Equação de transporte de poluentes miscíveis

PEF-3304 Poluição do Solo

EPUSP

Engenharia Ambiental

Fluxo de massa

$$J = \frac{M_{soluto}}{At}$$

J: fluxo de massa de soluto
M_{soluto}: massa de soluto
A: área da seção transversal
t: tempo

$$M_{soluto} = JAt$$

Concentração

$$c = \frac{M_{soluto}}{V_w}$$

c: concentração
M_{soluto}: massa de soluto
V_w: volume de água

$$n = \frac{V_v}{V} \qquad s = \frac{V_w}{V_v}$$

$$s = 1 \Rightarrow V_w = V_v$$

$$c = \frac{M_{soluto}}{nV}$$

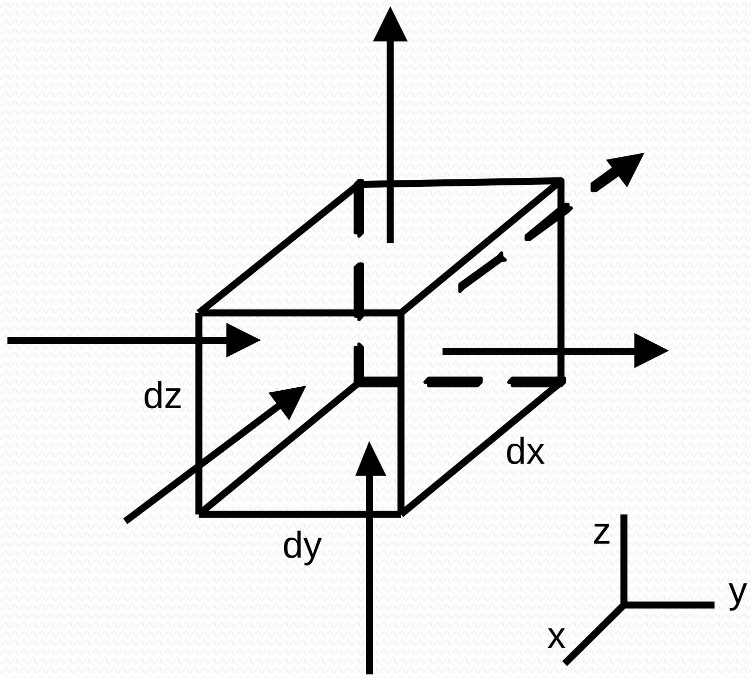
n: porosidade
V: volume

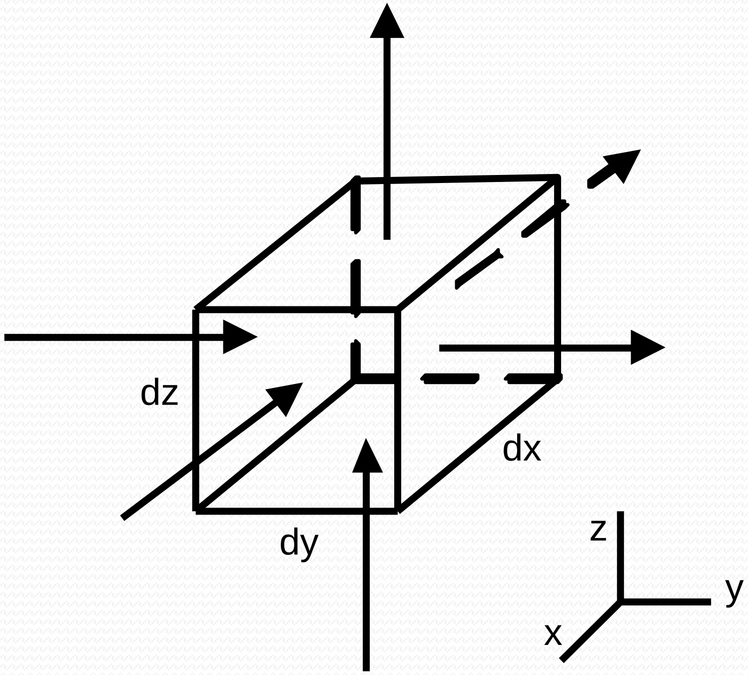
Solo saturado

$$M_{soluto} = cnV$$

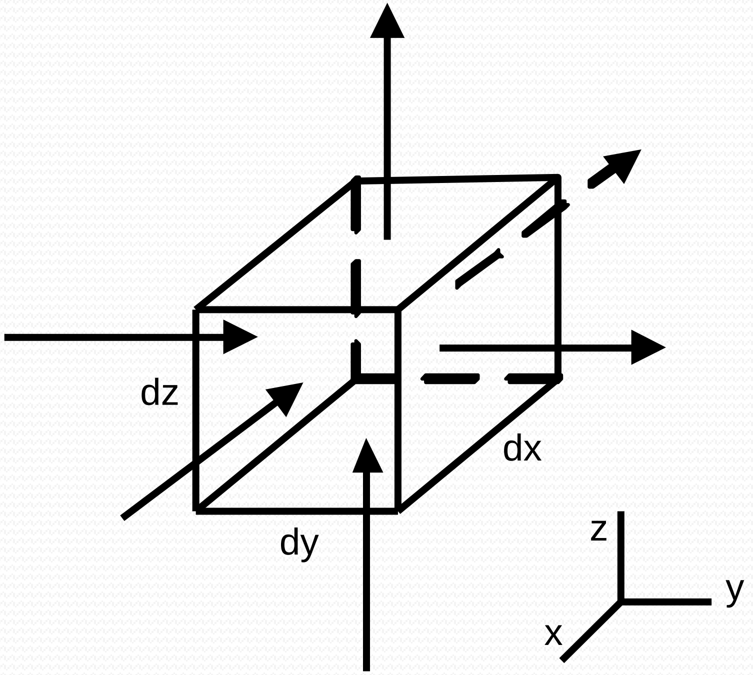
Equação da continuidade

Equação da conservação da massa



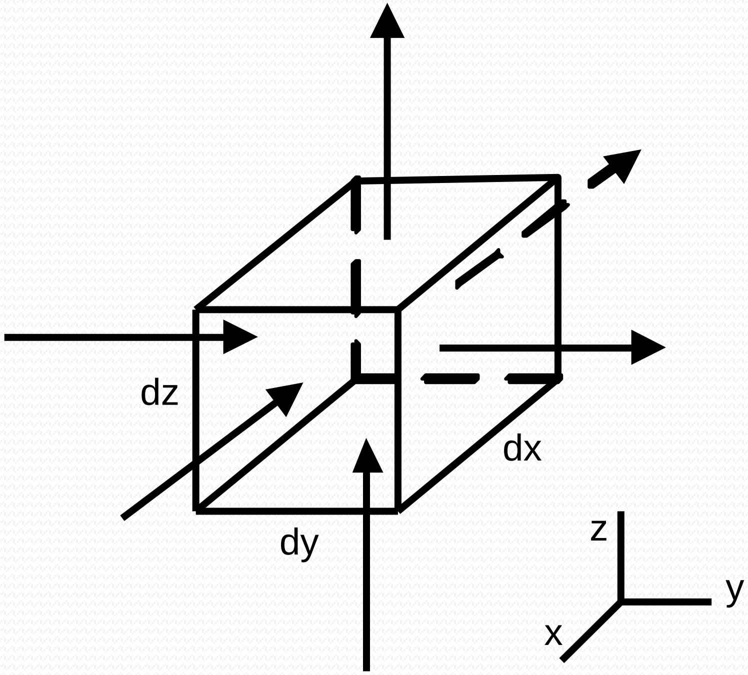


$$M_{soluto} = JAt$$



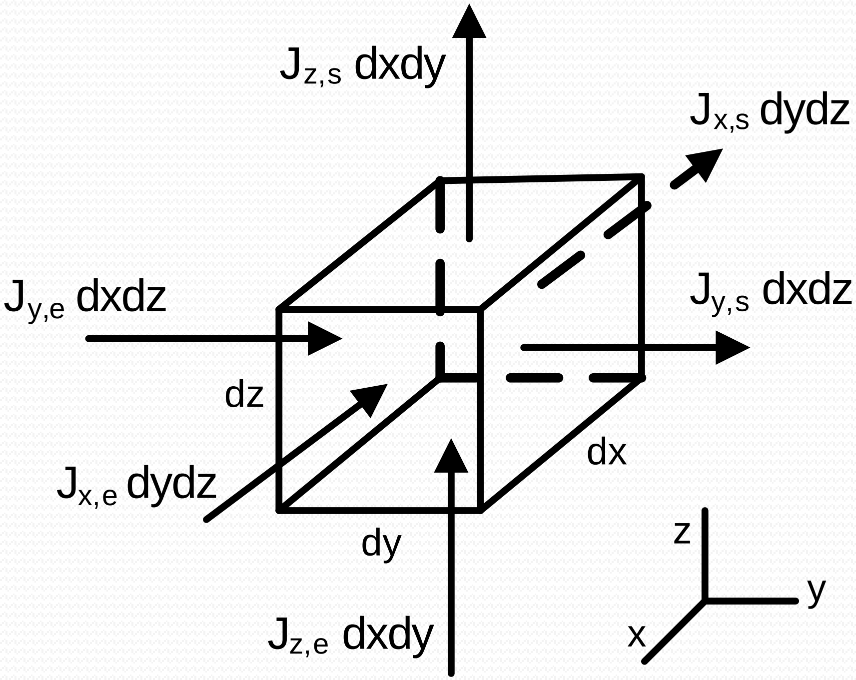
