Pennes' 1948 paper revisited

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Wissler, Eugene H. Pennes' 1948 paper revisited. *J. Appl. Physiol.* 85(1): 35–41, 1998.—A paper published by Harry H. Pennes in Volume 1 of the *Journal of Applied Physiology* defined the theoretical basis for a considerable body of analysis performed by many investigators during the ensuing half century. However, during the past decade, the Pennes' model of heat transfer in perfused tissue has been criticized for various reasons, one of which is that his own experimental data seemed to be at variance with the model. More specifically, the shape of the mean temperature-depth relationship measured by Pennes was distinctly different from the shape of the theoretical curve. In this paper, I show that Pennes used an inappropriate procedure to analyze his data and that, when the data are analyzed in a more rigorous manner, they support his theory. Additional support for Pennes' theory is provided by the experimental data of H. Barcroft and O. G. Edholm [*J. Physiol. (Lond.)* 102: 5–20, 1942 and 104: 366–376, 1946], who had previously studied cooling of the forearm during immersion in water at various temperatures.

bioheat equation; heat transfer in perfused tissue; theoretical model

IT CAN BE ARGUED that one of the most influential articles ever published in the *Journal of Applied Physiology* is the "Analysis of tissue and arterial blood temperatures in the resting human forearm" by Harry H. Pennes, which appeared in Volume 1, No. 2, published in August, 1948. Pennes measured the radial temperature distribution in the forearm by pulling fine thermocouples through the arms of nine recumbent subjects. He also conducted an extensive survey of forearm skin temperature and measured rectal and brachial arterial temperatures. The purpose of Pennes' study was "to evaluate the applicability of heat flow theory to the forearm in basic terms of the local rate of tissue heat production and volume flow of blood." An important feature of Pennes' approach is that his microscopic thermal energy balance for perfused tissue is linear, which means that the equation is amenable to analysis by various methods commonly used to solve the heat-conduction equation. Consequently, it has been adopted by many authors who have developed mathematical models of heat transfer in the human. For example, I used the Pennes equation to analyze digital cooling in 1958 (8, 9) and developed a whole body human thermal model in 1961 (10). The equation proposed by Pennes is now generally known either as the bioheat equation or as the Pennes equation.

Whereas the Pennes equation has gained widespread acceptance and has generally yielded results that agree with experimental observations, important questions about its validity remain unanswered. Those questions arise from three concerns. One is that Pennes' experimental data seem to be at variance with his theoretical results. This issue is discussed in detail in this paper. The second concern is that Pennes focused attention on heat transfer between capillary blood and tissue, but it is easily demonstrated that the temperature of blood in precapillary arterioles and postcapillary venules is close to the temperature of surrounding tissue. The third concern is that the perfusion effect is probably not isotropic. Because the small vessels often occur as countercurrent artery-vein pairs that carry blood at slightly different temperatures, their presence may augment heat transfer by conduction through the tissue when the direction of vessels has a component parallel to the temperature gradient. Those concerns have been addressed by many investigators who have proposed alternatives to the Pennes equation. The excellent summary (including 70 references) of those efforts prepared by Charny (3) can be consulted by those who are interested.

THEORY

Pennes' principal theoretical contribution was his suggestion that the rate of heat transfer between blood and tissue is proportional to the product of the volumetric perfusion rate and the difference between the arterial blood temperature and the local tissue temperature. He expressed that relationship as follows

$$h_b = V\,\rho_b\,C_b(1-\kappa)\,(T_a - T) \tag{1}$$

where h_b is the rate of heat transfer per unit volume of tissue, V is the perfusion rate per unit volume of tissue, ρ_b is the density of blood, C_b is the specific heat of blood, κ is a factor that accounts for incomplete thermal equilibrium between blood and tissue, T_a is the temperature of arterial blood, and T is the local tissue temperature. Pennes assumed that $0 \le \kappa \le 1$, although he set $\kappa = 0$ when he computed his theoretical curves, as have most subsequent investigators. In the rest of this paper, we define $w = V\rho_b C_b$.

Following Pennes' suggestion, the thermal energy balance for perfused tissue is expressed in the following form

$$\rho\,C\,\frac{\partial T}{\partial t} = k\,\nabla^2 T + h_m + h_b \tag{2}$$

where ρ and C refer to tissue, k is the thermal conductivity of tissue, and h_m is the rate of metabolic heat production per unit volume of tissue.

