Shs 5896

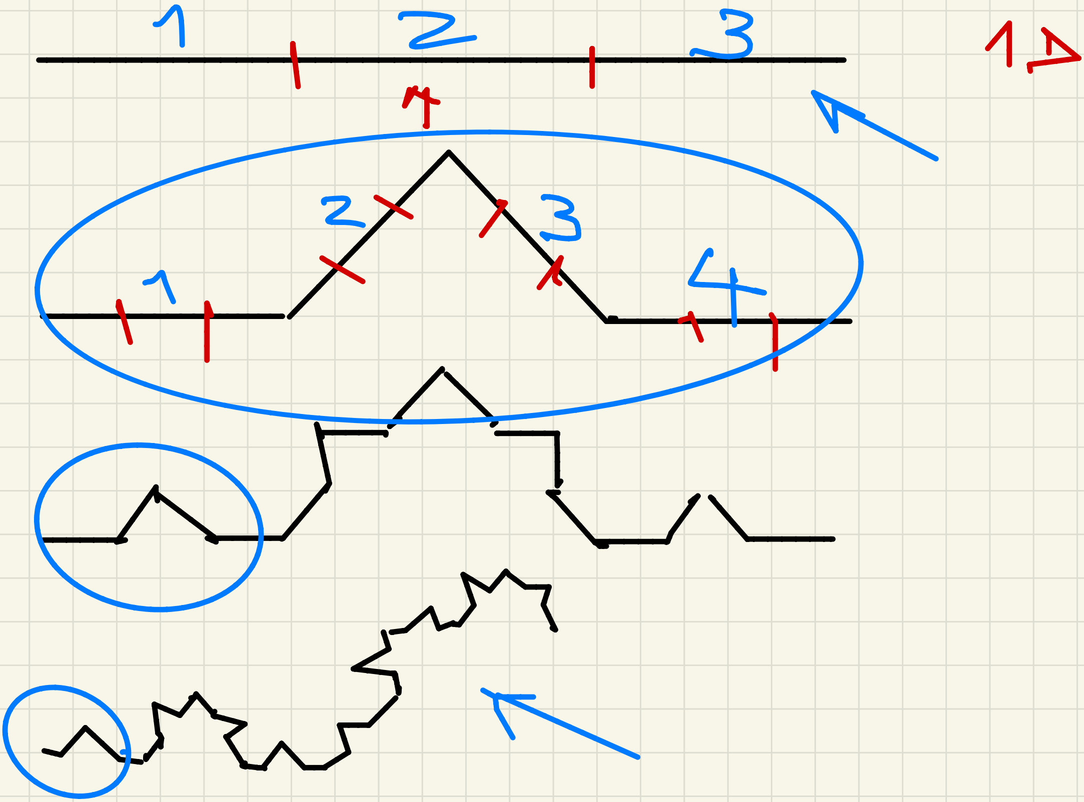
2020

Edson Wendland

EESC / USP

número fractal

auto-similaridade

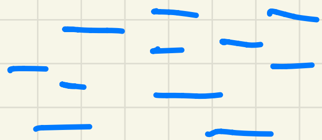


$$N = \frac{4}{3} = 1{,}33\ D$$

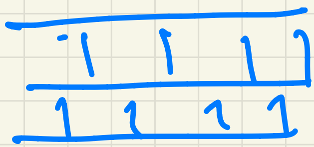
Simbología

arenosos

lutita = arcilla.



