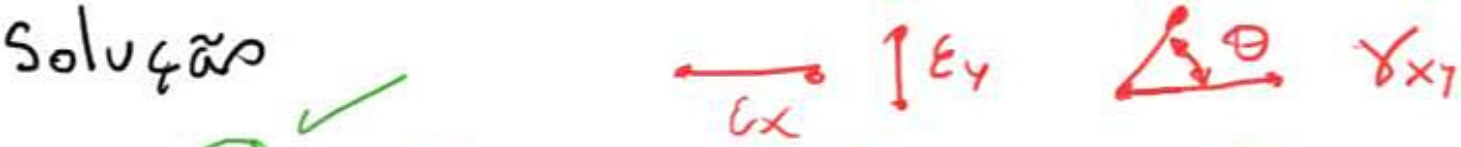
Solução



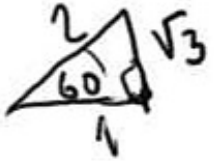
$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta \cos\theta_a$$

$$60\times10^{-6} = \varepsilon_x \cos^2 0 + \varepsilon_y \sin^2 0 + \gamma_{xy} \sin 0 \cos 0$$

$$135\times10^{-6} = \varepsilon_x \cos^2 60 + \varepsilon_y \sin^2 60 + \gamma_{xy} \sin 60 \cos 60$$

$$264\times10^{-6} = \varepsilon_x \cos^2 120 + \varepsilon_y \sin^2 120 + \gamma_{xy} \sin 120 \cos 120$$

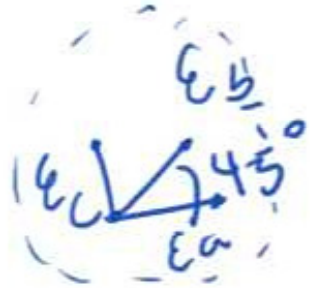
$$10^{-6}\begin{bmatrix}60\\135\\264\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 1/4 & 3/4 & \sqrt{3}/4\\ 1/4 & 3/4 & -\sqrt{3}/4\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \rightarrow \varepsilon_x = 60\times10^{-6}$$



$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 2/3 & 2/3 \\ 0 & 1{,}154 & -1{,}154 \end{bmatrix} \begin{bmatrix} 60 \\ 135 \\ 264 \end{bmatrix} .10^{-6}$$

$$\downarrow$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 60 \times 10^{-6} \\ 246 \times 10^{-6} \\ -150 \times 10^{-6} \end{bmatrix}$$



ROSETA

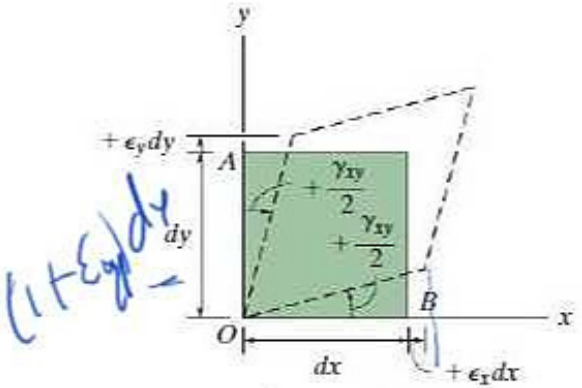
45° → $\varepsilon_x = \varepsilon_a$

$\varepsilon_y = \varepsilon_c$

$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_c + \varepsilon_a)$

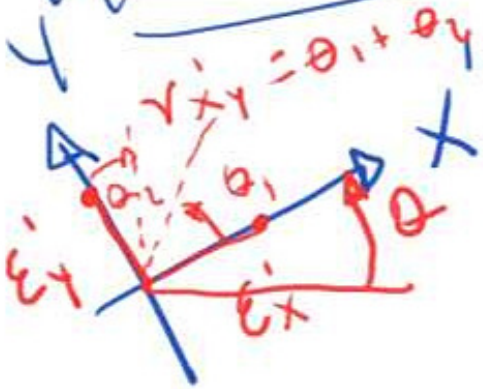
60° → $\varepsilon_x = \varepsilon_a$; $\varepsilon_y = \frac{1}{3}(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a)$; $\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$

