HEAT TRANSFER MODEL FOR PREDICTING SURVIVAL TIME IN COLD WATER IMMERSION

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ABSTRACT

In the present paper a heat transfer (HT) model to estimate survival time of individual stranded in cold water such as at sea is proposed. The HT model was derived based on the assumption that the body specific heat capacity and thermal conductance are not time dependent. The solution to the HT model simulates expected survival time as a function of water temperature, metabolism rate, skin, muscle and fat thickness, insulation thermal conductivity and thickness, height and weight of the subject. Although, these predictions must be considered approximate due to the complex nature of the variables involved, the proposed HT model can be employed to determine supplemental body insulation such as personal protective clothing to meet a predefined survival time in any given water temperature. In particular, the results obtained are useful as a decision aid in search and rescue mission in predicting survival time for shipwreck victims at sea.

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1. INTRODUCTION

The survival of an individual in prolonged cold water immersion are governed by pertinent factors including exposure time in water, roughness of the sea, water temperature, age, gender, fatigue and biophysical parameters of the individual [1-12]. For example, shipwreck victims immersed in deep cold water and without proper insulation or protection could die of respiratory or circulatory failure at body temperatures below 28° C [8]. Death in this pretext is associated with hypothermia-induced fatty degeneration [13] which is accompanied by series of internal complications, leading to cardiorespiratory arrest due to afterdrop (i.e. continued cooling following removal from cold stress) [14-18].

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Although hypothermia is most common in patients exposed to extreme cold environment, it can develop secondary to toxin exposure, metabolic derangements, infections, and dysfunction of the central nervous and endocrine systems [13, 14, 19-21]. The clinical presentation of hypothermia includes a spectrum of symptoms and is grouped into the following three categories: mild, moderate and severe. Table I describes the stages of hypothermia that ultimately leads to death if not treated immediately [6]. Alternatively, the uncontrollable hyperventilation [22] or the initial cold-shock response [23] seen on immersion can cause serious incapacitation of an individual to swim or float, thus leading to drowning long before onset of hypothermia.

An individual's rate of body cooling is governed by the difference between heat loss and heat production, and this in turn is influence by the size of body habitus and body fat thickness. For example, Nuckton *et al.* [15, 24] concluded that individual with large body habitus have higher probability of survival and that surface-to-volume (A/V) ratio is likely the biophysical variable most closely associated with decreased in body cooling. Body fat has long been

Hypothermia zone	Body temperature	Clinical features
Mild	32.2°C to 35°C	* Initial excitation phase to combat cold:
		- Hypertension
		- Shivering
		- Tachycardia
		- Tachypnea
		- Vasoconstriction
		* With time and onset of fatigue:
		- Apathy
		- Ataxia
		- Cold diuresis—kidneys lose
		concentrating ability
		- Hypovolemia
		- Impaired judgment
Moderate	28°C to 32.2°C	* Atrial dysrhythmias
		* Decreased heart rate
		* Decreased level of consciousness
		* Decreased respiratory rate
		* Dilated pupils
		* Diminished gag reflex
		* Extinction on shivering
		* Hyporeflexia
		* Hypotension
Severe	< 28°C	* Apnea
		* Coma
		* Decreased or no activity on
		electroencephalography
		* Nonreactive pupils
		* Oliguria
		* Pulmonary edema
		* Ventricular dysrhythmias/asystole

Table I. Stages of hypothermia and clinical features [14].

associated with cooling rate and the overall body size as reflected by A/V ratio is an important factor controlling the rate of cooling of an individual [25]. Sloan and Keatinge [26] reported that the rate of cooling correlated well regardless of age or gender with the individual's overall surface fat thickness on both trunk and limbs. They found that fat thickness was less and fall in body temperature more rapidly in young than older swimmers, and in boys than girls, even after correction for surface area to mass (A/M)ratio. In addition, Sloan and Keatinge pointed out that regional differences in fat distribution were less important, but older individuals and boys of given trunk fat thickness generally have less limb fat and cooled faster than younger individuals and girls of similar trunk fat thickness and A/M ratio. Castellani et al. [27] who studied the thermoregulatory responses of human to cold water at different times of the day inferred that man are at greater risk of hypothermia in the early morning because of a lower initial core

temperature.