Pennes solved *Eq. 2* for a steady-state cylindrical system in which $T = T(r)$, where r is radius. His solution can be expressed as follows

$$T = \frac{\left(T_s - \frac{b}{a}\right) I_0(\sqrt{a}\, r)}{I_0(\sqrt{a}\, R)} + \frac{b}{a} \tag{3}$$

where T_s is the surface temperature, and R is the radius of the forearm

$$a = \frac{w}{k} \tag{4}$$

$$b = \frac{w\, T_a + h_m}{k} \tag{5}$$

and I_0 is the modified Bessel function of order zero.

It follows from *Eq. 3* that

$$\frac{T(r) - T_s}{T_o - T_s} = \frac{I_0(\sqrt{a}\, r) - I_0(\sqrt{a}\, R)}{I - I_0(\sqrt{a}\, R)} \tag{6}$$

where T_o is the centerline temperature. *Equation 6* can also be expressed in the alternative form

$$\tilde{T}(\tilde{r}) = \frac{I_0(\omega\, \tilde{r}) - I_0(\omega)}{1 - I_0(\omega)} \tag{7}$$

in which

$$\tilde{T} = \frac{T(r) - T_s}{T_o - T_s} \tag{8}$$

$$\omega = \sqrt{\frac{w\, R^2}{k}} \tag{9}$$

and

$$\tilde{r} = \frac{r}{R} \tag{10}$$

If the Pennes model describes heat transfer in the human forearm, *Eq. 7* implies that the dimensionless temperature ratio $\tilde{T}$ is a function of the dimensionless radial coordinate $\tilde{r}$ and the dimensionless parameter ω.

EXPERIMENTAL RESULTS

Pennes measured depth-temperature distributions along the transverse axis of the proximal forearm. The results of those measurements, which were made during a 4- to 6-h period while the subject was recumbent in a room where the temperature was maintained between 25 and 27°C, are plotted in Fig. 15 of his article (see p. 26).

Because there is considerable variation in the individual depth-temperature distributions measured by Pennes, all subsequent investigators have compared their theoretical results with the mean experimental curve, which is shown in Pennes' Fig. 16 (see p. 32), together with theoretical curves that he derived for three different perfusion rates. Therefore, it is reasonable to inquire about the procedure Pennes used to derive the mean experimental curve from the individual distributions. Unfortunately, Pennes' statement that "The mean curve of all the data except curves 3 and 9 is plotted in Fig. 16 for a forearm of average radius of 4.0 cm." can be interpreted in several different ways.

It follows from *Eq. 7* that Pennes might have normalized his data by multiplying each value of r by the ratio $4/R$, in which R is the radius of the particular individual's forearm, before computing the mean temperatures. If that is what he did, the mean lateral (negative r) and medial (positive r) surface temperatures would be the means of the respective measured surface temperatures. However, when I tested that assumption by reading the surface temperatures from Pennes' Fig. 15 and computing the arithmetic means (excluding the values for curves 3 and 9), I obtained values of T = 32.9°C for $r = -4.0$ cm and T = 33.3°C for $r = 4$ cm. The corresponding values computed by Pennes were T = 33.8°C for $r = -4.0$ cm and T = 34.1°C for $r = 4$ cm, which are 0.9°C higher than the mean surface temperatures. Hence, we must conclude that Pennes did not normalize the radial distance before he computed the mean temperatures. Indeed, the fact that his mean temperatures at $r = \pm 4$ cm are higher than the mean surface temperatures suggests that some of the temperatures entering into those means are, in fact, subcutaneous temperatures.

Next, I assumed that Pennes simply averaged the measured temperatures at particular values of r. For example, at $r = -4$ cm, four temperatures (curves 1, 4, 5, and 7) were measured, and the mean of those four values is T = (34.6 + 34.8 + 32.9 + 32.7)/4 = 33.8°C, which is the value reported by Pennes. Similarly, at r = 4 cm, T = (34.35 + 33.3 + 34.5 + 33.84 + 34.0)/5 = 34.0°C, which also agrees with Pennes' value. The author attempted to reconstruct Pennes' data by reading individual values from his Fig. 15. When those values were used to compute a mean temperature profile in the manner described in this paragraph, the results shown in Fig. 1 were obtained; the solid curve is Pennes' profile replotted from his Fig. 16, and the points were computed by the author. Although we will never know for sure, it is reasonable to conclude that Pennes computed his mean temperature profile as described, which is rather disturbing. In particular, the end-point temperatures are not even surface temperatures, as we have always assumed they were; they are simply the mean tissue temperatures at an arbitrary distance of 4 cm from the center of those forearms that happened to be >8 cm in thickness.

