Fenômenos de Transporte I

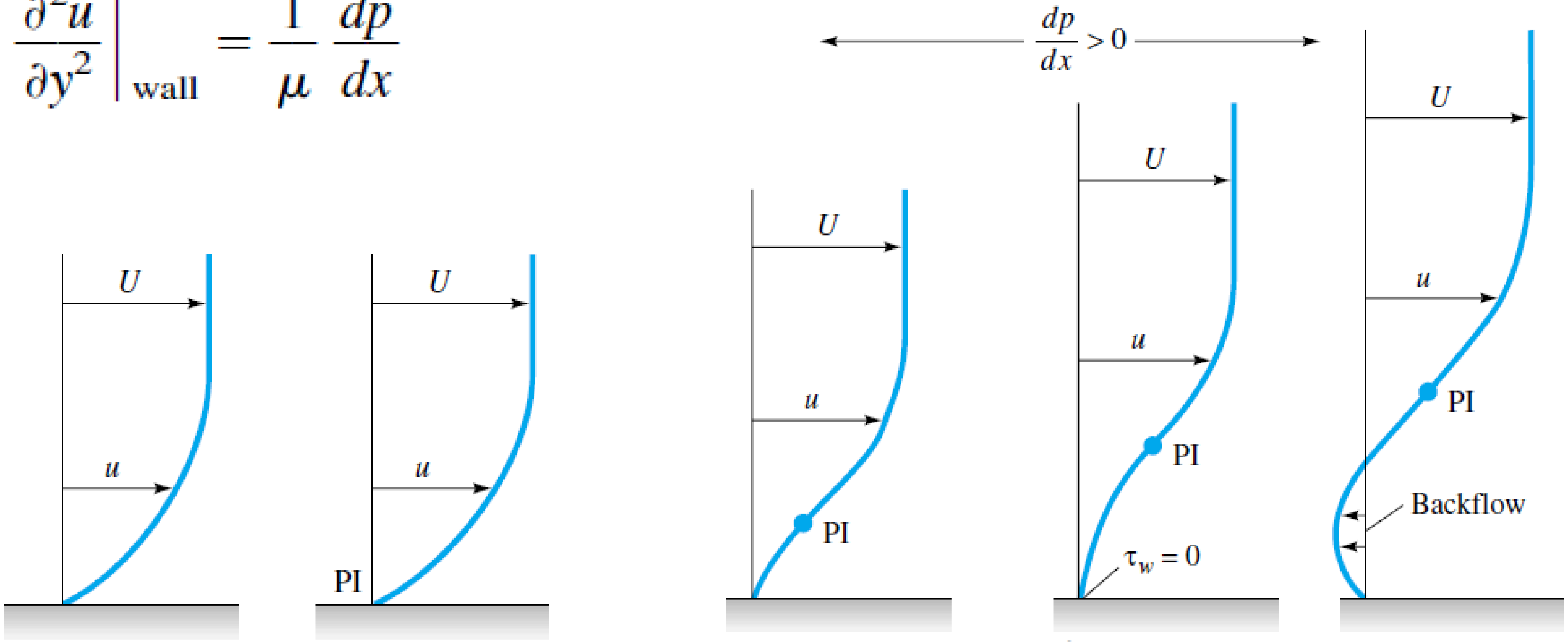
Fluid Mechanics – 4th ed. -Frank M. White

ESCOAMENTO EXTERNO

EFEITO DO GRADIENTE DE PRESSÃO NO PERFIL DE VELOCIDADES

$$\left.\frac{\partial \tau}{\partial y}\right|_{\text{wall}} = \mu \left.\frac{\partial^2 u}{\partial y^2}\right|_{\text{wall}} = -\rho U \frac{dU}{dx} = \frac{dp}{dx}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{\text{wall}} = \frac{1}{\mu}\frac{dp}{dx}$$



(*a*) Favorable gradient:

$\frac{dU}{dx} > 0$

$\frac{dp}{dx} < 0$

No separation, PI inside wall

(*b*) Zero gradient:

$\frac{dU}{dx} = 0$

$\frac{dp}{dx} = 0$

No separation, PI at wall

(*c*) Weak adverse gradient:

$\frac{dU}{dx} < 0$

$\frac{dp}{dx} > 0$

No separation, PI in the flow

(*d*) Critical adverse gradient:

Zero slope at the wall:

Separation

(*e*) Excessive adverse gradient:

Backflow at the wall:

Separated flow region

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EFEITO DO GRADIENTE DE PRESSÃO NO PERFIL DE VELOCIDADES

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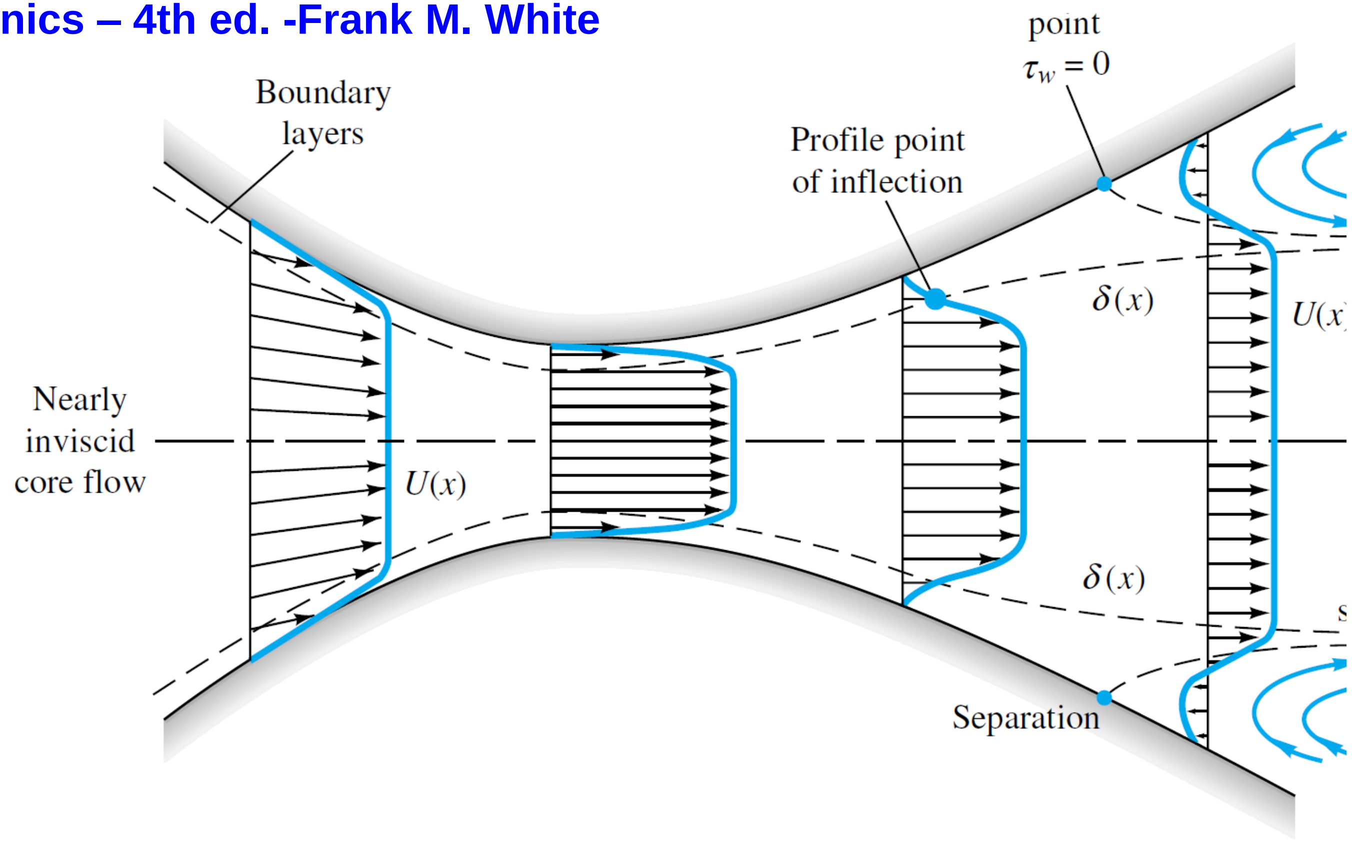

Nozzle:	*Throat*:	*Diffuser*:
Decreasing pressure and area	Constant pressure and area	Increasing pressure and area
Increasing velocity	Velocity constant	Decreasing velocity
Favorable gradient	Zero gradient	Adverse gradient (boundary layer thickens)

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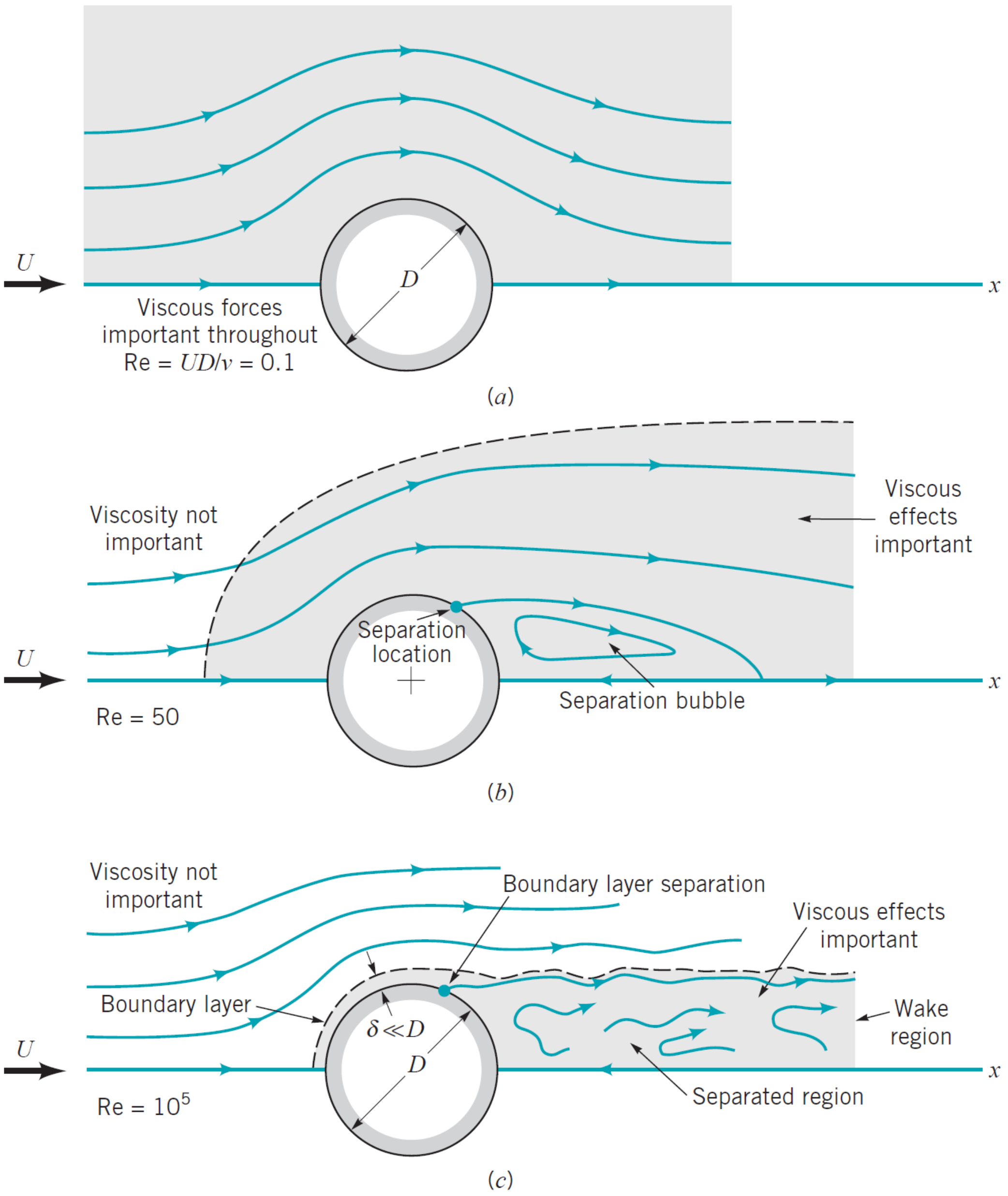
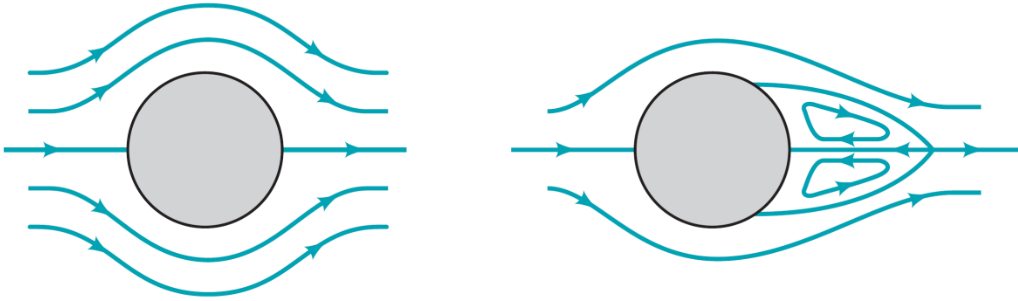