• CÍRCULO DE MOHR

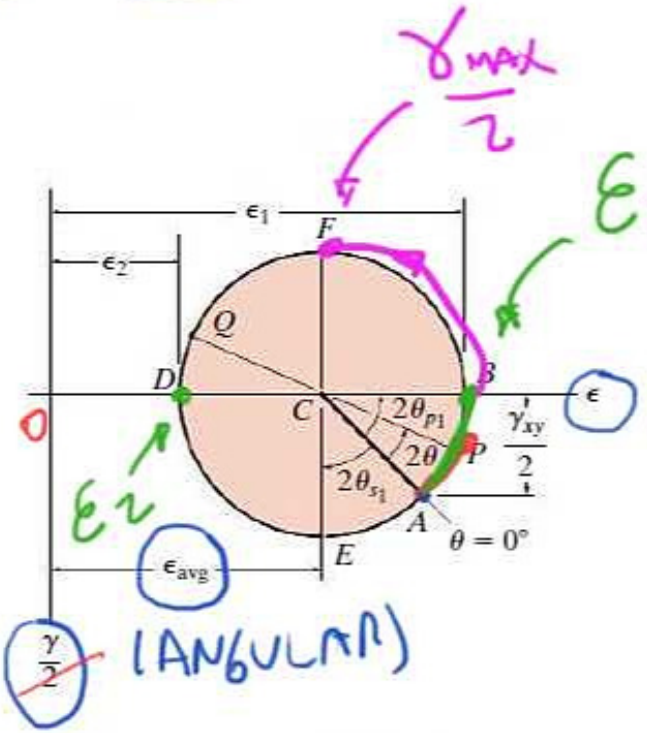


$(1+\epsilon_y)dy$

(+)



$(1+\epsilon_x)dx$



$\gamma_{MAX}/2$

ε_1

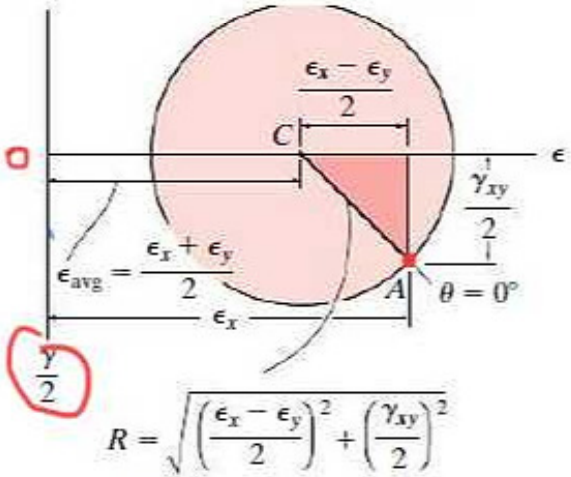
ε_2

(NORMAL)

(ANGULAR)

