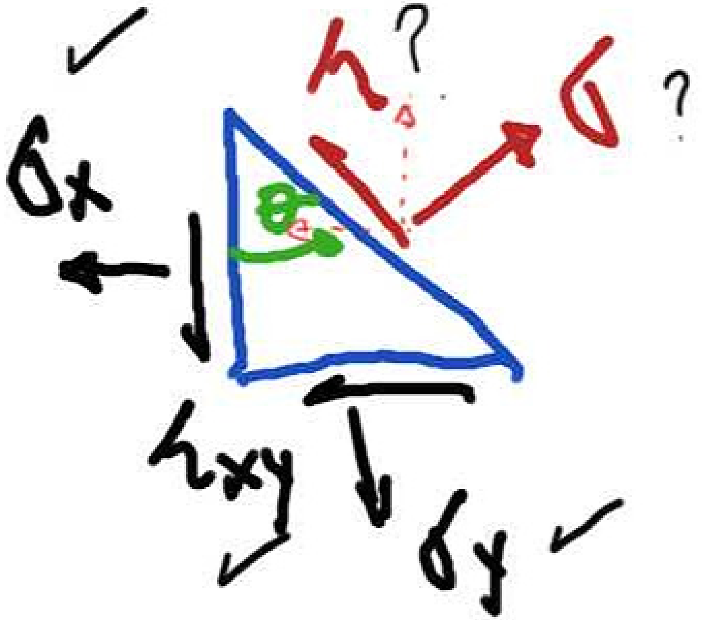
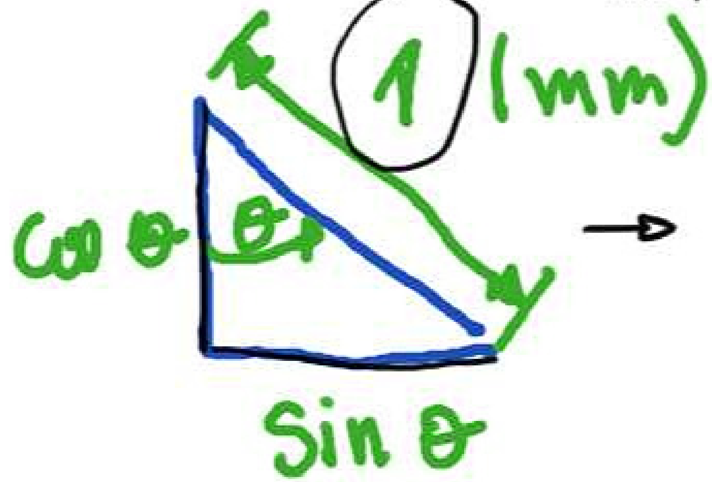
CIRCULO DE MOHR (2D) EPT