FIGURE 9.6 Character of the steady, viscous flow past a circular cylinder: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.

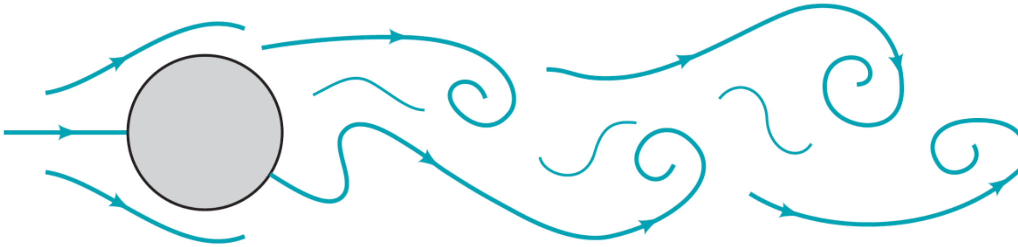


No separation

(*A*)

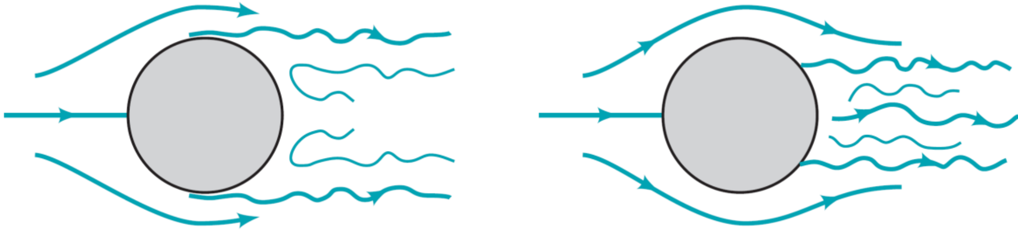
Steady separation bubble

(*B*)



Oscillating Karman vortex street wake

(*C*)



Laminar boundary layer, wide turbulent wake

(*D*)

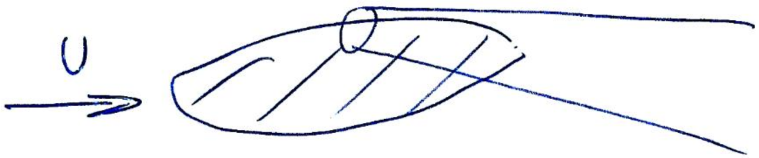
Turbulent boundary layer, narrow turbulent wake

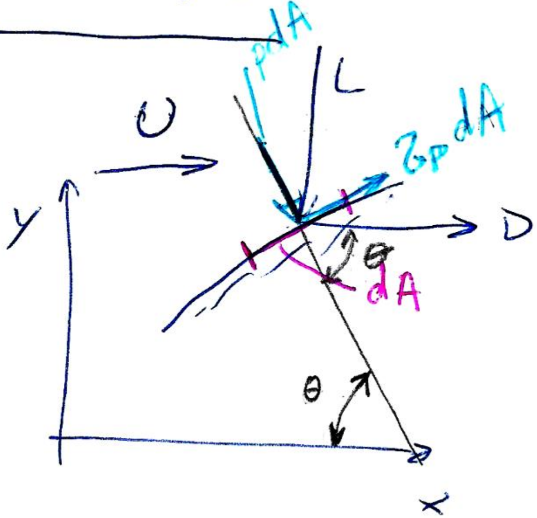
(*E*)

■ FIGURE 9.21 (*a*) Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere. (*b*) Typical flow patterns for flow past a circular cylinder at various Reynolds numbers as indicated in (*a*).

ESCOAMENTO SOBRE CORPOS IMERSOS

Arrasto e Sustentação.





$$dF_x = p\,dA(\cos\theta) + \tau_p dA \operatorname{sen}\theta$$

$$dF_y = -(p\,dA)\operatorname{sen}\theta + (\tau_p dA)\cos\theta$$

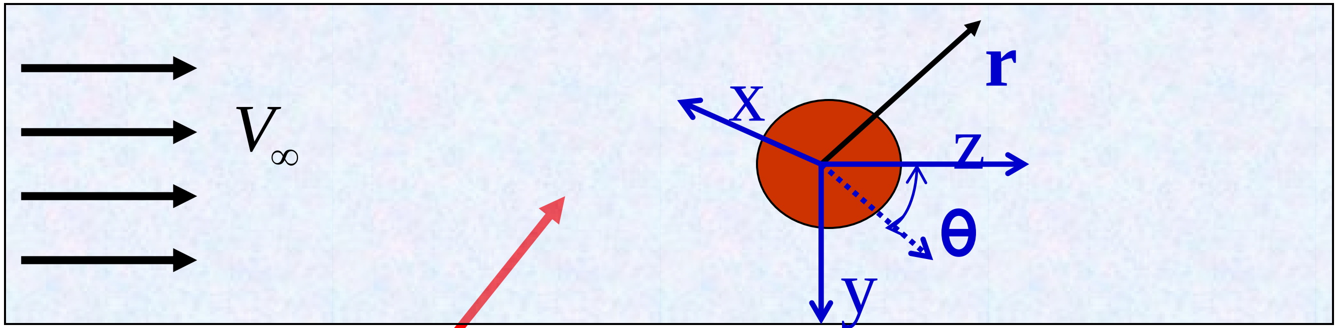
$$D = \int dF_x = \int p\cos\theta\, dA + \int \tau_p \operatorname{sen}\theta\, dA$$

$$L = \int dF_y = -\int p\operatorname{sen}\theta\, dA + \int \tau_p \cos\theta\, dA$$

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A} \quad \text{e} \quad C_D = \frac{D}{\frac{1}{\rho}\rho U^2 A}$$

A - área característica. Normalmente é a área frontal (i.e) a área projetada.

”Lei” de Stokes – Meio Contínuo



Fluido => Equação de Navier-Stokes – pressão e velocidades

$$\rho\left(\cancel{\frac{\partial\vec{V}}{\partial t}}+\cancel{\vec{V}.\nabla\vec{V}}\right)=\nabla P+\mu\nabla^2\vec{V}$$

Regime permanente

Re <<1

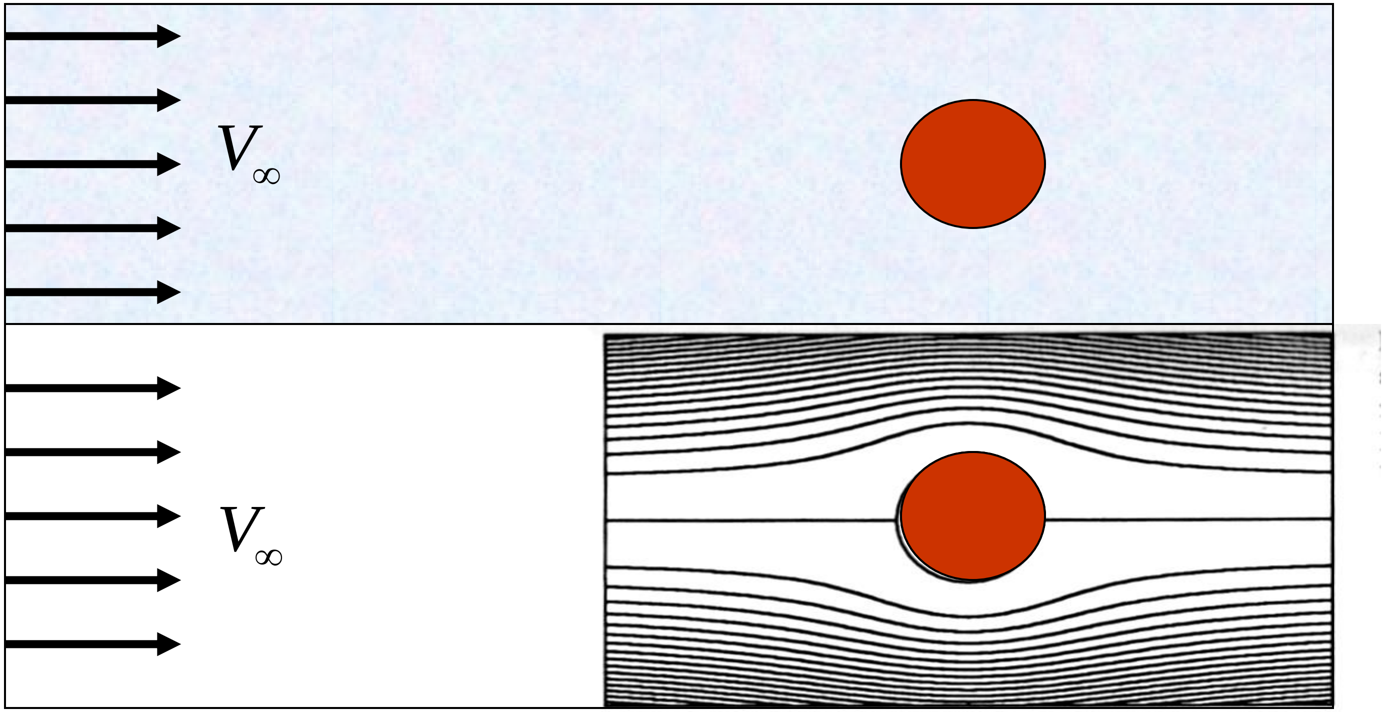
•Cond. de contorno: $\begin{cases} r\to\infty\ ,\ \vec{V}=V_\infty \\ r=R_P\ ,\ \vec{V}=0 \\ r\to\infty\ ,\ p=p_0 \end{cases}$

$$\boxed{p=p_0-\frac{3\mu V_\infty}{2R_P}\left(\frac{R_P}{r}\right)^2\cos\theta}$$

$$\boxed{V_r=V_\infty\cos\theta\left[1-\frac{3}{2}\left(\frac{R_P}{r}\right)+\frac{1}{2}\left(\frac{R_P}{r}\right)^3\right]}$$

$$\boxed{V_\theta=-V_\infty sen\theta\left[1-\frac{3}{4}\left(\frac{R_P}{r}\right)-\frac{1}{4}\left(\frac{R_P}{r}\right)^3\right]}$$