ALTERNATIVE ANALYSIS OF PENNES' DATA

Because the method employed by Pennes to compare measured and theoretical temperature distributions was inappropriate, we need to devise a more meaningful comparison. *Equation 7* suggests that the ratio $\tilde{T} = [T(r) - T_s]/(T_o - T_s)$ should be a function of the normalized radius $\tilde{r} = r/R$ and the dimensionless parameter ω. That representation should be independent of T_a, the ambient temperature (T_e), the heat transfer coefficient (h), and h_m. It is important to note that only the shape of the normalized tempera-

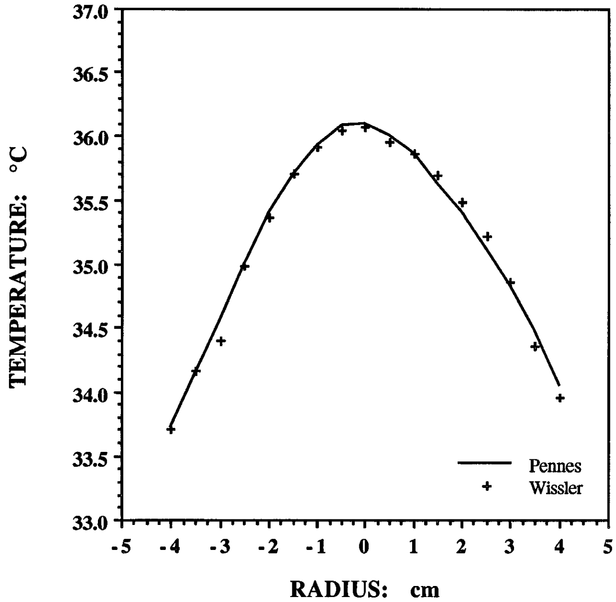

Fig. 1. Mean experimental curves published by Pennes and computed from Pennes' data by the author.

ture profile is affected by ω, because $\tilde{T}(0,\omega) = 1$, and $\tilde{T}(1,\omega) = 0$. Consequently, there is additional information to be obtained from the actual temperature distribution.

When Pennes' data are plotted as $\tilde{T}(\tilde{r})$, the curves shown in Fig. 2 are obtained. Also shown in Fig. 2 are points read from Pennes' theoretical curve for $w = 0.0003\ \mathrm{cal \cdot cm^{-3} \cdot s^{-1} \cdot {}^\circ C^{-1}}$. We see that agreement between the measured and theoretical values is good when they are represented in this way, and it would be difficult to argue that Pennes' model is seriously flawed. However, before a final judgment is made, a more careful analysis should be performed. In particular, we need to make certain that the parameters used by Pennes to compute his theoretical curves are reasonable. Moreover, it is not sufficient to establish that the depth-temperature distribution has the correct shape; we also need to establish that the magnitude of the computed deep tissue temperature is consistent with measured values.

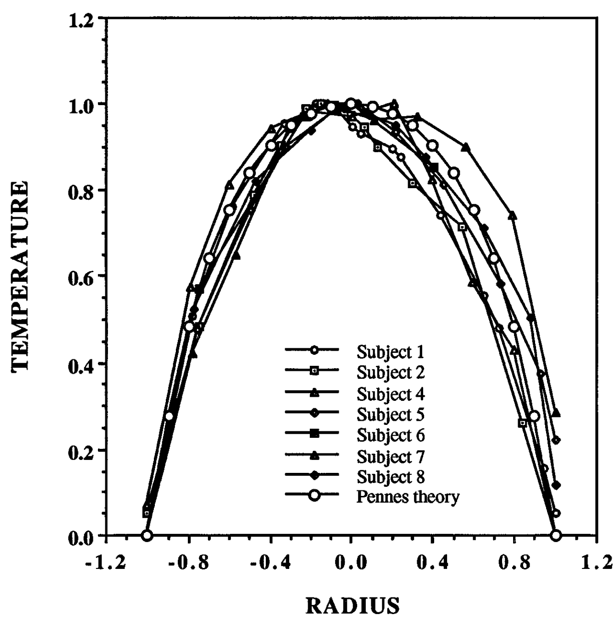

Fig. 2. Experimental data from Fig. 1 and Pennes' theoretical values from Fig. 2 plotted as $[T(r) - T_s]/(T_o - T_s)$ vs. r/R.