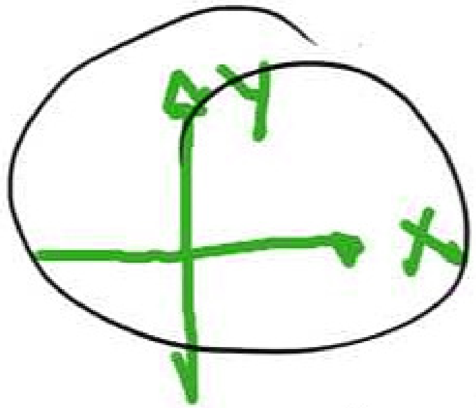


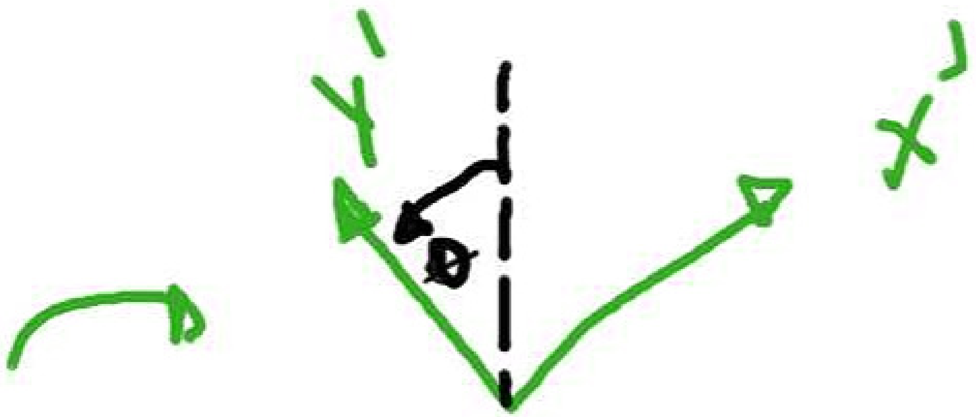


DEFINIMOS COMPRIMENTO UNITÁRIO

→ DEFINIR ÁREA

- CONSIDERANDO
  - ESPESSURA UNITÁRIA
    $B = 1$

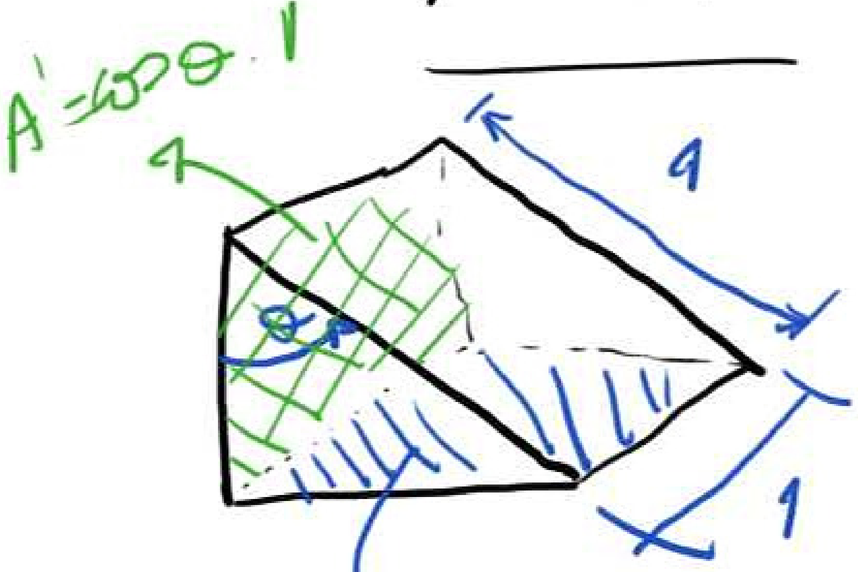


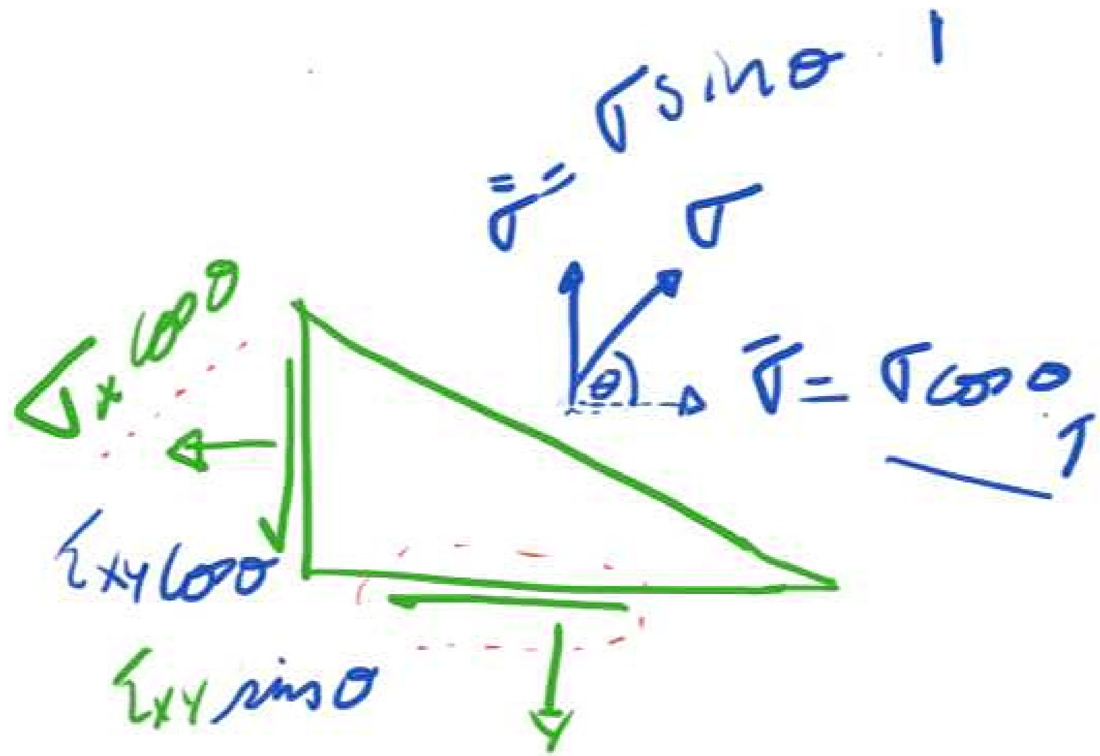


$\left.\sigma_x, \sigma_y, \tau_{xy}\right\}$ conhecidos $\longrightarrow$ σ, τ ?

$\cdot\ \sigma \rightarrow F$

$$\boxed{F = \sigma \cdot A}$$





· EQUILÍBRIO

$\cdot \sum F_x = 0 \rightarrow \sigma\cos\theta - \tau\sin\theta - \sigma_x\cos\theta - \tau_{xy}\sin\theta = 0$

$\sigma\sin\theta + \tau\cos\theta - \sigma_y\sin\theta - \tau_{xy}\cos\theta = 0 \leftarrow \cdot \sum F_y = 0$

· Fazendo a derivada

$$\frac{d}{d\theta}\sigma = 0$$

$$\frac{d}{d\theta}\left(\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right) = 0$$

$$\downarrow$$

$$\sigma = f(\theta, \dots) \longleftarrow \boxed{\tan 2\theta = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}} \quad (3)$$

$$\downarrow$$

σ_{max} ✓

· Fazendo a derivada

$$\frac{d}{d\theta}\sigma = 0$$

$$\frac{d}{d\theta}\left(\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \right) = 0$$

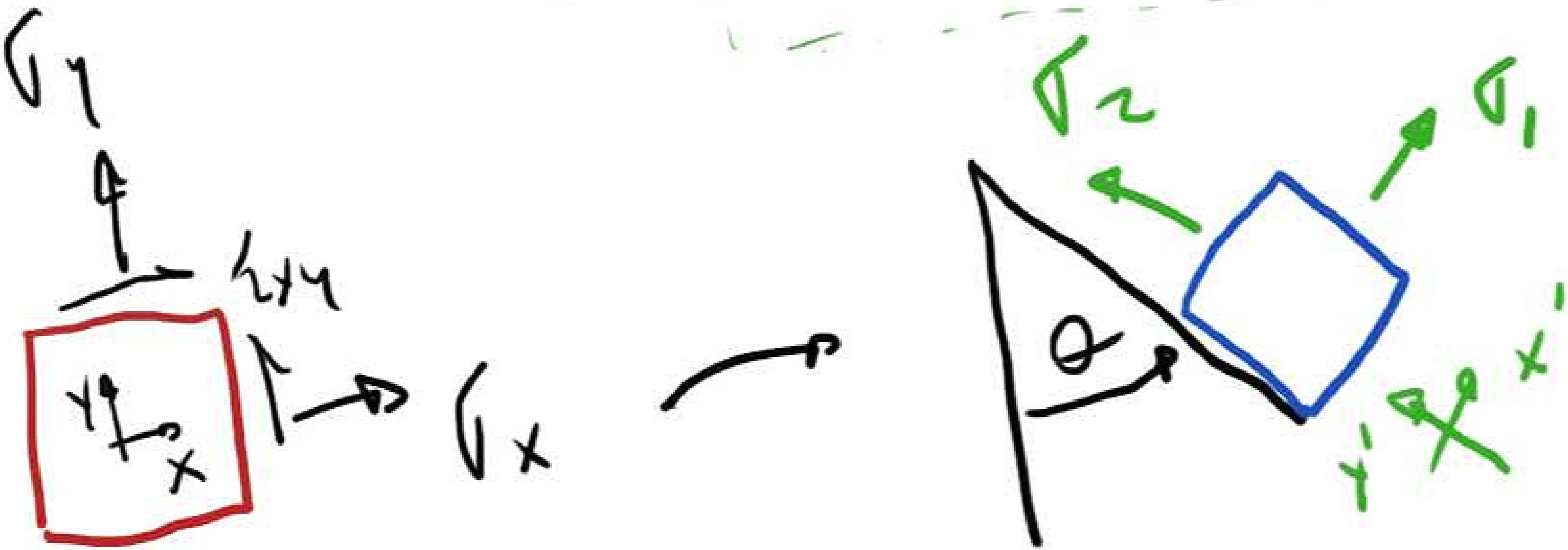
$$\downarrow$$

$$\sigma = f(\theta, \ldots) \longleftarrow \boxed{\tan 2\theta = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}} \quad (3)$$

$$\downarrow$$

σ_{max} ✓

(4) $\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$



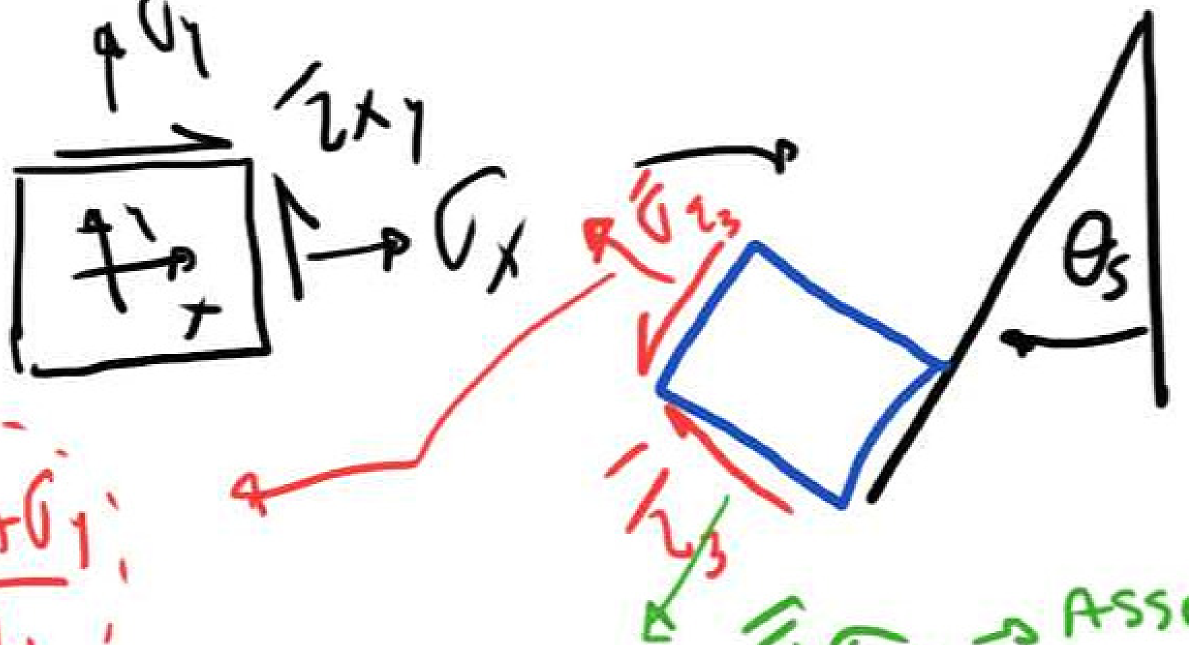
· CISALHAMENTO (MÁXIMO)