”Lei” de Stokes – Re < 1

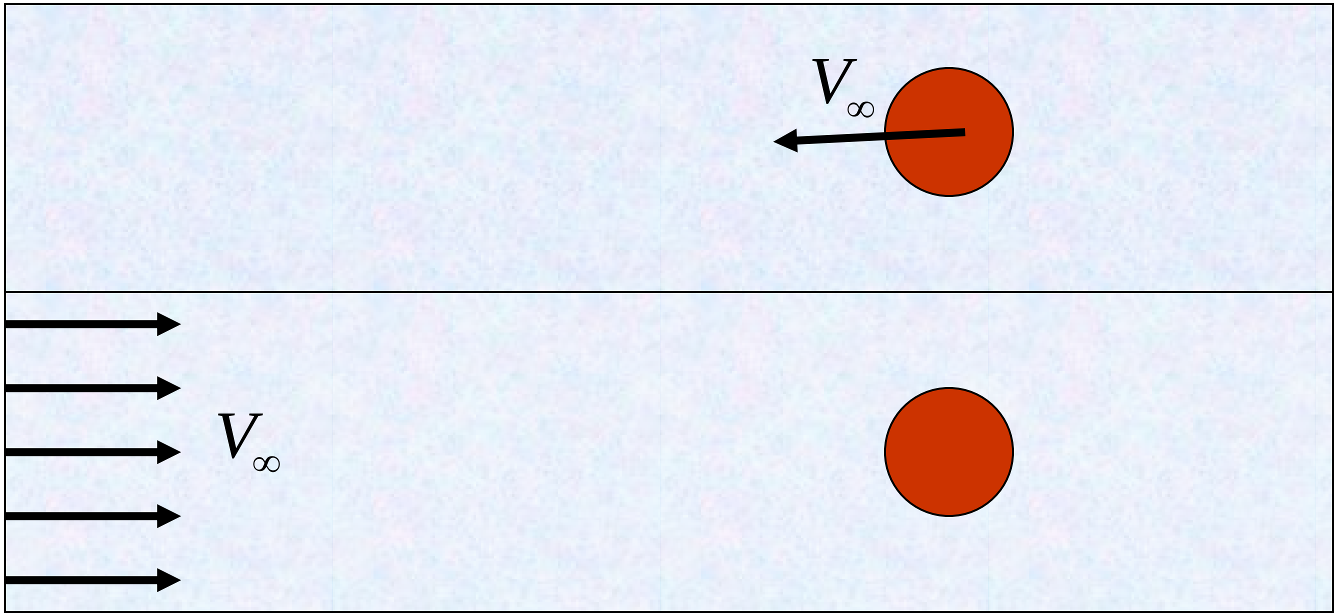


$$p = p_0 - \frac{3\mu V_\infty}{2R_P}\left(\frac{R_P}{r}\right)^2 \cos\theta$$

$$V_r = V_\infty \cos\theta\left[1 - \frac{3}{2}\left(\frac{R_P}{r}\right) + \frac{1}{2}\left(\frac{R_P}{r}\right)^3\right]$$

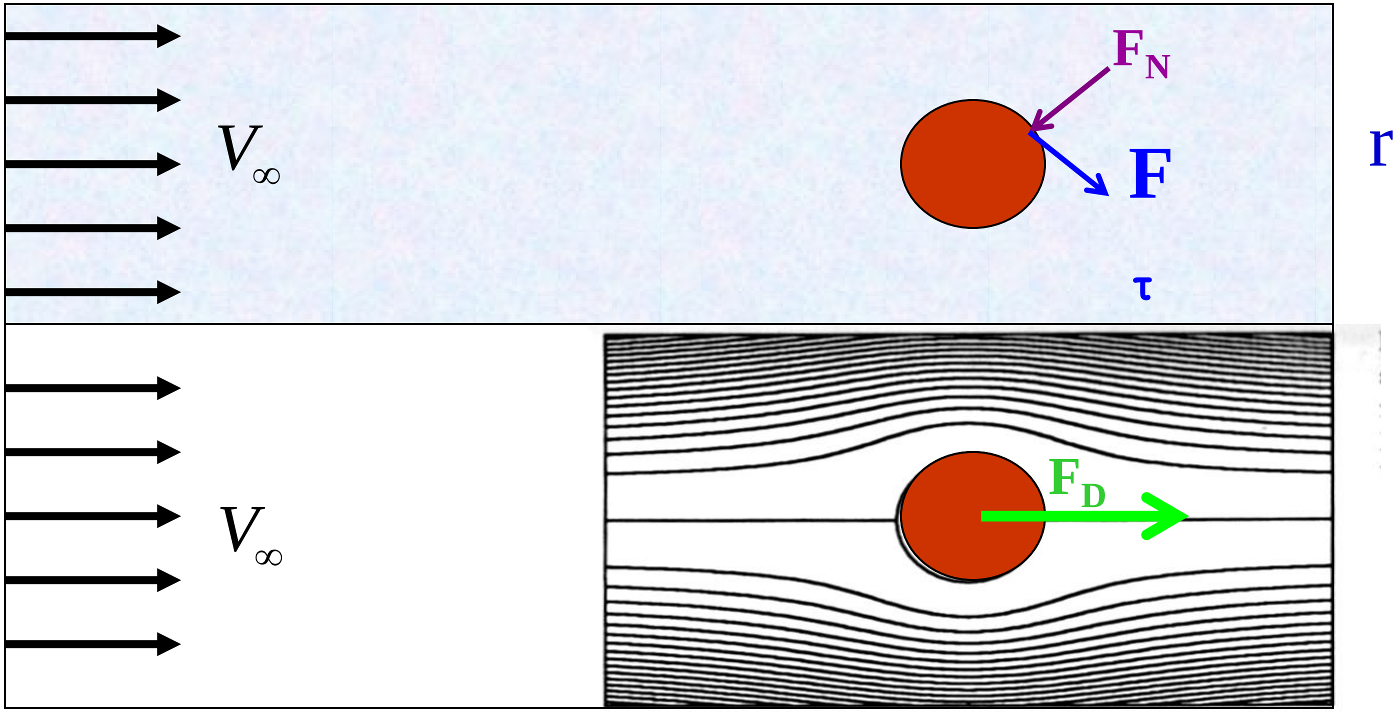
$$V_\theta = -V_\infty sen\theta\left[1 - \frac{3}{4}\left(\frac{R_P}{r}\right) - \frac{1}{4}\left(\frac{R_P}{r}\right)^3\right]$$

”Lei” de Stokes – Meio Contínuo



- Regime permanente
- Meio infinito
- Termos convectivos desconsiderados => Re <<1
- Fluido newtoniano
- Escoamento incompressível

”Lei” de Stokes – Força de Arraste (Drag)



$r = R_P$

$$\boxed{p = p_0 - \frac{3\mu V_\infty}{2R_P}\cos\theta} \Rightarrow \boxed{F_n = \int^A - p\, d\vec{A} = 2\pi\mu V_\infty R_P}$$

$$+$$

$$\boxed{\tau = \frac{3\mu V_\infty}{2R_P}\left(\frac{R_P}{r}\right)^4 sen\theta} \Rightarrow \boxed{F_\tau = \int^A \tau\, d\vec{A} = 4\pi\mu V_\infty R_P}$$

$$\boxed{F_D = 6\pi\mu V_\infty R_P}$$

Força de Arraste – Equacionamento Geral Coeficiente de Arrasto - CD

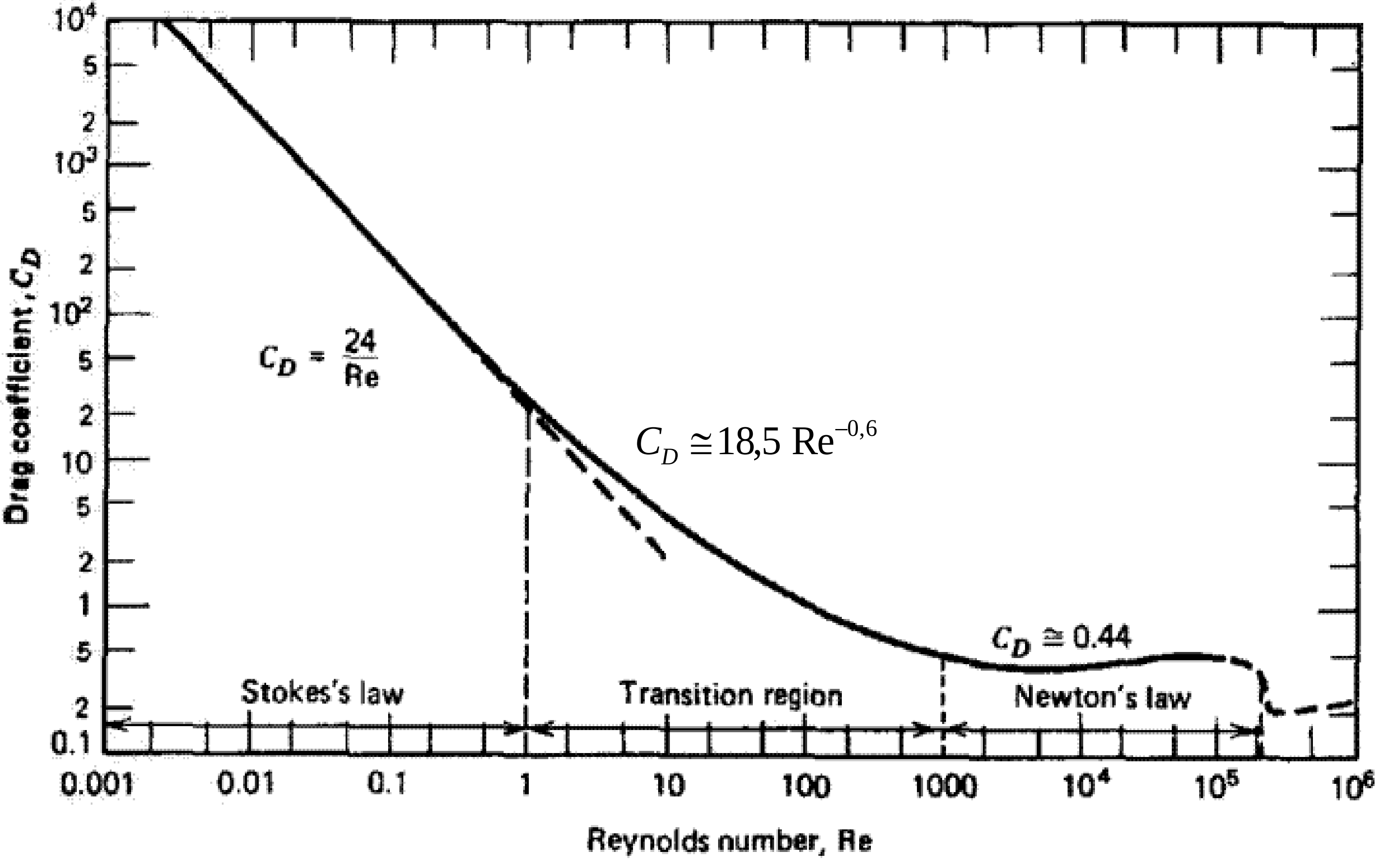
$$\boxed{C_D = \frac{F_D / A_P}{\rho V_\infty^2 / 2}}$$

$$\boxed{\Rightarrow F_D = \frac{1}{2} C_D A_P \rho V_\infty^2 = \frac{1}{8} \pi C_D \rho D_P^2 V_\infty^2}$$

$$\boxed{\mathrm{Re} = \frac{\rho V_\infty D_P}{\mu} < 1} \Rightarrow \boxed{F_D = 6\pi\mu V_\infty R_P} \Rightarrow \boxed{C_D = 24/\mathrm{Re}}$$