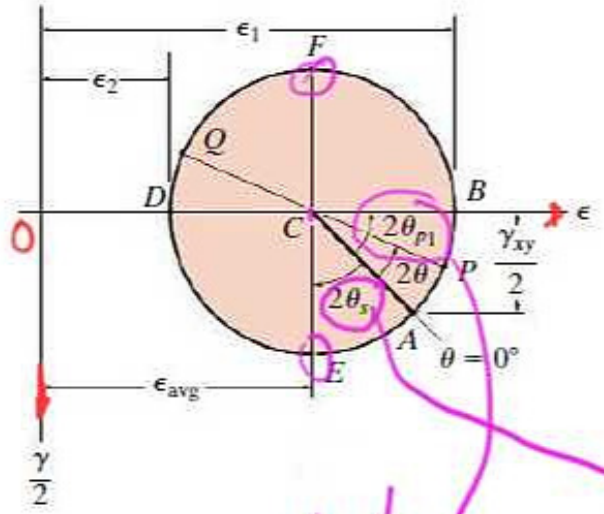
$(\varepsilon_x, \gamma_{xy}/2)$

$(\varepsilon_y, \gamma_{xy}/2)$

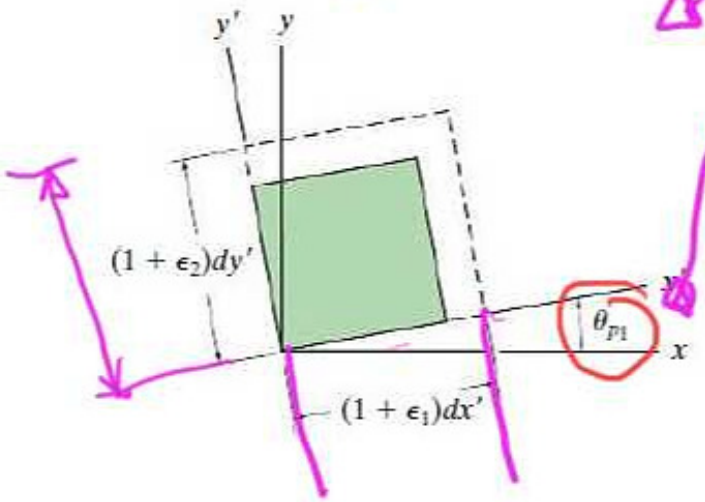


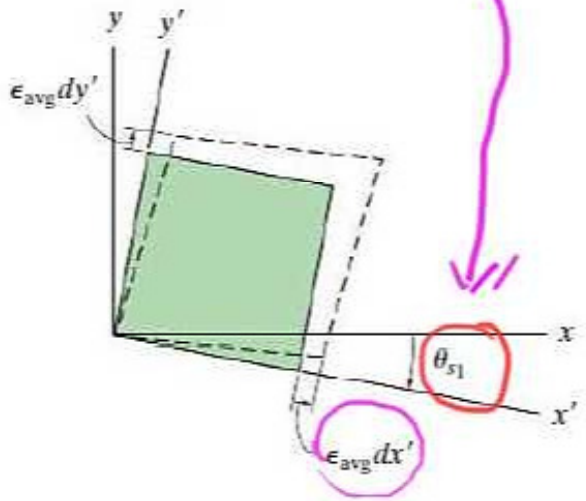
$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_x & \gamma_{xy}/2 & \gamma_{xz}/2 \\ & \varepsilon_y & \gamma_{yz}/2 \\ & & \varepsilon_z \end{bmatrix}$$

· CÍRCULO DE MOHR

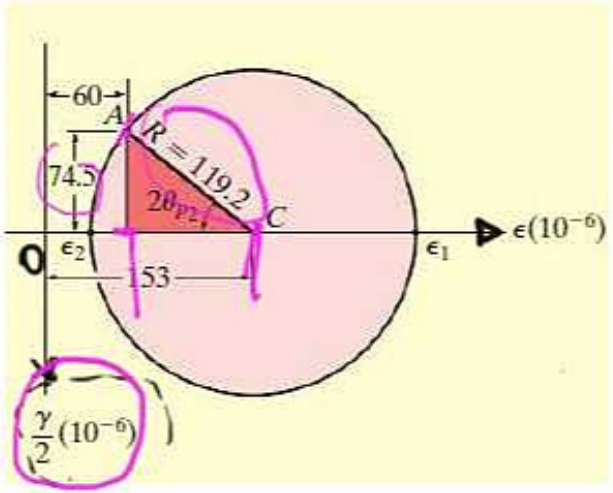


θ não é const.





· CÍRCULO DE MOHR (STRAIN)



$\varepsilon_x = 60 \times 10^{-6}$

$\varepsilon_y = 246 \times 10^{-6}$

$\gamma_{xy} = -149 \times 10^{-6}$

$A \equiv (\varepsilon_x, \gamma_{xy}) \rightarrow$ Centro Círculo

$A' \equiv (\varepsilon_y, -\gamma_{xy})$

$\varepsilon_{average} = \frac{\varepsilon_x + \varepsilon_y}{2}$

· RAIO $= \sqrt{(153-60)^2 + 74,5^2}$

RAIO $= 119.1 \times 10^{-6}$ //

$\varepsilon_{average} = \frac{60+246}{2} = 153\,\mu$

· Deformações principais (linear)

$$\varepsilon_{1,2} = R \pm \varepsilon_{average}$$

$$\varepsilon_1 = R + \varepsilon_{average} \qquad \varepsilon_2 = \varepsilon_{average} - R$$

$$\varepsilon_1 = 119.4 + 153 \ (10^{-6}) \qquad \varepsilon_2 = 153 - 119.1$$

$$\varepsilon_1 = 272 \times 10^{-6} \qquad \varepsilon_2 = 33{,}9 \times 10^{-6}$$

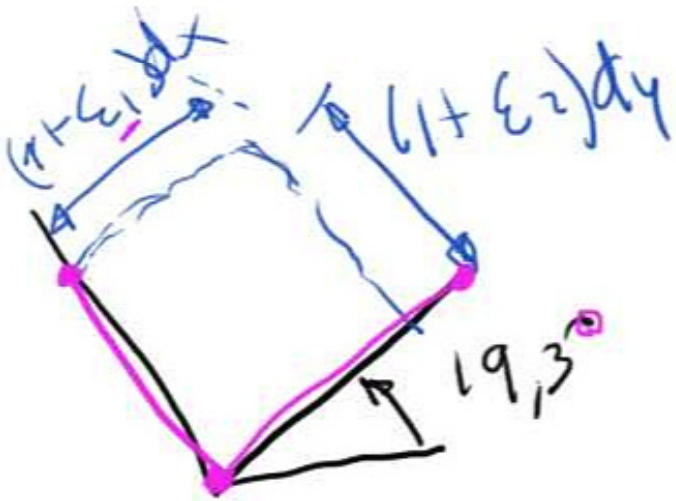
Def. plana

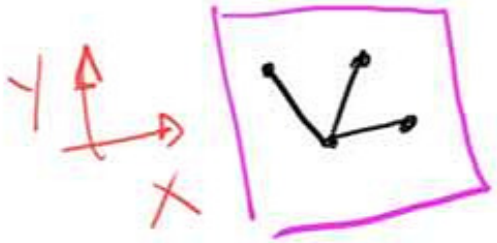
$$tg\,2\theta_{p_2} = \frac{\frac{\gamma_{xy}}{2}}{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)} = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$Tg\,2\theta_{p2} = \frac{74,5}{(133-60)}$$