$$\frac{d}{d\theta}\tau = 0 \rightarrow \frac{d}{d\theta}\left(-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right) = 0$$

$$\tan 2\theta_{SHEAR} = -\frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}} \quad (3)$$

$$\Downarrow$$

$$\tau_3 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



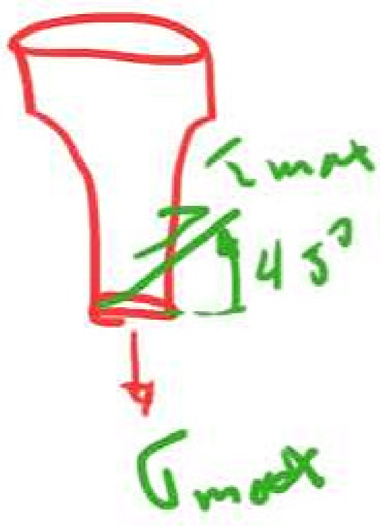
$$\bar{\sigma}_{\tau_3} = \frac{\sigma_x + \sigma_y}{2}$$

$\bar{\sigma}_{(\tau_3)}$ → ASSOCIADO AO PLANO τ_3!

$$\left.\begin{matrix} 2\theta_n \\ 2\theta_s \end{matrix}\right\} \pm 90^\circ$$