EVALUATION OF PARAMETERS

Thermal conductivity. Pennes used a thermal conductivity of $0.0005\ \mathrm{cal \cdot s^{-1} \cdot cm^{-1} \cdot {}^\circ C^{-1}}$, which is one-third of the currently accepted value of $0.0015\ \mathrm{cal \cdot s^{-1} \cdot cm^{-1} \cdot {}^\circ C^{-1}}$ for tissue (and water). He referred to two sources for his value of k; the tissues were beef muscle and fat (4), and skin (5).

Heat transfer coefficient. Pennes used a heat transfer coefficient of $0.0001\ \mathrm{cal \cdot s^{-1} \cdot cm^{-2} \cdot {}^\circ C^{-1}}$. Data from McAdams (6) for $h = h_c + h_r$ for horizontal cylinders in still air suggest that a value of $0.0002\ \mathrm{cal \cdot s^{-1} \cdot cm^{-2} \cdot {}^\circ C^{-1}}$ is probably more reasonable; the subscripts, c and r, refer to natural convection and radiation, respectively. Pennes stated that air movement was imperceptible to the subjects and was always <20 feet/s, as measured by a sensitive anemometer. Because a velocity of 10 feet/s gives a Reynolds number of 20 for the forearm in air, it is reasonable to assume that the heat transfer coefficient was somewhat higher than the value quoted by McAdams for still air. Although it seems quite clear that the value of h used by Pennes is too small, the available data are insufficient to evaluate an accurate value.

Arterial blood temperature. In separate determinations, Pennes measured the temperature of blood in the brachial artery at the elbow, with the forearm in complete supination to facilitate arterial puncture. Deep tissue temperatures were also measured with a needle thermocouple, and Pennes found that the arterial blood temperature was never less than the maximum deep tissue temperature. Because the mean difference between the arterial blood temperature and the deep tissue temperature was 0.16°C, Pennes used an arterial blood temperature of 36.25°C to compute his theoretical curves. In the computations discussed in the next section, I use $T_a = 36.8°C$, which is close to the mean arterial blood temperature measured by Pennes.

Ambient temperature. Ambient temperature T_e was said to be close to 26.6°C for all of Pennes' subjects, and there is no justification for using a different value.

Blood perfusion rate. Pennes referred to a paper by Barcroft and Edholm (2) when he cited a range of $0.00025–0.0005\ \mathrm{g \cdot ml^{-1} \cdot s^{-1}}$ for $V\rho_b$, which corresponds to $1.5–3.0\ \mathrm{ml \cdot 100\ ml^{-1} \cdot min^{-1}}$. Actually, he used 0.00020, 0.00025, and $0.00030\ \mathrm{g \cdot ml^{-1} \cdot s^{-1}}$ (1.2, 1.5, and $1.8\ \mathrm{ml \cdot 100\ ml^{-1} \cdot min^{-1}}$) in computing his values, which is the lower end of the cited range. The data reported by Barcroft and Edholm are plotted in Fig. 3, and we see that a value of $V = 3.0\ \mathrm{ml \cdot 100\ ml^{-1} \cdot min^{-1}}$ is more reasonable than the values Pennes used. Indeed, Barcroft and Edholm mention explicitly that $V = 3.1\ \mathrm{ml \cdot 100\ ml^{-1} \cdot min^{-1}}$ is typical of the forearm when the skin temperature is 33°C.

Density and specific heat. We assume that the density and specific heat of blood and tissue are equal to those

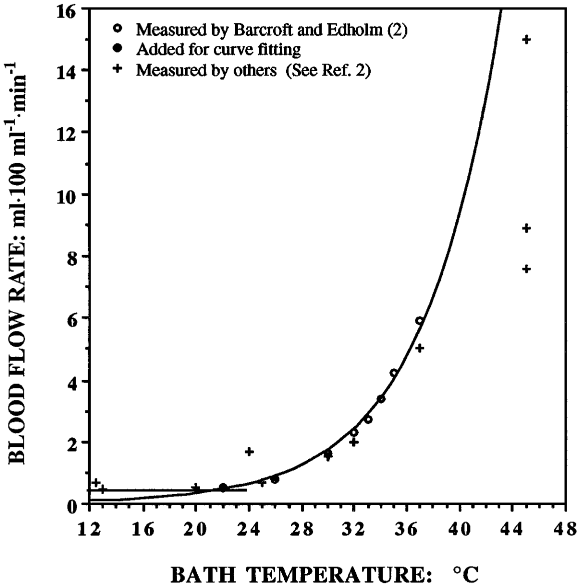

Fig. 3. Mean blood flow rate in forearm, plotted as a function of bath temperature. Solid curve represents a least squares fit of data represented by ○ or ●.

