Física IV

14 setembro 2020

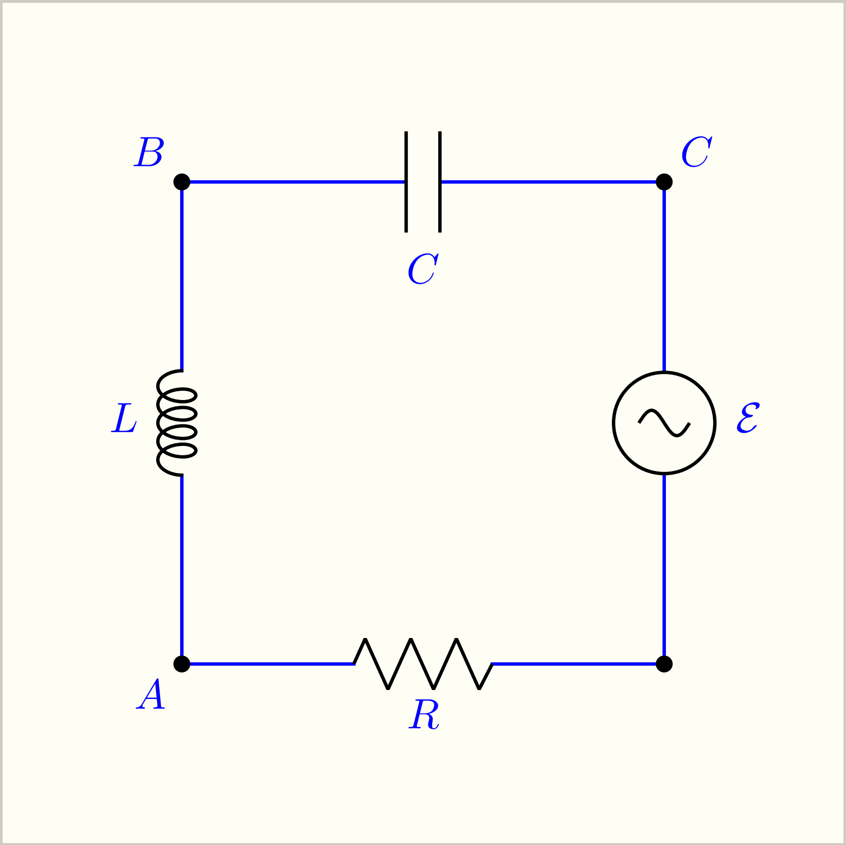
Circuitos de Corrente Alternada

Circuitos de corrente alternada

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

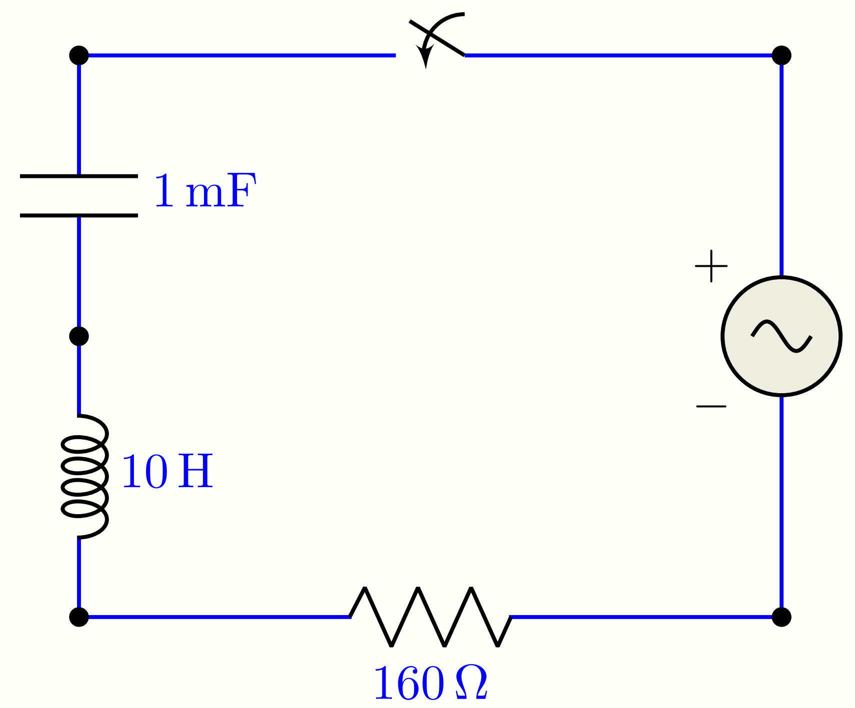
$$Q(0) = Q_0$$

$$I(0) = I_0$$



Circuitos de corrente alternada

$$10\frac{d^2Q}{dt^2} + 160\frac{dQ}{dt} + 10^3 Q = 1.6\cos(10t)$$



$$Q(0) = -10\,\mathrm{mC}$$

$$I(0) = 0$$

Solução estacionária Q(t) => q(t)

$$10\frac{d^2q}{dt^2} + 160\frac{dq}{dt} + 10^3 q = 1.6\cos(10t)$$

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$$10\frac{d^2z}{dt^2} + 160\frac{dz}{dt} + 10^3 z = 1.6\exp(i10t)$$

$$10\frac{d^2q}{dt^2} + 160\frac{dq}{dt} + 10^3 q = 1.6\cos(10t)$$

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$$\boxed{z(t) = z_0\exp(i10t)}$$

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$$\boxed{z(t) = z_0\exp(i10t)}$$

$$-10^3 z_0 + 1600iz_0 + 10^3 z_0 = 1.6$$

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$$z_0 = \frac{1}{10^3 i}$$

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$$z_0 = \frac{1}{10^3 i} \qquad \Rightarrow z = -10^{-3}i\Big(\cos(10t) + i\sin(10t)\Big)$$

$$10\frac{d^2q}{dt^2} + 160\frac{dq}{dt} + 10^3 q = 1.6\cos(10t)$$

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$$\boxed{z(t) = z_0\exp(i10t)}$$

