Correção da Prova 3 - Física I para IO.

Exercício 1

a) $x = R\varphi$.

b) mola 1 (da esquerda)

$$F_1 = -k\,[x-(-a)] = -k(x+a)$$

$$F_2 = -k\,[-x+a]$$

$$\tau_1 = F_1 . R \qquad \tau_R = \tau_1 + \tau_2 = [-k(x+a) + k\,[-x+a]]\,R$$

$$\tau_2 = -F_2 . R \qquad \tau_R = [-kx - ka - kx + ka]\,R$$

$$\tau_R = -2kxR = -2kR^2\varphi$$

c) $I . \dfrac{d^2\varphi}{dt^2} = -2kR^2\varphi \Rightarrow I . \dfrac{d^2\varphi}{dt^2} + 2kR^2\varphi = 0$

$$\omega_0^2 = \frac{2kR^2}{I} = \frac{2kR^2}{\frac{1}{2}MR^2} = \frac{4k}{M} = 4\,(rad/s)^2$$

$$T = \pi = \frac{2\pi}{\omega_0} \qquad \omega_0 = 2\,rad/s \qquad \therefore \quad M = k = 10\,kg$$

d) $\varphi(0) = 0 \qquad \varphi(t) = A\cos(\omega_0 t + \delta) \qquad t = 0$

$$0 = A\cos\delta \Rightarrow \delta = \pm\pi/2$$

$$\frac{d\varphi}{dt} < 0 \quad \rightsquigarrow \frac{d\varphi}{dt} = -A\omega_0\,\text{sen}(\omega_0 t + \delta) \qquad t = 0 \quad \delta = \pi/2$$

$$\frac{d\varphi}{dt} = -A\omega_0\,\text{sen}\frac{\pi}{2} = -A\omega_0 < 0$$

credeal

d) $I = \frac{1}{2} MR^2 = \frac{10}{2} \cdot \frac{1}{\pi^2} \sim I = \frac{5}{\pi^2}$

e) $\left(\frac{d\varphi}{dt}\right)^2 = A^2 \omega_0^2 \Rightarrow \frac{1}{2} I A^2 \omega_0^2 = \frac{5}{8} J \Rightarrow A^2 = \frac{5}{8} \cdot \frac{2}{I \omega_0^2}$

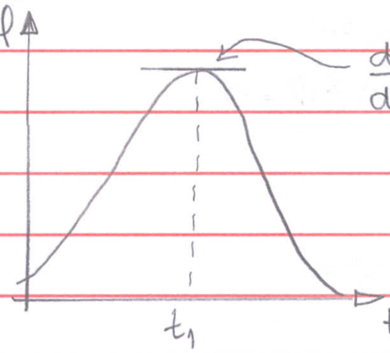
$\Rightarrow A^2 = \frac{5}{8} \cdot \frac{2}{4} \cdot \frac{\pi^2}{5} = \frac{\pi^2}{16} \Rightarrow A = \frac{\pi}{4}$

② EQUAÇÃO DIFERENCIAL: $\frac{d^2\varphi}{dt^2} + \gamma \frac{d\varphi}{dt} + \omega_0^2 \varphi = 0$

$\varphi(t) = (A + Bt)\, e^{-\gamma/2\, t}$ $\quad \gamma = 3/e$

a) $\varphi(0) = A e^0 = 0 \quad \therefore A = 0$

b)



$\frac{d\varphi}{dt} = 0$

$\varphi(t) = B t\, e^{-\gamma/2\, t}$

$\left.\frac{d\varphi}{dt}\right|_{t_1} = B e^{-\gamma/2\, t_1} + B t_1 (-\gamma/2)\, e^{-\gamma/2\, t_1}$

$0 \neq \left[B\, e^{-\gamma/2\, t_1} (1 - \gamma/2\, t_1)\right] = 0$

$0 \neq$

então $1 - \gamma/2\, t_1 = 0$

$1 = \gamma/2\, t_1$

$t_1 = 2/\gamma = \frac{2e}{3}$

) $\varphi_{max} = B t_1\, e^{-\gamma/2\, t_1}$

$\frac{\pi}{3} = B \cdot \frac{2e}{3} \cdot e^{-\gamma/2 \cdot 2/\gamma} = B \cdot \frac{2}{3} \cdot \frac{e}{e}$ $\quad B = \pi/2$

d) $\varphi = \frac{\pi}{2} t \cdot e^{-\gamma/2\, t} = \frac{\pi}{2} 4e \cdot e^{-\frac{3}{2e} \cdot 4e} = \frac{\pi}{2} 4e \cdot e^{-6} \Rightarrow \varphi = 2\pi e^{-5}$

) $\frac{d\varphi}{dt} = B e^{-\gamma/2\, t} (1 - \gamma/2\, t)$ $\quad t = 0 \quad B = \left.\frac{d\varphi}{dt}\right|_0 = \pi/2$

) $\omega_0 = \frac{\gamma}{2} = \frac{3}{2e} = 0{,}55$ rad

$f = \frac{\omega_0}{2\pi} = \frac{3}{4\pi e}$ Hz