Thus, the findings reported in the literature clearly indicate the complex nature of the variables involve in determining the body cooling rate and this in turn makes the prediction of survival time of an individual during cold water immersion more difficult.

In general, the human body is divided into a warm internal core and a cooler outer shell. The outer shell is basically a combination of skin, fat and inner tissue (muscle). Hypothermia and afterdrop happens when there is a decrease in the core body temperature to a level at which normal muscular and cerebral functions are impaired. In the case when the water temperature is exceptionally low, the body's maximum rate of shivering heat production is exceeded by the rate of heat loss, and the body will gradually acquire the surrounding temperature [28]. However, if the water temperature is not sufficiently cold so that the individual can achieve a steady state of thermogenesis that balances the rate of heat loss, then survival time can be prolonged and the chances of survival can be increased [29]. The attainment of heat balance and body temperatures characteristics of this state forms the basis of the present research.

The objective of this work is to develop a heat transfer mathematical model that is useful in predicting survival time of a victim immersed in cold water based on equivalent body fat thickness, insulation thickness, water temperature and A/V ratio.

2. HEAT TRANSFER MODEL

Body heat is lost to the environment via five mechanisms: radiation, conduction, convection, evaporation and respiration. However, under immersion incidents, conduction and convection are the two major heat transfer mode that controls the rate of body cooling. In contrast to air, which is considered an effective thermal insulator, water is an excellent conductor. Specific heat of water is about 4,000 times that of air, and the thermal conductivity of water is approximately 25 times that of air. As a result, heat loss may be 25-30 times faster when immersed in water such as during swimming than during other activities such as cycling or running at equivalent ambient temperatures [30]. In general, the greater the temperature gradient between the skin and the environment, the greater the rate of heat loss.

Another important factor to consider is the thickness of the subcutaneous fat layer. Because fat is an effective thermal insulator, the thicker the subcutaneous fat layer, the better the preservation of body heat, particularly in the water [31, 32]. This is particularly true when the cutaneous blood supply is shut down, thus markedly reducing convection of heat from body core to the periphery. The rate of heat loss from the body is then limited by the slow conduction through the fat layer.

To develop the heat transfer mathematical model, a general lumped capacitance analysis is adopted [33]. In this analysis it is assumed that the temperature of the core is spatially uniform at any instant during the transient process. This assumption implies that the temperature gradient within the core is negligible. In neglecting the temperature gradient within the core, the transient temperature response is determined by formulating an overall energy balance on the core. It is also assumed that the heat flow will be in one direction only (one dimensional flow). The basic energy balanced used in this model is depicted in equation (1).

Rate of heat change = heat produced - heat lost

$$E_{st} = Q - q$$
 (1)

Heat produced (Q) in equation (1) represents the

basal metabolism rate (BMR) whereas heat lost (q) is the heat removed from the core to the surroundings. Here it is assumed that the basal metabolism rate is independent of time and the core temperature. Numerous equations have been formulated for prediction of human BMR [34], however in the present work the equation proposed by the World Health Organization (WHO), where Q is expressed as a function of weight and height of the subject, was employed [35]. A schematic diagram depicting the heat lost path from the core to the water through the subcutaneous fat layer and skin layers, hereinafter is known as the system, is schematically shown in Figure 1. For easier understanding and conceptualizing the heat transfer problem due to heat loss, the concept of equivalent thermal circuit is adopted where the thermal resistance is analogous to electrical resistance. Thermal resistance is defined as the ratio of a driving potential to the corresponding heat transfer rate [33]. The thermal resistance $(R_{thermal})$ in this model is due to conduction and convection heat transfer. The generic equation that describes this thermal resistance is given by equation (2):

$$R_{thermal} = \frac{\Delta T}{q} \tag{2}$$

where ΔT is the change of temperature and the heat lost (q) based on conduction and convection is given by equations (3) and (4), respectively:

$$q = kA \tag{3}$$

$$q = hA \tag{4}$$

where A represents the body surface area whereas k is the thermal conductivity of the system comprising the various layers (see Figure 1) as given in Table II [36] and h is the convective heat transfer coefficient of the water which is taken as 12,500 Wm⁻²K⁻¹ [37]. The body surface area, A, can be calculated using equation (5) [15]:

$$A = 0.007184 \left(W_{eff} \right)^{0.425} \left(H_{eff} \right)^{0.725}$$
(5)

where W_{eff} and H_{eff} are the effective weight (in kg) and height (in cm) of subject immersed in cold water, respectively. These parameters are determined from the relationship proposed by Nigg and Herzog [38].