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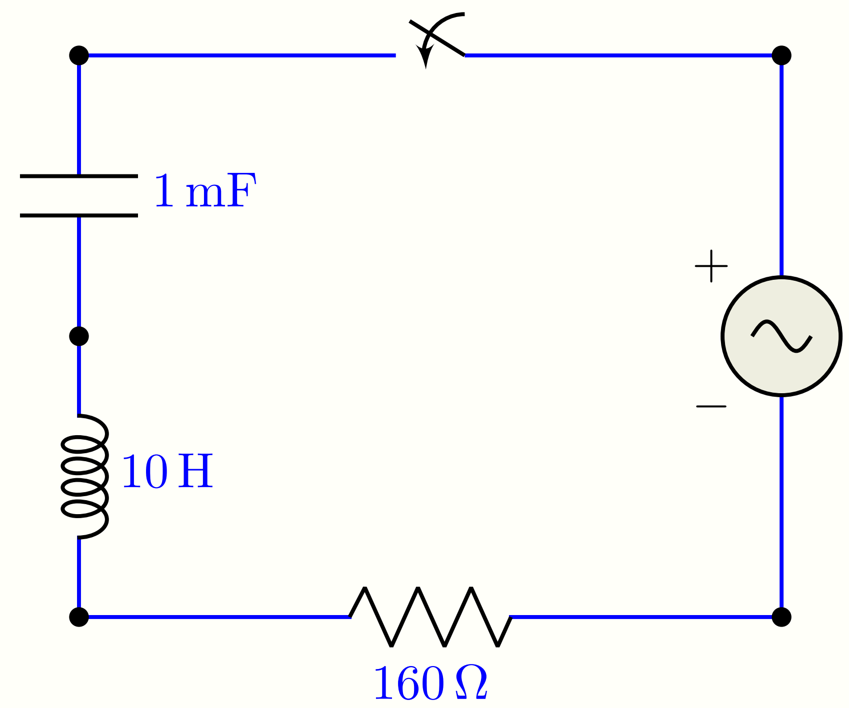
$$z_0 = \frac{1}{10^3 i}$$

$$\Rightarrow z = -10^{-3} i\Big(\cos(10t) + i\sin(10t)\Big)$$

$$\Rightarrow q = 10^{-3}\sin(10t)$$

Circuitos de corrente alternada

$$10\frac{d^2Q}{dt^2} + 160\frac{dQ}{dt} + 10^3 Q = 1.6\cos(10t)$$



$$Q(t) = q(t) + \Delta Q$$

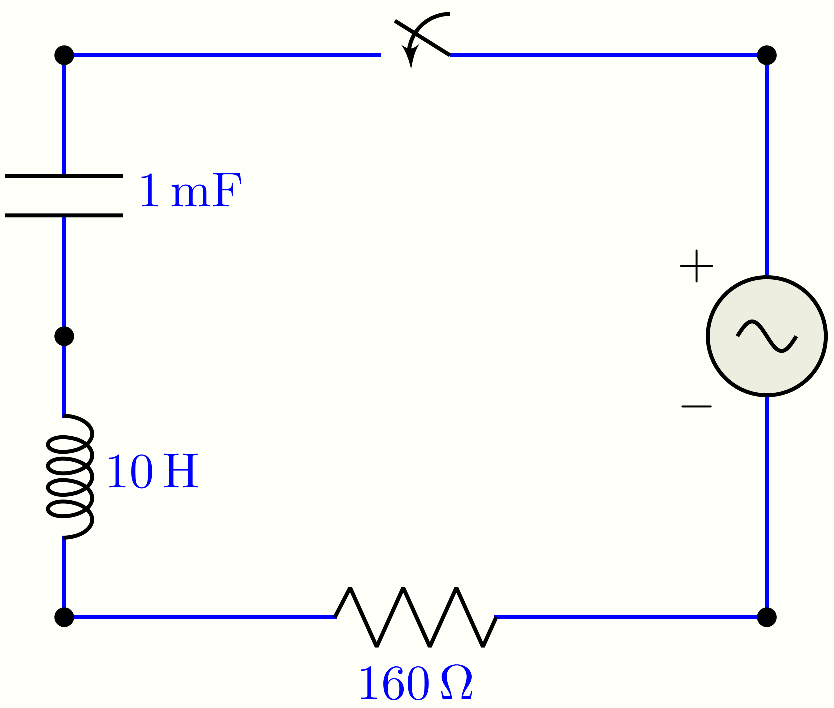
$$Q(0) = -\,10\,\mathrm{mC}$$

$$I(0) = 0$$

Circuitos de corrente alternada

$$q(t) = \sin(10t)\,\mathrm{mC}$$

$$10\frac{d^2Q}{dt^2} + 160\frac{dQ}{dt} + 10^3 Q = 1.6\cos(10t)$$



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Circuitos de corrente alternada

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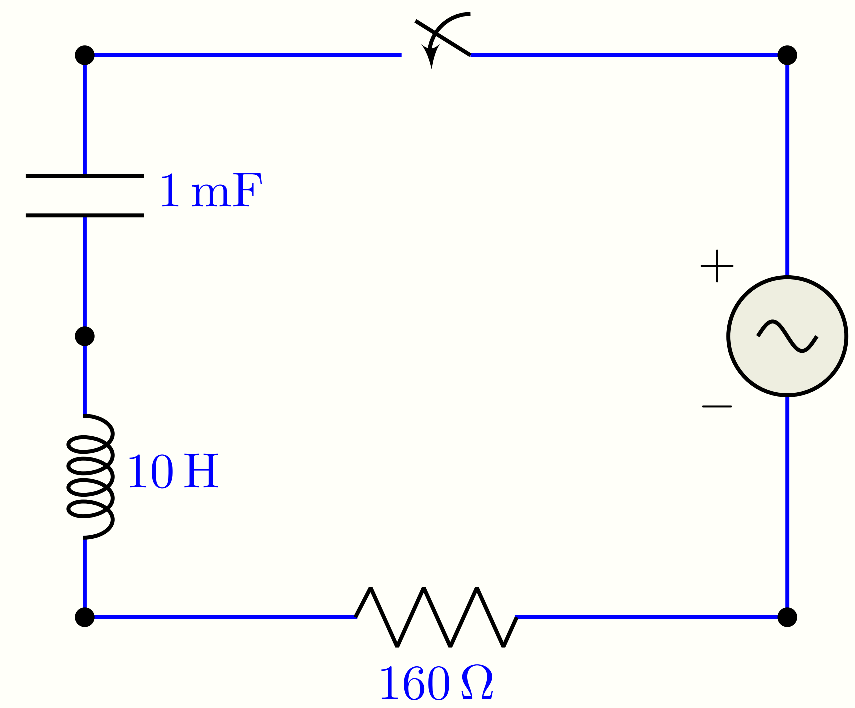

$$Q(t) = q(t) + \Delta Q$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3 \Delta Q = 0$$

$$Q(0) = -10\,\mathrm{mC}$$

$$I(0) = 0$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$\frac{d^2\Delta Q}{dt^2} + 16\frac{d\Delta Q}{dt} + 10^2\Delta Q = 0$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$\frac{d^2\Delta Q}{dt^2}+16\frac{d\Delta Q}{dt}10^2\Delta Q = 0$$

$$\tau = \frac{1}{8}\,\mathrm{s}$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$\frac{d^2\Delta Q}{dt^2} + 16\frac{d\Delta Q}{dt} + 10^2\Delta Q = 0$$

$$\tau = \frac{1}{8}\,\text{s}$$

$$\omega_0 = 10\,\text{rad/s}$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$\frac{d^2\Delta Q}{dt^2} + 16\frac{d\Delta Q}{dt} + 10^2\Delta Q = 0$$

$$\left.\begin{array}{l} \tau = \dfrac{1}{8}\,\mathrm{s} \\[2em] \omega_0 = 10\,\mathrm{rad/s} \end{array}\right\} \quad \omega_0\tau > 1 \qquad \text{(subamortecido)}$$

$$10\frac{d^2\Delta Q}{dt^2} + 160\frac{d\Delta Q}{dt} + 10^3\Delta Q = 0$$

$$\frac{d^2\Delta Q}{dt^2} + 16\frac{d\Delta Q}{dt} + 10^2\Delta Q = 0$$

$$\left.\begin{array}{l} \tau = \dfrac{1}{8}\,\mathrm{s} \\ \\ \omega_0 = 10\,\mathrm{rad/s} \end{array}\right\} \omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\mathrm{rad/s}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\text{rad/s}$$