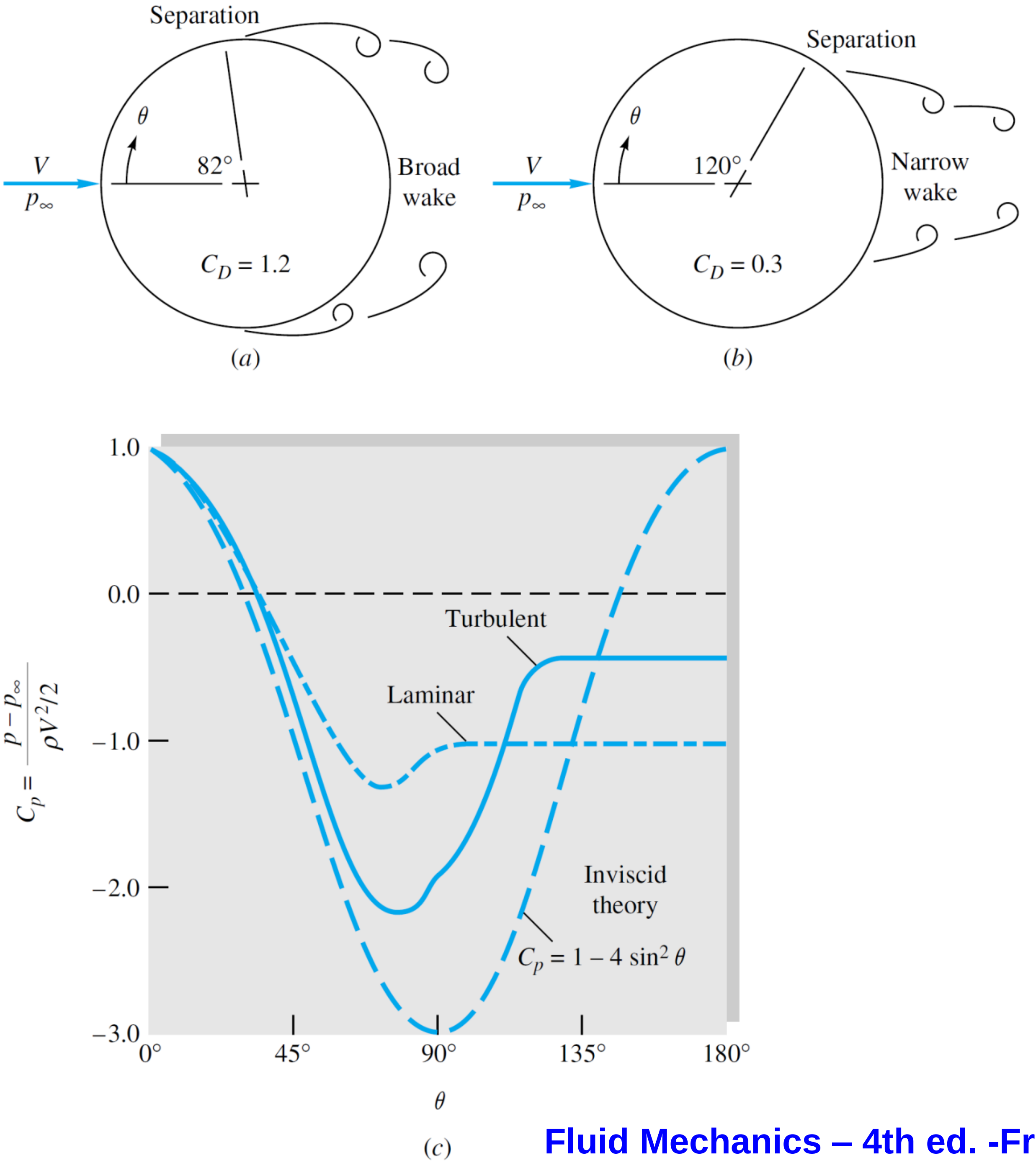
C_D	*Re*
$24/\mathrm{Re}$	*Re <1*
$18{,}5\ \mathrm{Re}^{-0{,}6}$	$1 < Re < 10^3$
$0{,}44$	$10^3 < Re < 10^5$

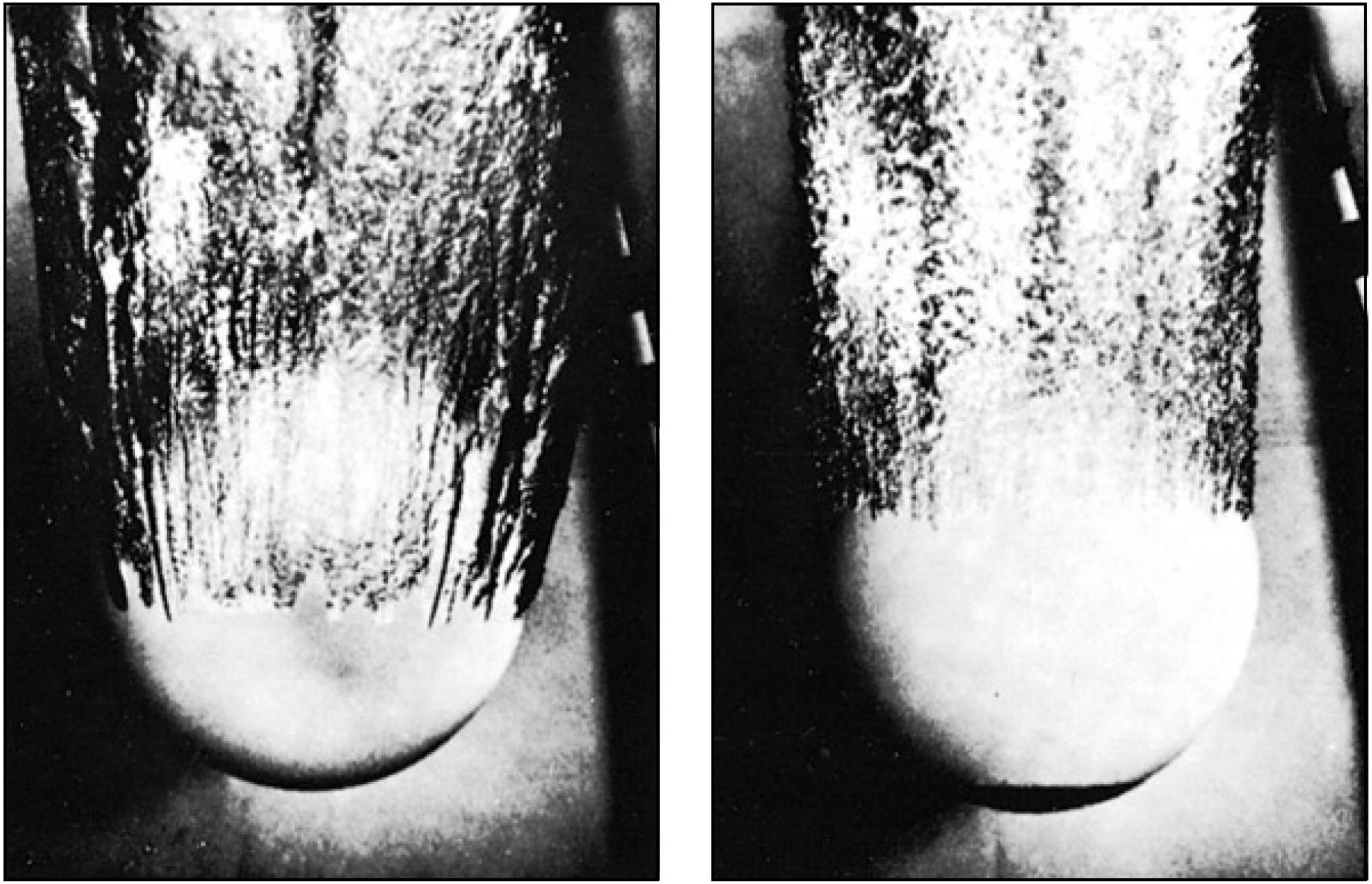
”Lei” de Stokes – Força de Arraste (Drag)



$$C_D \cong 18{,}5\ \text{Re}^{-0{,}6}$$

FIGURE 3.1 Drag coefficient versus Reynolds number for spheres.


Separation
θ
82°
V
p_∞
Broad wake
$C_D = 1.2$
(a)
Separation
θ
120°
V
p_∞
Narrow wake
$C_D = 0.3$
(b)
1.0
0.0
−1.0
−2.0
−3.0
$C_p = \frac{p - p_\infty}{\rho V^2/2}$
Turbulent
Laminar
Inviscid theory
$C_p = 1 - 4 \sin^2 \theta$
0°
45°
90°
135°
180°
θ
(c)

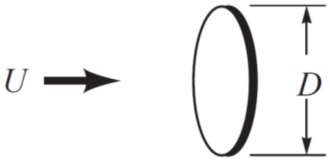
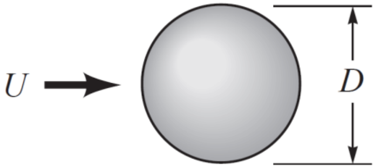
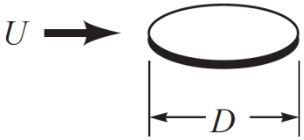
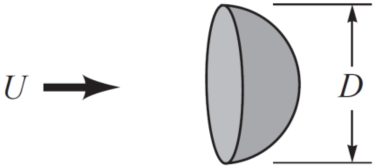


(*a*) (*b*)

