Aula 05 - Inequações e Noção Intuitiva de Limites

Inequações

$>$ maior que $\quad$ $<$ menor que

$\geq$ maior ou igual $\quad$ $\leq$ menor ou igual

$\neq$ diferente

Inequação de 1º grau

$ax + b > 0$
$ax + b < 0$
$ax + b \geq 0$
$ax + b \leq 0$

Cuidado! multiplicar ou dividir por um nº negativo

Exemplos:

$a)\ 3x + 19 < 40$

$3x < 40 - 19$

$3x < 21$

$x < \frac{21}{3} \Rightarrow x < 7$

$b)\ 15 - 7x \geq 2x - 30$

$-7x - 2x \geq -30 - 15$

$-9x \geq -45$

$x \leq \frac{-45}{-9} \Rightarrow x \leq 5$

ou

$15 + 30 \geq 2x + 7x$

$45 \geq 9x$

$\frac{45}{9} \geq x$

$x \leq 5$

$c)\ 3x - 21 < 0$

$3x - 21 = 0$

$x = \frac{21}{3}$

$x = 7$

$-$ $\quad$ 7 $\quad$ $+$

$x < 7$

$d)\ 5 \leq 2x - 3 < 7$

$2x - 3 < 7$

$a)\ 5 \leq 2x - 3 < 7$

$5 \leq 2x - 3$

$5 + 3 \leq 2x$

$2x \geq 8$

$x \geq 4$

$2x - 3 < 7$

$2x < 7 + 3$

$x < \frac{10}{2}$

$x < 5$

$S = \{x \in \mathbb{R} \mid 4 \leq x < 5\}$

Inequação de 2º grau

$ax^2 + bx + c > 0$
$ax^2 + bx + c < 0$
$ax^2 + bx + c \geq 0$
$ax^2 + bx + c \leq 0$

$32 = 4 \cdot 4 \cdot 2$

$\sqrt{32} = 4\sqrt{2}$

Exemplos:

$a)\ x^2 - 4x - 4 < 0$

$x^2 - 4x - 4 = 0$

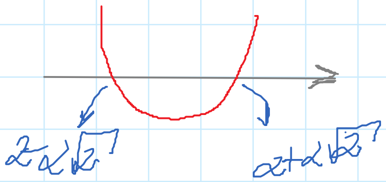
$\Delta = 16 - 4 \cdot 1 \cdot (-4)$

$\Delta = 32$

$x = \frac{4 \pm 4\sqrt{2}}{2}$

$x_1 = 2 + 2\sqrt{2} \cong 5$

$x_2 = 2 - 2\sqrt{2} \cong -1$



Noção gráfica!

$S = \{x \in \mathbb{R} \mid 2 - 2\sqrt{2} < x < 2 + 2\sqrt{2}\}$

$b)\ x^2 + x - 2 \leq 0$

$x^2 + x - 2 = 0$

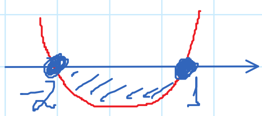
$\Delta = 1 - 4 \cdot 1 \cdot (-2)$

$\Delta = 9$

$x = \frac{-1 \pm 3}{2}$

$x_1 = -2$

$x_2 = 1$



$S = \{x \in \mathbb{R} \mid -2 \leq x \leq 1\}$

Inequação Produto e Inequação Quociente

$-2 \qquad -\frac{1}{2}$

$f(x) \quad - \quad - \quad +$

Exemplos:

$a)\ (2x + 1) \cdot (x + 2) \leq 0$

Exemplos:

$a)\ (2x+1)\cdot(x+2)\leq 0$

$f(x)$ $g(x)$

	-2		$-\frac{1}{2}$	
$f(x)$	$-$	$-$		$+$
$g(x)$	$-$	$+$		$+$
$f(x)\cdot g(x)$	$+$	$-$		$+$