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$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\mathrm{rad/s}$$

$$Q_x = \exp(-8t)\cos(6t)$$

$$Q_y = \exp(-8t)\frac{\sin(6t)}{6}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\text{rad/s}$$

$$Q(t) = q(t) + \alpha Q_x + \beta Q_y$$

$$Q_x = \exp(-8t)\cos(6t)$$

$$Q_y = \exp(-8t)\frac{\sin(6t)}{6}$$

$$q(t) = \sin(10t)\,\text{mC}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

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$$Q(t) = q(t) + \alpha Q_x + \beta Q_y$$

$$Q_x = \exp(-8t)\cos(6t)$$

$$Q_y = \exp(-8t)\frac{\sin(6t)}{6}$$

$$Q(0) = q(0) + \alpha Q_x(0) + \beta Q_y(0)$$

$$q(t) = \sin(10t)\,\text{mC}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\text{rad/s}$$

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$$Q(0) = \cancel{q(0)} + \alpha Q_x(0) + \cancel{\beta Q_y(0)}$$

$$q(t) = \sin(10t)\,\text{mC}$$

$$\Rightarrow \alpha = Q(0) = -10\,\text{mC}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

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$$Q(t) = q(t) + \alpha Q_x + \beta Q_y$$

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$$q(t) = \sin(10t)\,\text{mC}$$

$$\alpha = -\,10\,\text{mC}$$

$$\beta = -\,90\,\text{mC/s}$$

$$\Delta Q = \alpha Q_x + \beta Q_y$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} = 6\,\text{rad/s}$$

$$Q(t) = q(t) + \alpha Q_x + \beta Q_y$$

$$Q_x = \exp(-8t)\cos(6t)$$

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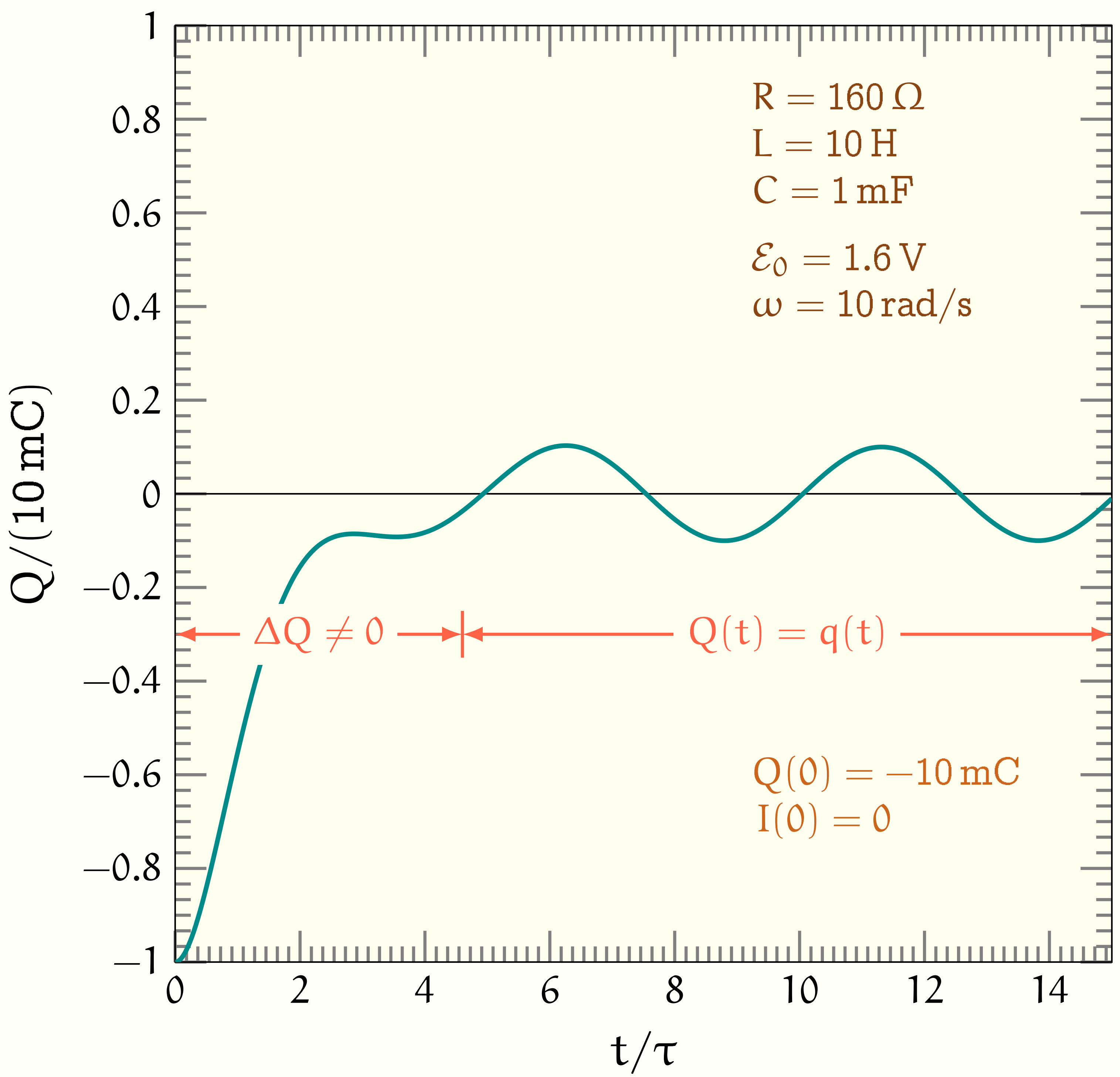
$$Q(0) = \cancel{q(0)} + \alpha Q_x(0) + \cancel{\beta Q_y(0)}$$