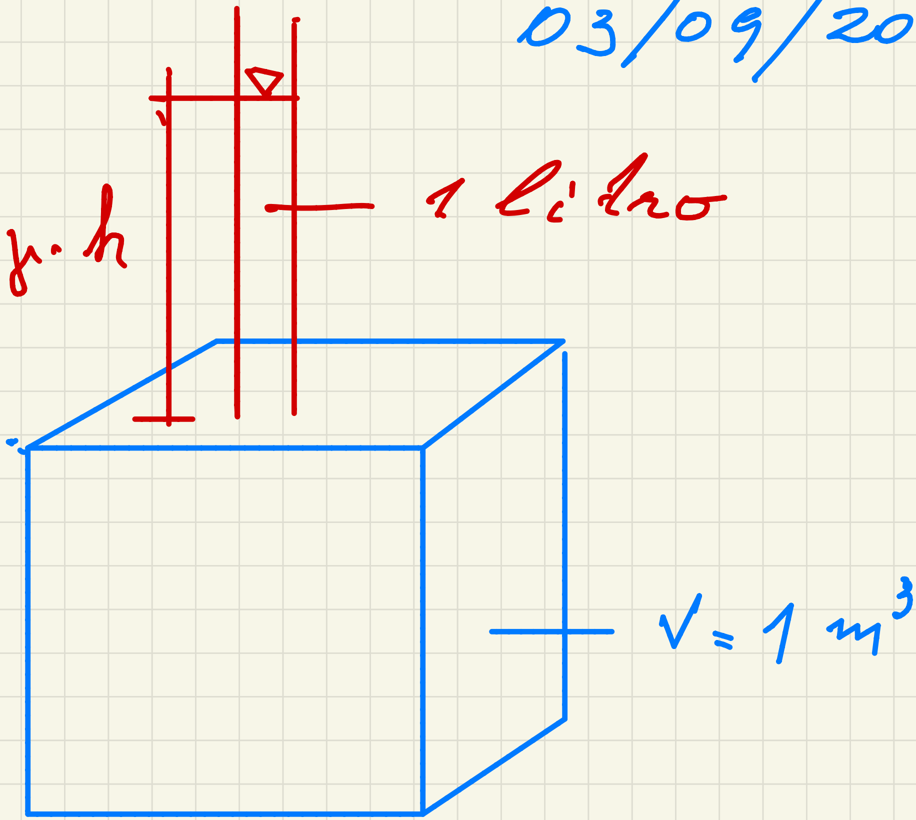
calcáreos
carbonatos



magmáticas

03/09/2020

$p = \gamma \cdot h$



$h = 20\ m \to 100\ m,\ 1000\ m$

$250\ m,\ 100\ m,\ 1000\ m$

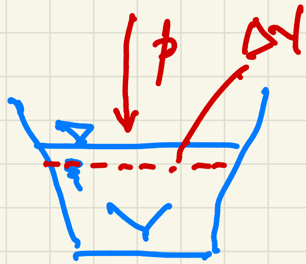
$500\ m$

$p = 1\ MPa = 10^6\ Pa$

Compressibilidade

$$\beta = 4,8 \cdot 10^{-10}\ m^2/N$$

$$\beta = -\frac{\Delta V}{V}\ \frac{1}{p}$$



$$\rightarrow\ \gamma = 9789\ N/m^3$$

$$V = 1\ m^3$$

$$\Delta V = -1\ \ell = -10^{-3}\ m^3$$

$$p = -\frac{\Delta V}{V} \cdot \frac{1}{\beta}$$

$$p = \frac{10^{-3}}{1} \cdot \frac{1}{4,8 \cdot 10^{-10}} \left[\frac{m^3}{m^3} \cdot \frac{N}{m^2}\right]$$

$$p = \frac{10 \cdot 10^{-4} \cdot 10^{10}}{4,8} \cong 2 \cdot 10^{6}\ \frac{N}{m^2}$$

$$p \cong 2\ MPa$$

$$p = \gamma \cdot h \ \therefore\ h = \frac{p}{\gamma} = \frac{2 \cdot 10^{6}}{9789} \cong 200\ mca$$