$f(x) \to 2x+1=0$
$x=-\frac{1}{2}$

$\ominus\ -\frac{1}{2}\ \oplus$

$S=\{x\in\mathbb{R} \mid -2\leq x\leq -\frac{1}{2}\}$

$g(x) \to x+2=0$
$x=-2$

$\ominus\ -2\ \oplus$

ou

$S=[-2\,;-\frac{1}{2}]$

$b)\ \dfrac{2x-3}{1-x}\leq 0$ $\to f(x)$, $\to g(x)$

$Dom(f)=\{x\in\mathbb{R} \mid x\neq 1\}$

$1-x\neq 0$

$x\neq 1$

$f(x) \to 2x-3=0$
$x=3/2$

$\ominus\ 3/2\ \oplus$

		1		$3/2$	
$f(x)$	$-$		$-$		$+$
$g(x)$	$+$		$-$		$-$
$f(x)/g(x)$	$-$		$+$		$-$

$g(x) \to 1-x=0$
$x=1$
$-x+1=0$

$\oplus\ 1\ \ominus$

reta decrescente

$S=\{x\in\mathbb{R} \mid x<1 \text{ ou } x\geq 3/2\}$

ou $S=(-\infty\,;1[\ \text{ou}\ [3/2\,;+\infty)$

$c)\ \dfrac{(x+1)\cdot(x+4)}{x-2}>0$

$f(x) \to x+1=0$
$x=-1$

$\ominus\ -1\ \oplus$

$g(x) \to x+4=0$
$x=-4$

$\ominus\ -4\ \oplus$

$h(x) \to x-2=0$
$x=2$

$\ominus\ 2\ \oplus$

obs:
$ax+b=0$
$a>0$ $a<0$

$Dom(h)=\{x\in\mathbb{R} \mid x\neq 2\}$

$Dom(f) = \{x \in \mathbb{R} \mid x \neq 2\}$

$S = \{x \in \mathbb{R} \mid -4 < x < -1 \text{ ou } x > 2\}$

	-4	-1	2	
$f(x)$	$-$	$-$	$+$	$+$
$g(x)$	$-$	$+$	$+$	$+$
$h(x)$	$-$	$-$	$-$	$+$
$\frac{f(x)\cdot g(x)}{h(x)}$	$-$	$+$	$-$	$+$

d) $(x^2 - 8x + 12) \cdot (x^2 - 5x) < 0$

$f(x) \rightarrow x^2 - 8x + 12 = 0$

$\Delta = 64 - 4 \cdot 1 (12)$

$\Delta = 64 - 48$

$\Delta = 16$

$x = \frac{8 \pm 4}{2}$ → $x_1 = 6$, $x_2 = 2$

$+$ 2 $-$ 6 $+$

Obs: $\frac{4}{0} \nexists$

$\frac{4}{x-5} \rightarrow x - 5 \neq 0$, $x \neq 5$

quociente denominador $\neq 0$

restrição: - quociente $\neq 0$
- logaritmo > 0
- raiz par ≥ 0

$g(x) \rightarrow x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ $x = 5$

$+$ 0 $-$ 5 $+$

	0	2	5	6	
$f(x)$	$+$	$+$	$-$	$-$	$+$
$g(x)$	$+$	$-$	$-$	$+$	$+$
$f(x)\cdot g(x)$	$+$	$-$	$+$	$-$	$+$

$S = \{x \in \mathbb{R} \mid 0 < x < 2 \text{ ou } 5 < x < 6\}$

Inequação Logarítmica

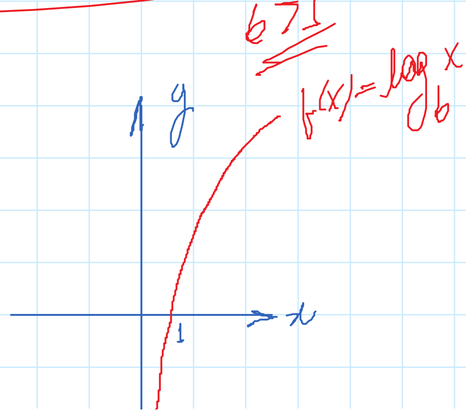
Exemplos:

a) $\log_5(2x - 3) < \log_5 x$

base > 1

bases iguais e maiores que 1

$2x - 3 < x$



$2x-3 < x$

$2x - x < 3$

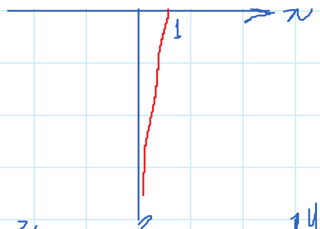
$x < 3$

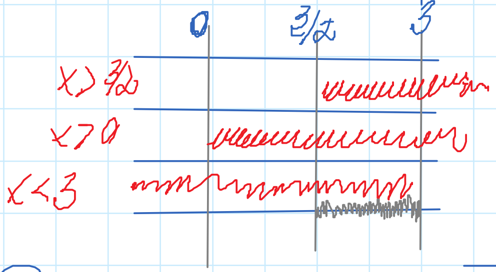
$\log(x) \rightarrow$ maior que zero

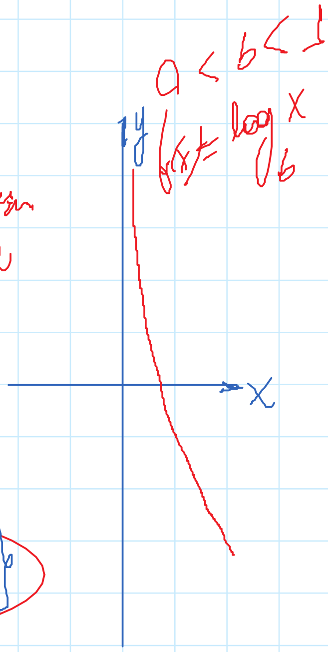
$\log_5(2x-3) \Rightarrow 2x-3>0$

$x > 3/2$

$\log_5 x \Rightarrow x>0$







$S = \{x \in \mathbb{R} \mid 3/2 < x < 3\}$

b) $\log_2(x+3) \geq 3$

$x+3>0$

$x>-3$

$\log_2(x+3) \geq 3$

base 2 > 1

$x+3 \geq 2^3$

$x \geq 8-3 \rightarrow x \geq 5$

$S = \{x \in \mathbb{R} \mid x \geq 5\}$

c) $\log_{\frac{1}{2}} 3x > \log_{\frac{1}{2}}(2x+5)$

$3x>0$

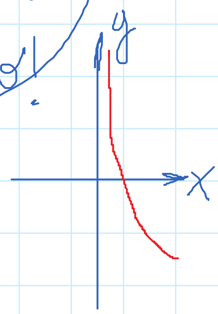
$x>0$

$2x+5>0$

$x > -5/2$

0 < base < 1

cuidado!



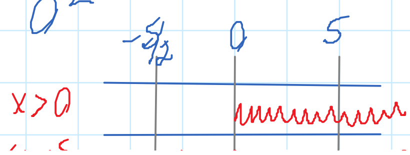
$\log_{1/2} 3x > \log_{1/2}(2x+5) \rightarrow$ bases iguais

$0<b<1$

$3x < 2x+5$

$3x-2x<5$

$x<5$



$3x - 2x < 5$

$x < 5$

$x > 0$

$x > -\frac{5}{2}$

$x < 5$

$S = \{x \in \mathbb{R} \mid 0 < x < 5\}$

Inequação Exponencial

$a^m = a^n$

$m = n$

Exemplos:

a) $4^x < 16$ → mesma base, base > 1

$a^m = b^m$

$a = b$

$4^x < 4^2$

$x < 2$

$S = \{x \in \mathbb{R} \mid x < 2\}$

b) $25^x - 6 \cdot 5^x + 5 < 0$

$S = \{x \in \mathbb{R} \mid 0 < x < 1\}$

$(5^2)^x - 6 \cdot 5^x + 5 < 0$

$(5^x)^2 - 6 \cdot 5^x + 5 < 0$

$h = 5^x$

$h^2 - 6h + 5 < 0$

P/ $h = 1$

$1 = 5^x$

$5^0 = 5^x$

$x = 0$

$\Delta = 36 - 4 \cdot 1 \cdot 5$

$\Delta = 16$

$h = \frac{6 \pm 4}{2}$ → $h_1 = 5$, $h_2 = 1$

$1 < h < 5$

P/ $h = 5$

$h = 5^x$

$5 = 5^x$

$5^1 = 5^x$

$x = 1$

1 − 5

Noção Intuitiva de Limite

2.1 Limite

2.1.1 Noção intuitiva de Limite

→ Tende à

Considere as seguintes sequências numéricas:

2.1 Limite

2.1.1 Noção intuitiva de Limite

$x = 1$

$\to$ tende à

Considere as seguintes sequências numéricas:

a. $\{1, 2, 3, 4, \ldots, 100, 101, \ldots\}$ $x \to +\infty$

b. $\{-1, -2, -3, -4, \ldots, -100, -101, \ldots\}$ $x \to -\infty$

c. $\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{1024}, \ldots\}$ $x \to 0$

Agora, se considerarmos uma função real:

$$f : \begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto y = f(x) \end{cases}$$

O conceito de limite refere-se a seguinte questão: qual é o valor para o qual y tende quando x tende a um valor específico?

Estude o limite das funções a seguir nos pontos indicados.

Exemplo 2.1 $y = 2x + 1$ quando $x \mapsto 10$.

$y \to 21$

$x \to 10$

x	9	9,9	9,99
y	19	20,8	20,98

$y \to 21$

$x \to 0^+$

$y \to -\infty$

$x \to 0^-$

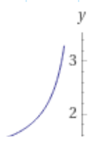
$y \to +\infty$

Exemplo 2.2 $y = 1 - \dfrac{1}{x}$ quando $x \mapsto \pm\infty$.

$x \to +\infty$

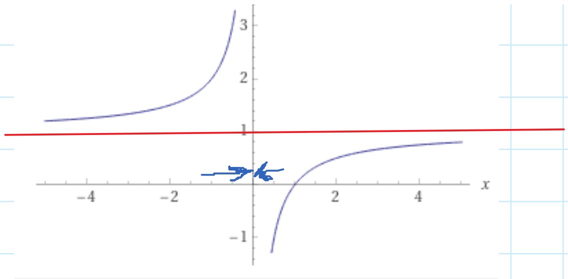
x	10	100	1000	10000
y	0,9	0,99	0,999	0,9999

$y \to 1^-$



y	0,9	0,99	0,999	0,9999

$y \to 1^-$



$x \to -\infty$

x	-10	-100	-1000	-10000
y	1,1	1,01	1,001	1,0001

$y \to 1^+$

$x \to 0^+$

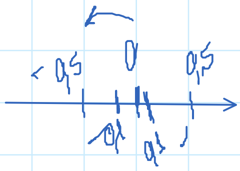
x	0,1	0,01	0,001
y	-9	-99	-999

$y \to -\infty$

$x \to 0^-$

x	$-0,1$	$-0,01$	$-0,001$
y	11	101	1001

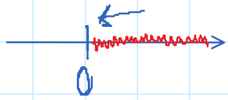
$y \to +\infty$



log

Exemplo 2.3 $y = \ln(x)$ quando $x \mapsto 0$.

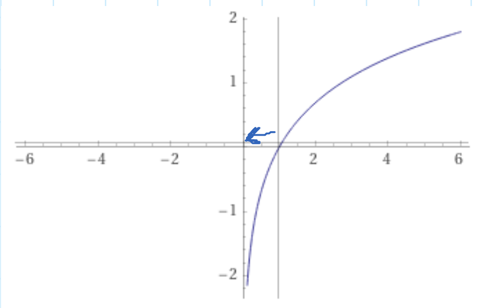
$\mathrm{Dom}(f) = \{x \in \mathbb{R} \mid x > 0\}$



$x \to 0^+$

x	0,1	0,01	0,001	0,0001
y	-1	-2	-3	-4

$y \to -\infty$



Exemplo 2.4 $y = \exp(x)$ quando $x \mapsto -\infty$.