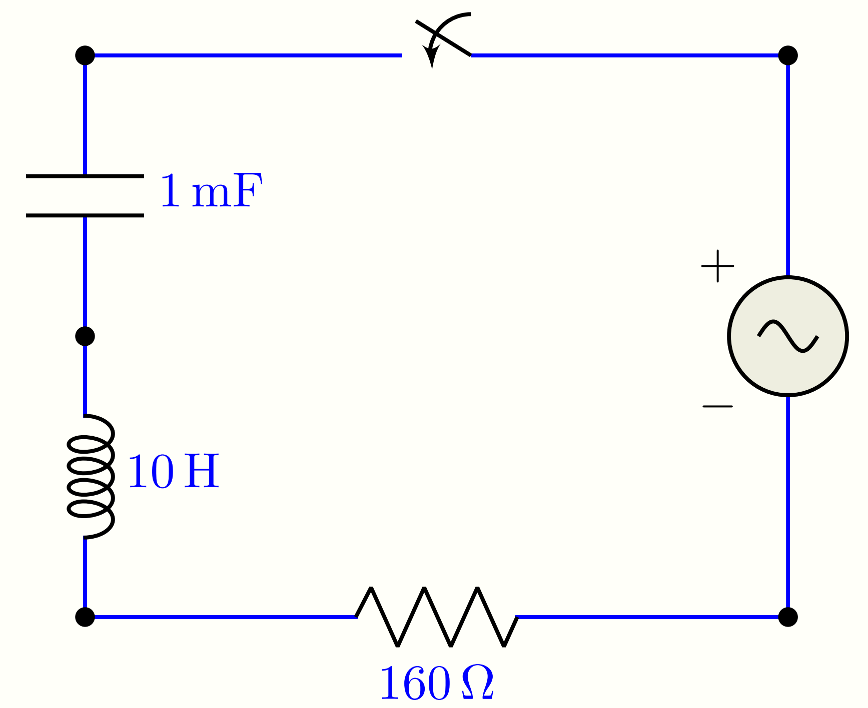
$$q(t) = \sin(10t)\,\text{mC}$$

$$Q(t) = \left(\sin(10t) - 10\exp(-8t)\left(\cos(6t) + \frac{9}{6}\sin(6t)\right)\right)\text{mC}$$

Transiente e regime estacionário

$$Q(t) = \left(\sin(10t) + 10\exp(-8t)\left(\cos(6t) + \frac{7}{6}\sin(6t)\right)\right) \text{mC}$$

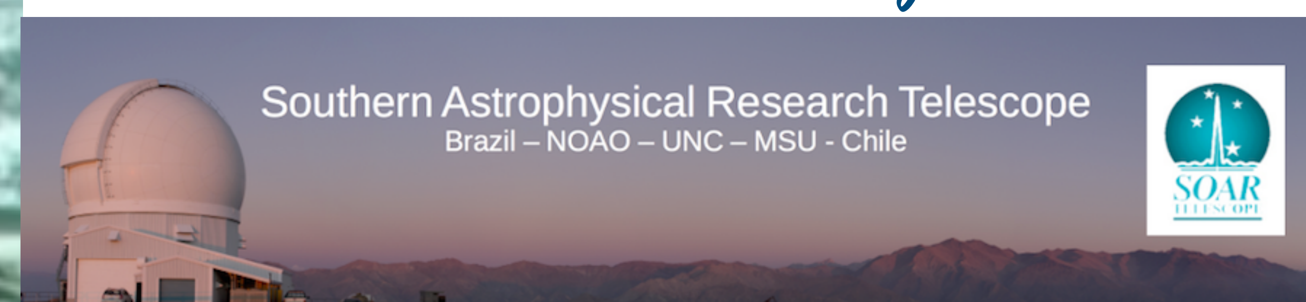




$Q(0) = -10\,\text{mC}$

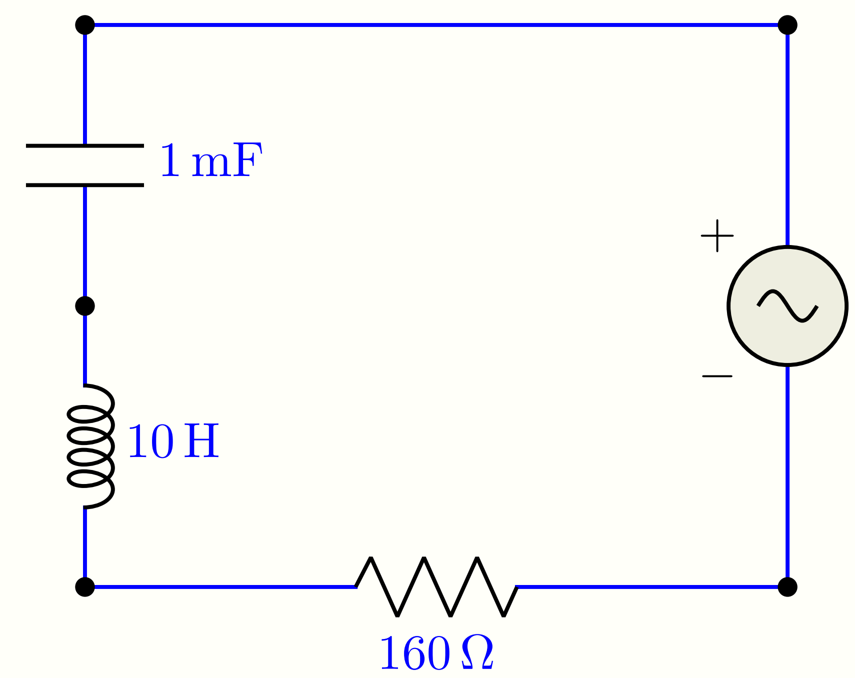
$I(0) = 0$

João Evangelista Steiner



Circuitos de corrente alternada
Corrente estacionária

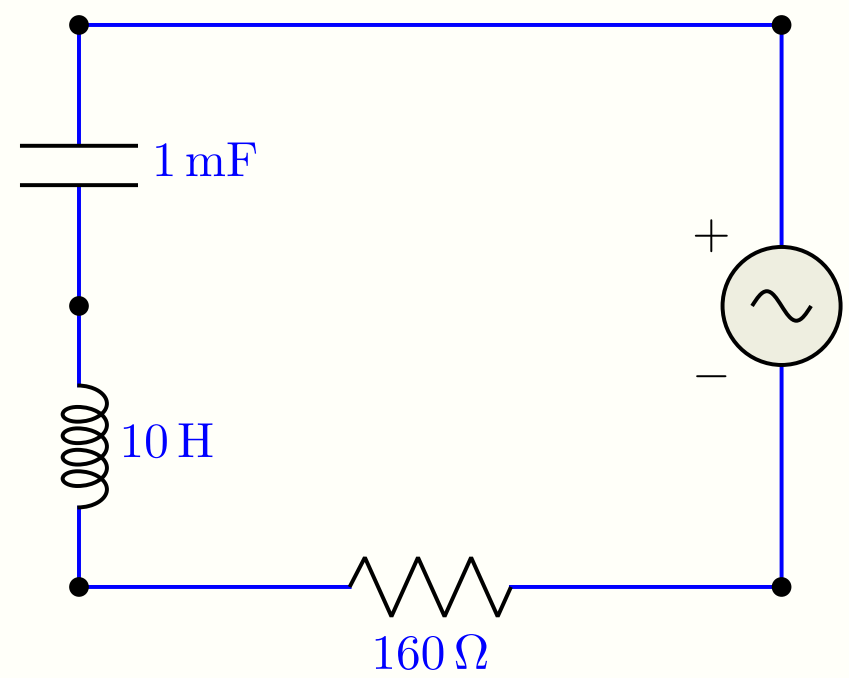
$$L\frac{d^2z}{dt^2}+R\frac{dz}{dt}+\frac{z}{C}=\mathscr{E}_0\exp(i\omega t)$$