$$\dot{m}_x = \rho \cdot u_x \cdot A_x$$

$$= \frac{kg}{\cancel{m^3}} \cdot \frac{\cancel{m}}{s} \cdot \cancel{m^2}$$

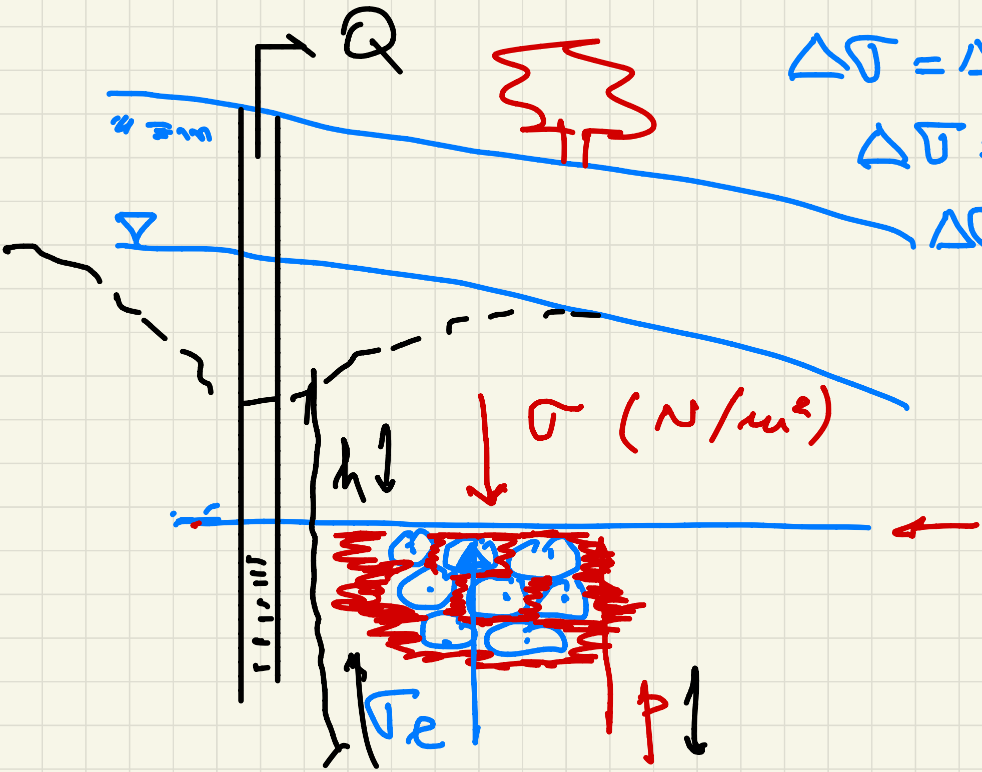
$$\dot{m}_x = kg/s \qquad \frac{\partial m}{\partial t} = \frac{kg}{s}$$

$$\sigma = \sigma_e + p$$

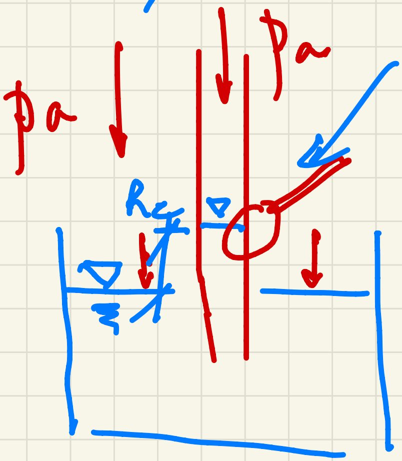
$$\Delta\sigma = \Delta\sigma_e + \Delta p$$