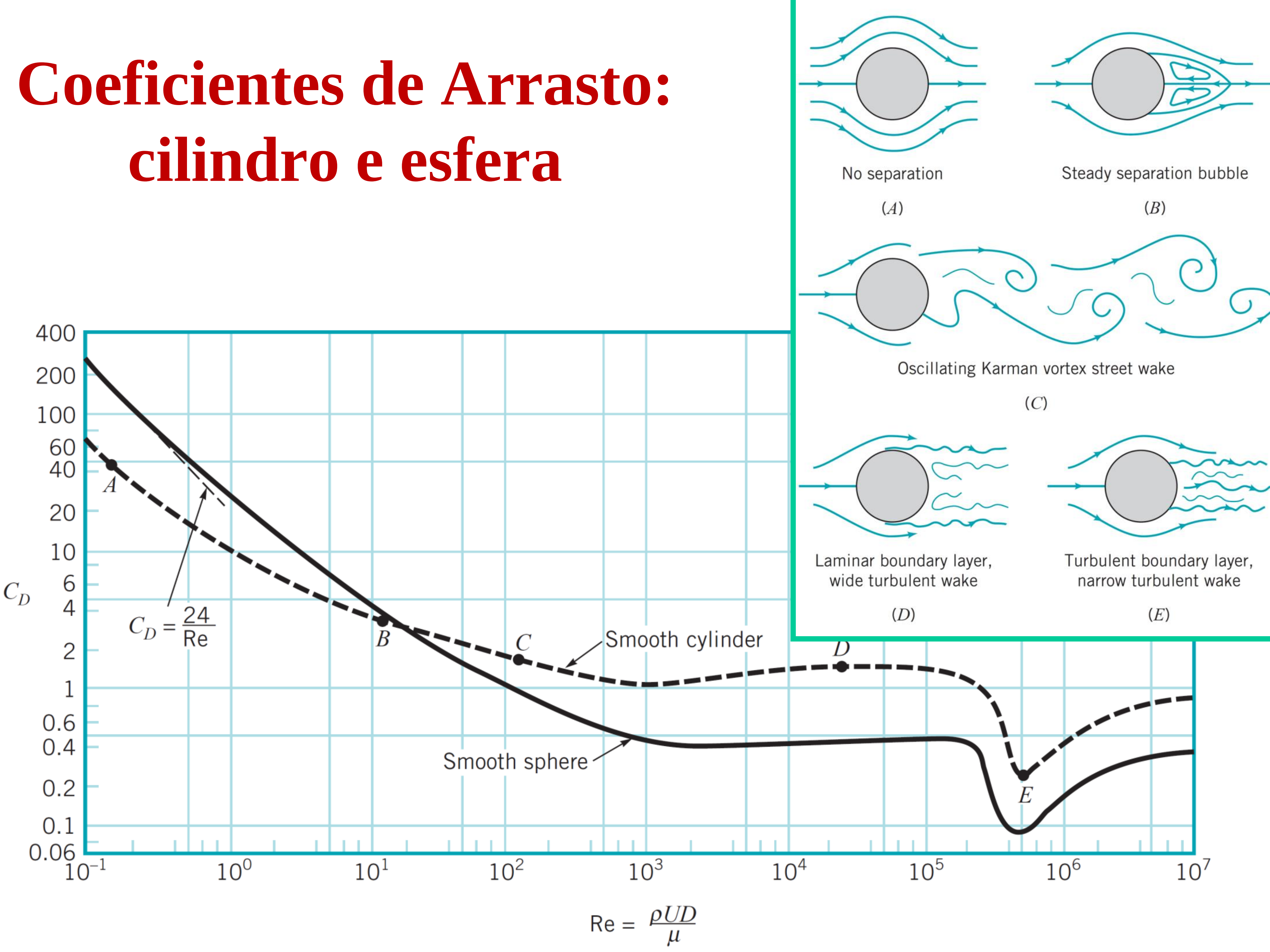
Strong differences in laminar and turbulent separation on an 8.5-in bowling ball entering water at 25 ft/s: (*a*) smooth ball, laminar boundary layer; (*b*) same entry, turbulent flow induced by patch of nose-sand roughness.

(*U.S. Navy photograph, Ordnance Test Station,Pasadena Annex.*)

Coeficientes de Arrasto

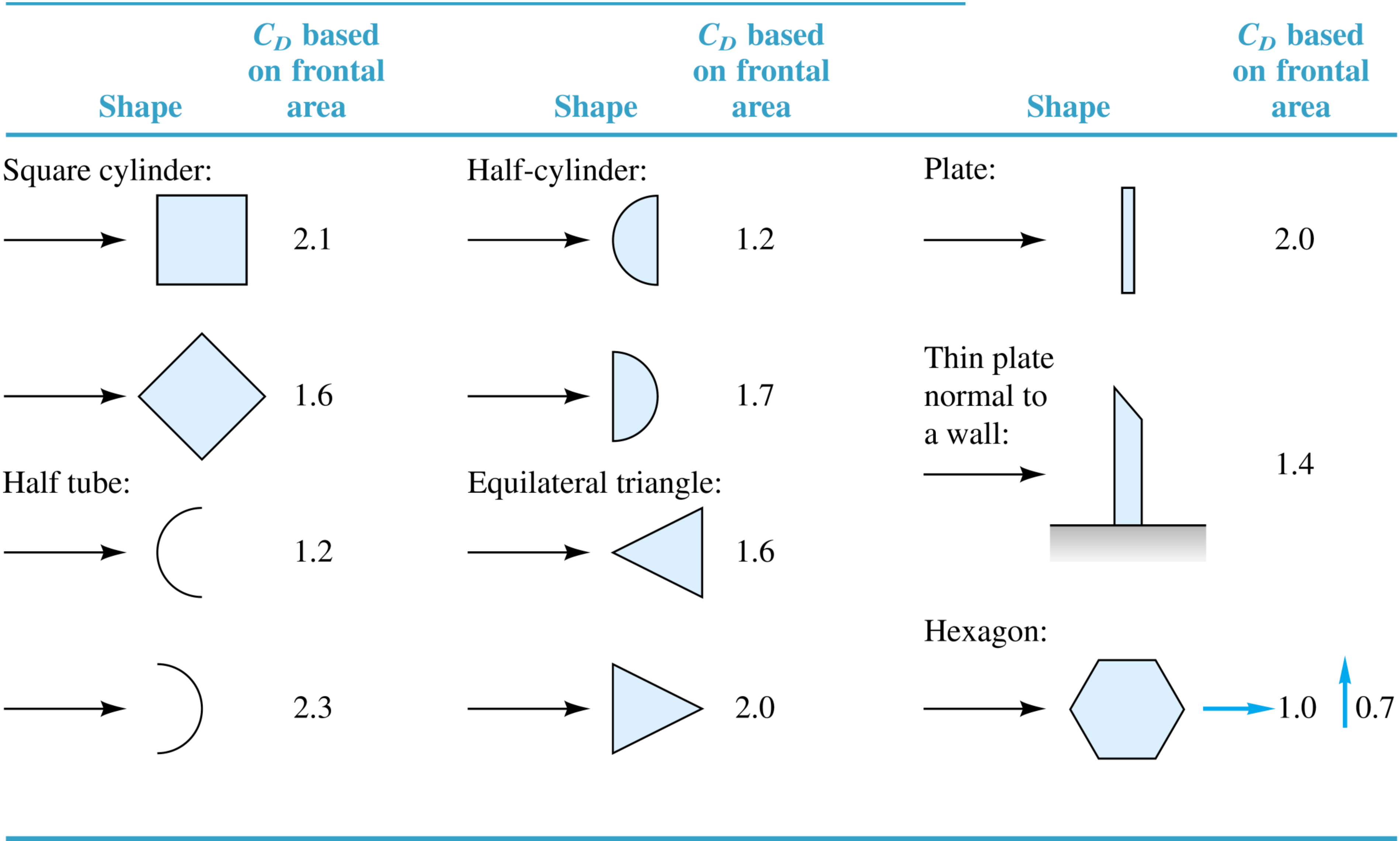
Object	$C_D = \mathscr{D}/(\rho U^2 A/2)$ (for Re ≲ 1)	Object	C_D
a. Circular disk normal to flow  	20.4/Re	c. Sphere  	24.0/Re
b. Circular disk parallel to flow  	13.6/Re	d. Hemisphere  	22.2/Re

Coeficientes de Arrasto: cilindro e esfera



Coeficientes de Arrasto

Table 7.2 Drag of Two-Dimensional Bodies at Re ≥ 10^4



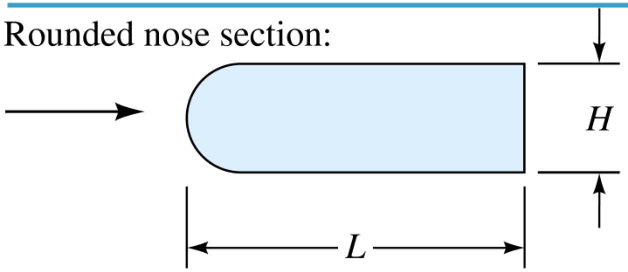
Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:	2.1	Half-cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	1.0 0.7

Coeficientes de Arrasto

Table 7.2 Drag of Two-Dimensional Bodies at Re $\geq 10^4$

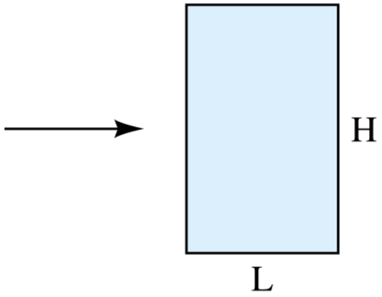
Shape	C_D based on frontal area

Rounded nose section:



L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

Flat nose section



L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:		Laminar	Turbulent
1:1		1.2	0.3
2:1		0.6	0.2
4:1		0.35	0.15
8:1		0.25	0.1

Coeficientes de Arrasto

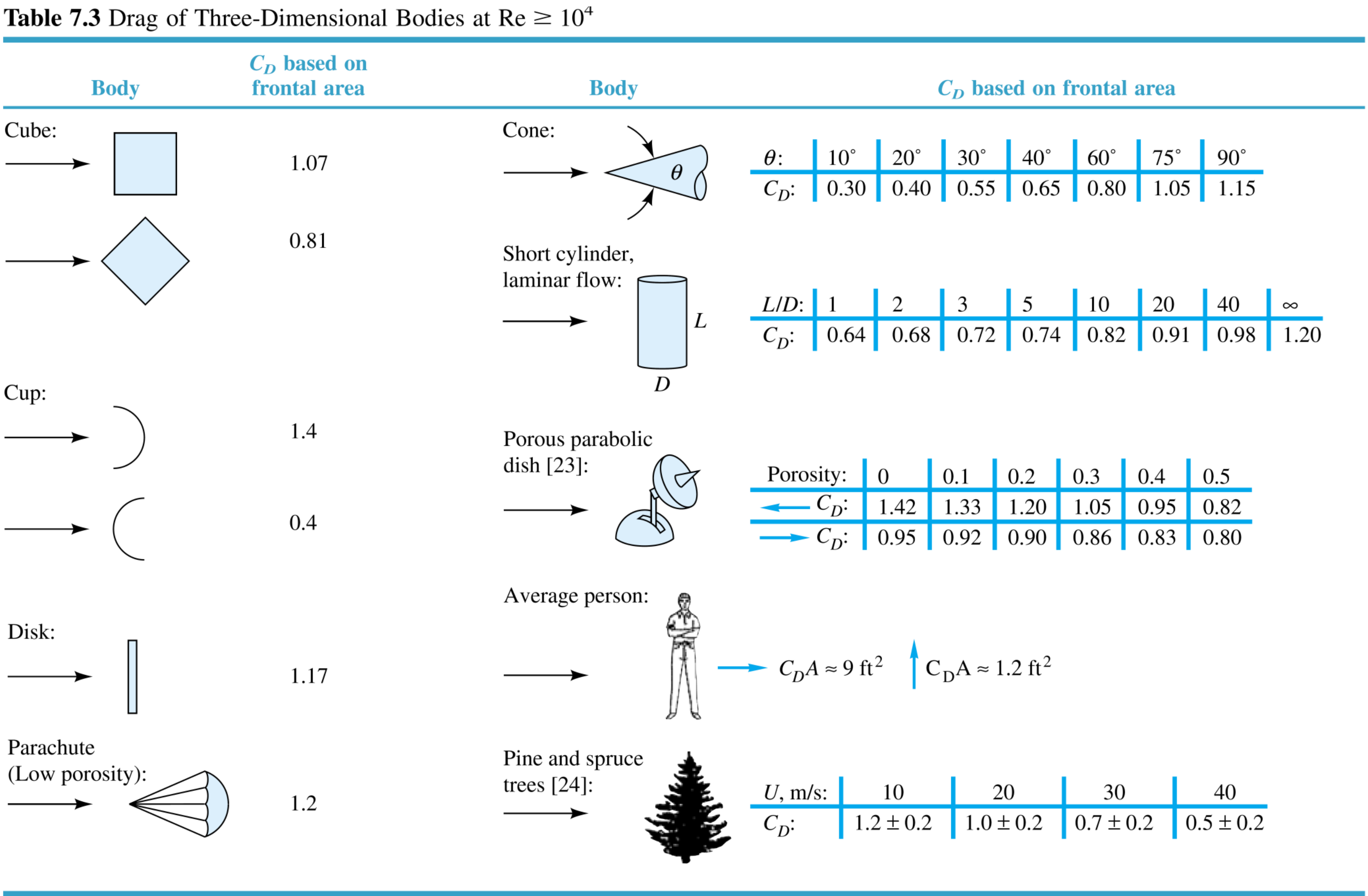


Table 7.3 Drag of Three-Dimensional Bodies at Re ≥ 10^4

Body	C_D based on frontal area
Cube:	1.07
	0.81
Cup:	1.4
	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Body	C_D based on frontal area