Circuitos de corrente alternada
Corrente estacionária

$$L\frac{d^2z}{dt^2} + R\frac{dz}{dt} + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

$$-L\omega^2 z + Ri\omega z + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

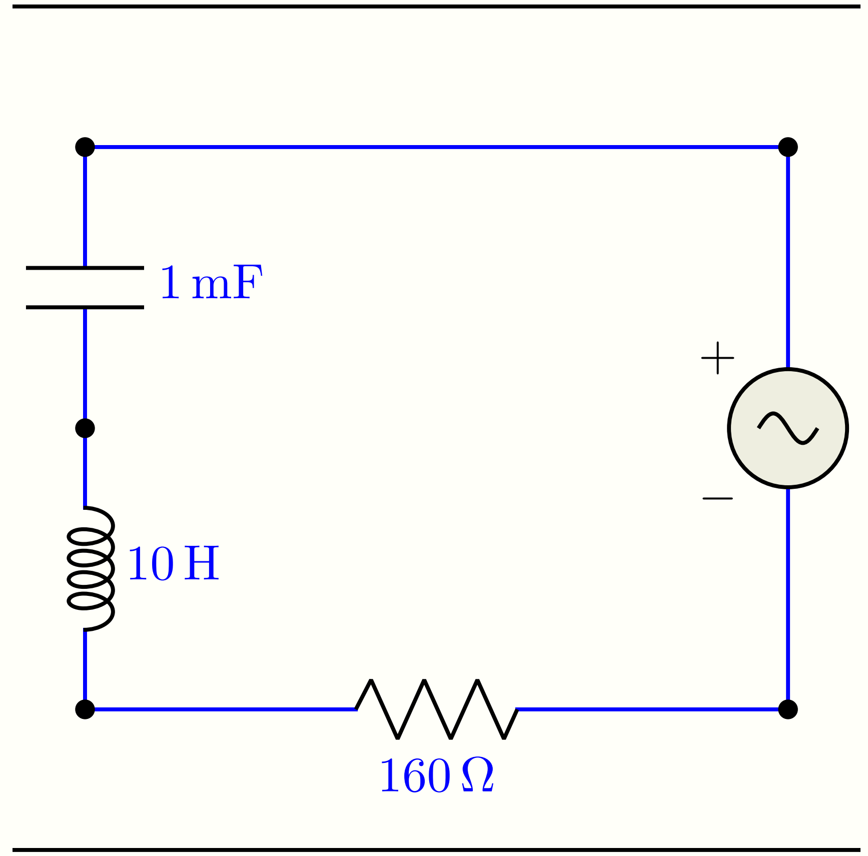


Circuitos de corrente alternada
Corrente estacionária

$$L\frac{d^2z}{dt^2} + R\frac{dz}{dt} + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

$$-L\omega^2 z + Ri\omega z + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

$$u \equiv \frac{dz}{dt} \Rightarrow I = \Re u$$



Circuitos de corrente alternada
Corrente estacionária

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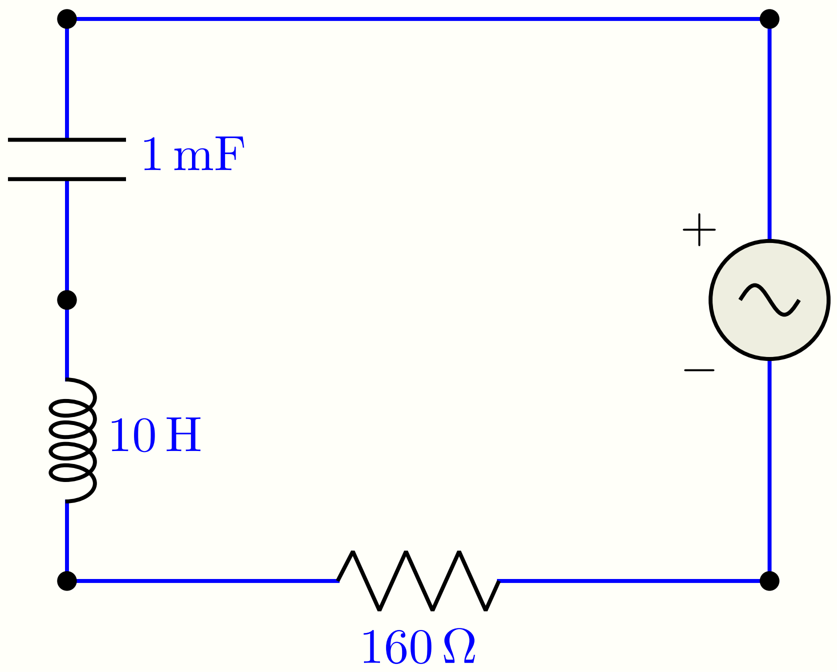

Circuitos de corrente alternada
Corrente estacionária

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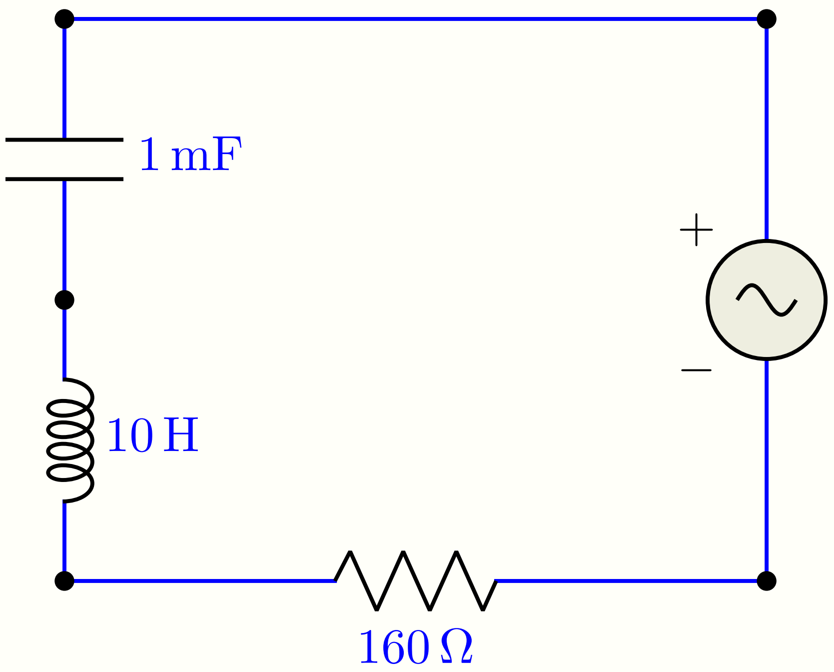