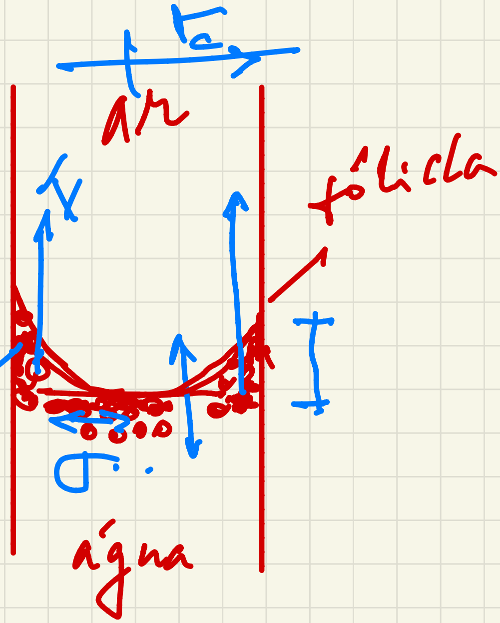
$$\Delta\sigma = 0$$

$$\Delta\sigma_e = -\Delta p$$

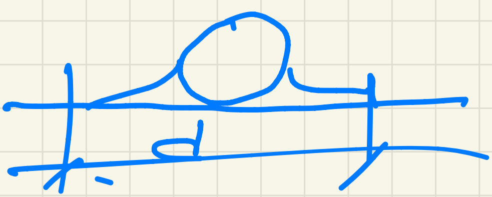


Capilaridade



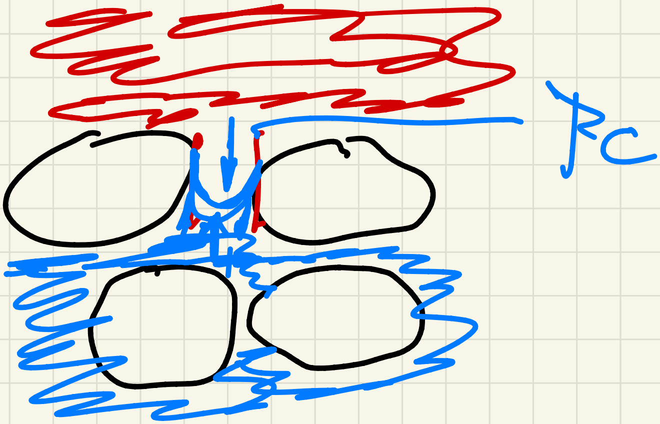


molhabilidade

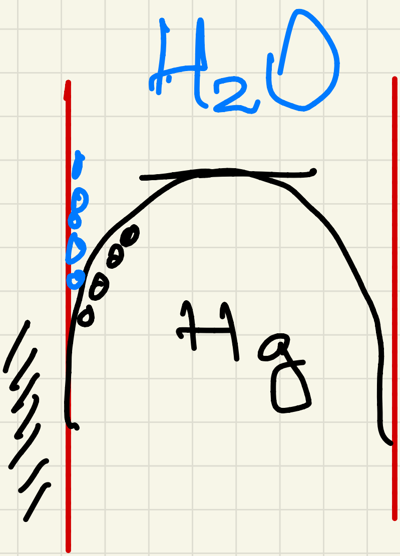


$$h_c = f(\sigma_e, t_e)$$

$$\gamma_c = \gamma_{H_2O} - \gamma_{ar}$$