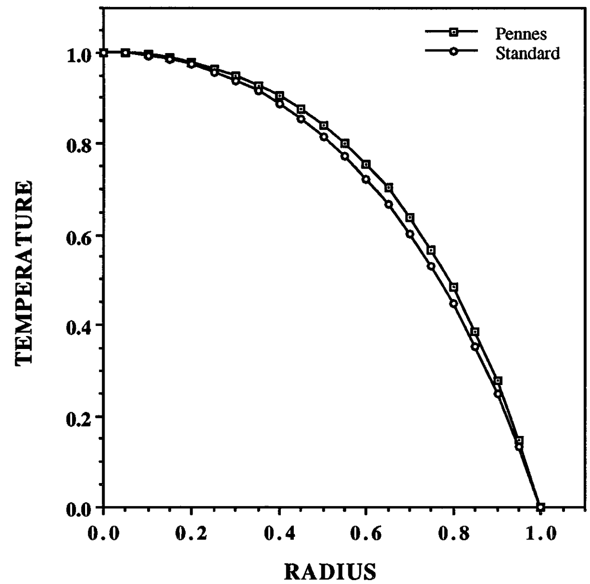

Fig. 4. Normalized temperature profiles for 2 values of ω: one computed by using Pennes' parameters and the other by using currently accepted values. $[T(r) - T_s]/(T_o - T_s)$ is plotted as a function of r/R.

of water, that is, 1 g/cm^3 and 1 cal·g^{-1}·°C^{-1}, respectively.

ALTERNATIVE THEORETICAL CURVES

We have established that the theoretical curves computed by Pennes represent the experimental data rather well when the normalized temperature $\tilde{T}$ is plotted against the normalized radius $\tilde{r}$, but some of the values he used for parameters are questionable. Therefore, we need to evaluate the effect of changing ω on the shape of the normalized temperature distribution. The parameters used by Pennes are compared with more reasonable values in Table 1, and the normalized temperature profiles computed by using those two sets of parameters are plotted in Fig. 4. It is apparent that the shape of the normalized temperature distribution is not strongly affected by the value of ω, at least for the range of ω appropriate for Pennes' measurements. The curve computed by using the "standard" parameters is slightly "thinner" than Pennes' curve, which may improve agreement between the computed and measured mean curves, but the difference is probably not significant.

Equation 3 can be expressed in a form that shows more clearly how various physiological and physical parameters affect the temperature distribution within the arm. It can be shown that

$$\Theta(\tilde{r}) = \left(\Theta_a + \frac{h_m}{w}\right) F(\tilde{r},\omega) \qquad (11)$$

in which $\Theta = T - T_s$, $\Theta_a = T_a - T_s$, and $F(\tilde{r},\omega) = \{1 - [I_0(\omega\tilde{r})/I_0(\omega)]\}$. When the tissue temperature is measured relative to the skin temperature, we see that the effect of physiological parameters is separated from the effect of the environmental parameters, T_e and h, although T_s depends on T_e and h.

The difference, $\Theta_o = \Theta(0)$, between the central temperature and the skin temperature is

$$\Theta_o = \left(\Theta_a + \frac{h_m}{w}\right) F(0,\omega) \qquad (12)$$

Three independent factors, Θ_a, h_m/w, and ω, affect that difference. For the conditions of Pennes' study, Θ_a varies from 3.6 to 4.5°C, whereas the ratio h_m/w has a value of ~0.2°C. Because h_m/w is only 5% of Θ_a, the internal temperature difference is not strongly dependent on the metabolic rate h_m. The dependence of Θ_o on ω occurs through the factor $\{1 - [1/I_0(\omega)]\}$. That dependence is shown in Fig. 5.