$$M_{soluto} = \mathrm{JAt}$$

$$\mathrm{JA} = \frac{\mathrm{M_{soluto}}}{t}$$



massa de soluto que entra no elemento segundo a direção x por unidade de tempo.

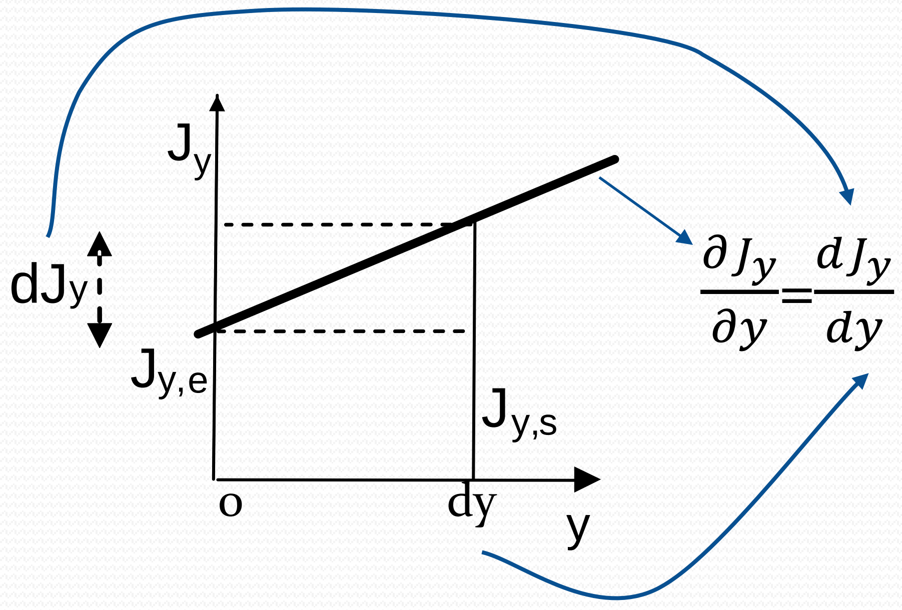
$$J_{x,e}dydz = \frac{dM_{x,e}}{dt}$$



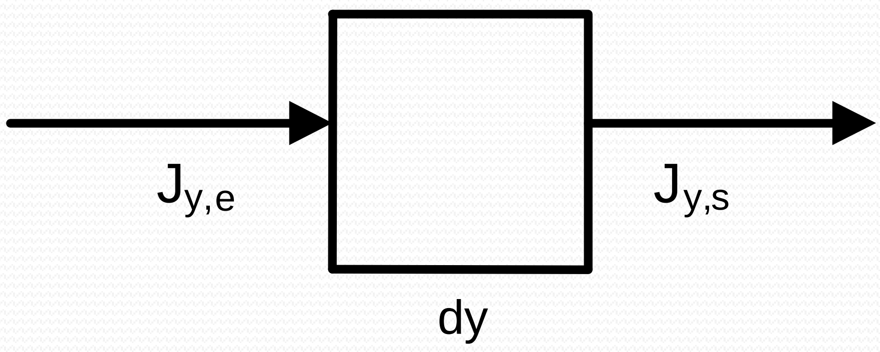
$$J_{x,e} dydz = \frac{dM_{x,e}}{dt}$$

massa de soluto que entra no elemento segundo a direção x por unidade de tempo.