$$p_c = p_a - p_0$$



EDP fluxo

10/09/2020

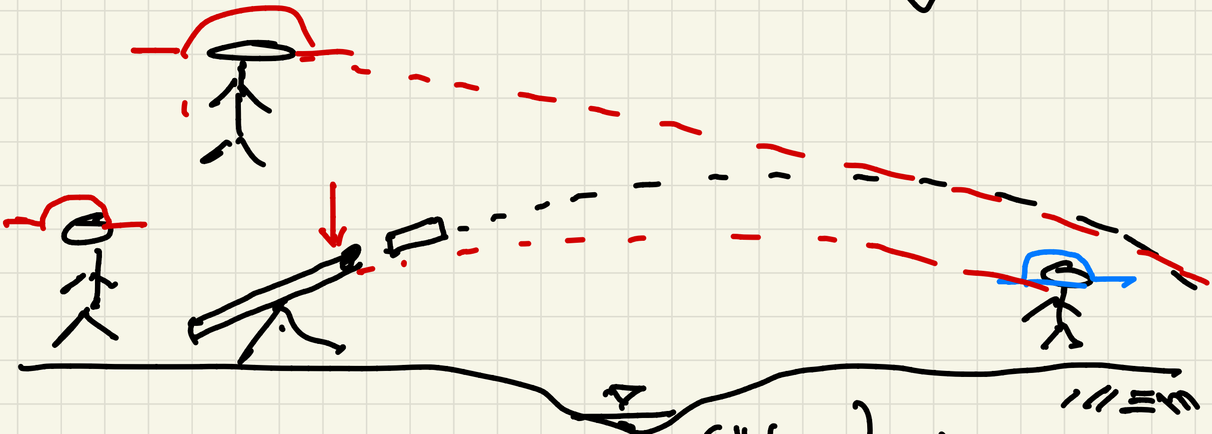
$$S_0 \frac{\partial h}{\partial t} = -\nabla (K \nabla h) + Q$$

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

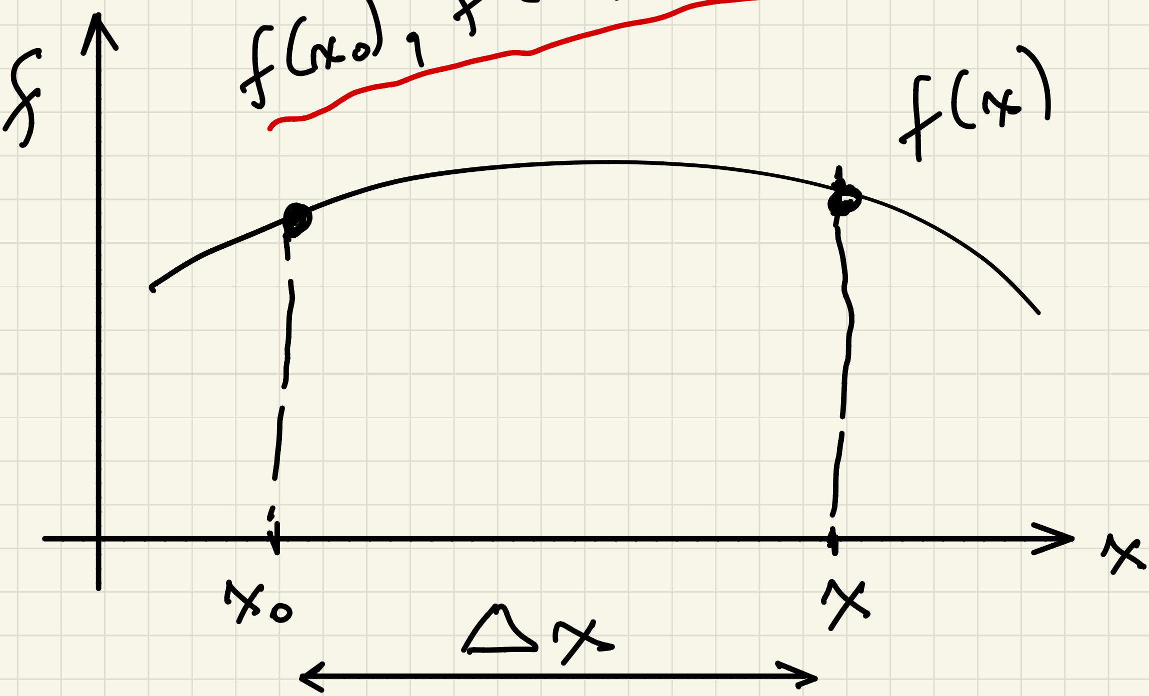
$$\bar{h} \cong \sum_{i=1}^{n} a_i \cdot \varphi^i$$