Circuitos de corrente alternada
Corrente estacionária

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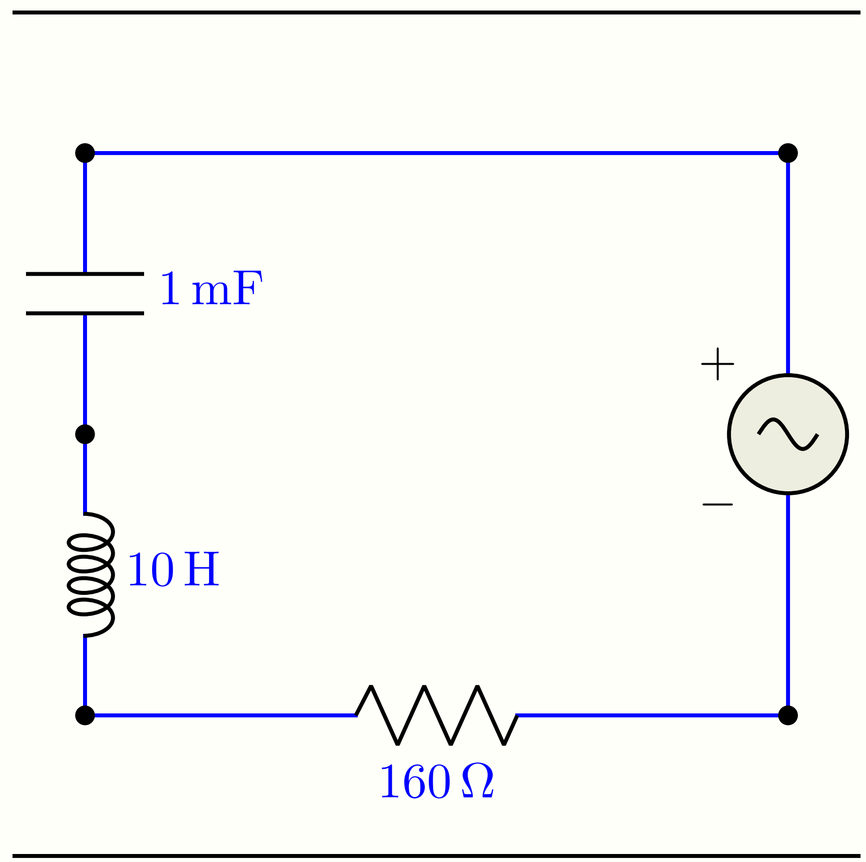


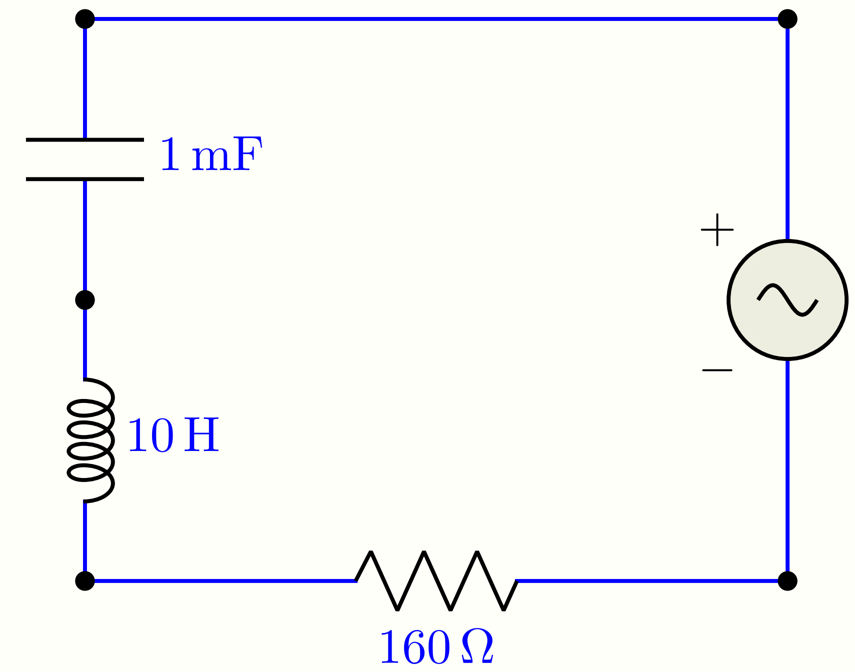
$$-L\omega^2 z + Ri\omega z + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

$$u \equiv \frac{dz}{dt} \Rightarrow u = i\omega z$$

$$i\omega L u + R u + \frac{u}{i\omega C} = \mathscr{E}_0 \exp(i\omega t)$$

$$\left(i\omega L + R + \frac{1}{i\omega C}\right) u = \mathscr{E}_0 \exp(i\omega t)$$

Circuitos de corrente alternada
Corrente estacionária

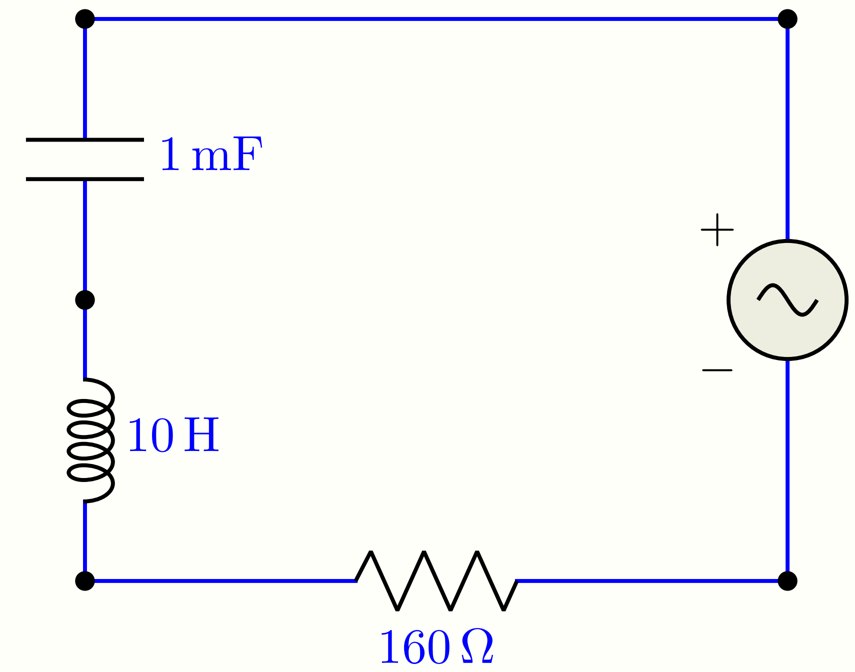


$$\left(i\omega L + R + \frac{1}{i\omega C}\right)u = \mathscr{E}_0 \exp(i\omega t)$$