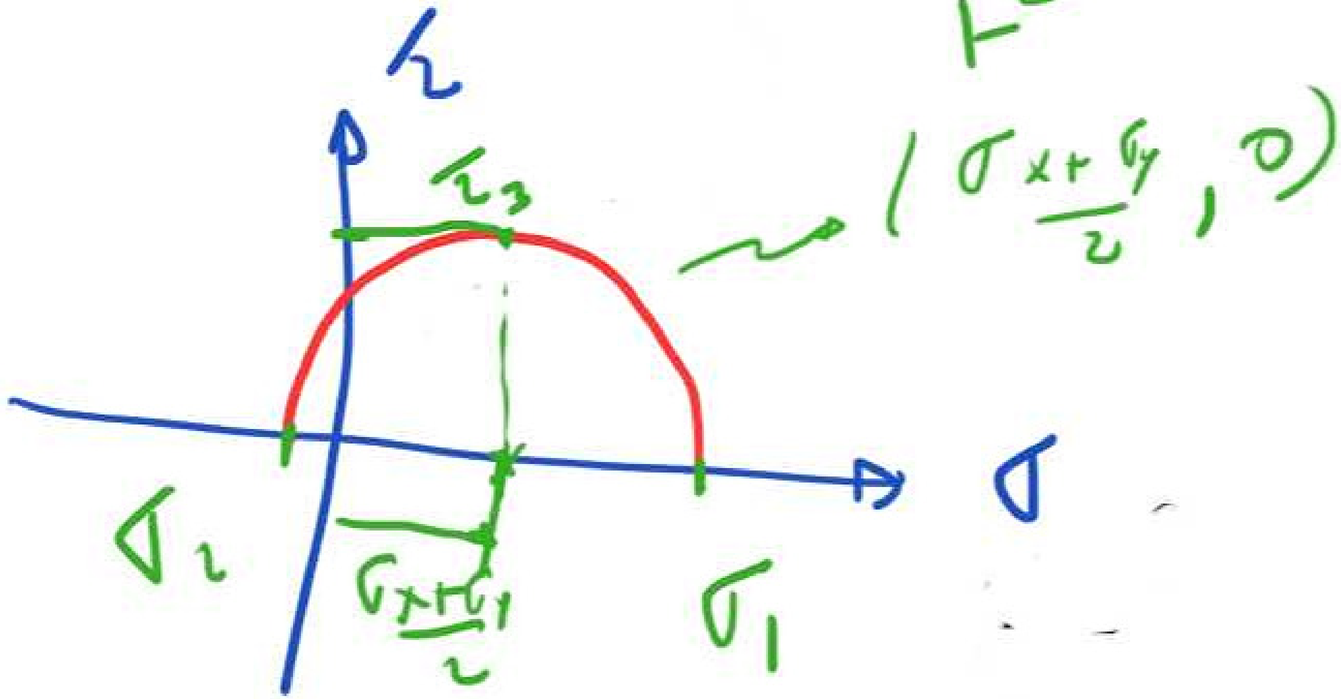
$$\downarrow$$

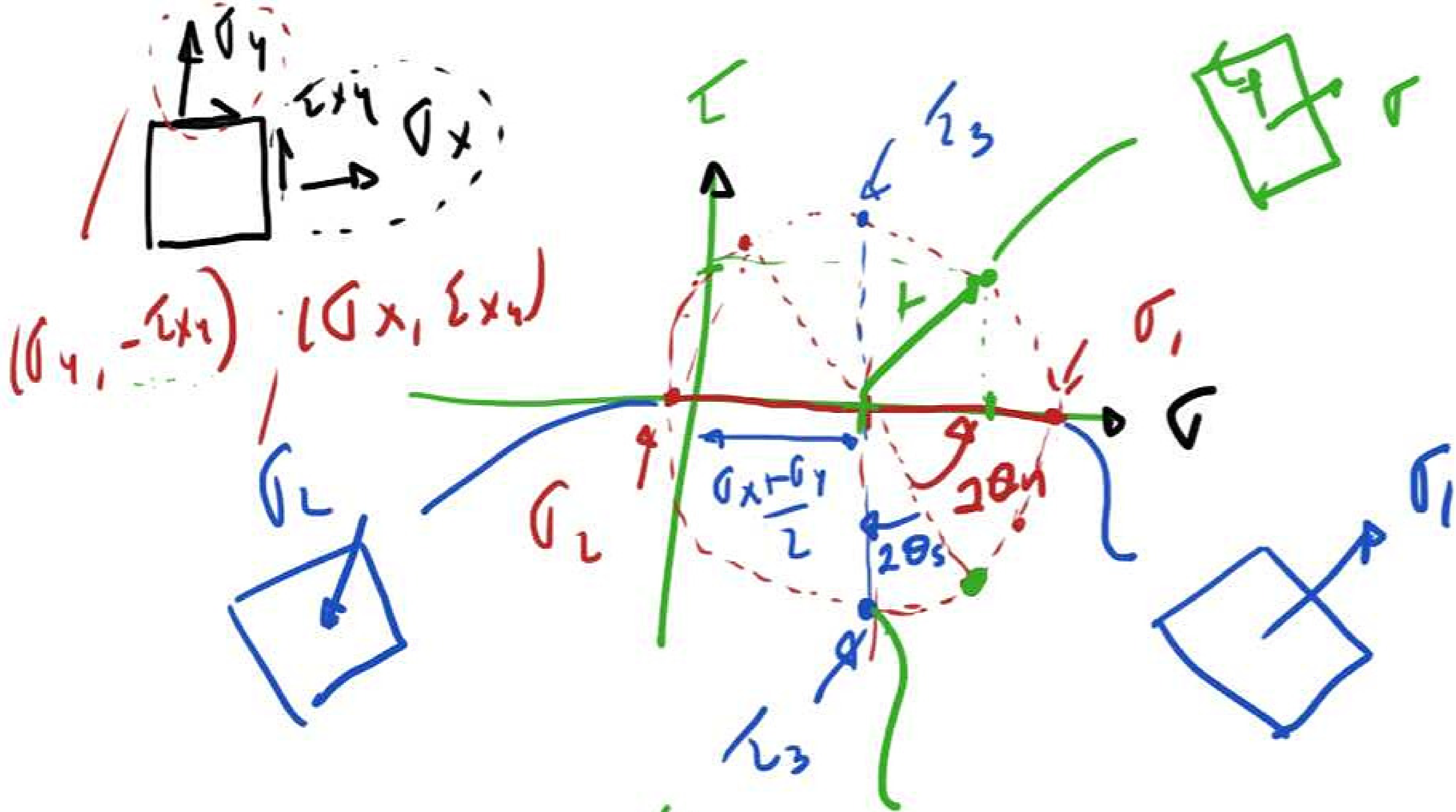
$$|\theta_n - \theta_s| = 45^\circ$$

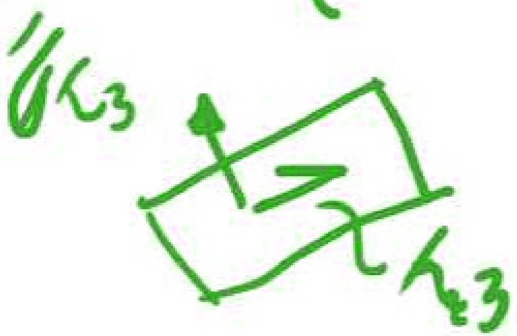


• ELIMINANDO 2θ DAS EQ (1) e (2)

$$\left(\sigma - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2 = \underbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}_{r^2}$$





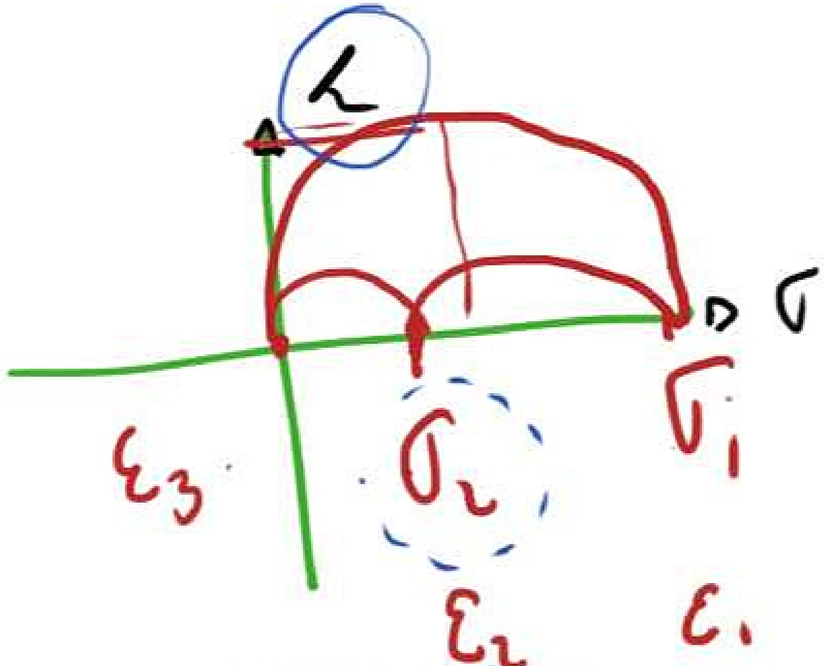


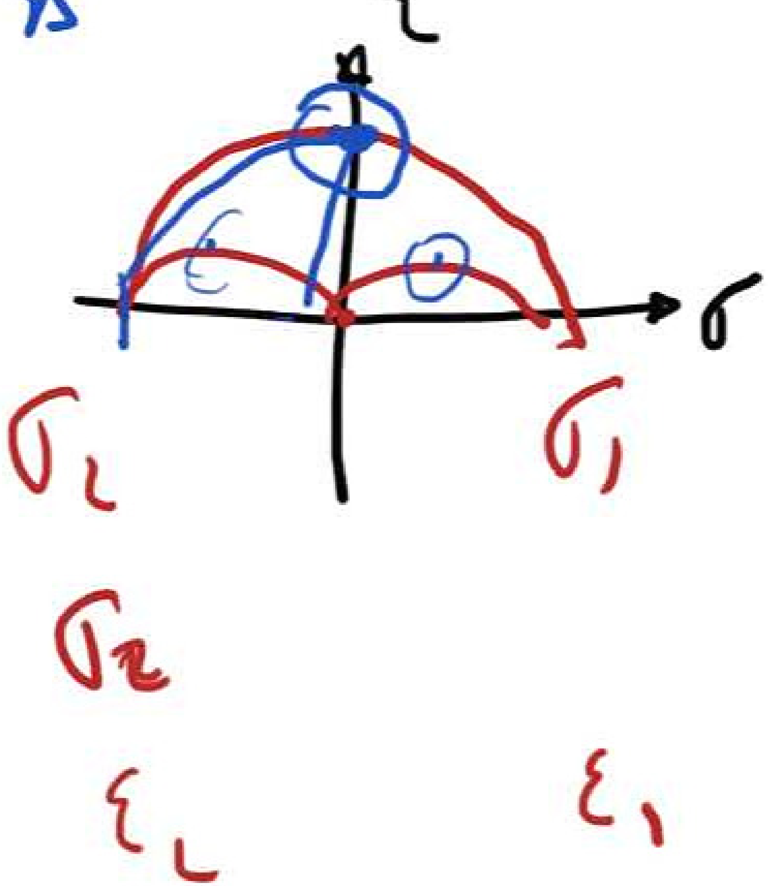
$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

EPT $\sigma_3 = 0$

$\sigma_1, \sigma_2 \neq 0$

$\overleftarrow{\sigma_1} \geq \overleftarrow{\sigma_2} \geq \overleftarrow{\sigma_3}$





- EQUAÇÕES DE TRANSFORMAÇÃO

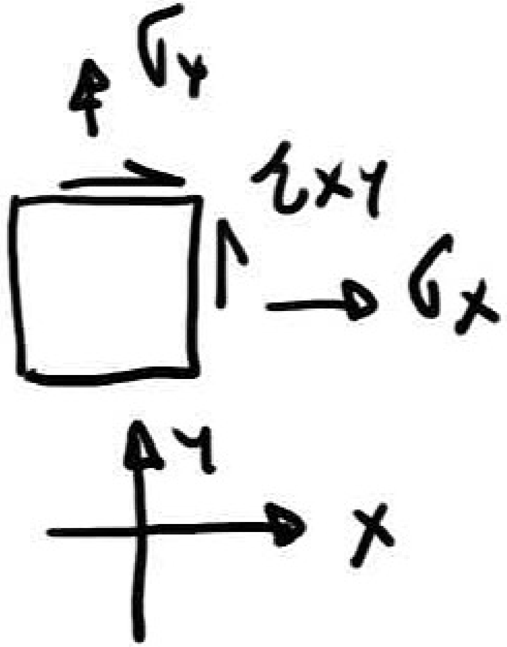
  - ROTAÇÃO DO SISTEMA DE COORDENADAS

EQUILIBRIO

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau' = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