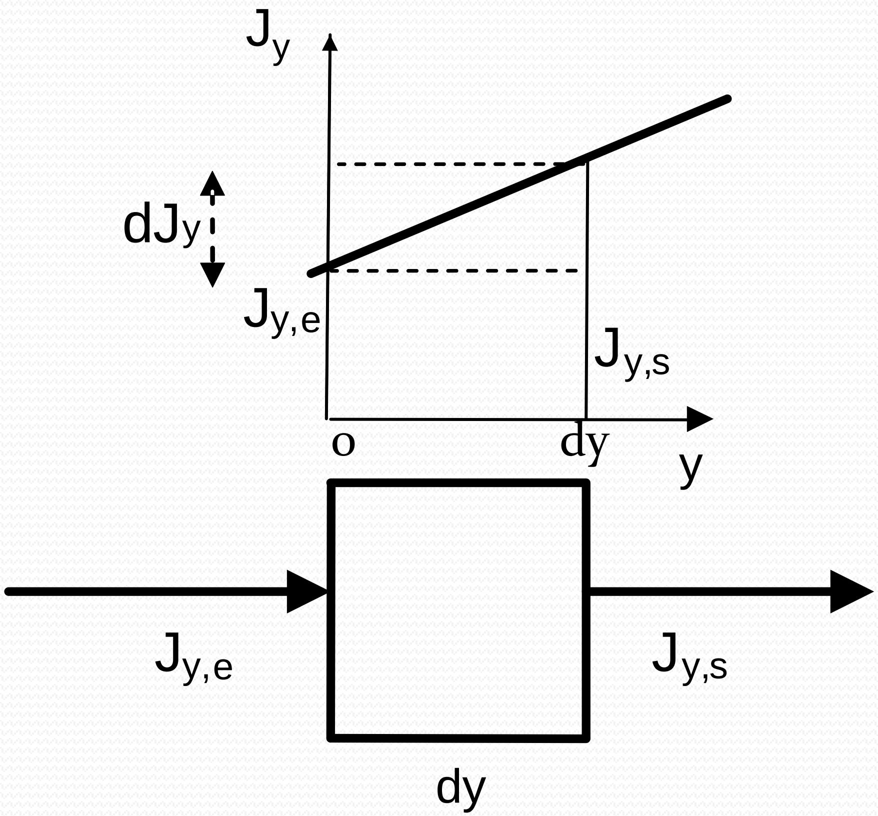
$$J_{x,e}dydz + J_{y,e}dxdz + J_{z,e}dxdy - J_{x,s}dydz - J_{y,s}dxdz - J_{z,s}dxdy = \frac{dM_{soluto,w}}{dt}$$



J_y
dJ_y
$J_{y,e}$
$J_{y,s}$
o
dy
y
$\frac{\partial J_y}{\partial y} = \frac{dJ_y}{dy}$