$$= a_1 \varphi^1 + a_2 \varphi^2 + a_3 \varphi^3 + \dots$$

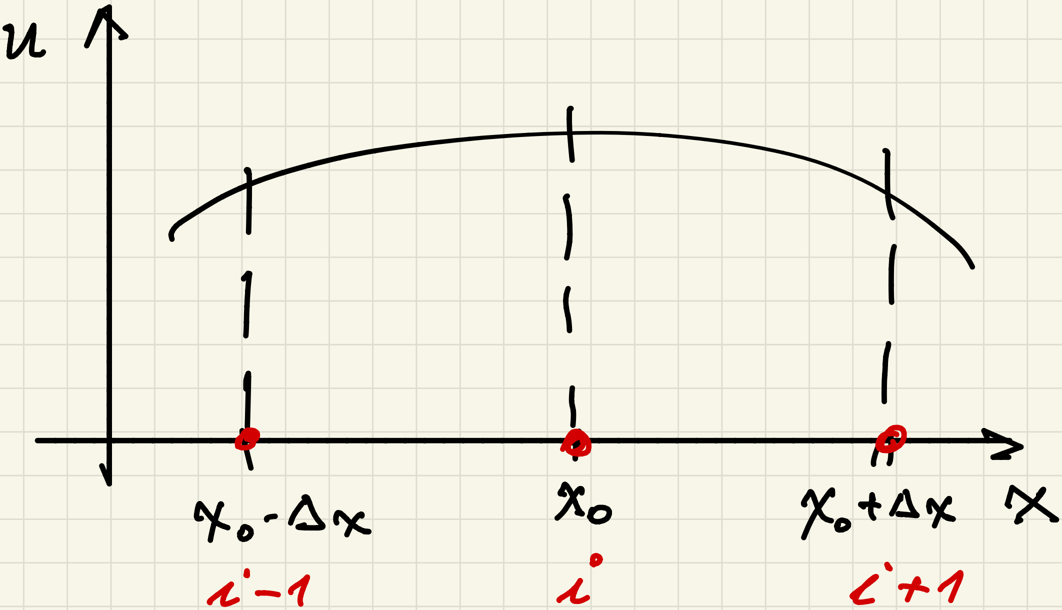
Séries de Taylor



$f(x_0),\ f'(x_0),\ f''(x_0) \ldots$



$$f(x) = f_{x_0} + f'_{x_0} \cdot \Delta x + f''_{x_0} \cdot \frac{\Delta x^2}{2!} + f'''_{x_0} \frac{\Delta x^3}{3!} + \ldots$$



$i = 1, 2, 3, 4 \ldots$

modelo mat.

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \Rightarrow u_x^{4} = u_t^{1}$$

$$\frac{\partial u}{\partial x} \cong \frac{u(x_0+\Delta x) - u(x_0)}{\Delta x} + O(\Delta x)$$

$$\frac{\partial u}{\partial x} \cong \frac{u_{i+1} - u_i}{\Delta x} + O(\Delta x)$$

$$u_t^{1} = \frac{u^{n+1} - u^{n}}{\Delta t} + O(\Delta t)$$

$$f(x_0+\Delta x) = f(x_0) + \cancel{\Delta x f'} + \frac{\Delta x^2}{2!} f'' + \dots$$

$$+\ f(x_0-\Delta x) = f_{x_0} - \cancel{\Delta x f'} + \frac{\Delta x^2}{2!} f'' - \dots$$

$$\underbrace{f_{x_0+\Delta x}}_{i+1} + \underbrace{f_{x_0-\Delta x}}_{i-1} = 2 \underbrace{f_{x_0}}_{i} + \cancel{2} \frac{\Delta x^2}{\cancel{2}!} f'' + \dots$$

$$f'' = \frac{f_{i+1} - 2 f_i + f_{i-1}}{\Delta x^2} + \frac{1}{\Delta \cancel{x^2}} \left[\frac{2 \Delta x^{\cancel{4}\,2}}{4!} f^{IV} \dots \right.$$

$$f'' = \frac{f_{i+1} - 2 f_i + f_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

central difference

$O(\Delta x^2)$

$$u_{xx} = \frac{u_{i+1} - 2 u_i + u_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

modelo mat.