Hence, the total resistance (in series) to heat lost comprises the inner tissue (muscle), subcutaneous fat, dermis and epidermis of the skin layers, the insulation (depends on type of clothing), and finally the

Table II. Thermal physical properties of the skin [34].

Structure	Thickness (mm)	Thermal Conductivity (Wm ⁻¹ K ⁻¹)
Epidermis	0.08	0.24
Dermis	2	0.45
Subcutaneous Fat	≤10	0.19
Inner tissue	30	0.5



Fig 1. Schematic diagram showing the thermal resistance and heat lost (q) path through the system.

resistance due to convection. The energy balance equation reduces to a linear, first order, nonhomogeneous differential equation of the form given in equation (6):

$$\rho V c \frac{d(T_{core})}{dt} = Q V - \frac{(T_{core} - T_{water})}{R_{total}}$$
(6)

where, ρ is the average density of the core (inner tissue), V is the representative core volume, c is the specific heat capacity of the core, T_{core} is the temperature of the core, T_{water} is the temperature of the water, t is time and R_{total} is the total thermal resistance to the heat lost component of the equation. For simplicity of calculation, the specific heat capacity of the core is taken to be similar to the inner tissue, i.e. 4,000 Jkg⁻¹K⁻¹ [36]

The solution to the above differential equation can be obtained by eliminating the nonhomogeneity by introducing a transformation equation, followed by separating of variables and integrating the equation to take the final form as shown in equation (7) [33]:

$$T_{core} = \left\{ \Delta T - \frac{Q}{\rho c} \right\} \exp\left[-\left(\frac{1}{\rho V c R_{Total}}\right) t \right] + V R_{total} + T_{water}$$
(7)

where ΔT is the difference between the initial temperature of the core i.e. the arithmetic difference between the body temperature (37 °C) and the temperature of water.

3. RESULTS AND DISCUSSION

The mathematical model derived in the previous section was used to simulate the survival time in cold-water immersion with respect to water temperatures (Figure 2), insulation thickness (Figure 3) and body fat thickness (Figure 4). Figure 5 shows the effect of body surface area to volume (A/V) ratio on the survival time of individual.

The curve in Figure 2 was obtained by assuming that the subject did not wear proper protection thermal insulation suit or use any floating device such as lifejacket. The model predicted that the survival time increase exponentially with water temperature, initially very slowly up to 15° C and thereafter increases very rapidly with increase in the water temperature. The result indeed shows that the survival time of individuals stranded in cold waters is dependent on the water temperature, with low probability of survival below 15° C water.

The probability of survival however can be increased by the use of external thermal insulation material such as wetsuit or life jacket as shown in Figure 3. In particular, the insulation thickness was found to be an important parameter controlling the survival rate of individuals. For instance, Figure 3



Fig 2. The effect of water temperature on the survival time of individuals immersed in cold water without having proper thermal protection or insulation.

shows that to maintain a core temperature of 28°C. increasing the insulation thickness from 2 to 5 mm can increase the survival time by about 7 hours. This result is useful to deep-sea divers where the water temperature drops significantly with the depth of the sea. In addition, the derived HT model can play a significant role in the design and development of new floating devices or thermal suit for a particular application such as to predict the required thickness and also to guide the designer in the selection of appropriate material/fabric for insulation.

The fat thickness beneath the skin was also found to have an influence on the rate of cooling as depicted in Figure 4. In general, it was found the survival time increases with fat thickness of individuals immersed in cold water. This result is in good agreement with that reported in the literature. It is normally perceived that individuals with more fat content can survive longer in cold water as the fats will take the role of a natural insulator to the body [24]. Additionally, Figure 4 also indicate that the survival time required to maintain a core temperature of 28°C in individuals having fat thickness of 10 mm can be increased by about 8 hours by wearing an insulating material (5 mm thick) such as thermal wetsuit or lifejacket. Hence, this result clearly shows the importance of using external thermal insulation material including floatation devices in the event of sudden immersion in cold water at sea.