Cone:

θ:	10°	20°	30°	40°	60°	75°	90°
C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Short cylinder, laminar flow:

L/D:	1	2	3	5	10	20	40	∞
C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Porous parabolic dish [23]:

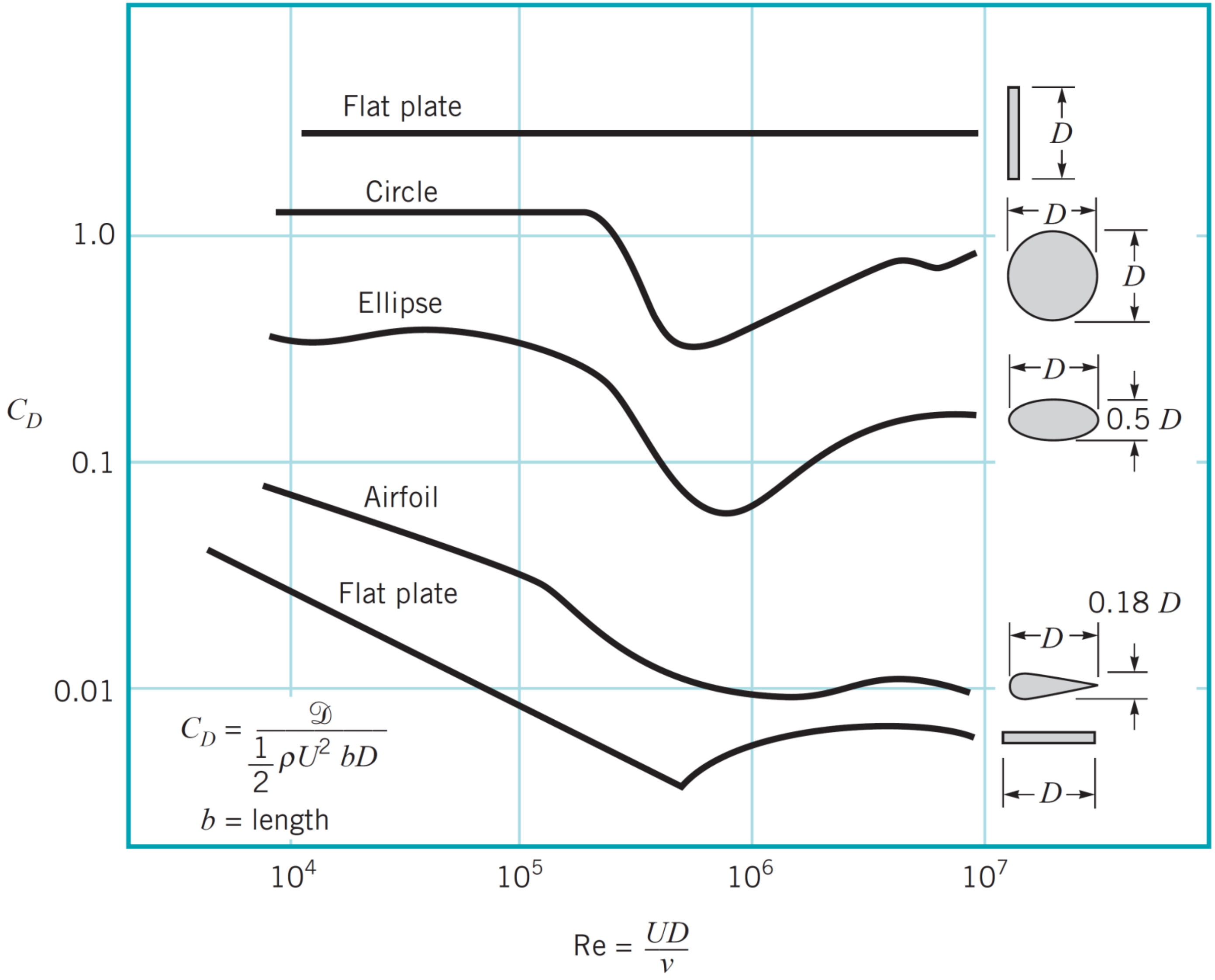
Porosity:	0	0.1	0.2	0.3	0.4	0.5
← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Average person:

→ $C_DA \approx 9\ \text{ft}^2$ ↑ $C_DA \approx 1.2\ \text{ft}^2$

Pine and spruce trees [24]:

U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2


Flat plate
Circle
Ellipse
Airfoil
Flat plate
C_D
1.0
0.1
0.01
10^4
10^5
10^6
10^7
$\mathrm{Re} = \frac{UD}{\nu}$
$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho U^2\, bD}$
b = length
D
D
D
D
0.5 D
0.18 D
D
D

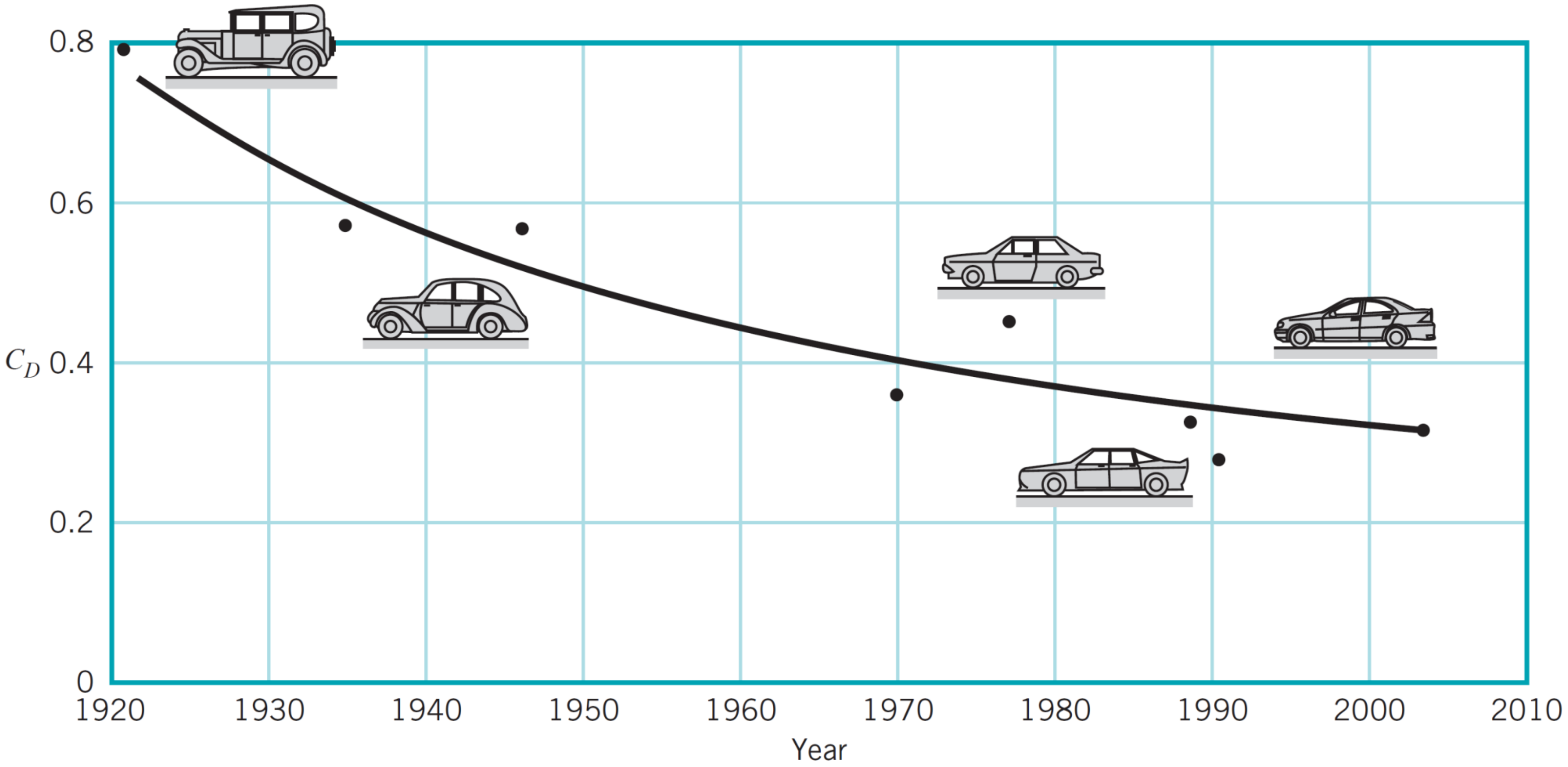


FIGURE 9.27 **The historical trend of streamlining automobiles to reduce their aerodynamic drag and increase their miles per gallon (adapted from Ref. 5).**