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \Rightarrow u''_x = u'_t$$

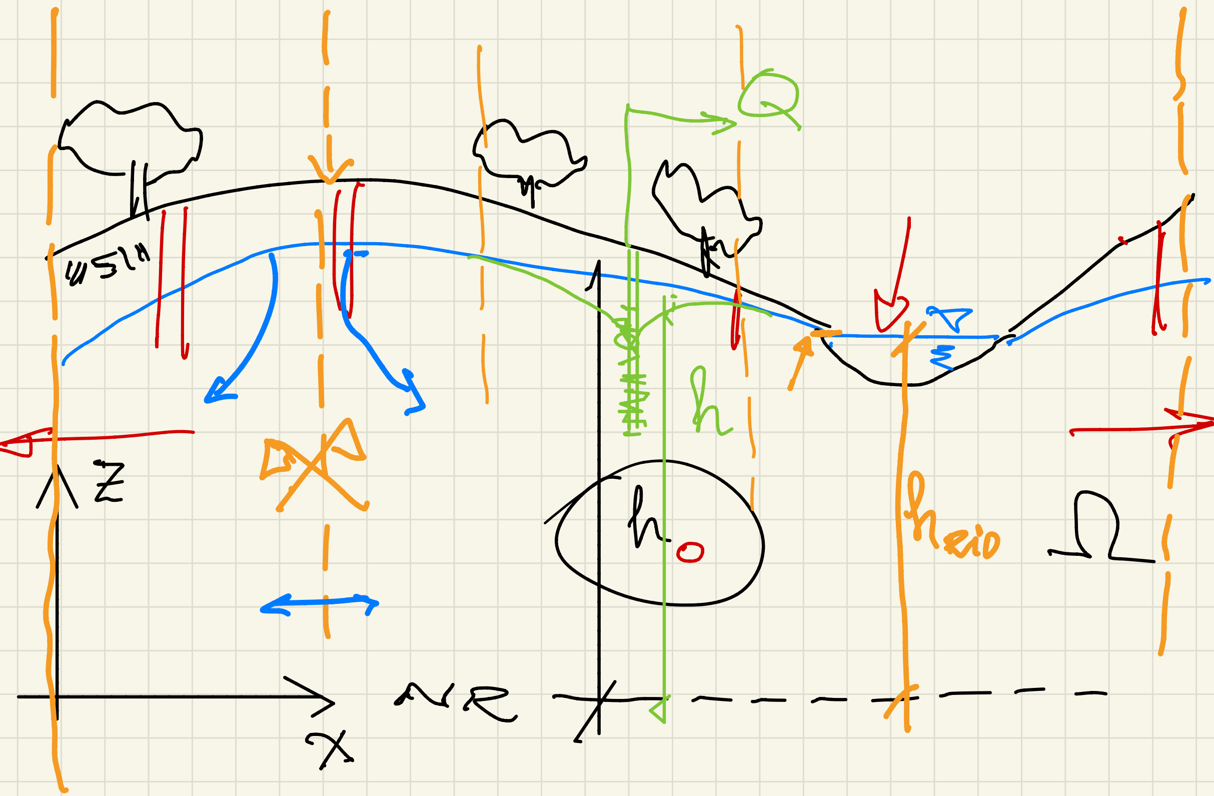
$$u''_x = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + O(\Delta x^2)$$

$$u'_t = \frac{u^{n+1} - u^n}{\Delta t} + O(\Delta t)$$

$$\boxed{\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \simeq \frac{u^{n+1} - u^n}{\Delta t} + O(\Delta x^2, \Delta t)}$$