2D

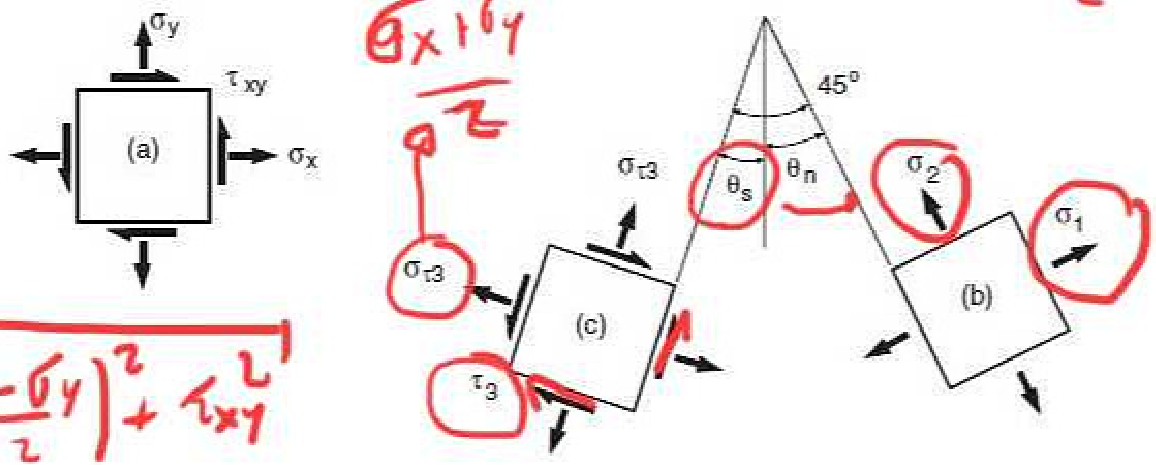


ROTAÇÃO

θ

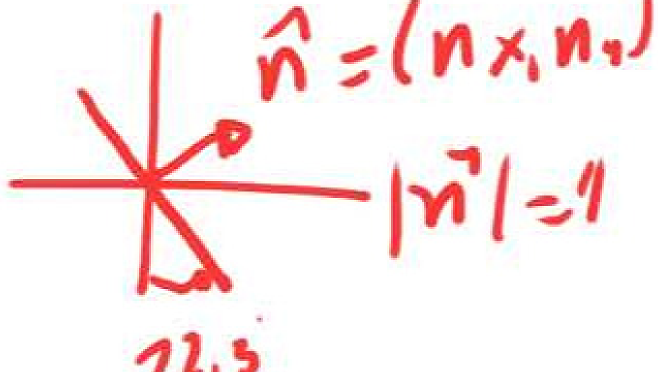
$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\frac{\sigma_x+\sigma_y}{2}$$



$$\tau_3 = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$Tg\,2\theta_n = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}$$



ε_1

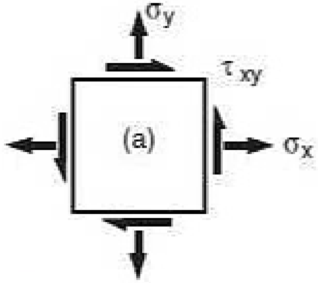
ε_2

ε_3

$$\varepsilon_{ij} = \begin{bmatrix} & \\ & \end{bmatrix}_{3\times3}$$

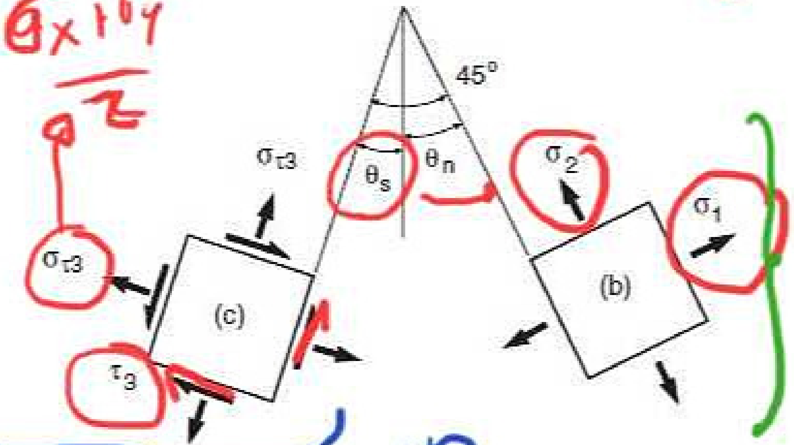
$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

ESTADO PLANO DE TENSÃO



$$\tau_3 = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\frac{\sigma_x+\sigma_y}{2}$$



TENSÕES NORMAIS PRINCIPAIS

CISALHAMENTO MÁXIMO

τ_{xy}

$\mathrm{Tg}\,2\theta_n =$

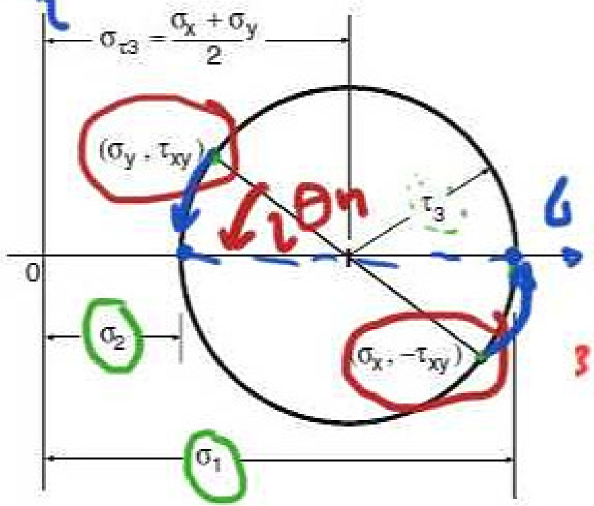
$(\sigma_x - \sigma_y)$

* REPRESENTAÇÃO GRÁFICA DO ESTADO DE TENSÃO NO PONTO

$\hat{n} = (n_x, n_y)$

Eq. CÍRCULO

$\varepsilon \quad \sigma$



3x3

$$(\sigma - a)^2 + \tau^2 = \underbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}_{r^2}$$

