cp cte

$$\hat{T}=\frac{T-T_0}{T_s-T_0}=\frac{T-T_0}{\Delta T}$$

$$\frac{1}{Sr}\frac{\partial\hat{\rho}\hat{T}}{\partial\hat{t}}+div\,\hat{\rho}\left(\hat{\vec{v}}\hat{T}-\frac{1}{Pe}\hat{grad}\,\hat{T}\right)=\frac{\dot{\sigma}_{\forall\phi}L}{\rho_0\,c_p\Delta T\,v_0}$$

Para dissipação viscosa:

$$\dot{\sigma}_{\forall T}=\mu\Phi_\forall$$

$$\Rightarrow\frac{\mu\Phi_\forall L}{\rho_0\,c_p\Delta T\,v_0}=\frac{1}{Pe}\frac{\mu\Phi_\forall L^2}{k\,\Delta T}=\frac{Br}{Pe}$$

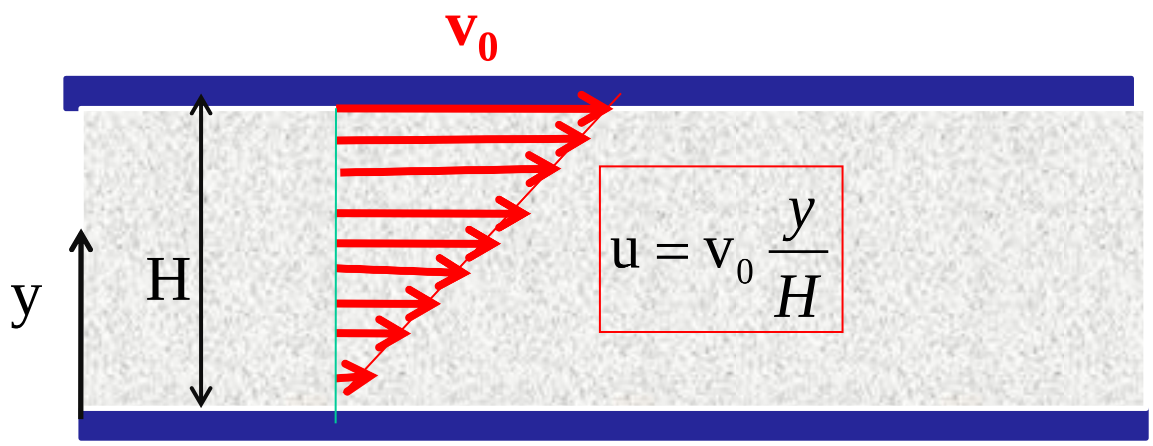
BRINKMAN

$$Br=\frac{\mu\Phi_\forall L^2}{k\,\Delta T}$$

Equação de Energia - BRINKMAN

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\, T}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{T} - \frac{1}{Pe}\,\mathrm{gr\hat{a}d}\,\hat{T}\right) = \frac{Br}{Pe}$$

Caso particular: perfil linear de velocidade



$$\mu\Phi_{\forall} = \mu\left(\frac{\partial v}{\partial y}\right)^2 = \mu\left(\frac{v_0}{L}\right)^2$$

$$L = H$$

$$Br = \frac{\mu\Phi_{\forall} L^2}{k\,\Delta T} = \frac{\mu\, v_0^2}{k\,\Delta T}$$

Não depende de L !!

Navier-Stokes

Pressão Modificada- Piezométrica

Navier-Stokes:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v}.\mathrm{gr\vec{a}d}\,\vec{v} = \rho \vec{g} - \mathrm{gr\vec{a}d}\,p + \mu\,\mathrm{lap}\,\vec{v}$$

z orientado "para cima"

Força de campo, gradiente do potencial U

$$\vec{g} = \vec{f} = -gr\vec{a}d\,U$$

$$\rho\,\vec{g} = -\rho\,gr\vec{a}d\,U$$

$$\rho\,\vec{g} = -gr\vec{a}d\,\rho U \quad (gr\vec{a}d\,\rho = 0)$$

$$U = g\,z + cte$$

$$gr\vec{a}d\,U = -\vec{g}$$

$$\rho\vec{g} - gr\vec{a}d\,p = -gr\vec{a}d\,\rho U - gr\vec{a}d\,p = -gr\vec{a}d\left(\rho U + p\right)$$

Pressão modificada:

$$P = \rho U + p = \rho\,g\,z + p$$

$$gr\vec{a}d\,P = -\rho\,\vec{g} + gr\vec{a}d\,p$$

Navier-Stokes - Adimensionalização

$$grad\vec{}\, P = -\rho\,\vec{g} + grad\vec{}\; p$$

Substituindo-se na Navier-Stokes:

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho\,\vec{v}.grad\vec{}\,\vec{v} = \underbrace{\rho\,\vec{g} - grad\vec{}\,p}_{-grad\vec{}\,P} + \mu\,lap\,\vec{v}$$