Circuitos de corrente alternada
Corrente estacionária
Impedância

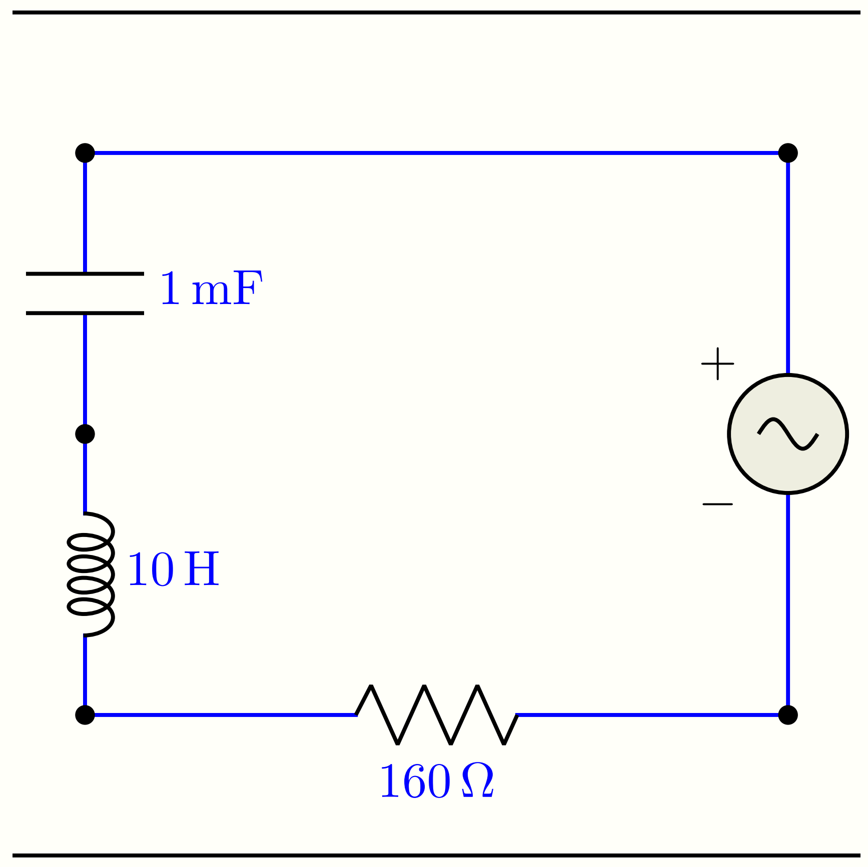
$$\underbrace{\left(i\omega L + R + \frac{1}{i\omega C}\right)}_{Z} u = \mathscr{E}_0 \exp(i\omega t)$$

$$\mathscr{E}_0 \exp(i\omega t) = Z u$$



Circuitos de corrente alternada
Corrente estacionária
Impedância

$$\underbrace{\left(i\omega L + R + \frac{1}{i\omega C}\right)}_{Z} u = \mathscr{E}_0 \exp(i\omega t)$$



$$\mathscr{E}_0 \exp(i\omega t) = Z u$$

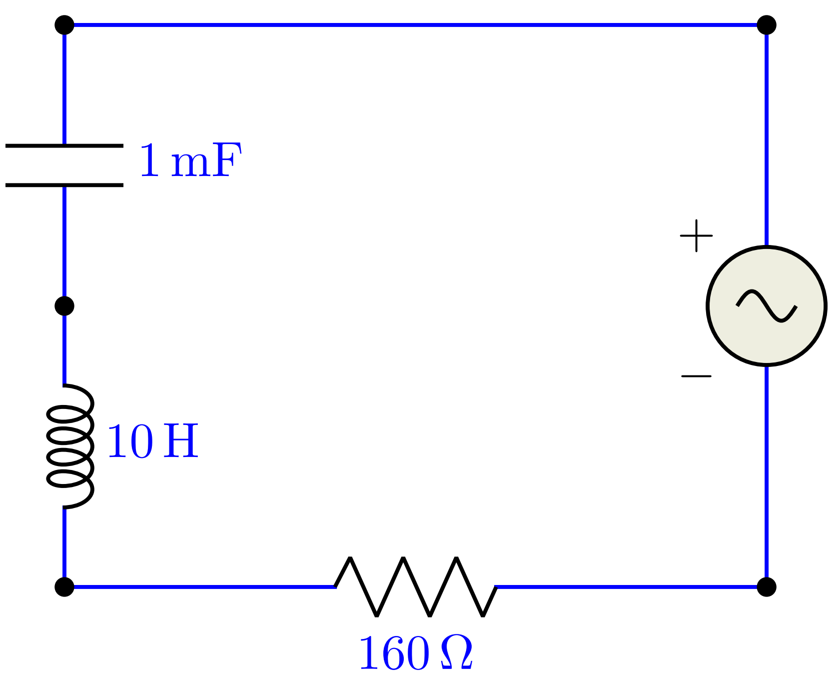
$$Z_R = R$$

$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C}$$

Pratique o que aprendeu

$$\mathscr{E}_0 \exp(i\omega t) = Z u$$



$$Z_R = R$$

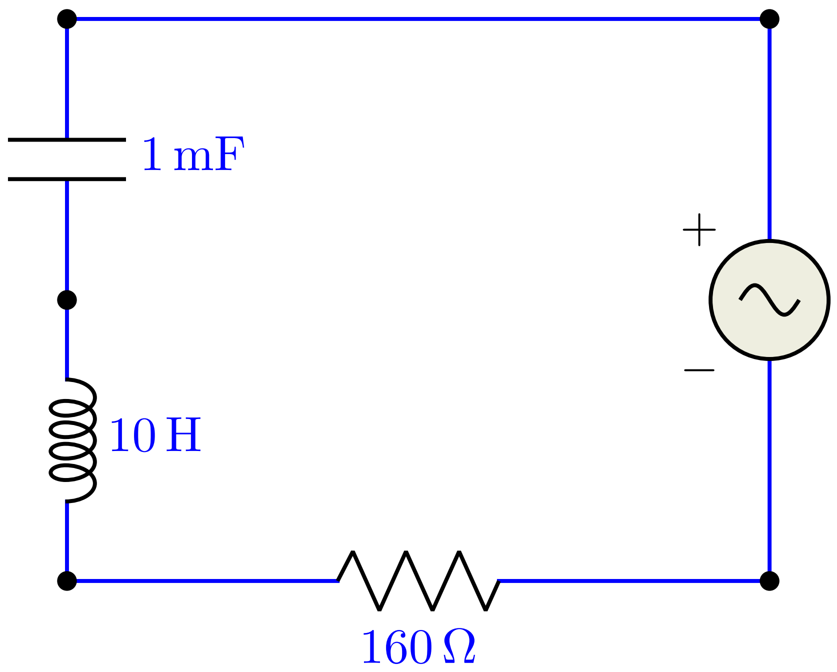
$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C}$$

Pratique o que aprendeu

$$\mathscr{E}_0 \exp(i\omega t) = Z u$$

$$\left(10i\omega + 160 - i\frac{10^3}{\omega}\right)u = \mathscr{E}_0 \exp(i\omega t)$$