$$a = \frac{\sigma_x+\sigma_y}{2}$$

$$\sigma_{1,2} = a \pm r$$

• CONVENÇÃO SINAIS

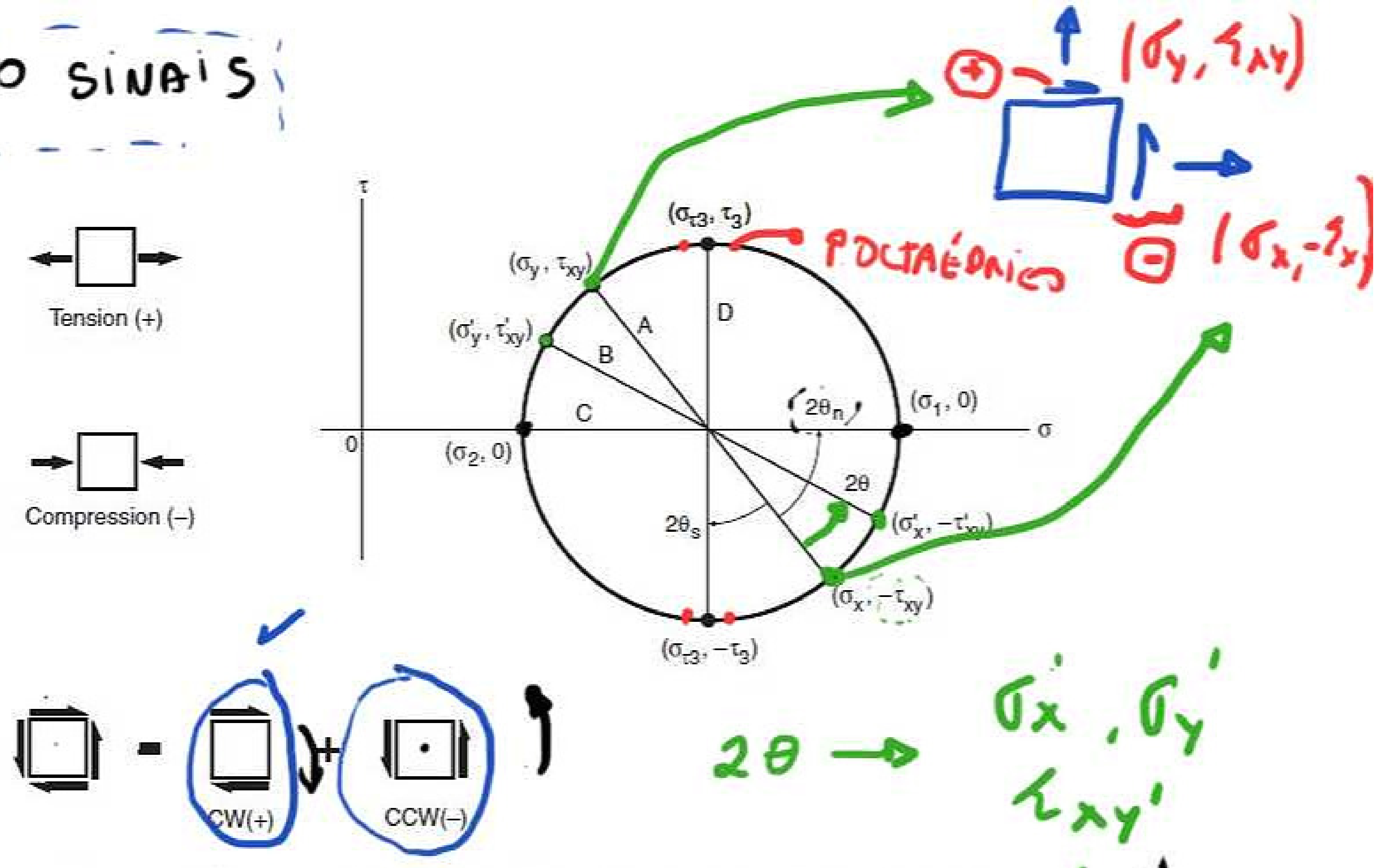
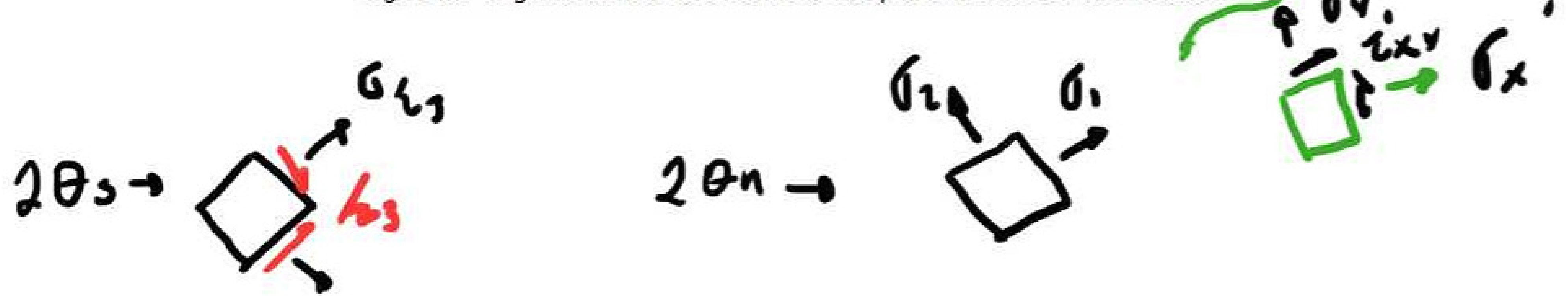


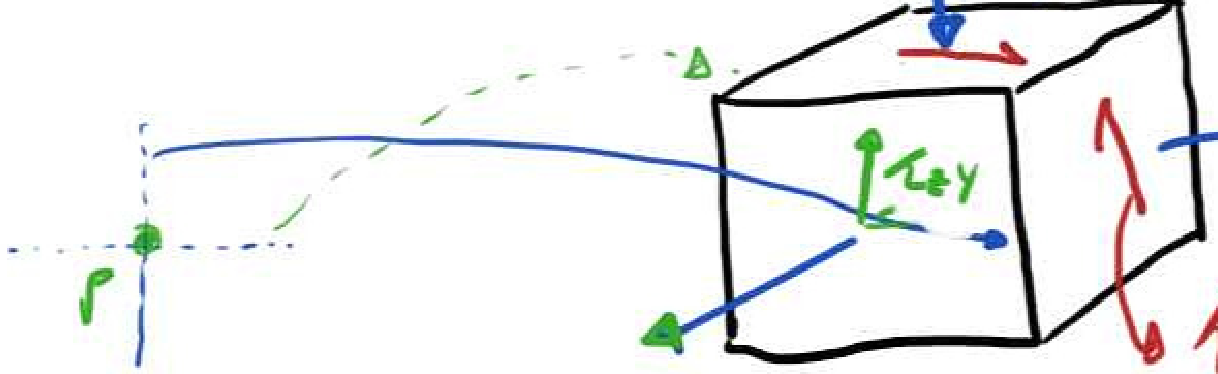
Figure 6.6 Sign convention and diameters of special interest for Mohr's circle.



· EXEMPLO

· PARA O SEGUINTE ESTADO DE TENSÃO

$\sigma_y = -60$ MPa $(-60, 80)$



$\sigma_x = 100$ MPa

$(100, -80)$

$\tau_{xy} = \tau_{yx} = 80$ MPa

$\sigma_z = 40$ MPa

$\tau_{yz} = \tau_{zx} = 0$

Determine:

a) $\sigma_1, \sigma_2, \sigma_3$ ✓

b) τ_1, τ_2, τ_3 ✓

c) DIREÇÕES PRINCIPAIS ✓

$$\sigma_{ij} = \begin{bmatrix} 100 & 80 & 0 \\ 80 & -60 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