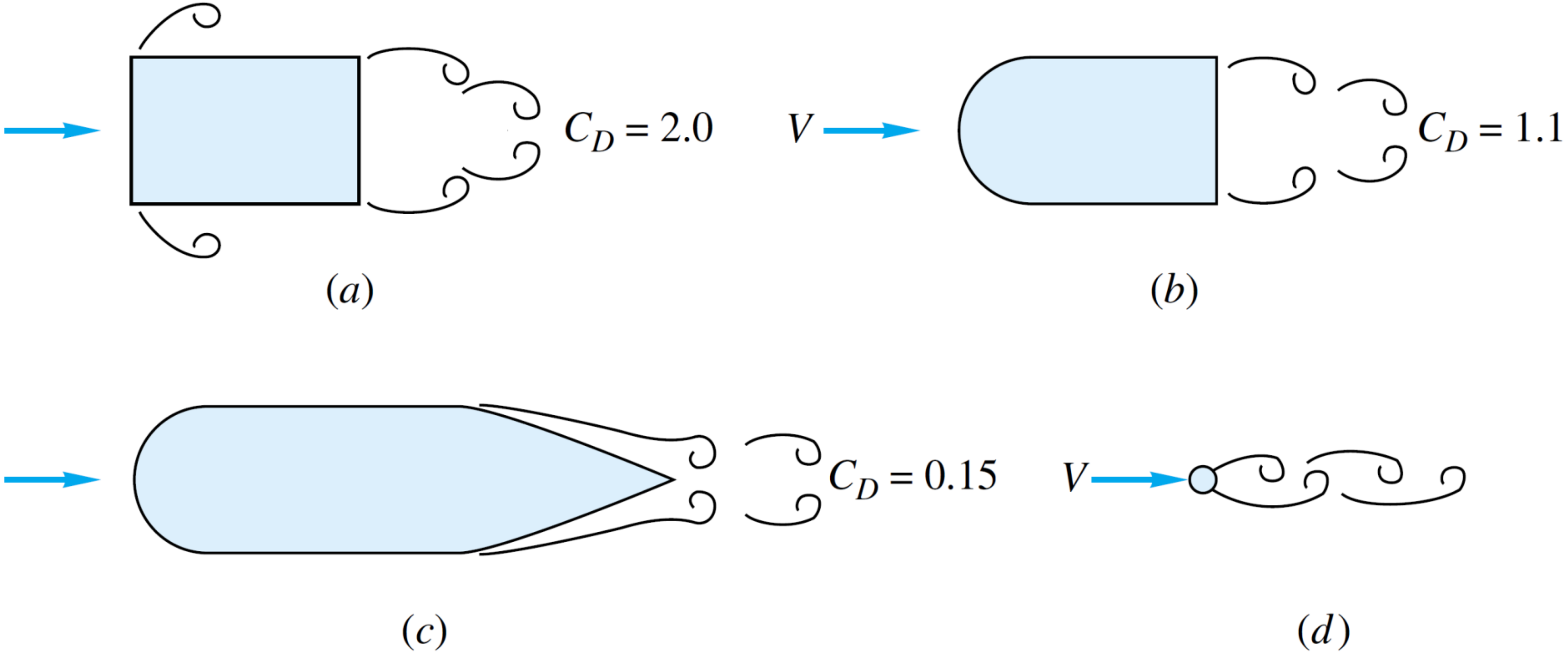


Fig. 7.15 The importance of streamlining in reducing drag of a body (C_D based on frontal area): (*a*) rectangular cylinder; (*b*) rounded nose; (*c*) rounded nose and streamlined sharp trailing edge; (*d*) circular cylinder with the same drag as case (*c*).

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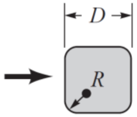
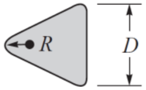
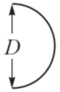
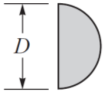
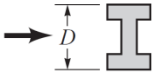
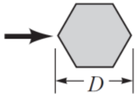
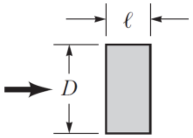
Shape	Reference area A (b = length)	Drag coefficient $C_D = \dfrac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}$	Reynolds number $\mathrm{Re} = \rho UD/\mu$
 Square rod with rounded corners	$A = bD$	R/D C_D 0 2.2 0.02 2.0 0.17 1.2 0.33 1.0	$\mathrm{Re} = 10^5$
 Rounded equilateral triangle	$A = bD$	R/D C_D → ← 0 1.4 2.1 0.02 1.2 2.0 0.08 1.3 1.9 0.25 1.1 1.3	$\mathrm{Re} = 10^5$
 Semicircular shell	$A = bD$	→ 2.3 ← 1.1	$\mathrm{Re} = 2 \times 10^4$
 Semicircular cylinder	$A = bD$	→ 2.15 ← 1.15	$\mathrm{Re} > 10^4$
 T-beam	$A = bD$	→ 1.80 ← 1.65	$\mathrm{Re} > 10^4$
 I-beam	$A = bD$	2.05	$\mathrm{Re} > 10^4$
 Angle	$A = bD$	→ 1.98 ← 1.82	$\mathrm{Re} > 10^4$
 Hexagon	$A = bD$	1.0	$\mathrm{Re} > 10^4$
 Rectangle	$A = bD$	ℓ/D C_D ≤0.1 1.9 0.5 2.5 0.65 2.9 1.0 2.2 2.0 1.6 3.0 1.3	$\mathrm{Re} = 10^5$

■ **FIGURE 9.28** **Typical drag coefficients for regular two-dimensional**

Força de Arraste – Equacionamento Geral Coeficiente de Arrasto - CD

$$C_D = \frac{F_D / A_P}{\rho V_\infty^2 / 2}$$

$$\Rightarrow F_D = \frac{1}{2} C_D A_P \rho V_\infty^2 = \frac{1}{8} \pi C_D \rho D_P^2 V_\infty^2$$

$$\mathrm{Re} = \frac{\rho V_\infty D_P}{\mu} < 1$$

⇨

$$F_D = 6 \pi \mu V_\infty R_P$$

⇨

$$C_D = 24 / \mathrm{Re}$$

C_D	**_Re_**
$24 / \mathrm{Re}$	$Re < 1$
$18{,}5\ \mathrm{Re}^{-0{,}6}$	$1 < Re < 10^3$
$0{,}44$	$10^3 < Re < 10^5$

Partículas de areia de 0,1 mm de diâmetro sedimentam em água (20 C) . Calcule a velocidade terminal, adotando-se uma densidade de 2300 kg/m³ .

$Re < 1{,}0$

$F_D = 6\pi\mu v R$

areia — $\rho = 2300\ kg/m^3$

$D = 0{,}1\ mm$

água 20°C

$$P = E + F_D$$

$$\rho_P \cdot \frac{\pi D^3}{6} g = \rho_f \frac{\pi D^3}{6} g + 3\pi\mu v D$$

$$(\rho_P - \rho_f) g \frac{\pi D^2}{6} = 3\pi\mu v$$

$$\boxed{v = \frac{(\rho_P - \rho_f) g D^2}{18\mu}} \qquad Re < 1$$

$$\boxed{v = \frac{(\rho_p - \rho_f)\, g\, D^2}{18\, \mu}} \qquad Re \leq 1$$

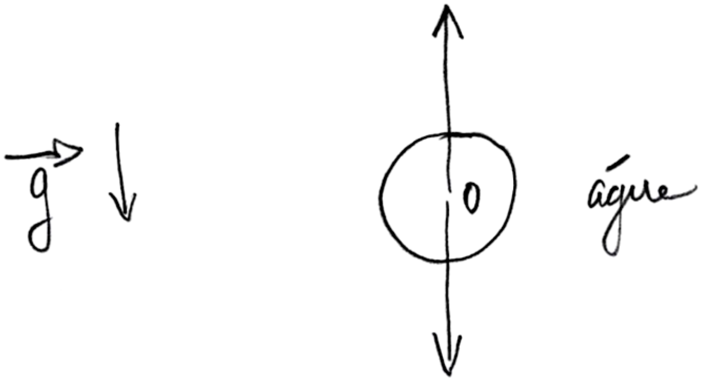
$$\boxed{C_D = \frac{F_D}{A_p \frac{1}{2} \rho v^2} = \frac{3\pi \mu v D}{\frac{\pi D^2}{4} \frac{\rho}{2} v^2} = \frac{24 \mu}{\rho v D} = \frac{24}{Re}}$$

$$v = \frac{(2300 - 1000).9,8.(0,1.10^{-3})^2}{18 \cdot 10^{-3}} = 7.10^{-3}\, m/s$$

$$Re = \frac{1000.7.10^{-3}.10^{-4}}{10^{-3}} = 0,7$$

$$\uparrow\!\!\!\lrcorner \; OK \rightarrow Re < 1,0$$

Pequenas gotas de óleo de 0,4 cm de diâmetro são separadas de da água, por gravidade. Calcule a velocidade terminal dessas gotas, adotando-se uma densidade de 850 kg/m^3 para o óleo e temperatura de 20ºC.



$$F_D + P = E \implies F_D = (E - P) = g\forall(\rho_A - \rho_O)$$

$$\forall = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\, 0{,}2^3 = 0{,}0335 \text{ cm}^3$$

$$F_D = 980 \times 0{,}0335\,(1{,}0 - 0{,}85) = 4{,}93 \text{ dinas}$$

$$\underline{\text{Stokes:}} \quad \underline{F_D} = 6\underset{\nwarrow 0{,}01 \text{ poise}}{\mu}\pi R v = 6.0{,}01.\pi.0{,}2\, v = \underline{0{,}038\, v}$$

$$0{,}038\, v = 4{,}93 \longrightarrow \underline{v = 130 \text{ cm/s}}$$

”Lei” de Stokes – Força de Arraste (Drag)

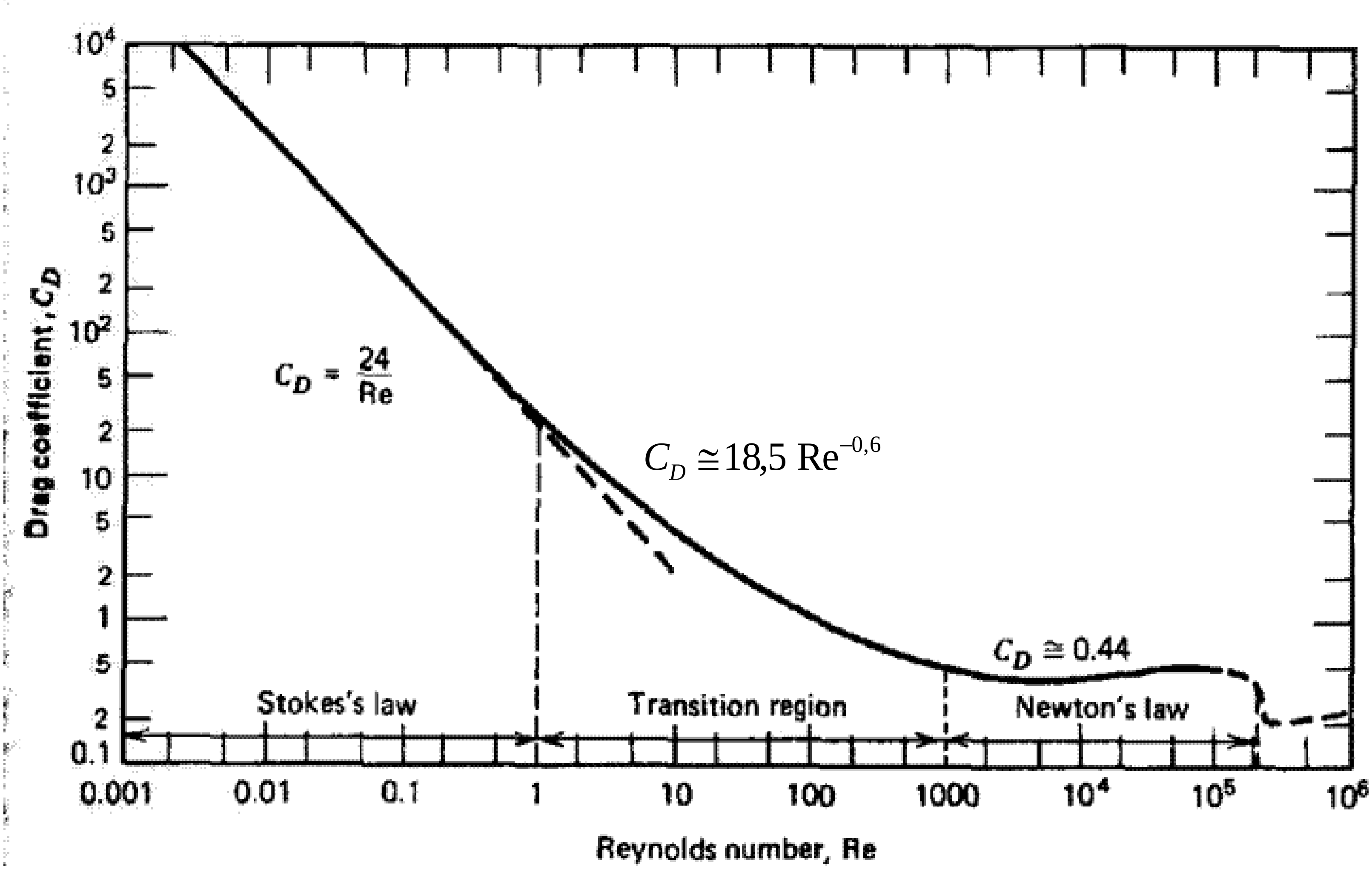


FIGURE 3.1 Drag coefficient versus Reynolds number for spheres.