17/09/2020

Condições de contorno



EDP

$$S_o \frac{\partial h}{\partial t} = -\nabla (K \nabla h) + Q \quad , \Omega, t$$

$$h = f(h_o, x) \quad , \forall \Omega, \quad t = t_o$$

↳ condição inicial

Condições de contorno

1 - carga hid. conhecida

$h = h_{RIO}$ → condição do 1º tipo (Dirichlet)

→ variável de interesse é conhecida

2 → derivada normal é conhecida

$$\frac{\partial h}{\partial n}$$

$q_0 = -K \frac{\partial h}{\partial n}$ ⇒ fluxo através do contorno (fronteira) é conhecido

$q_0 = 0$ (fluxo nulo)

01/10

$$S_0 \cancel{\frac{\partial h}{\partial t}} = -\nabla(K\nabla h) + \cancel{Q}$$

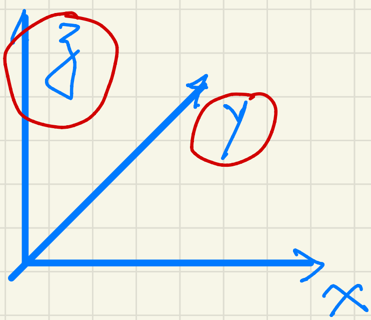
$$0 = K\nabla^2 h$$

$$\nabla^2 h = 0 \quad \text{(Eq. Laplace)}$$

$$\frac{\partial^2 h}{\partial x^2} + \cancel{\frac{\partial^2 h}{\partial y^2}} + \cancel{\frac{\partial^2 h}{\partial z^2}} = 0$$

$q_z = 0$

$q_y = 0$



$$\boxed{\frac{\partial^2 h}{\partial x^2} = 0}$$

EDO

$$\frac{\partial^2 h}{\partial x^2} = 0 \quad \overset{\sim}{\Rightarrow} \quad \frac{\Delta^2 h}{\Delta x^2} = 0$$

$$h_{i-1} - 2h_i + h_{i+1} = 0 \Leftarrow$$

$i=2$ $$h_1 - 2h_2 + h_3 = 0$$

$$+2h_2 = +h_1 + h_3$$

$$h_2 = \frac{h_1 + h_3}{2}$$

$$h_1 = 50\,m$$

$$h_2 = \frac{50 + h_3}{2}$$

$$\boxed{h_i^{v} = \frac{h_{i-1}^{v-1} + h_{i+1}^{v-1}}{2}}$$ Jacobi

Iteração $v = 1$

$$h_3^{v} = \frac{h_2^{v-1} + h_4^{v-1}}{2}$$

$$\boxed{h_i^{\nu} = \frac{h_{i-1}^{\nu-1} + h_{i+1}^{\nu-1}}{2}}$$ Jacobi

$$h_i^{\nu} = \frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2}$$

Gauss - Seidel

SOR (Successive Over-relaxation)

p/ nó i $\qquad \Delta_i = h_i^{\nu} - h_i^{\nu-1}$

$$h_i^{\nu} = h_i^{\nu-1} + \omega\,\Delta$$

$\omega \rightarrow$ coef. de relaxação

$$h_i^{\nu} = h_i^{\nu-1} + \omega\left(h_i^{\nu} - h_i^{\nu-1}\right)$$

$$h_i^{\nu} = \underline{h_i^{\nu-1}} + \underline{\omega}\left(h_i^{\nu} - \underline{h_i^{\nu-1}}\right)$$

$$h_i^{\nu} = h_i^{\nu-1}(1-\omega) + \omega \cdot \underline{h_i^{\nu}}$$

Gauss - Seidel

$$h_i^{\nu} = \frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2}$$

$$\boxed{h_i^{\nu} = (1-\omega)\, h_i^{\nu-1} + \omega\left(\frac{h_{i-1}^{\nu} + h_{i+1}^{\nu-1}}{2}\right)}$$

SOR

$\omega > 1 \Rightarrow$ over relaxat.

$\omega < 1 \Rightarrow$ under relaxation

$\omega = 1 =$ Gauss-Seidel