If we assume that k = 0.0015 cal·s^{-1}·cm^{-1}·°C^{-1}, h_m = 0.0001 cal·s^{-1}·cm^{-3}, and T_a = 36.8°C, values of ω and F can be computed for each experimental curve. We have

$$\omega = \sqrt{\frac{V\,\rho_b\,C_b}{k}}\,R \qquad (13)$$

and

$$F = \frac{T_0 - \overline{T}_s}{T_a - \overline{T}_s + (h_m/V\,\rho_b C_b)} \qquad (14)$$

Table 1. *Parameters used by Pennes compared with accepted values*

Parameter	Pennes	Standard	Units
V	1.5	3.0	ml·min^{-1}·100 ml^{-1}
k	0.0005	0.0015	cal·s^{-1}·cm^{-1}·°C^{-1}
T_a	36.25	36.8	°C
R	4.5	4.5	cm
ω	3.18	2.64	Dimensionless

V, perfusion rate per unit volume of tissue; k, thermal conductivity in tissue; T_a, temperature of arterial blood; *R*, radius of the forearm; ω, dimensionless parameter.

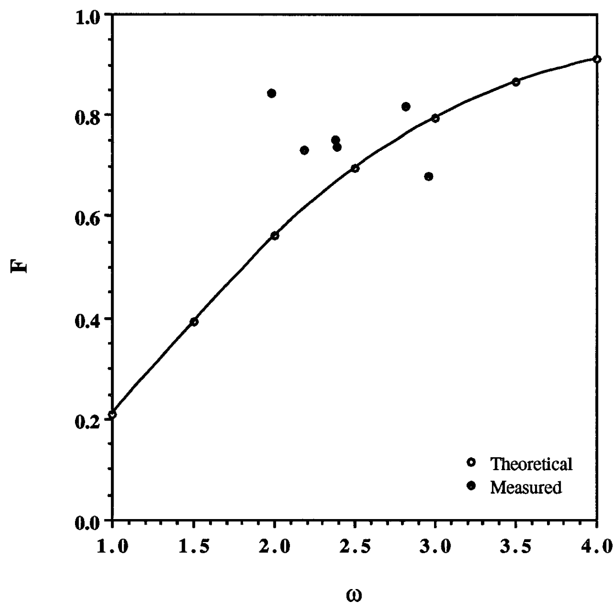

Fig. 5. Values of function $F(0,\omega)$ computed by using *Eq. 11* and from Pennes' experimental data.

in which $\overline{T}_s$ is the arithmetic mean of the lateral and medial skin temperatures, and the correlation shown is Fig. 3 is used to compute V as a function of $\overline{T}_s$. The values computed for six of Pennes' subjects (curves 3, 6, and 9 were excluded) are also plotted in Fig. 5. With the exception of one point, agreement between the theoretical and measured values of F is reasonable.

THE BARCROFT AND EDHOLM STUDIES

Two papers published before the Pennes' paper by Barcroft and Edholm (1, 2) deal with blood flow and the deep temperature in the human forearm. The forearm blood flow rate was measured plethysmographically, whereas the deep temperature was measured with a needle thermocouple, 2.5 cm in length, that was pushed through the brachioradialis muscle until it reached the bone, from which point it was withdrawn ~1 mm. A second thermocouple inserted obliquely into the forearm measured a subcutaneous temperature at some poorly defined depth. Measurements were made with the arm exposed either to air or to water at a controlled temperature that ranged from 12 to 41°C. In this section, we analyze some of the immersion data.

Equation 2 describes transient heating or cooling, as well as defining the steady-state depth-temperature distributions measured by Pennes. When we assume that the time-dependent temperature profile depends only on radial position, *Eq. 2* reduces to the following form

$$\rho C \frac{\partial T}{\partial t} = k \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + h_m + V s(T_a - T) \qquad (15)$$

where we have assumed that $\kappa = 0$ and $s = \rho_b C_b$. *Equation 15* must be solved subject to an initial condition of the form

$$T(0,r) = T_i(r) \qquad (16)$$

and two boundary conditions, which have the following form

$$\frac{\partial T}{\partial r} = 0 \qquad \text{at } r = 0 \qquad (17)$$

and

$$-k \frac{\partial T}{\partial r} = h(T - T_e) \qquad \text{at } r = R \qquad (18)$$

Transient temperature profiles were computed for the following conditions: $R = 4.0$ cm, $k = 0.0015$ $\text{cal}\cdot\text{s}^{-1}\cdot\text{cm}^{-1}\cdot{}^\circ\text{C}^{-1}$, $h_m = 0.0001$ $\text{cal}\cdot\text{s}^{-1}\cdot\text{cm}^{-3}$, and $hR/k = 20.0$, and V is the function of T_s shown in Fig. 3. The measured deep muscle temperature is compared with the temperature computed at $r = 1.5$ cm in Figs. 6–9. Also compared in Figs. 6–9 are the measured subcutaneous temperature and the computed temperature at $r = 3.0$ cm, i.e., 1 cm below the skin. Those comparisons are made at four different water temperatures: 12, 20, 30, and 41°C.