The body surface area to volume ratio (A/V) is also an important factor in determining the probability of survival of an individual stranded at sea. The body A/V ratio is inversely related to size, therefore individuals with small A/V ratios are large in size whereas those with large A/V such as infants and children are generally small size [15]. As it is known

> 37 35 Thickness = 10 mm 33 31 29 28°C Thickness = 5 mn 27 25 Thickness = 2 mm

Fig 3. The influence of thermal insulation thickness on the survival time.

12

Survival Time (h)

16

20

24

Increase of survival time by 7 h due to increase of insulation thickness

163

that body heat loss is proportional to surface area (equations 3 and 4), thus a person with small A/V ratio cools more slowly and thus has a longer survival time than those exhibit larger ratios, as clearly indicated in Figure 5. Based on this geometric difference alone, assuming no differences in physiological responses, one would predict that a child would have a faster heat loss (via convection, conduction and radiation) to a cool environment when compared to an adult. As a result children are more vulnerable to hypothermia. particularly when the child is small and lean.

In order to validate the HT model developed in the present work, a case study reported by Nuckton et al. [15] was examined. In addition, the equation to determine the survival time of persons accidentally immersed in cold water as proposed by Hayward et al. [1] was employed for comparison purposes. Nuckton



Fig 4. The effect of fat thickness on the survival time of individuals immersed in cold water.



Fig 5. The influence of different body surface to volume ratio (A/V) on the survival time of individuals immersed in cold water.

37

33

31

29

27

25

23

21

19

17

28°C

Core Temperature (°C)

et al. reported that a shipwreck victim, 36 years old. was rescued following prolonged immersion in cold water and suffers from hypothermia. The patient survived for over 9 hours in seawater, after his vessel capsized and sank in the Pacific Ocean. The water temperature at that time was 14.4°C. The rectal temperature (core temperature) of the victim at time of rescue was 30°C. The victim was reportedly not wearing the thermal suit but instead he used the suit as a floatation device by holding it closely against his chest. Based on the information provided in the literature, the heat transfer model derived in this paper was used to predict the time needed for the core temperature to reach 30°C. After taking into consideration the victim's age and biophysical parameters, the present model predicted that it would take about 8.5 hours for the core temperature to reached 30°C. On the other hand, Hayward et al. [1] model predicted that the victim could not have survived for more than 5 hours in the Pacific Ocean. Therefore, based on the assumptions taken, the HT model developed in the present work is acceptable and can be employed to predict the survival time of victim immersed in cold water. While these predictions must be considered approximate and subject to change as latest information becomes available, the model can be useful as a decision aid or guide in search and rescue mission.

4. CONCLUSIONS

A simple heat transfer mathematical model was developed in this work to simulate core temperature as a function of time, when the subject is immersed in cold water. A good correlation was obtained between the core temperature and survival time, insulation thickness, fat thickness and body surface to volume ratio, respectively. In particular, the results indicated that the use of thermal insulation is very important in increasing the survival time and hence the probability of survival of victims immersed in cold water. Additionally, it was found that because of their higher surface area to volume ratio, children's rate of body heat loss is faster than that of adults and thus children are more vulnerable to hypothermia.

The heat transfer model was also verified by examining a reported case of survival following prolonged immersion in the Pacific Ocean due to shipwreck and hypothermia. The model was used to determine the survival time of the victim. The result obtained is in good agreement with the reported survival time in the literature involving the immersion of victim in 14.4°C water. Therefore, the heat transfer model developed in the present work can be useful to predict the survival time of shipwreck victims in cold water based on the core temperature and water temperature. In addition, the derived HT model has the potential to be used to estimate supplemental body insulation needed such as lifejacket for cold water survival and this can be a useful guide to authorities in regulating the thickness and type of material that should be employed in the design of safety devices or thermal suits for use at sea.

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