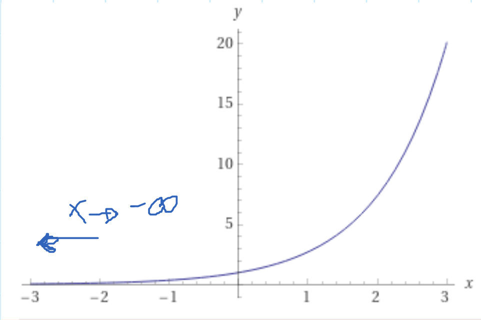
Exemplo 2.4 $y = \exp(x)$ quando $x \mapsto -\infty$.

$x \to -\infty$

x	-10	-100	-1000	-10000
y	$0{,}0005$	$3{,}7 \times 10^{-44}$	0	0

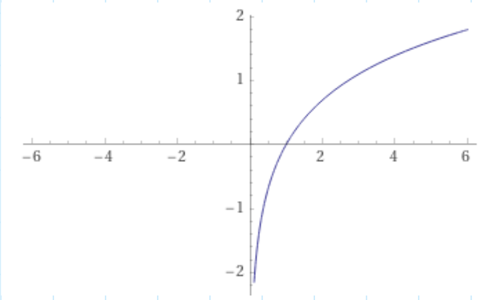
$y \to 0^+$

$\underbrace{0{,}00000\ldots}_{43 \text{ zeros}}37$



Verifique graficamente o que acontece com as funções:

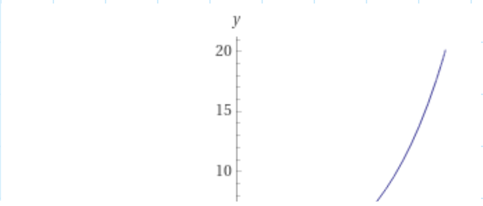
Exemplo a) $f(x) = \log(x)$



$\begin{cases} x \to +\infty \\ y \to +\infty \end{cases}$

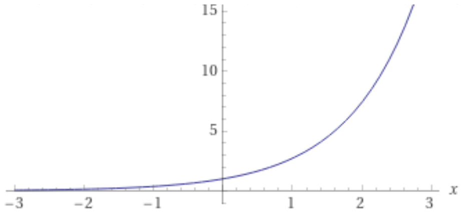
$\begin{cases} x \to 0^+ \\ y \to -\infty \end{cases}$

Exemplo b) $f(x) = e^x$



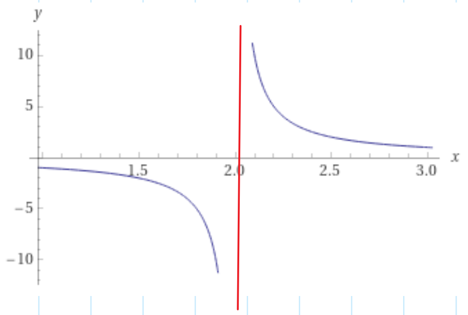
$\begin{cases} x \to -\infty \\ y \to 0^+ \end{cases}$

$\{ x \to +\infty$



$$\begin{cases} x \to +\infty \\ y \to +\infty \end{cases}$$

Exemplo c) $f(x) = \frac{1}{x-2}$ $\to$ $Dom(f) = \{x \in \mathbb{R} \mid x \neq 2\}$



$$\begin{cases} x \to +\infty \\ y \to 0^+ \end{cases} \qquad \begin{cases} x \to 2^+ \\ y \to +\infty \end{cases}$$

$$\begin{cases} x \to -\infty \\ y \to 0^- \end{cases} \qquad \begin{cases} x \to 2^- \\ y \to -\infty \end{cases}$$