Adimensionalizando-se:

$$\hat{P} = \frac{P}{P_0}$$

$$P_0 = \rho g z_0 + cte + p_0$$

$$\frac{1}{Sr}\frac{\partial \hat{\vec{v}}}{\partial \hat{t}} + \hat{\vec{v}}.gr\hat{\vec{a}}d\,\hat{\vec{v}} = -\frac{1}{Ru}gr\hat{\vec{a}}d\,\hat{P} + \frac{1}{Re}l\hat{a}p\,\hat{\vec{v}}$$

RUARK

EULER

$$Ru = \frac{\rho\,v_0^2}{P_0}$$

Navier-Stokes - Adimensionalização

$$Ru = \frac{\rho v_0^2}{P_0}$$

$$Re = \frac{v_0 L}{\nu}$$

Regime permanente

$$\hat{\vec{v}}.\mathrm{gr\hat{\vec{a}}d}\,\hat{\vec{v}} = -\frac{1}{Ru}\,\mathrm{gr\hat{\vec{a}}d}\,\hat{P} + \frac{1}{Re}\,\mathrm{l\hat{a}p}\,\hat{\vec{v}}$$

Euler – invíscido

$$Re \to \infty$$

Escoamento externo (camada limite)

$$\hat{\vec{v}}.\mathrm{gr\hat{\vec{a}}d}\,\hat{\vec{v}} = -\frac{1}{Ru}\,\mathrm{gr\hat{\vec{a}}d}\,\hat{P}$$

“Creeping flow”

$$Re \to 0$$

Leito poroso (filtração, solo)

$$-\frac{1}{Ru}\,\mathrm{gr\hat{\vec{a}}d}\,\hat{P} + \frac{1}{Re}\,\mathrm{l\hat{a}p}\,\hat{\vec{v}} = 0$$

Incompressível

$$\mathrm{l\hat{a}p}\,\hat{P} = 0$$

Equação da Continuidade para espécie A:

$$\frac{L}{v_0 t_0}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\mathrm{gr\hat{a}d}\;\hat{\varphi}\right) = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow$$

$\varphi = \omega_A$ *fração mássica* $\hat{\varphi} = \hat{\omega}_A = \dfrac{\omega_A - \omega_{A0}}{\omega_{AS} - \omega_{A0}}$

Reação química

Equação Cinética:

$$\dot{\sigma}_{\forall T} = r_A \quad \left(\frac{\mathrm{kg\,de\,A}}{\mathrm{m^3 s}}\right)$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \mathrm{di\hat{v}}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{\omega}_A - \frac{1}{Pe}\,\mathrm{gr\hat{a}d}\;\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

Equação da Continuidade para espécie A – Escalas de tempos:

$$\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{D_{AB}}{v_0 L}\,\hat{\mathrm{grad}}\;\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{D_{AB}}{v_0 L}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{v_0}{L}\hat{\mathrm{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{D_{AB}}{L^2}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{r_A}{\rho_0\,\Delta\omega_A}$$

$$\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{1}{t_C}\hat{\mathrm{div}}\,\hat{\rho}\hat{\vec{v}}\hat{\omega}_A = \frac{1}{t_D}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{1}{t_R}$$

Convecção

$$t_C = \frac{L}{v_0}$$

Difusão

$$t_D = \frac{L^2}{D_{AB}}$$

Reação

$$t_R = \frac{\rho_0\,\Delta\omega_A}{r_A}$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\,\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\,\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\,\hat{\omega}_A \right) = \frac{r_A L}{\rho_0\, \Delta\omega_A\, v_0}$$

$$\frac{1}{Pe} = \frac{L}{v_0}\frac{D_{AB}}{L^2} = \frac{t_C}{t_D}$$

$$\frac{r_A L}{\rho_0\, \Delta\omega_A\, v_0} = \frac{t_C}{t_R} = Da_1$$

DAMKÖHLER 1

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\,\hat{\omega}_A \right) = Da_1$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\;\hat{\omega}_A\right) = \frac{r_A L}{\rho_0 \Delta\omega_A v_0}$$

x Pe:

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(Pe\;\hat{\vec{v}}\hat{\omega}_A - \hat{grad}\;\hat{\omega}_A\right) = \frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}}$$

Tempo de Reação:

$$t_R = \frac{\rho_0 \Delta\omega_A}{r_A}$$

Tempo de Difusão:

$$t_D = \frac{L^2}{D_{AB}}$$

$$\frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}} = \frac{t_D}{t_R} = Da_2$$

DAMKÖHLER 2

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(Pe\;\hat{\vec{v}}\,\hat{\omega}_A - \hat{grad}\;\hat{\omega}_A\right) = Da_2$$