Although one would like to see better agreement between the computed and measured values, it is probably as good as can be expected. The largest difference between computed and measured muscle temperatures occurs when $T_e = 12$°C. In that case, the initial rate of cooling is much greater than the computed rate, which is difficult to understand, because during the first 30 min of cooling the computed curve with perfusion shown in Fig. 6 is <0.5°C above the lower limiting case of cooling without perfusion. The large difference between the computed and measured values could be caused by several factors unrelated to the Pennes model. For example, if the measurement site were close to a vein that returned blood from the hand, one might expect more rapid cooling. Another

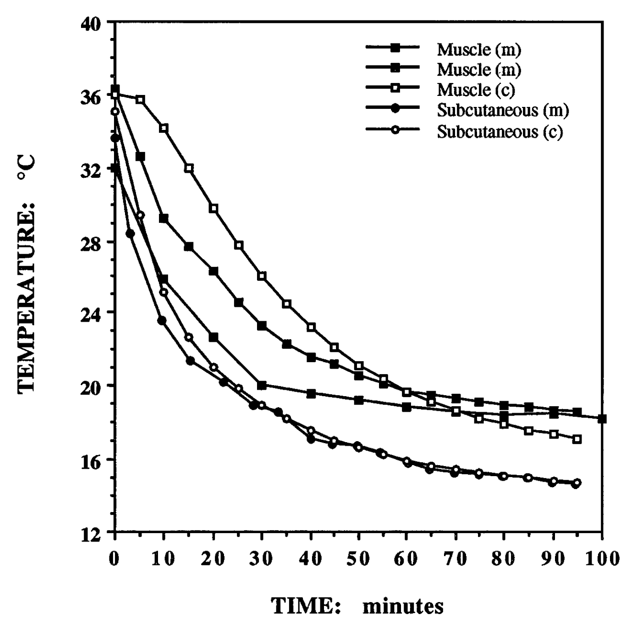

Fig. 6. Comparison of deep muscle and subcutaneous temperatures measured (m) in forearm by Barcroft and Edholm (2) with values computed (c) by using the Pennes model. Water temperature is 12°C.

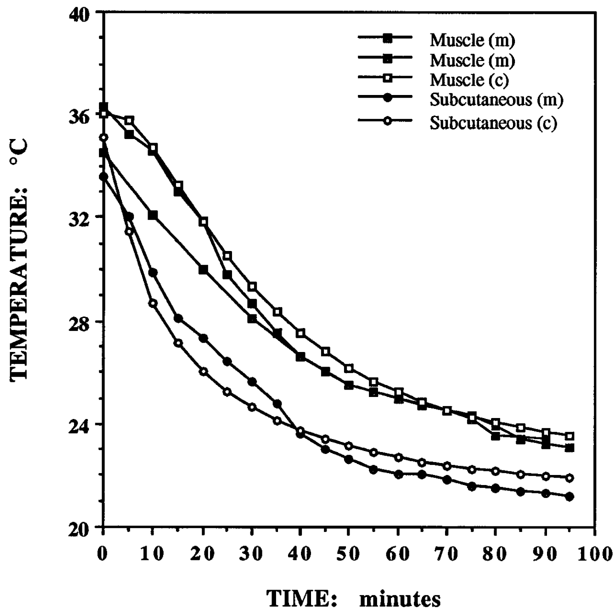

Fig. 7. Comparison of deep muscle and subcutaneous temperatures measured in forearm by Barcroft and Edholm (2) with values computed by using the Pennes model. Water temperature is 20°C.

possibility is that conduction of heat along the needle might have introduced an artifact. In any event, the discrepancy does not necessarily reflect unfavorably on the Pennes model.

It is interesting to note that, when the bath temperature is higher than the arterial blood temperature, the temperature gradient within the forearm is reversed, and the subcutaneous temperature lies above the deep muscle temperature. In that case, perfusion cools the tissue instead of heating it, which is correctly predicted by the Pennes model.