Pequenas gotas de óleo de 0,4 cm de diâmetro são separadas de da água, por gravidade. Calcule a velocidade terminal dessas gotas, adotando-se uma densidade de 850 kg/m^3 para o óleo e temperatura de 20ºC.

$$0{,}038\,v = 4{,}93 \longrightarrow \underline{v = 130\ \text{cm/s}}$$

$$Re = \frac{\rho v 2R}{\mu} = \frac{1{,}0 . 130 . 2 . 0{,}2}{0{,}01} = 5227 >> 1{,}0$$

$$Re = 5200 \longrightarrow C_D \simeq 0{,}44$$

$$A = \pi R^2 = 0{,}125\ \text{cm}^2$$

$$\frac{F_D}{A . \frac{1}{2} \rho v^2} = C_D \Longrightarrow v^2 = \frac{2 F_D}{\rho A . C_D} = \frac{2 . 4{,}93}{0{,}126 . 1{,}0 . 0{,}44} = 177{,}8$$

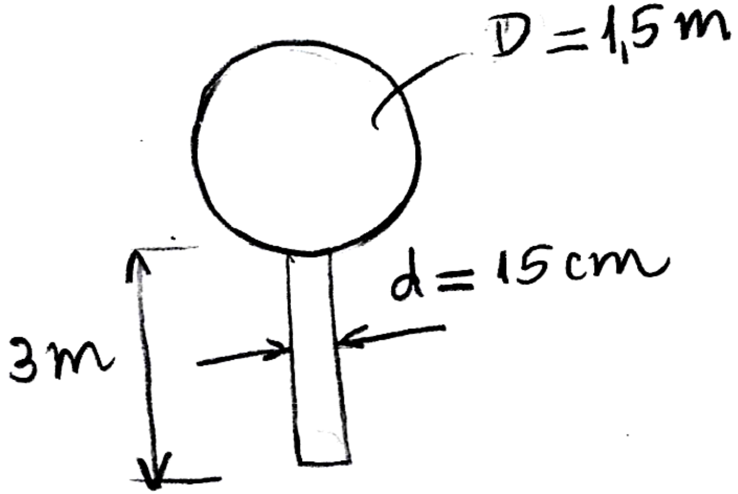
$$v = 13{,}34\ \text{cm/s}$$

$$\text{Novo } Re = \frac{1 \times 13{,}34 \times 0{,}4}{0{,}01} = 533 \longrightarrow C_D = 0{,}566$$

$$v^2 = \frac{2 \times 4{,}93}{0{,}126 . 1{,}0 . 0{,}566} = 138 \longrightarrow v = 11{,}8\ \text{cm/s}$$

$$Re = 470 \longrightarrow C_D = 0{,}588 \longrightarrow v = 11{,}6\ \text{cm/s}$$

Uma placa de sinalização tem o formato de um disco de 1,5 m de diâmetro e é montada em um suporte (poste) de 15 cm de diâmetro e 3 m de altura. Qual a força exercida por um vento de 13,4 m/s soprando normalmente à placa. O ar está a 1 atm e temperatura de 26 ºC (viscosidade cinemática = 1,57.10^{-5} m^2/s).



$v_{NORMAL} = 13{,}4\ m/s$

$\nu = 1{,}57.10^{-5}\ m^2/s$

$\rho = \frac{PM}{RT} = 1{,}18\ kg/m^3$

Coeficientes de Arrasto

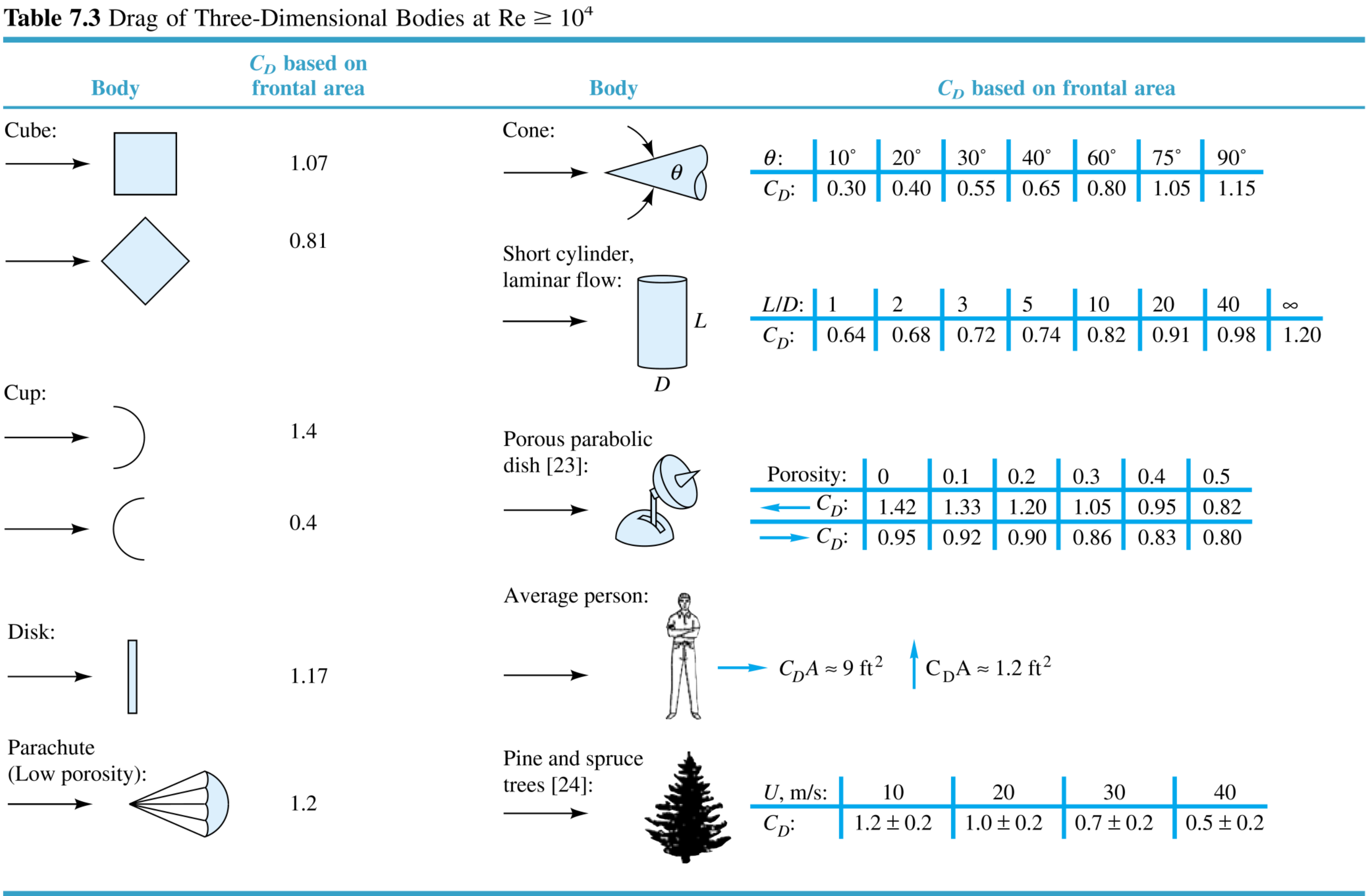


Table 7.3 Drag of Three-Dimensional Bodies at Re ≥ 10^4

Body	C_D based on frontal area
Cube:	1.07
	0.81
Cup:	1.4
	0.4
Disk:	1.17
Parachute (Low porosity):	1.2

Body	C_D based on frontal area
Cone:	
Short cylinder, laminar flow:	
Porous parabolic dish [23]:	
Average person:	→ $C_DA \approx 9$ ft^2 ↑ $C_DA \approx 1.2$ ft^2
Pine and spruce trees [24]:	

Cone:

θ:	10°	20°	30°	40°	60°	75°	90°
C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Short cylinder, laminar flow:

L/D:	1	2	3	5	10	20	40	∞
C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

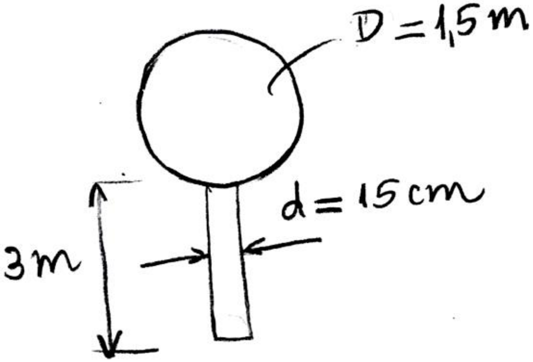
Porous parabolic dish [23]:

Porosity:	0	0.1	0.2	0.3	0.4	0.5
← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Pine and spruce trees [24]:

U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Uma placa de sinalização tem o formato de um disco de 1,5 m de diâmetro e é montada em um suporte (poste) de 15 cm de diâmetro e 3 m de altura. Qual a força exercida por um vento de 13,4 m/s soprando normalmente à placa. O ar está a 1 atm e temperatura de 26 ºC (viscosidade cinemática = $1{,}57.10^{-5}$ m^2/s).



$$Re_{poste} = \frac{v d}{\nu} = \frac{13{,}4 . 0{,}15}{1{,}57.10^{-5}} = 1{,}3.10^{5}$$

$$L/d = 20 \text{ longo} \rightarrow C_D = 0{,}91$$

$$Re_{placa} = \frac{v D}{\nu} = \frac{13{,}4 \times 1{,}5}{1{,}57.10^{-5}} = 1{,}3.10^{6}$$

$$C_D = 1{,}17$$

$$F_D = \frac{1}{2}\rho v^2 \left((C_D \underset{\overset{\uparrow}{d.L}}{A})_{poste} + (C_D \underset{\overset{\uparrow}{\pi D^2/4}}{A})_{PLACA} \right)$$

$$F_D = \frac{1}{2} \times 1{,}18 \times (13{,}4)^2 \left(0{,}91 \times 0{,}15 \times 3 + 1{,}17 . \frac{\pi \times 1{,}5^2}{4} \right)$$

$$\boxed{F_D = 250 \text{ N}}$$