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$$Z_L = i\omega L$$

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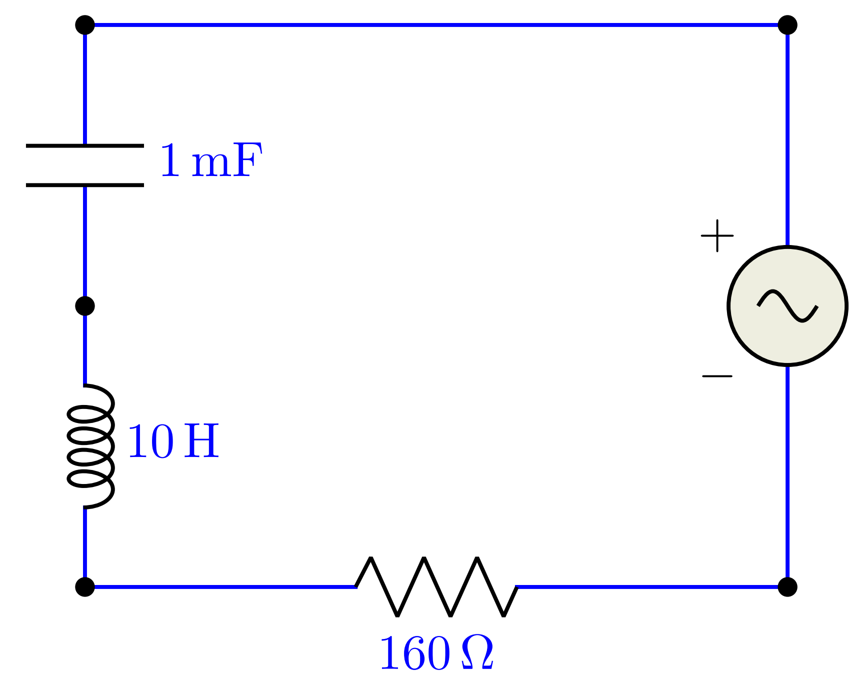
Pratique o que aprendeu

$$\mathscr{E}_0 \exp(i\omega t) = Z u$$

$$\left(10i\omega + 160 - i\frac{10^3}{\omega}\right)u = \mathscr{E}_0 \exp(i\omega t)$$

$$\omega = 10 \qquad \mathscr{E}_0 = 1.6$$

$$160u = 1.6 \exp(10it)$$



$$Z_R = R$$

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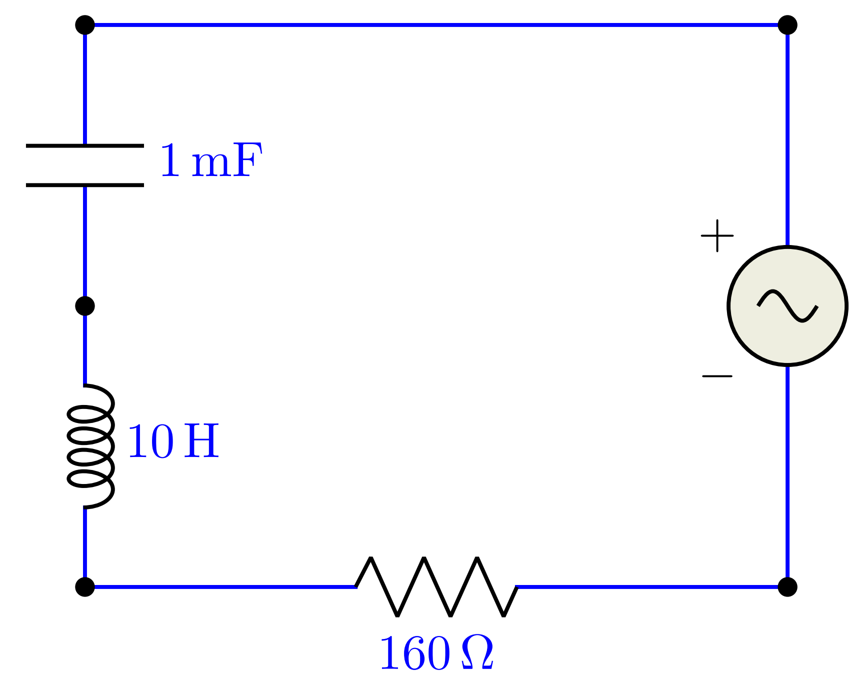


$$\omega = 10 \qquad \mathscr{E}_0 = 1.6$$

$$160u = 1.6 \exp(10it)$$

$$u = 10^{-2} \exp(10it)$$

$$Z_R = R$$

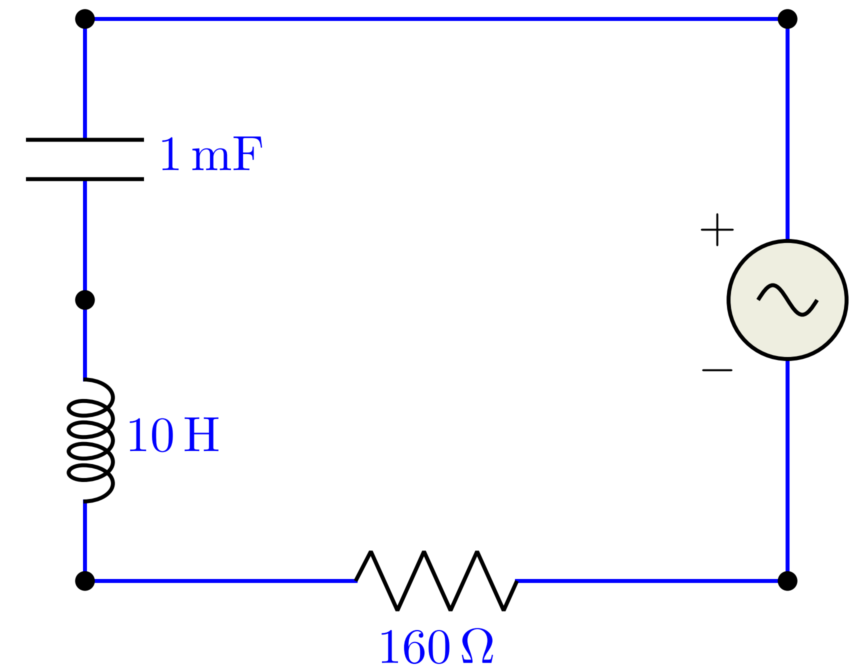
$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C}$$

Pratique o que aprendeu

$$\mathscr{E}_0 \exp(i\omega t) = Z u$$

$$\left(10i\omega + 160 - i\frac{10^3}{\omega}\right)u = \mathscr{E}_0 \exp(i\omega t)$$



$$\omega = 10 \qquad \mathscr{E}_0 = 1.6$$

$$160u = 1.6 \exp(10it)$$

$$u = 10^{-2} \exp(10it)$$

$$I = 10^{-2} \cos(10t)$$

$$Z_R = R$$

$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C}$$