DATA: ____________________ Q3 ______
Q4 ______
TOTAL

① $\vec{v}(t) = A e^{-\alpha t}\hat{x} + B\cos(\omega t)\hat{y} + Ct\hat{z}$

(a) $\vec{F}_R = m\frac{d\vec{v}}{dt} = m\left(-\alpha A e^{-\alpha t}\hat{x} - \omega B \sin(\omega t)\hat{y} + C\hat{z}\right)$

$\vec{F}_R = -m\alpha A e^{-\alpha t}\hat{x} - m\omega B \sin(\omega t)\hat{y} + mC\hat{z}$

(b) $\vec{r} - \vec{r}_0 = \int_0^t \vec{v}(t')\,dt' \qquad \vec{r}_0 = 0$

$\vec{r} = \int_0^t A e^{-\alpha t'}\hat{x}\,dt' + \int_0^t B\cos(\omega t')\hat{y}\,dt' + \int_0^t Ct'\hat{z}\,dt$

$\vec{r}(t) = \frac{A}{-\alpha}\left(e^{-\alpha t} - 1\right)\hat{x} + \frac{B}{\omega}\sin(\omega t)\hat{y} + \frac{C}{2}t^2\hat{z}$

(c) $\vec{J} = \Delta\vec{p} = m\Delta\vec{v} \qquad (\vec{p} = m\vec{v})$

$\vec{J} = m\vec{v}(t) - m\vec{v}(0) \qquad \vec{v}(0) = A\hat{x} + B\hat{y}$

$\vec{J}(t) = A\left(e^{-\alpha t} - 1\right)\hat{x} + B\left(\cos(\omega t) - 1\right)\hat{y} + Ct\hat{z}$

(d) $W_{ab} = \Delta K_{ab} = \frac{1}{2}m\left(v_b^2 - v_a^2\right) \qquad v_b = v(1)$
$v_a = v(0)$

$v_1^2 = A^2 e^{-2\alpha} + B^2\cos^2(\omega) + C^2$

$W_{ab} = \frac{1}{2}m\left[A^2\left(e^{-2\alpha} - 1\right) + B^2\left(\cos^2\omega - 1\right) + C^2\right]$

$W_{ab} = -5\,J$

② $M = \int_0^{\ell} \left(\frac{dM}{dx}\right) dx \qquad I = \int_0^{\ell} x^2 \left(\frac{dM}{dx}\right) dx$

(a)

$$M = \int_0^{\ell} \alpha x^2 dx = \frac{\alpha \ell^3}{3} = 1 \text{ kg}$$

$$I = \int_0^{\ell} \alpha x^4 dx = \frac{\alpha \ell^5}{5} = \frac{3}{5} \text{ kg m}^2$$

$$\frac{I}{M} = \frac{3}{5} = \frac{3}{5}\ell^2 \Rightarrow \ell = 1 \text{ m}$$

$$\Rightarrow \alpha = 3 \text{ kg/m}^3$$

(b) $x_{cm} = \frac{1}{M}\int_0^{\ell} x \left(\frac{dM}{dx}\right) dx = \frac{1}{M}\int_0^{\ell} \alpha x^3 dx$

$x_{cm} = \frac{\alpha}{M}\frac{\ell^4}{4} = \frac{3}{4} \text{ m} \qquad I_{cm} = I - M x_{cm}^2 = \frac{3}{80} \text{ kg m}^2$

(c) Cons. K : $\frac{1}{2} m v^2 = \frac{1}{2} I \omega^2 \Rightarrow m v^2 = I \omega^2$

Cons. L : $m v \ell = I \omega \Rightarrow m^2 v^2 \ell^2 = I^2 \omega^2$

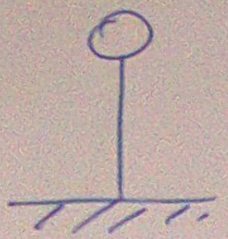
$$\frac{m^2 v^2 \ell^2}{m v^2} = \frac{I^2 \omega^2}{I \omega^2} = I = \frac{3}{5} \text{ kg m}^2$$

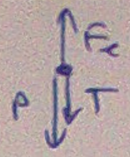
$$\Rightarrow m \ell^2 = \frac{3}{5} \text{ kg m}^2$$

$$m = \frac{3}{5} \text{ kg}$$

NOME:
PROFESSOR:
DATA:

③ (a)





$F_e = T + P$

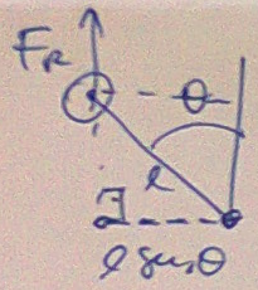
$T = 10000\,N$

$P = mg = 1000\,N$

$F_e = 10000 + 1000 = 11000\,N$

$F_e = \rho V g \qquad V = \frac{F_e}{\rho g} = \frac{11000}{10^3 \times 10} = 1{,}1\,m^3$

(b) $\tau_R = 2\times 10^5\,\theta \qquad F_R = F_e - P = 10000\,N$



$\tau_R = F_R\,\ell \operatorname{sen}\theta \approx \underbrace{F_R\,\ell}\,\theta$

$F_R\,\ell = 2\times 10^5$

$\ell = \frac{2\times 10^5}{10000} = 20\,m$

(c) $A^2 = \frac{(F_0/m)^2}{(\omega^2-\omega_0^2)^2 + 4\omega^2\gamma^2} \qquad \frac{d^2\theta}{dt^2} + \underbrace{0{,}25}_{\gamma^2}\frac{d\theta}{dt} + \underbrace{4}_{\omega_0^2}\theta = \underbrace{0{,}1}_{F/m}\cos(\omega t)$

$\gamma = 0{,}25 = \frac{1}{4}; \quad \omega_0 = 2; \quad \frac{F_0}{m} = 0{,}1$

$A^2 = \frac{0{,}01}{16\times\frac{1}{16}} = 0{,}01 \qquad A = 0{,}1\ rad$