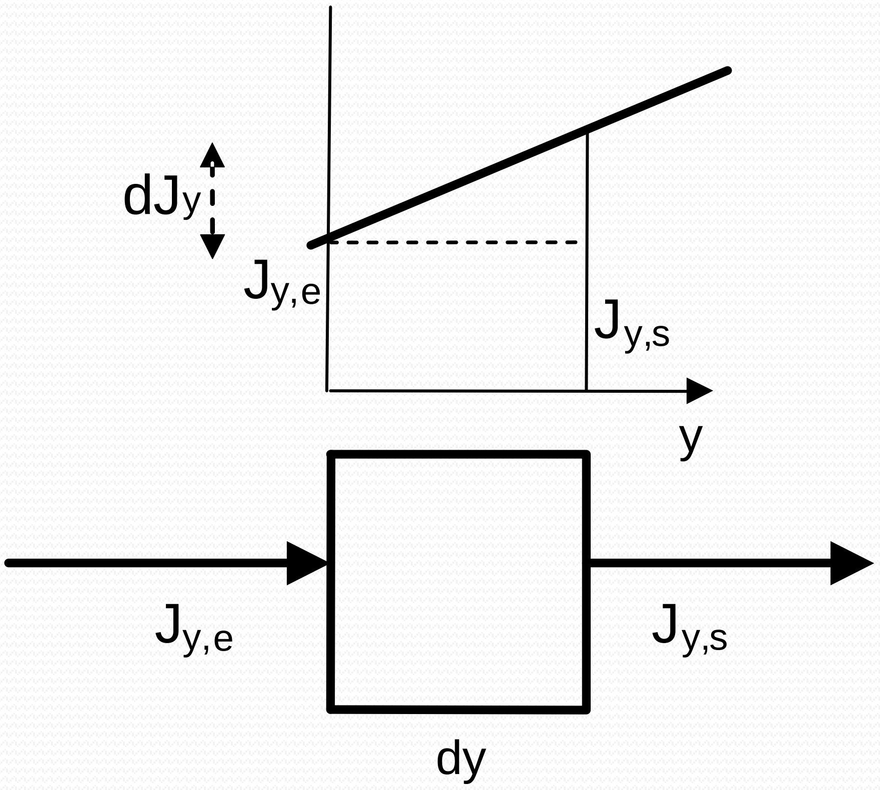


$J_{y,e}$
$J_{y,s}$
dy



$$dJ_y = \frac{\partial J_y}{\partial y} dy$$

$$J_{y,s} = J_{y,e} + \frac{\partial J_y}{\partial y} dy$$



$$J_{x,e}dydz + J_{y,e}dxdz + J_{z,e}dxdy - J_{x,s}dydz - J_{y,s}dxdz - J_{z,s}dxdy = \frac{\partial M_{soluto,w}}{\partial t}$$

$$J_{x,e}dydz + J_{y,e}dxdz + J_{z,e}dxdy - (J_{x,e} + \frac{\partial J_x}{\partial x}dx)dydz - (J_{y,e} + \frac{\partial J_y}{\partial y}dy)dxdz$$

$$-(J_{z,e} + \frac{\partial J_z}{\partial z}dz)dxdy = \frac{\partial M_{soluto,w}}{\partial t}$$

Variação linear do fluxo em função da distância

$$-\frac{\partial J_x}{\partial x}dxdydz - \frac{\partial J_y}{\partial y}dxdydz - \frac{\partial J_z}{\partial z}dxdydz = \frac{\partial M_{soluto,w}}{\partial t}$$

$$-\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right)dxdydz = \frac{\partial M_{soluto,w}}{\partial t}$$

$$-\left(\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}+\frac{\partial J_z}{\partial z}\right)dxdydz=\frac{\partial M_{soluto,w}}{\partial t}$$

$$\frac{\partial M_{soluto,w}}{\partial t} = \frac{\partial (cnV)}{\partial t}$$

Volume não muda com o tempo

Porosidade não muda com o tempo

$$\frac{\partial M_{soluto,w}}{\partial t} = nV \frac{\partial c}{\partial t}$$

$$-\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right) dxdydz = \frac{\partial M_{soluto,w}}{\partial t}$$

$$-\left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right) dxdydz = nV \frac{\partial c}{\partial t}$$

$$-\frac{\partial J_z}{\partial z} dxdydz = nV \frac{\partial c}{\partial t}$$

Fluxo unidimensional

$$n \frac{\partial c}{\partial t} = -\frac{\partial J_z}{\partial z}$$

- fluxo por advecção

$$J_{advecção} = cv = cnu$$

Fluxo de água permanente e uniforme

- fluxo por difusão

$$J_{difusão} = -nD_d \frac{\partial c}{\partial z}$$

- fluxo por dispersão hidrodinâmica

$$J_{dispersão} = -nD_{dh} \frac{\partial c}{\partial z}$$

$$D_{dh} = D_d + D_{dm} = D_d + \alpha u = D_d + \alpha \frac{v}{n}$$

$$n\frac{\partial c}{\partial t} = -\frac{\partial J_z}{\partial z}$$

$$n\frac{\partial c}{\partial t} = -\frac{\partial J_{advecção}}{\partial z} - \frac{\partial J_{difusão}}{\partial z} - \frac{\partial J_{dispersão\ mecânica}}{\partial z}$$