TENSÃO PRINCIPAL σ_3

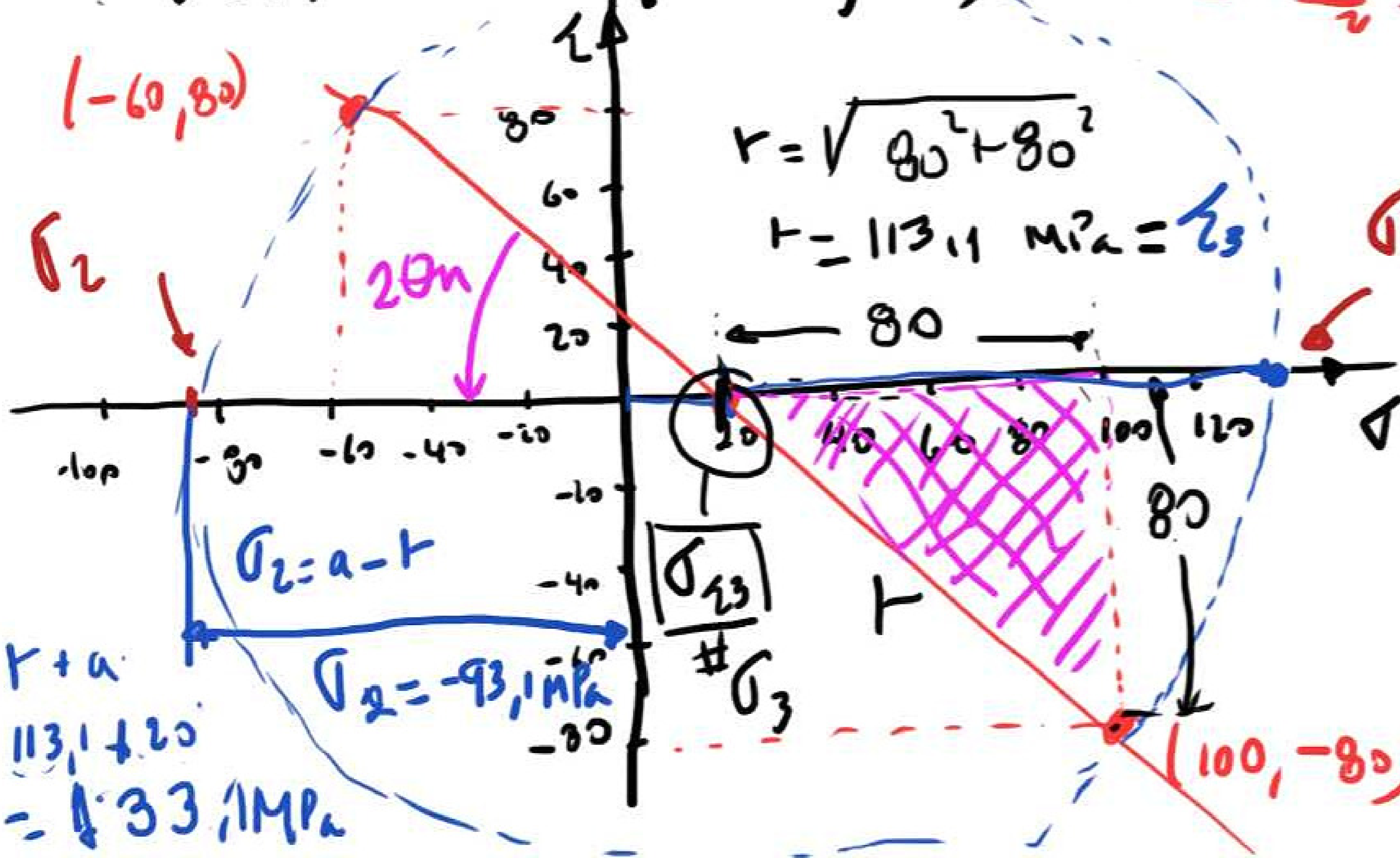
Solução

$$(\sigma_x, -\tau_{xy}) = (100, -80)$$

$$(\sigma_y, \tau_{xy}) = (-60, 80)$$

$$\rightarrow a = \frac{\sigma_x + \sigma_y}{2}$$

$$a = \frac{100 - 60}{2} = 20$$



$$r = \sqrt{80^2 + 80^2}$$

$$r = 113{,}1 \text{ MPa} = \tau_3$$

$$\sigma_2 = a - r$$

$$\sigma_2 = -93{,}1 \text{ MPa}$$

$$^{*}\sigma_1 = r + a$$

$$\sigma_1 = 113{,}1 + 20$$

$$\sigma_1 = 133{,}1 \text{ MPa}$$

$\sigma_1 = 133{,}1 \text{ MPa}$

$\sigma_2 = -93{,}1 \text{ MPa}$ $\longrightarrow$

$\sigma_3 = 40 \text{ MPa}$