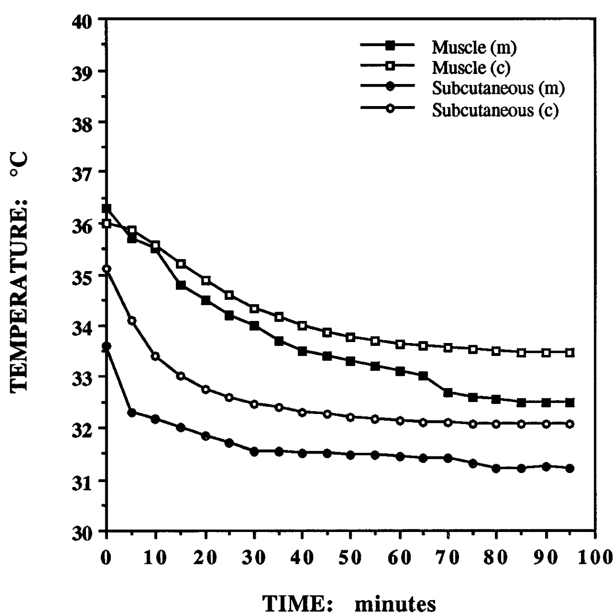

Fig. 8. Comparison of deep muscle and subcutaneous temperatures measured in forearm by Barcroft and Edholm (2) with values computed by using the Pennes model. Water temperature is 30°C.

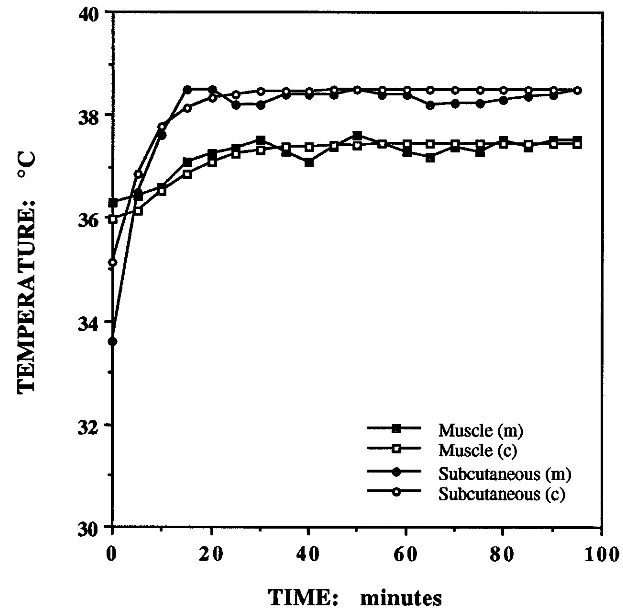

Fig. 9. Comparison of deep muscle and subcutaneous temperatures measured in forearm by Barcroft and Edholm (2) with values computed by using the Pennes model. Water temperature is 41°C.

DISCUSSION

Many factors affect the temperature distribution in the forearm. For example, the forearm is not a circular cylinder composed of homogeneous, uniformly perfused tissue. It contains two large bones and an irregular layer of subcutaneous fat, all of which have thermal properties that are significantly different from those of aqueous tissue. Pennes demonstrated clearly that the temperature field is affected by heat transfer between blood large vessels and the surrounding tissue. Moreover, it is now recognized that heat transfer between smaller vessels that supply the capillary beds affects the temperature of blood entering those beds, and, therefore, T_a in the Pennes equation is not a constant. Consequently, many potentially significant factors have been ignored both in Pennes' paper and in this paper. Some of them, such as the geometry and inhomogeneous structure of the forearm, are relatively easy to correct, because very effective numerical methods exist for solving the heat-conduction equation. Others, such as heat transfer between thermally significant vessels and tissue, are not so easily resolved. However, further progress is probably more severely limited by the paucity of definitive experimental data than by lack of theoretical methods for determining more realistic temperature fields.

Summary. In conclusion, the purpose of this paper is to show that much of the criticism directed toward the Pennes model is not justified. Experimental data reported by Pennes are probably as good as we will ever have, unless a noninvasive technique is developed for measuring deep tissue temperatures. The principal criticism of Pennes' study is his analysis of the depth-temperature distributions, and that is a serious criticism. The author has attempted to resolve that difficulty in this paper, and the result is that temperature profiles computed by using the Pennes model agree with the measured profiles as well as can be expected. Therefore, those who base their theoretical calculations on the Pennes model can be somewhat more confident that their starting equations are valid.

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