$$n\frac{\partial c}{\partial t} = -\frac{\partial J_{advecção}}{\partial z} - \frac{\partial J_{dispersão\ hidrodinâmica}}{\partial z}$$

$$n\frac{\partial c}{\partial t} = -\frac{\partial (ucn)}{\partial z} - \frac{\partial (-nD_d\frac{\partial c}{\partial z})}{\partial z} - \frac{\partial (-nD_{dm}\frac{\partial c}{\partial z})}{\partial z}$$

$$n\ \frac{\partial c}{\partial t} = -n\frac{\partial (uc)}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2}$$

Porosidade, Dd e Dm não variam com z ⇒ solo homogêneo

$$n\ \frac{\partial c}{\partial t} = -n\frac{\partial (uc)}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2}$$

Velocidade não varia com z ⇒ fluxo uniforme

$$n\ \frac{\partial c}{\partial t} = -nu\frac{\partial c}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2}$$

$$\boldsymbol{\frac{\partial c}{\partial t} = D_{dh}\frac{\partial^2 c}{\partial z^2} - u\frac{\partial c}{\partial z}}$$

Equação da advecção-dispersão

Soluto não reativo

Adsorção

Alteração de massa de soluto na água do solo

Alteração de massa de soluto na superfície dos grãos

$$\frac{\partial(nVc)}{\partial t} = -\frac{\partial(Sm_s)}{\partial t}$$

Transferência de soluto para a fase sólida por adsorção

$$S = \frac{m_{soluto\ adsorvido}}{m_{solo\ seco}}$$

$$\frac{\partial(nVc)}{\partial t} = -\frac{\partial(S\rho_d V)}{\partial t}$$

$\rho_d = m_s/V$

$$\frac{\partial(nVc)}{\partial t} = -\frac{\partial(K_d c\rho_d V)}{\partial t}$$

Adsorção linear

$$n\frac{\partial c}{\partial t} = -K_d\rho_d\frac{\partial c}{\partial t}$$

n, K_d e ρ_d não variam com o tempo

$$n \frac{\partial c}{\partial t} = -un\frac{\partial c}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2}$$

$$n \frac{\partial c}{\partial t} = -un\frac{\partial c}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2} \boldsymbol{- K_d\rho_d\frac{\partial c}{\partial t}}$$

$$(n + K_d\rho_d)\frac{\partial c}{\partial t} = -un\frac{\partial c}{\partial z} + nD_d\frac{\partial^2 c}{\partial z^2} + nD_{dm}\frac{\partial^2 c}{\partial z^2}$$

$$(1 + \frac{K_d\rho_d}{n})\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial z} + D_d\frac{\partial^2 c}{\partial z^2} + D_m\frac{\partial^2 c}{\partial z^2}$$

$$(1 + \frac{K_d\rho_d}{n})\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial z} + D_{dh}\frac{\partial^2 c}{\partial z^2}$$

$$(1+\frac{K_d\rho_d}{n})\frac{dc}{dt} = -u\frac{\partial c}{\partial z} + D_d\frac{\partial^2 c}{\partial z^2} + D_m\frac{\partial^2 c}{\partial z^2}$$

$$(1+\frac{K_d\rho_d}{n})\frac{dc}{dt} = -u\frac{\partial c}{\partial z} + D_{dh}\frac{\partial^2 c}{\partial z^2}$$

$$\boldsymbol{R_d\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial z} + D_{dh}\frac{\partial^2 c}{\partial z^2}\mathbf{c}}$$

$$R_d = 1 + \frac{K_d\rho_d}{n}$$

$$\left(1 + \frac{K_d \rho_d}{n}\right) \frac{dc}{dt} = -u \frac{\partial c}{\partial z} + D_{dh} \frac{\partial^2 c}{\partial z^2}$$

$$R_d \frac{dc}{dt} = -u \frac{\partial c}{\partial z} + D_{dh} \frac{\partial^2 c}{\partial z^2} \mathrm{c}$$

Equação do transporte unidimensional de soluto em solo homogêneo e isotrópico, com fluxo de água saturado, permanente e uniforme, com adsorção linear