$\sigma_1 = 133{,}1 \text{ MPa}$

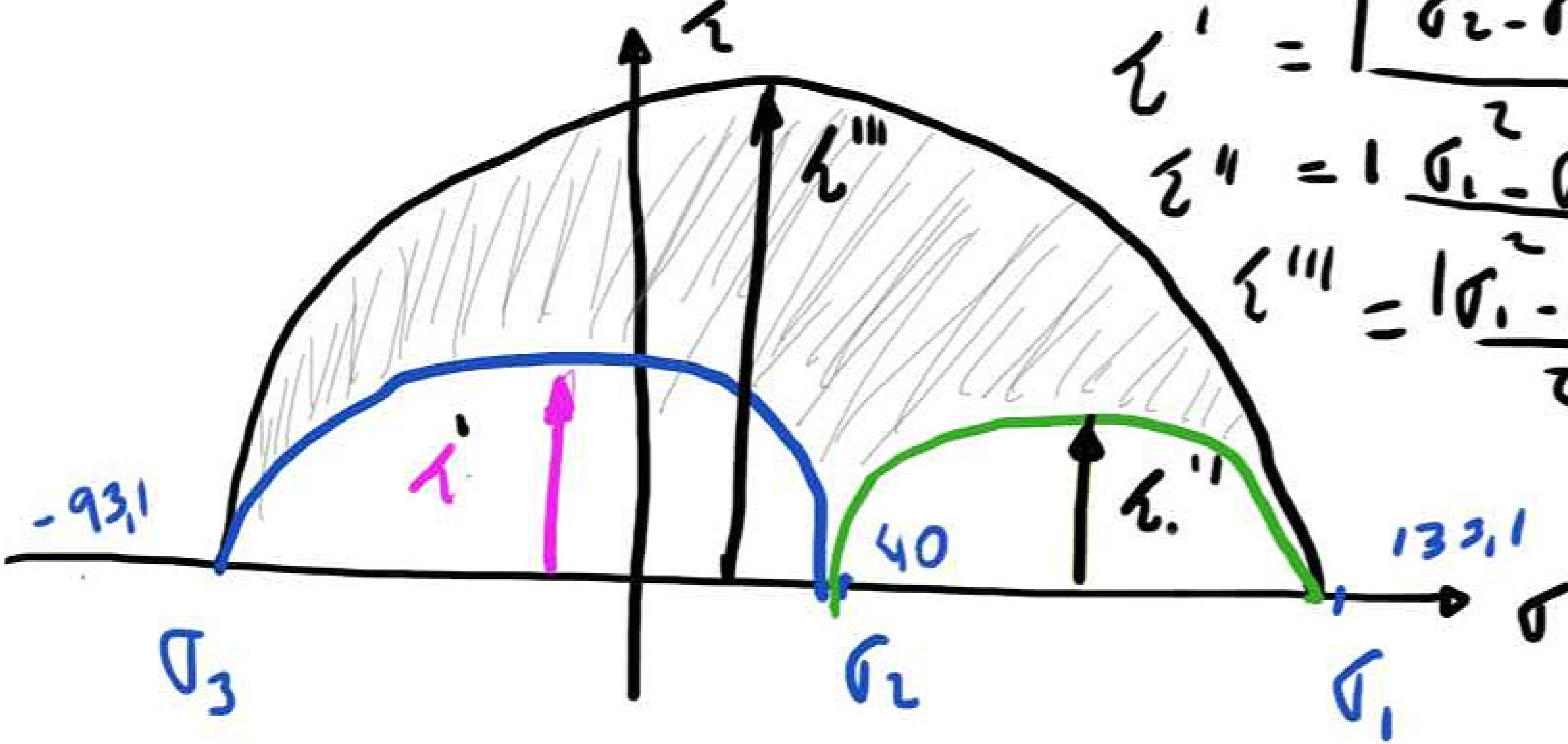
$\sigma_2 = 40 \text{ MPa}$

$\sigma_3 = -93{,}1 \text{ MPa}$

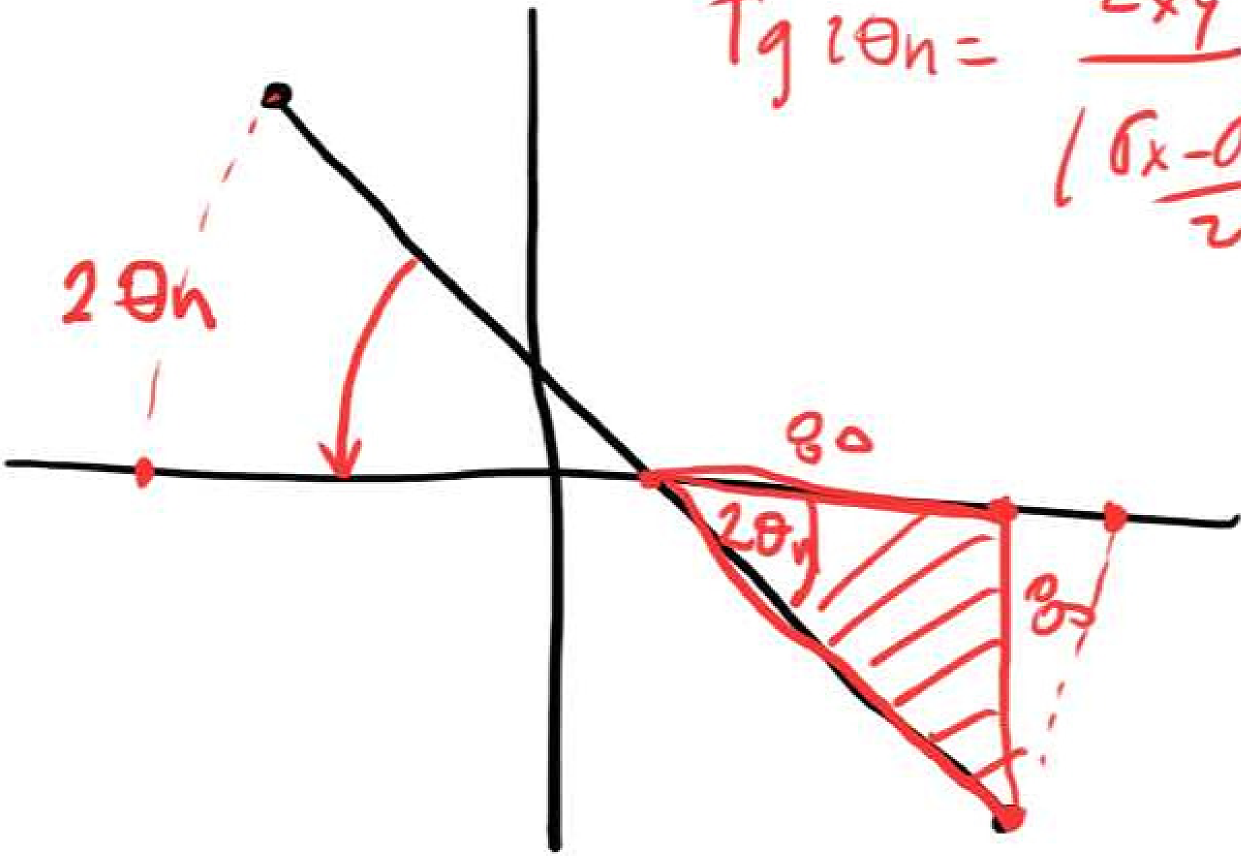
$$\tau' = \frac{|\sigma_2 - \sigma_3|}{2}$$

$$\tau'' = \frac{|\sigma_1 - \sigma_2|}{2}$$

$$\tau''' = \frac{|\sigma_1 - \sigma_3|}{2}$$

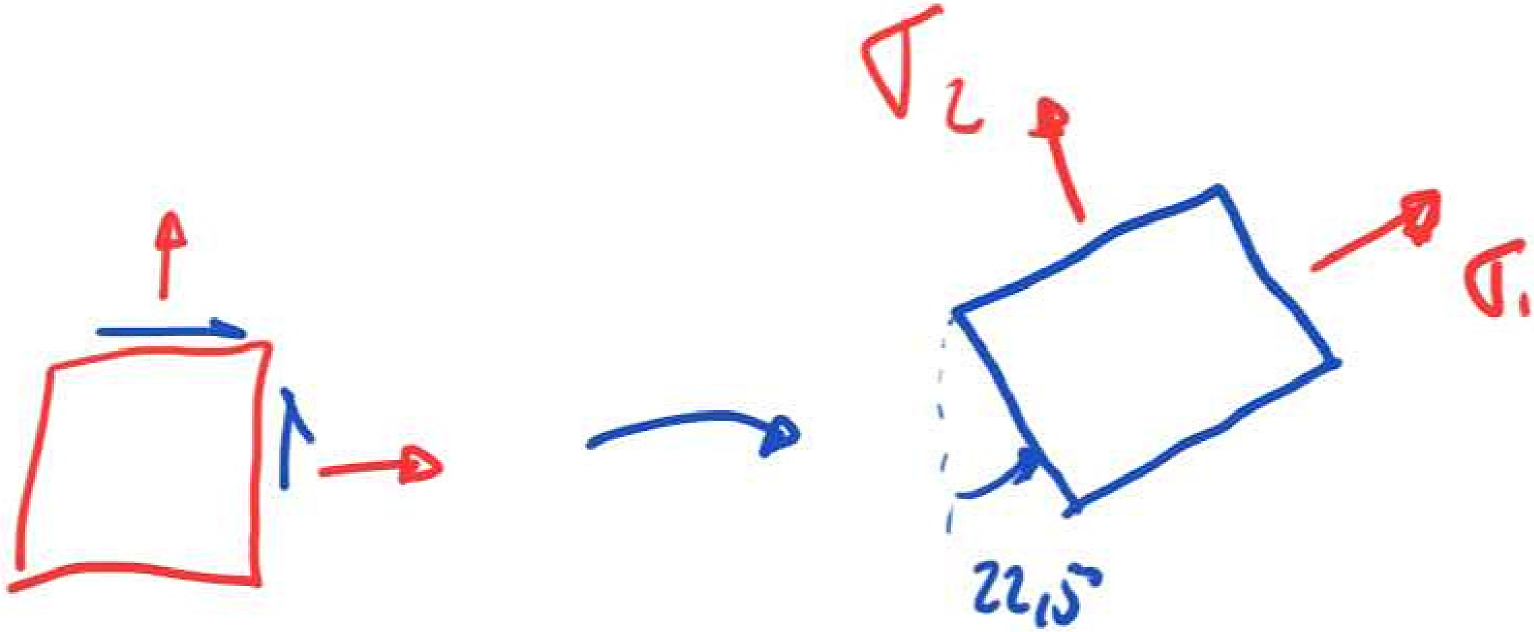


$$Tg\,2\theta_n = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$



$$Tg\,2\theta_n = \frac{80}{80} \rightarrow \quad 2\theta_n = 45^\circ$$

$$\theta_n = 22{,}5^\circ$$



σ_2
σ_1
22,5°