$$2\theta_{p_2} = 38,7°$$

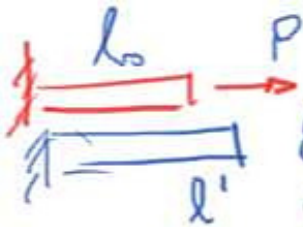
$$\theta_{p_2} = 19,3°$$





ε_x ✓
ε_y ✓
γ_{xy}

$\varepsilon_z \neq 0$?



ε_x ✓ $\neq 0$
ε_y ✓ $\neq 0$
ε_z

. Deformação ε_z (fora do plano) out-of-plane

$\equiv$

(LEI DE HOOKE) 3D

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[\underbrace{(1-\nu)\varepsilon_z + \nu(\varepsilon_x+\varepsilon_y)}_{=0}\right]$$

$\downarrow$

na superfície $(\sigma_z = 0)$

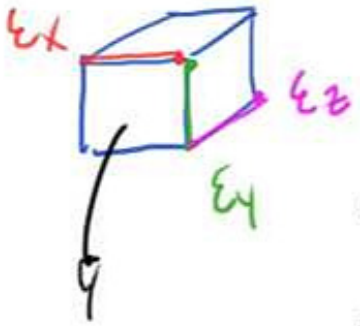
$\downarrow$

$$\varepsilon_z = \left(\frac{\nu}{1-\nu}\right)\underbrace{(\varepsilon_x+\varepsilon_y)}_{\text{no plano}}$$

$\equiv$

Invariante!

$I_1 = \sigma_x + \sigma_y + \sigma_z$



$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$$

$\varepsilon_x' + \varepsilon_y'$

$$\varepsilon_z = \frac{0.3}{1-0.3}\left(60 + 246\right) 10^{-6}$$

$$\varepsilon_z = \frac{0.3}{0.7}\left(303\right) \times 10^{-6}$$

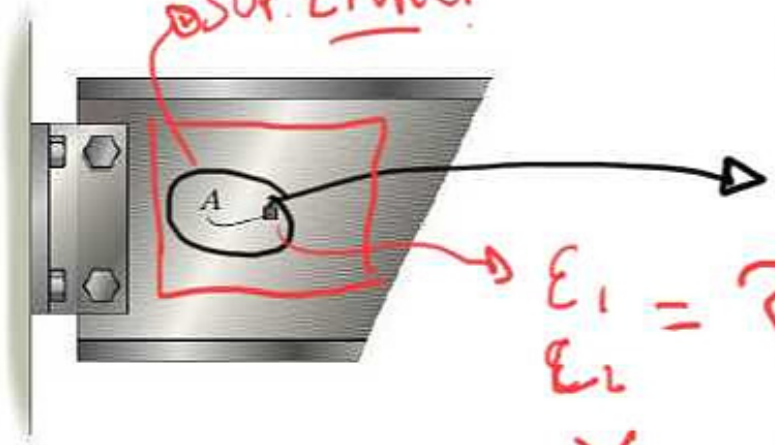
Uma roseta 60° é montada em uma viga. As seguintes leituras foram obtidas devido ao carregamento aplicado:

epsilon_a = 60 10^-6
epslion_b = 135 10^-6
epslion_c = 264 10^-6

← ESFORÇOS EXTERNOS

Determine:
a) Deformações principais no plano
b)Deformações angulares máximas
c) Direções principais

} → CÍRCULO MOHR

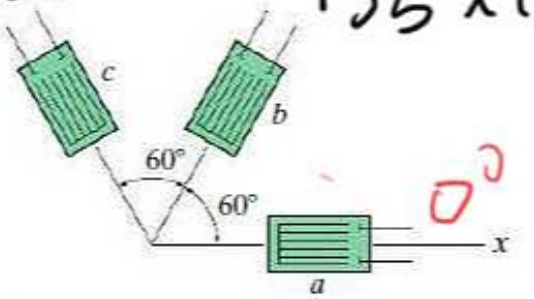


DSUP. LIVRE!

$\varepsilon_1 = ?$
ε_2
$\gamma_{max} = ?$

264×10^{-6}

133×10^{-6}



60 μstrains

VIGA + CARREGAMENTOS