Reação/difusão em poros de catalisador sólido –

Módulo de Thiele = Th = $(Da_2)^{0,5}$

Coeficientes Convectivos

ADIMENSIONALIZAÇÃO

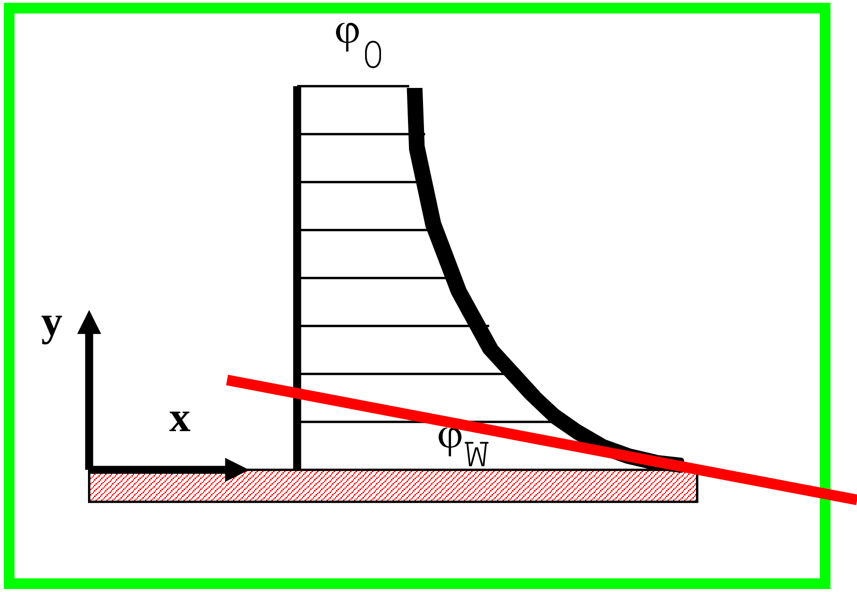
$$\frac{1}{\mathrm{Sr}}\frac{\partial\hat{\rho}\hat{\varphi}}{\partial\hat{t}}+\mathrm{div}\,\hat{\rho}\underbrace{\left(\hat{\vec{v}}\hat{\varphi}-\frac{1}{\mathrm{Pe}}\,\mathrm{gr\hat{a}d}\,\hat{\varphi}\right)}=\frac{\dot{\sigma}_{\forall_\Phi}L}{\rho_0\,\Delta\varphi\,v_0}$$

ESCOAMENTO + DIFUSÃO

INTERFACE / PAREDE

COEFICIENTE DE CONVECÇÃO

$$\vec{j}_{\phi,\,parede}=CO_{\varphi}\left(\varphi_S-\varphi_0\right)$$

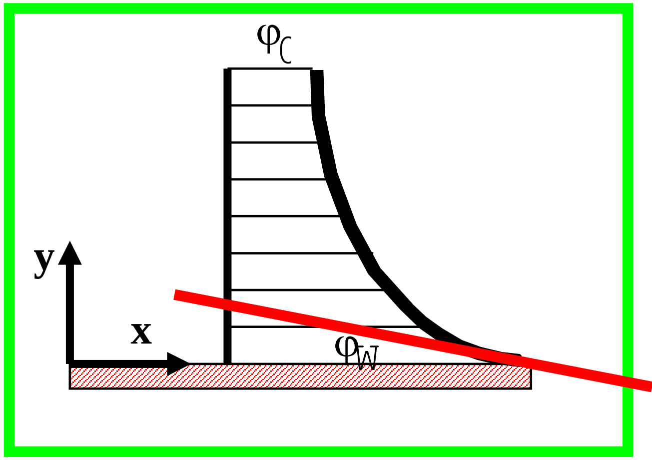


grad φ

Coeficientes Convectivos

ADIMENSIONAIS

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\underbrace{\left(\hat{\vec{v}}\hat{\varphi} - \frac{1}{Pe}\,\mathrm{gr\hat{a}d}\;\hat{\varphi}\right)}_{NA\ PAREDE\ \hat{\vec{v}}=0} = \frac{\dot{\sigma}_{\forall_\Phi} L}{\rho_0\,\Delta\varphi\, v_0}$$



grad φ

NUSSELT

$$Nu = \left(\mathrm{gr\hat{\vec{a}}d}\,\hat{T}\right)_{\hat{r}=0} = \frac{h\,L}{k}$$

CALOR

SHERWOOD

$$Sh = \left(\mathrm{gr\hat{\vec{a}}d}\,\hat{w}_i\right)_{\hat{r}=0} = \frac{k\,L}{D_{AB}}$$

MASSA

FATOR DE ATRITO

$$f = \frac{1}{Re}\left(\mathrm{gr\hat{\vec{a}}d}\,\hat{v}\right)_{\hat{r}=0}$$

QUANTIDADE DE MOVIMENTO

ADIMENSIONAIS –ANALOGIA de REYNOLDS
NÚMERO DE STANTON

St = NÚMERO DE STANTON

QUANTIDADE DE MOVIMENTO

$$St = \frac{f}{2}$$

FATOR DE FANNING

CALOR

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re\,Pr} = \frac{h}{\rho c_P V}$$

MASSA

$$St = \frac{Sh}{Pe} = \frac{Sh}{Re\,Sc} = \frac{k}{V}$$

ADIMENSIONAIS –ANALOGIA DE COLBURN

FATOR j

QUANTIDADE DE MOVIMENTO

$$\frac{f}{2} = j_H = j_M$$

FATOR DE FANNING

CALOR

$$j_H = \frac{Nu}{Re\,Pr^{1/3}}$$

MASSA

$$j_M = \frac{Sh}{Re\,Sc^{1/3}}$$