On $\gamma_R \Rightarrow f_2(z) = \dfrac{R^{-a}e^{-ia\theta}}{Re^{i\theta}+1}$; $\theta_0 \leq \theta \leq 2\pi$

On $\gamma_\rho \Rightarrow f_2(z) = \dfrac{\rho^{-a}e^{-ia\theta}}{\rho e^{i\theta}+1}$; $\theta_0 \leq \theta \leq 2\pi$

On $L \Rightarrow f_2(z) = \dfrac{r^{-a}e^{-ia\theta_0}}{re^{i\theta_0}+1}$; $\theta = \theta_0$; $\rho \leq r \leq R$

On $z = x \Rightarrow f_2(z) = \dfrac{r^{-a}e^{-ia2\pi}}{r+1}$; $\theta = 2\pi$; $\rho \leq r \leq R$, $dz = dr$

Besides, the residue theorem gives: $\displaystyle\int_{C_2} f_2(z)dz = 0$

$$\int_{C_2} f_2(z)dz = \int_{\gamma_R} f_2(z)dz - \int_\rho^R \frac{r^{-a}e^{-ia2\pi}}{r+1}dr + \int_{\gamma_\rho} f_2(z)dz - \int_L f_2(z)dz = 0$$

c) On analysing the behavior of each branch $f_1(z)$ and $f_2(z)$ over each portion of the contours, it is pretty obvious that such behavior is promptly replicated by the proposed branch

$$f(z) = \frac{z^{-a}}{z+1}, \quad z = re^{i\theta};\ |z|>0;\ 0<\theta<2\pi$$

Then on adding up the two integrals, we get

$$\int f(z)dz = \int f_1(z)dz + \int f_2(z)dz$$

$$\int_\rho^R \frac{r^{-a}}{r+1}dr + \underbrace{\int_{C_R} f(z)dz}_{\to 0\ (R\to\infty)} + \cancel{\int_L f(z)dz} + \underbrace{\int_{-C_\rho} f(z)dz}_{\to 0\ (\rho\to 0)} - \int_\rho^R \frac{r^{-a}e^{-ia2\pi}}{r+1}dr - \cancel{\int_L f(z)dz} =$$

$$= 2\pi i e^{-ia\pi}$$

$$\int_0^\infty \frac{r^{-a}}{r+1}dr = \frac{2\pi i e^{-ia\pi}}{(1-e^{-ia2\pi})} = \frac{\pi}{\sin(a\pi)}$$

according to eq. (6) on page 195